

# A Novel Numerical Approach to Investigate Shaft Diameter Effect on Dynamic Behaviour of Multi-Disc Rotors

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#### **1. Introduction**

Multi-disc rotor systems are extensively utilized in rotating machinery for different industrial applications. Their dynamic behavior is essentially based on rotor-bearing characteristics. Specifically, the diameter of the rotating shaft and bearing stiffness play a major role in this respect. In the design of any of these machines, precise prediction of the system dynamic properties is a major step [1-5]. The physical model parameters of rotating machines exhibit uncertainty in different forms because of the asymmetrically distribution of the materials used in parts such as shafts and discs. Thus, the geometries of the system will show certain degrees of uncertainty. A comprehensive dynamic characteristics of rotor systems under uncertainty are reviewed by Chao *et al.*, [1]. However, the work scope of rotor systems in this review is mostly dealt with by the classical shaft-disk-bearing rotor systems. Shuaijun *et al.*, [2] reported a new modeling method for bearing-rotor systems by introducing a semi-flexible body element. A typical ball-bearing-rotor system was utilized as an example to generalize the model. Hongyang *et al.,* [3] proposed a dynamic model of the bearing-

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rotor system considering the supporting force and stiffness of bearings with and without inner ring dynamic misalignment. Jalali *et al.*, [4] studied the dynamic characteristics of a high speed rotor with certain geometrical and mechanical properties using 3D finite element model, one-dimensional beam-type model and experimental modal test. In their work, the critical speeds and unbalance response of the rotor were determined to investigate the dynamic behavior of the rotating system. Tiwari *et al.,* [5] investigated numerically the dynamic response of the rotor system considering the effect of variation in radial internal clearance on the bearing stiffness. The bearing stiffness was determined experimentally.

The most extensive part of the literature on rotating shaft dynamic behavior is concerned with identifying the natural frequencies and critical speeds. The effect of shaft diameter effect on the associated dynamic performance of rotor systems has been addressed by many researchers, for example [6-12]. Research conducted by Shooshtari *et al.,* [6] investigated the influence of diameter on the dynamic behavior of centrifugal pumps by applying an inverse dynamic methodology. The results obtained in their study demonstrate that as the pump impeller diameter increased, the vibration amplitude was increased by 52% at the rotor critical speeds. Moreover, they found that the diameter of impeller has to be considered as a major factor when selecting centrifugal pumps, besides the hydraulic condition and efficiency. Houxin *et al.,* [7] carried out numerical simulation to investigate the influence of disc position and flexibility on critical speeds and natural frequencies of a rotor system. The results obtained indicate that flexibility and position of the disc have a significant impact on natural frequencies and critical speeds. Firouzi and Hassan [8] studied whirling speeds under various conditions considering the influence of the shaft diameter and torsional stiffness. They found that the variation in critical speed resulting from an increase in the diameter of the shaft was higher than the variation in critical speed resulting from increasing shaft torsional stiffness. In addition, an increase in propeller shaft diameter and torsional stiffness resulted in an increase in the shaft natural frequency. Furthermore, they reported that as the diameter of the shaft increased, the variation in whirling speed was observed at high level of frequencies, but it was not sensitive at lower frequencies. The results obtained by Jinli and Luo [9] indicate that, at the given speed, improving the bearing stiffness can reduce the vibration response of the rotor system. However, when the bearing stiffness increased to a critical value, the effects of bearing stiffness on the vibration reduction became insignificant. The effect of changing the diameter of a composite shaft on critical speed under a dynamic load was investigated by Adil *et al.,* [10]. The results in this study show that the critical speed value increases when moving from the first mode to the second mode and the critical speed relies on the condition of fixation. In their works, Metsebo *et al.,* [11] proposed a mathematical model to study the influence of rotating shaft on the dynamics of rotor-ball bearings system by considering the shaft as rotating Timoshenko beam model. In this study, the bifurcation diagram and Lyapunov exponent were demonstrated to present the state of the system as a function of rotational speed. Balasaheb [12] analyzed theoretical and practical frequencies of different rotating shaft diameters. In this study, frequencies at different speeds for various shaft diameters are evaluated and a relation between natural frequencies and shaft diameters is established using graphical approach.

To sum up, the problem of modeling dynamic behavior of rotating machines considering rotor parameters was addressed by a number of studies. However, a dynamic model that satisfactorily addresses the effect of rotating shaft diameter taking into account the variation in rotor bearing stiffness for multi-disc rotors is still highly demanded. In the current study, a displacement-based finite element model considering variation in rotor bearing stiffness was proposed to investigate the influence of the diameter of rotating shafts on the natural frequencies and critical speeds of multidisc rotor systems. The gyroscopic moment due to disc flexibility was also considered in the developed model. The importance of the work presented in the current paper was manifested

through deepening the understanding of the effect of rotor shaft diameter on dynamic behavior of rotating machinery with bearing stiffness variation. Bearing support stiffness is a significant factor in the design stage to predict critical speeds and natural frequency. Therefore, the effect of support stiffness on the dynamic characteristics of a rotating machine was also considered. Natural frequencies and critical speeds over a wide range of shaft diameters were identified under different bearing stiffness conditions using the numerical model developed in this paper. Experimental verification was also performed to validate the simulation results. The novel modeling approach proposed in this paper provides theoretical support for the analysis of multi-bearing rotor systems.

# **2. Modelling of Rotor System**

Finite element (FE) method is an effective tool to model rotating machines dynamics taking into account the flexibilities of all rotating components and has been largely used for rotor dynamics design to provide mathematical representation of rotating machines. This section presents the development and assembling of the global mass, damping, and stiffness matrices of multi-disc rotary systems. The dynamic responses can then be comprehensively identified via determining natural frequencies and mode shapes using the global matrices developed for the rotor-bearing system, Friswell *et al.,* [13].

In the modeling procedure, the rotating shaft was discretized into a number of Timoshenko beam finite elements, Lalanne and Guy [14]. Each beam element was expressed as an element with two nodes. Four degrees of freedom, in the lateral direction, two translations, and two rotations, were introduced for each node as demonstrated in Figure 1.



**Fig. 1.** Beam element representation

where, *u* and *v* refer to translations in direction of *<sup>x</sup>* and *y* respectively while α and β refer to rotation about *<sup>x</sup>* and *y* axes respectively. The mass, stiffness, and gyroscopic matrices for the beam element are provided from Friswell *et al.,* [13].

The beam element stiffness matrix was identified by Friswell *et al.,* [13]:

$$
\mathbf{K}_{ele} = \frac{EI}{(1+\varphi)l^3} \begin{bmatrix} 12 & 0 & 0 & 6l & -12 & 0 & 0 & 6l \\ 0 & 12 & -6l & 0 & 0 & -12 & -6l & 0 \\ 0 & -6l & (4+\varphi)l^2 & 0 & 0 & 6l & (2-\varphi)l^2 & 0 \\ 6l & 0 & 0 & (4+\varphi)l^2 & -6l & 0 & 0 & (2-\varphi)l^2 \\ -12 & 0 & 0 & -6l & 12 & 0 & 0 & -6l \\ 0 & -12 & 6L & 0 & 0 & 12 & 6l & 0 \\ 0 & -6l & (2-\varphi)l^2 & 0 & 0 & 6l & (4+\varphi)l^2 & 0 \\ 6l & 0 & 0 & (2-\varphi)l^2 & -6l & 0 & 0 & (4+\varphi)l^2 \end{bmatrix}
$$
(1)

In the above matrix equation,  $\varphi$  refers to shear constant which reflects the shear flexibility of the beam, *l* is length of element of the beam, *E* expresses the modulus of elasticity and *I* represents the second moment for the beam cross-section area. The generalized coordinate of each shaft element is expressed as in the following sequence of the DoFs:

$$
[u_1\;v_1\;\alpha_1\;\beta_1\;u_2\;v_2\;\alpha_2\;\beta_2].
$$

The mass matrix for beam element was determined from Friswell *et al.,* [13]:

$$
\mathbf{M}_{e} = \frac{\rho A l}{840(1+\varphi)^{2}} \begin{bmatrix} M_{e1} & 0 & 0 & M_{e2} & M_{e3} & 0 & 0 & M_{e4} \\ 0 & M_{e1} & -M_{e2} & 0 & 0 & M_{e3} & -M_{e4} & 0 \\ 0 & -M_{e2} & M_{e5} & 0 & 0 & M_{e4} & M_{e6} & 0 \\ M_{e2} & 0 & 0 & M_{e5} & -M_{e4} & 0 & 0 & M_{e6} \\ M_{e3} & 0 & 0 & -M_{e4} & M_{e1} & 0 & 0 & -M_{e2} \\ 0 & M_{e3} & M_{e4} & 0 & 0 & M_{e1} & M_{e2} & 0 \\ 0 & -M_{e4} & M_{e6} & 0 & 0 & M_{e2} & M_{e5} & 0 \\ M_{e4} & 0 & 0 & M_{e6} & -M_{e2} & 0 & 0 & M_{e5} \end{bmatrix}
$$
(2)

In the above matrix:

$$
M_{e1} = 312 + 588\varphi + 280\varphi^2 \tag{3}
$$

$$
M_{e2} = (44 + 77\varphi + 35\varphi^2)l
$$
\n(4)

$$
M_{e3} = 108 + 252\varphi + 140\varphi^2 \tag{5}
$$

$$
M_{e4} = -\left(26 + 63\varphi + 35\varphi^2\right)l\tag{6}
$$

$$
M_{e5} = (8 + 14\varphi + 7\varphi^2)l^2 \tag{7}
$$

$$
M_{e6} = -\left(6 + 14\varphi + 7\varphi^2\right)l^2\tag{8}
$$

The matrix of gyroscope for the beam element was calculated from Friswell *et al.,* [13]:

$$
\mathbf{G}_{e} = \frac{\rho I}{15 I (1 + \phi)^{2}} \begin{bmatrix} 0 & -G_{e1} & G_{e2} & 0 & 0 & G_{e1} & G_{e2} & 0 \\ G_{e1} & 0 & 0 & G_{e2} & -G_{e1} & 0 & 0 & G_{e2} \\ -G_{e2} & 0 & 0 & -G_{e3} & G_{e2} & 0 & 0 & -G_{e4} \\ 0 & -G_{e2} & G_{e3} & 0 & 0 & G_{e2} & G_{e4} & 0 \\ 0 & G_{e1} & -G_{e2} & 0 & 0 & -G_{e1} & -G_{e2} & 0 \\ -G_{e1} & 0 & 0 & -G_{e2} & G_{e1} & 0 & 0 & -G_{e2} \\ -G_{e2} & 0 & 0 & -G_{e4} & G_{e2} & 0 & 0 & -G_{e3} \\ 0 & -G_{e2} & G_{e4} & 0 & 0 & G_{e2} & G_{e3} & 0 \end{bmatrix}
$$
\n
$$
(9)
$$

In the above matrix equation:

$$
G_{\rm el} = 36\tag{10}
$$

$$
G_{e2} = (3 - 15\varphi)l \tag{11}
$$

$$
G_{e3} = (4 + 5\varphi + 10\varphi^2)l^2
$$
\n(12)

$$
G_{e4} = \left(-1 - 5\varphi + 5\varphi^2\right)l^2\tag{13}
$$

where  $\rho$  refers to material density of the shaft,  $\rho$  is the cross sectional area of the rotating shaft and *φ* refers to shear coefficient.

To model the bearings of the rotor system, the stiffness of bearing was introduced as a spring like element at a single node in direction of bearing support. The outer damping was involved in the system using bearing support damping. Computing the matrices of mass, stiffness, and gyroscopic of the element facilitated assembling of the rotor system global matrices mass, damping and stiffness matrices respectively. The contributions of the damping and stiffness from the bearings were then involved in the global matrices of the system.

The natural frequencies and mode shapes were then determined using the assembled global  $\{M, D, K\}$  matrices. The matrix of state space companion of the system can be arranged as shown below, Lalanne and Guy [14]:

$$
\mathbf{A} = \begin{bmatrix} \mathbf{0} & \mathbf{I} \\ -\mathbf{M}^{-1}\mathbf{K} & -\mathbf{M}^{-1}\mathbf{D} \end{bmatrix} \tag{14}
$$

The rotor system natural frequencies and corresponding mode shapes were determined from the imaginary part of the eigenvalues and eigenvectors of the above matrix respectively. The rotor critical speeds were then determined accordingly, Lalanne and Guy [14]. It is well known that the critical rotational speed of a rotating shaft is the angular velocity that excites the natural frequency of the rotor. This critical speed (whirling) is relying on the shaft dimensions, material, and loads.

#### **3. Numerical Case Study**

The computational model developed in the current study was applied on a case study of multidisc rotor for the sake of verifying the reliability of the model. The finite element model was completely implemented using MATLAB. This rotating machine comprises of a horizontal flexible

shaft supported on four bearing supports with four rigid discs attached to the rotor as indicated in Figure 2.



**Fig. 2.** A case study: multi-disc rotating machine model

The Finite Element model of this rotor system is illustrated in Figure 3. The rotating shaft length is 860 mm. It is divided into 12 shaft elements. There are 13 nodes with 4DoFs per node. The properties of the rotor system model and material properties of the shaft are provided in Table 1 and Table 2 respectively.



**Fig. 3.** Finite element model for the multi-disc rotating machine





Three different types for rotor bearing supports were considered. These were rigid support, semi rigid support and flexible support. For each one of these bearing types, a range of shaft diameter was investigated. The developed numerical model was employed to quantify the natural frequencies and critical speeds of the rotating machinery. The simulation results obtained were then validated experimentally.

# **4. Experimental Setup**

The experimental setup is constructed according to the numerical multi-disc rotor system model and is depicted in Figure 4. The rig comprises of a 22 mm diameter rotating shaft supported on four bearings supports with four attached discs having a diameter of 140 mm and thickness of 10 mm. The rotor was driven using a motor as shown in Figure 4 below.



**Fig. 4.** Experimental arrangement of multi-disc rotor system

Impact hammer test procedure was conducted to extract natural frequencies of the rotor system. This tap testing was performed using impact hammer as the excitation source and an accelerometer as the sensor as well as using a suitable amplifier and oscilloscope, Ewins [15]. The accelerometer was installed at different locations on the test rig. The experiments were implemented to measure first natural frequencies of the above multi-disc rotor. The corresponding critical speeds were then found accordingly.

Three different bearing supports namely, rigid, semi rigid and flexible bearing were arranged. The flexible bearing support was provided via utilizing a rubber bearing (bonded rubber bushes). Both first natural frequencies and critical speeds were measured for the three types of bearing supports considered in this work. Five shafts with different diameters of d, 1.25d, 1.5d, 1.75d, and 2d were tested for each type of the above bearing supports.

## **5. Results and Discussion**

This section discusses the results obtained numerically, from the simulation provided from the numerical model, and experimentally from the test rig.

The computational results obtained are demonstrated in Figure 5 and Figure 6. It can be seen from the results shown in these figures that the increase in the diameter of the shaft has a significant impact on both natural frequencies and critical speed of the rotor bearing system. In addition, this impact was found to be very sensitive to the rigid bearing support type, less sensitive for the semi rigid bearing and very little sensitive for the flexible bearing support type. When different bearing types is used, the stiffness of the supporting bearing changes and the natural frequency and critical speed of the rotor system varies accordingly. The more flexible bearing, the lower natural frequency and critical speed, the more rigid bearing, the higher the natural frequency and critical speed. Moreover, the increase in shaft diameter resulted in an increase in both natural frequencies and critical speeds. However, the results show that this increase was up to certain increase in shaft diameter, depending on bearing support type, after which both natural frequencies and critical speeds start decrease. Furthermore, the results show that the maximum increase in natural frequencies and critical speeds of the multi-disc rotor system was found to be corresponding to an increase in shaft diameter by 1.2 %, 1.5% and 1.8% for rigid, semi-rigid and flexible bearing supports respectively. And finally, this reflects clearly the influence of bearing support condition on the dynamics of the rotor system. It is also worth mentioning that the results obtained in this work were very good in compliance with the results found from other researchers' findings, for example, Firouzi *et al.,* [8] and Balasaheb [12]. However, the current study presented the impact of the wider variation effect of shaft diameter on the natural frequencies of the rotary machines. The numerical model developed in this research was implemented utilizing MATLAB as this software are widely used in this field and others applications, for example, studies conducted by Hamim *et al.,* [17], Annasaheb *et al.,* [18], Munimus *et al.,* [19], and Fitriadhy *et al.,* [20].



**Fig. 5.** First natural frequency of the multi-disc rotor over a range of shaft diameter



**Fig. 6.** First critical speed of the multi-disc rotor system over a range of shaft diameter

The experimental results obtained are illustrated in Figure 7 and Figure 8. Examining the results indicated in these figures confirms the same features resulted from the simulation results. In addition, by comparing the numerical results obtained using the developed computational model, shown in Figure 5 and Figure 6, with the experimental results shown in Figure 7 and Figure 8, one may deduce that the simulation results show very close agreement with the experimental results and consequently verifies the methodology developed in this study.



**Fig. 7.** The measured first natural frequencies of the rotating machine



**Fig. 8.** The measured first critical speed of the rotating machine

# **6. Conclusions**

In this study, the influence of shaft diameter of a multi-disc rotor system supported on three different bearing supports on their dynamic characteristics was investigated numerically and experimentally. The work involves developing a displacement-based finite element model to tackle this issue. The computational model was applied on a typical numerical example. The numerical model was then validated experimentally through a test rig. The results obtained in this study show that the increase in shaft diameter resulted in an increase in both natural frequencies and critical speeds up to a certain amount of increase in diameter of the shaft depending on the flexibility of the rotor system.

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## **References**

- [1] Fu, Chao, Jean-Jacques Sinou, Weidong Zhu, Kuan Lu, and Yongfeng Yang. "A state-of-the-art review on uncertainty analysis of rotor systems." *Mechanical Systems and Signal Processing* 183 (2023): 109619. <https://doi.org/10.1016/j.ymssp.2022.109619>
- [2] Shuaijun Ma, Yanjing Yin, Ben Chao, Ke Yan, Bin Fang, and Jun Hong. "A real-time coupling model of bearing-rotor system based on semi-flexible body element." *International Journal of Mechanical Sciences* 245 (2023): 108098. <https://doi.org/10.1016/j.ijmecsci.2022.108098>
- [3] Hongyang Xu, Yang Yang, Hui Ma, Zhong Luo, Xiaopeng Li, Qingkai Han, and Bangchun Wen. "Vibration characteristics of bearing-rotor systems with inner ring dynamic misalignment." *International Journal of Mechanical Sciences* 230 (2022): 107536. <https://doi.org/10.1016/j.ijmecsci.2022.107536>
- [4] Mohammad Hadi Jalali, Mostafa Ghayour, Saeed Ziaei-Rad, and Behrooz Shahriari. "Dynamic analysis of a high speed rotor-bearing system." *Measurement* 53 (2014): 1-9. <https://doi.org/10.1016/j.measurement.2014.03.010>
- [5] Tiwari, M., K. Gupta, and O. Prakash. "Effect of radial internal clearance of a ball bearing on the dynamics of a balanced horizontal rotor." *Journal of sound and vibration* 238, no. 5 (2000): 723-756. <https://doi.org/10.1006/jsvi.1999.3109>
- [6] Shooshtari, Alireza, Mahdi Karimi, Mehrdad Shemshadi, and Sareh Seraj. "Effect of impeller diameter on dynamic response of a centrifugal pump rotor." *Vibration* 4, no. 1 (2021): 117-129. <https://doi.org/10.3390/vibration4010010>
- [7] She, Houxin, Chaofeng Li, Qiansheng Tang, and Bangchun Wen. "Influence mechanism of disk position and flexibility on natural frequencies and critical speeds of a shaft-disk-blade unit." *Journal of Sound and Vibration* 469 (2020): 115156. <https://doi.org/10.1016/j.jsv.2019.115156>
- [8] Firouzi, Jabbar, and Hassan Ghassemi. "The effect of the shaft diameter and torsional stiffness on the whirling speed of the ship propeller shafting system." *Journal of Mechanical Design and Vibration* 5 (2017): 37-42. <https://doi.org/10.12691/jmdv-5-1-5>
- [9] Jinli, Xu, Wan Lei, and Luo Wenxin. "Influence of Bearing Stiffness on the Nonlinear Dynamics of a Shaft-Final Drive System." *Shock and Vibration* 2016, no. 1 (2016): 3524609. <https://doi.org/10.1155/2016/3524609>
- [10] Nayeeif, Adil Abed, Hayder Abdual khaliq Ali, Fadhel Abbas Abdulla, and Zahra Khalid. "Effect of diameter on the critical speed of a composite shaft under dynamic load." *Journal of Mechanical Engineering Research and Developments* 44, no. 6 (2021): 169-176.
- [11] Metsebo, J., Upadhyay, N., Kankar, P. K., & Nana Nbendjo, B. R. "Modelling of a rotor-ball bearings system using Timoshenko beam and effects of rotating shaft on their dynamics." *Journal of Mechanical Science and Technology* 30 (2016): 5339-5350. <https://doi.org/10.1007/s12206-016-1101-x>
- [12] Takle, B. K. "Experimental investigation of shafts on whirling of shaft apparatus." *Int J Sci, Eng Technol Res (IJSETR)* 3, no. 8 (2014).
- [13] Friswell, M. I. *Dynamics of Rotating Machines*. Cambridge University Press, 2010. <https://doi.org/10.1017/CBO9780511780509>
- [14] Lalanne, Michel, and Guy Ferraris. "Rotordynamics prediction in engineering." *England, John Wiley and Sons*, 1998.
- [15] Ewins, David J. *Modal testing: theory, practice and application*. John Wiley & Sons, 2009.
- [16] Fan, Li, Tsuyoshi Inoue, Akira Heya, and Yusuke Watanabe. "Predicting stability of synchronous nonlinear vibration in vertical rotating shaft system with journal bearing." *Journal of Sound and Vibration* 572 (2024): 118191. <https://doi.org/10.1016/j.jsv.2023.118191>
- [17] Hamim, Muhammad Zikri, Suriana Salimin, and Afarulrazi Abu Bakar. "Analysis of Variable Frequency Drive for Induction Motor using Matlab Software." *Journal of Advanced Research in Applied Mechanics* 116, no. 1 (2024): 117-129. <https://doi.org/10.37934/aram.116.1.117129>
- [18] Annasaheb, Kardile Balasaheb, Khizar Ahmed Pathan, Abhijeet Bhikashet Auti, and Sher Afghan Khan. "Simulation of Energy Dissipation in BLDC Motor and Analysis of Speed-Acoustic Characteristics." *Journal of Advanced Research in Applied Mechanics* 111, no. 1 (2023): 38-61. <https://doi.org/10.37934/aram.111.1.3861>
- [19] Salam, Syed Munimus, and Muhammad Mahbubur Rashid. "Performance Analysis of a Designed Prototype of a Motor Coupled Variable Inertia Flywheel System." *Malaysian Journal on Composites Science and Manufacturing* 12, no. 1 (2023): 62-72. <https://doi.org/10.37934/mjcsm.12.1.6272>
- [20] Fitriadhy, Ahmad, Nur Amira Adam, and Kong Wai Sheng. "Computational Prediction of a Propeller's Performance in Open Water Condition." *Journal of Ship and Marine Structures* 2, no. 1 (2023): 1-12.