



Finite Element Analysis of a Repaired Cracked Aluminium Plate with Piezoelectric Patches under Mechanical and Thermal Loading

Mohammed Abdulla¹, Meftah Hrairi^{1,*}, Abdul Aabid², Nur Azam Abdullah¹

¹ Department of Mechanical and Aerospace Engineering, Kulliyah of Engineering, International Islamic University Malaysia, Kuala Lumpur, Malaysia

² Department of Engineering Management, College of Engineering, Prince Sultan University, Riyadh, Saudi Arabia

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ABSTRACT

In thin plate structures, the use of piezoelectric actuators for active repairs can significantly decelerate the progression of crack damage. The prevalent mode-I fractures, wherein tension causes the opening displacement leading to failure, undermine the structural integrity required for load support. This current research endeavors to explore how piezoelectric (PZT) actuators influence the repair efficacy of fractured structures. The ANSYS software was harnessed to conduct the study, employing the finite element method. Additionally, the study delved into the repercussions of thermal heating and performed a parametric analysis to gauge their impact on the restoration efficiency of the compromised structure. The outcomes unveiled that the application of a negative electric field through the PZT actuator effectively diminishes the stress intensity factor (SIF) at the crack tip. Moreover, it was determined that thermal stresses contribute to a 57% augmentation in SIF, posing a heightened risk of structural failure. The study's deductions emphasize the desirability of utilizing a slender actuator alongside optimally adjusted adhesive thickness to attain a more pronounced reduction in SIF.

1. Introduction

Amid cyclic dynamic loading conditions, the presence of fracture defects hastens the degradation of structural components. This poses a challenge as maintaining the integrity of a cracked structure is both essential and financially demanding. To mitigate this, the practice of repairing cracks has emerged as an effective strategy for prolonging component life [1]. A well-recognized and widely adopted method involves the application of metallic or composite patches for mending cracks. While the success of patch composites in restoring fractured structures is remarkable, certain limitations curtail their application [2]. Notably, the design of composite patch necessitates thorough structural analysis under specific external loading patterns. However, this design's effectiveness can be undermined by changes in external loading conditions. Additionally, a persistent challenge remains in developing a robust model for evaluating the repaired structure [3].

* Corresponding author.

E-mail address: meftah@iium.edu.my

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Structural health monitoring, a common non-destructive method, has found practical application in this context. In recent years, the use of piezoelectric (PZT) actuators has gained prominence [4]. These devices have been investigated for their electromechanical response in the active control and repair of damaged structures [4-5]. The emergence of diverse numerical and analytical methodologies has facilitated such research endeavors, highlighting the advantages of PZT-based techniques. These advantages encompass cost-efficient maintenance strategies and the extension of the lifecycle of repaired structures[7].

A PZT patch exhibits two unique outcomes: passive outcomes that change the structure's mass and stiffness and active outcomes caused by applied voltage and stress. In comparison to its initial form, the passive outcomes affect the structure's rigidity & weight together, while the active outcomes enable the versatile application of PZT materials as actuators, sensors, or both [8]. Notably, PZT films are lightweight, thin, and easily attached to damaged structures [9]. This robust electro-mechanical property ensures the effectiveness of repairs even amidst external loading variations or changes in the damaged state, enabling the application of varying voltages. In prior studies, Fesharaki *et al.*, [10] employed a PZT actuator to decrease the stress in the structures which were subjected to tensile load. They discovered that PZT material helped them in minimizing the amount of stress from the plate. Hai *et al.*, [11] carried out an analytical approach to study the influence of PZT patches for restoring multiple cracked beams and found that with the application of PZT patches, the performance of beams was enhanced and fatigue life was increased. Similarly, Abuzaid *et al.*, [12] employed PZT actuators to decrease the stress concentration factor (SCF) on a structure with semicircular notches. Findings showed there was a significant fall in SCF with the application of voltage on PZT patches. An edge-cracked structure was repaired using PZT patch and the XFEM approach was utilized to evaluate the SIF at the fracture point [13]. The author found that the fatigue life of the structure was significantly increased with the application of PZT patch, and it was concluded that more improvement of the fatigue life of the structure can be attained by increasing the value of applied voltage and optimizing the thickness and shape of the patch. Abbood *et al.*, [14] discussed the applications of PZT materials in energy conversion and vibration field. The impact of notch defect under differing thermal load on electrical impedance of PZT was studied in another application of PZT materials [15].

Badrin *et al.*, [16] In their study of plate repairing under mechanical loading, Kumar *et al.*, [17] contributed by conducting an experimental evaluation of damage durability in both passive and active modes. Their findings revealed a linear reduction in stress intensity factor (SIF) as voltage increases. This relationship between voltage and SIF exhibited a linear pattern. Kumar *et al.*, [18] utilized the extended finite element method (XFEM) to repair structures with straight and angular cracks using PZT materials. They observed a 37.28% reduction in maximum displacement at the free end of the plate for the active patch in comparison to the condition of the unrepaired plate. Using PZT-based transducers, the effectiveness of Electromechanical Impedance technique was investigated for structural health monitoring purposes [19].

Abuzaid *et al.*, [20] made a significant contribution by conducting experimental and numerical analysis of a damaged structure repaired with a PZT actuator subjected to mode I stress. Their study discovered that the application of higher voltage leads to improvement in repair performance. Furthermore, increasing the patch area with lesser thickness resulted in more reduction of SIF at the fracture point. Aabid *et al.*, [21] employed PZT actuators and a composite patch for the repair of a center-cracked plate, and a reduction of 85% in SIF was recorded. Kumar *et al.*, [22] engaged in repairing cracked composite plates using PZT patches. Their efforts yielded an 81.21% reduction in stress at the crack tip compared to an unrepaired plate. Benyahia *et al.*, [23] highlighted the influence of thermal residual stresses on stress intensity factors. They noted that these stresses cause an

increase in SIF, subsequently leading to decreased fatigue life in repaired aeronautical structures. Notably, optimizing curing temperatures and adhesive qualities emerged as a strategy to lessen the impact of thermal residual strains on repair performance. Interestingly, the existing literature has yet to thoroughly explore the effects of curing temperature on the efficiency of repairs in actively repaired structures. Thus, this study fills this gap by investigating the repair of a center-cracked plate using finite element analysis, involving a PZT patch under thermo-mechanical loading conditions. Moreover, the study incorporates various adhesives to identify the most effective one for reducing stress intensity factors. Essential parameters influencing repair performance are also comprehensively examined.

2. Geometric Model of the Specimen

A structural plate with dimensions: Height $H = 200$ mm, width $2W = 80$ mm, and a thickness $T = 1$ mm, contains a central crack of length $2a$. Under mechanical loads of 1MPa at both ends of the plate, the PIC151 PZT actuator is employed. This actuator measures height $h_p = 20$ mm, width $w_p = 40$ mm, and thickness $t_p = 0.5$ mm. Accompanying the actuator is adhesive with dimensions: height $h_{ad} = 20$ mm, width $w_{ad} = 40$ mm, and adhesive thickness $t_{ad} = 0.03$ mm. This arrangement, illustrated in Figure 1, situates the PZT patch at a distance ' S ' of 1 mm away from the crack. Material properties for the damaged structure, the PZT patch, and the adhesive material are detailed in Table 1.

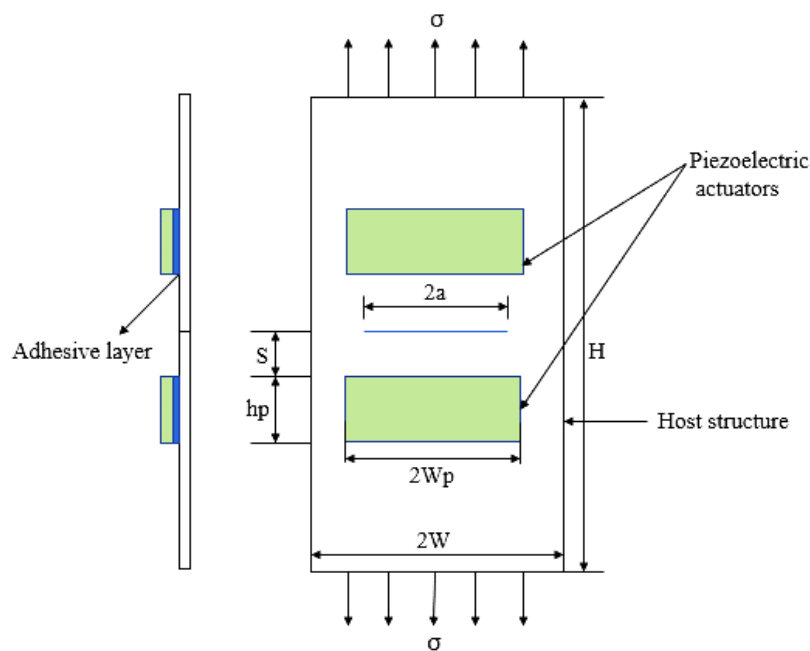


Fig. 1. A rectangular plate integrated with PZT actuators.

Table 1
 Material properties

Parameter	Host plate	PIC151 patch	Adhesive
Density	2715 kg/m ³	7800 kg/m ³	1160kg/m ³
Poisson's ratio	0.33		0.345
Young's modulus	68.95 GPa	$S_{11} = 15.0 \times 10^{-12} \text{ m}^2/\text{N}$	5.1 GPa
Compliance matrix		$S_{33} = 19.0 \times 10^{-12} \text{ m}^2/\text{N}$	
Electric permittivity coefficient		$\epsilon_{11}^T = 1977$	
		$\epsilon_{33}^T = 2400$	

3. Finite Element Modelling

For the sake of simulation, finite element modeling (FEM) is carried out using the ANSYS Mechanical APDL software. To capture this behavior accurately, an approach involving displacement extrapolation is employed. This approach utilizes the nodal displacements in the proximity of the fracture tip. This choice is made due to the substantial gradient of stress and strain fields near the crack front. Singular elements are included to ensure a thorough representation of the crack tip displacement, as shown in Figure 2(a). Specific nodes shown in Figure 2(b) are used to determine the equation's necessary fracture tip displacements.

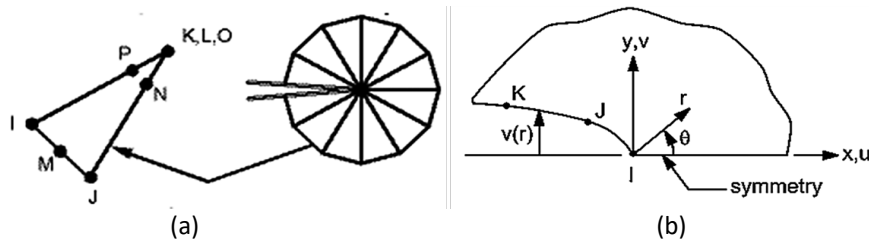


Fig. 2. (a) The singular element near the fracture tip and (b) nodes used to roughly measure the displacements of the crack tips

3.1 Meshing of the Structure

In this study, the modeling of the PZT patch utilizes ANSYS' Solid226 element. Each of the Solid226 element's 20 nodes possesses up to 5 degrees of freedom. For enhanced accuracy, the bonding layer (with a thickness of 0.03 mm) is also incorporated into the model. Both the primary plate and the bonding layer are constructed using the Solid186 element. The Solid186 element, a higher-order 20-node element, is recommended for linear applications and suits the analysis of solid structures. Figure 3 presents a representative finite element (FE) model, illustrating the integration of a center-cracked plate and a PZT actuator. Because of symmetry considerations, the modeling focuses on a single quadrant. The replication of the fracture front involves the utilization of a total of 16 separate elements. To model the host plate, PZT patch, and adhesive material, a combined total of 83,184 elements are employed.

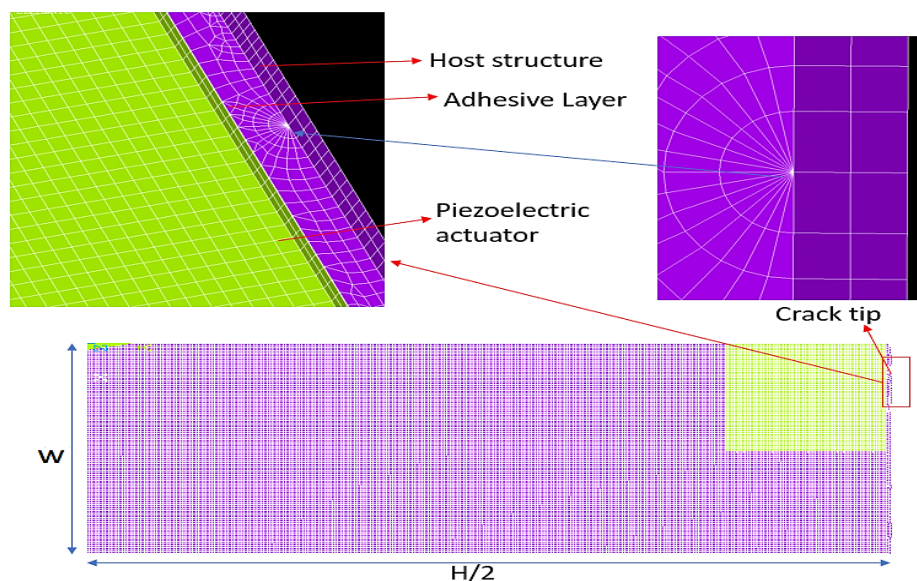


Fig. 3. Mesh model

3.2 Boundary Condition

Figure 4 illustrates the boundary conditions that have been applied to our current model. Due to symmetry, one-fourth of the PZT patch is modeled. The x-plane's interface nodes were constrained in the x-direction, which is in the horizontal direction, whereas the y-plane's interfacial nodes were constrained in the y-direction i.e., in the vertical direction.

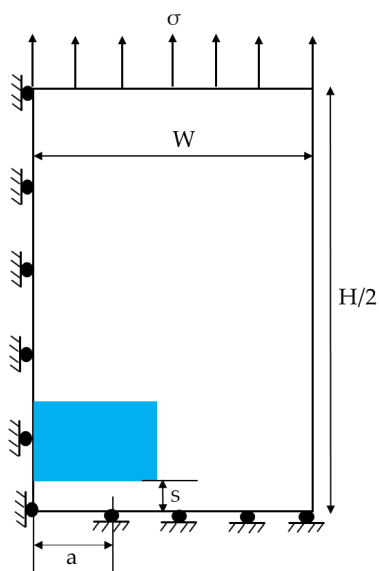


Fig. 4. Boundary conditions for centre cracked plate.

3.3 Validation of Cracked Plate for Repaired and Unrepaired Case

The evaluated cracked plate has the following dimensions: $H = 200$ mm, $2W = 80$ mm, and a thickness of 1 mm. The centre crack lengths encompass a range of values: 5 mm, 10 mm, 15 mm, and 20 mm. The host structure's material properties employed for validation are identical to those listed in Table 1. The displacement extrapolation technique is employed to calculate the SIF. The computed mode-I SIF outcomes from finite element (FE) analysis are then compared with the findings of Abuzaid *et al.*, [24]. Their study involved active repair via a PZT actuator. The outcomes of our simulation and the literature correspond well, showcasing an agreement with a percentage error of 1.55%, as evidenced in Table 2.

Table 2
 Validation of present simulation work with previous literature

Case	Present work SIF (MPa√m)	Literature work [24] (MPa√m)	Percentage error (%)
Unrepaired plate	0.1223	0.12070	1.316
Repaired plate	0.07931	0.07812	1.50

4. Results

In this section, the modelled plate along with adhesive and patch is solved using the SOLVE command. After the problem has been solved, the crack tip region is zoomed to define the crack face

path to acquire the SIF. Three nodes are also used to build the local coordinate system. Finally, the KCALC command is used to determine the Mode-I SIF.

4.1 Effect of Positive and Negative Voltage on SIF

As indicated in Figure 5, the impact of applying a positive electric field to the PZT patch is evident. This application leads to a rise of SIF, surpassing the SIF recorded for unrepaired scenarios. Conversely, the utilization of a negative electric field results in a reduction of SIF compared to the SIF value observed in the unrepaired state. The negative electric field prompts positive strain in the patches, leading to an altered stress distribution around the fracture tip and consequent SIF reduction. In contrast, when a positive electric field is employed, the PZT actuators generate a negative strain, elevating the SIF and subsequently escalating the fracture's severity, potentially causing additional damage. Consequently, the selection of the applied electric field becomes crucial in achieving the objective of SIF reduction. This can be accomplished by aligning the stress developed by the actuator in a direction opposing external stresses. Consequently, for subsequent investigations, a negative electric field is adopted as the applied voltage to attain a reduction in SIF.

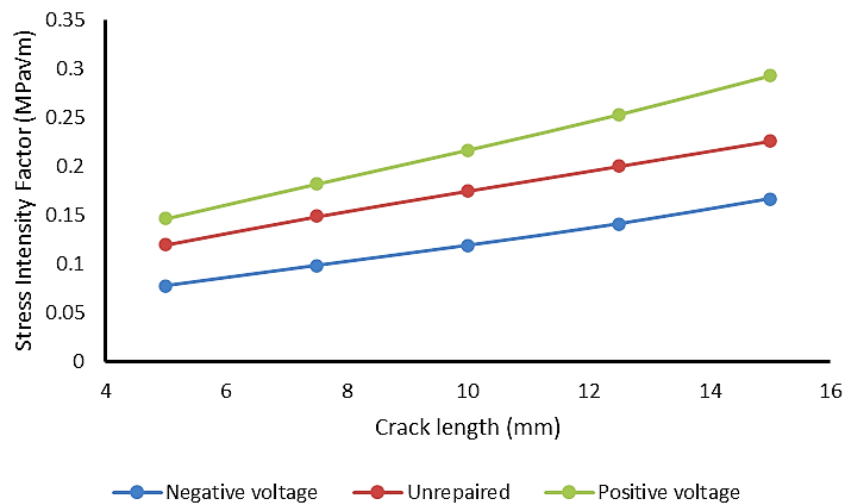


Fig. 5. Effect of negative and positive voltage on SIF

4.2 Effect of Crack Length on SIF

The impact of the PZT patch on SIF is illustrated in Figure 6. Introducing the PZT patch for repairing the damaged plate yielded a significant SIF reduction of approximately 42.65% through the application of 100 V. This reduction is attributed to the voltage-induced mechanical strain around the crack tip, effectively mitigating SIF in its vicinity. The trend of SIF's relationship with crack length holds for both repaired and unrepaired cases, showing an increase with longer cracks. In the context of repaired plates, a more pronounced reduction in SIF demands a higher negative electric field. Consequently, amplifying the voltage can yield a more substantial reduction in SIF.

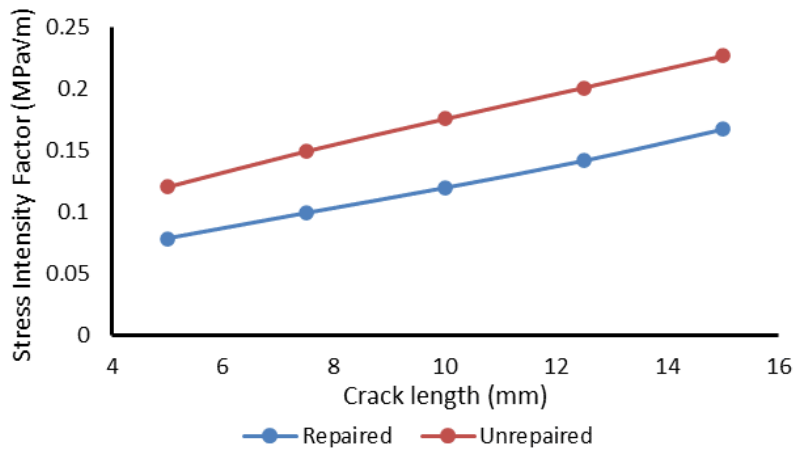


Fig. 6. Effect of crack length on SIF

4.3 Effect of Varying Voltage on SIF

Figure 7 presents the influence of varying applied voltage on the stress intensity factor (SIF). Analysis findings demonstrate a consistent linear decline in SIF as the applied voltage increases. This behavior can be attributed to the direct relationship between the narrowing stress caused by the PZT actuator and the applied voltage. Within the voltage range of 0 V to 140 V, a notable reduction of 54.5% in SIF is achieved. Additionally, an increment of approximately 8% in SIF reduction is noted for every 20 V increase in voltage. Consequently, to effectively diminish SIF and enhance the operational longevity of the repaired structure, it is advisable to escalate the applied voltage.

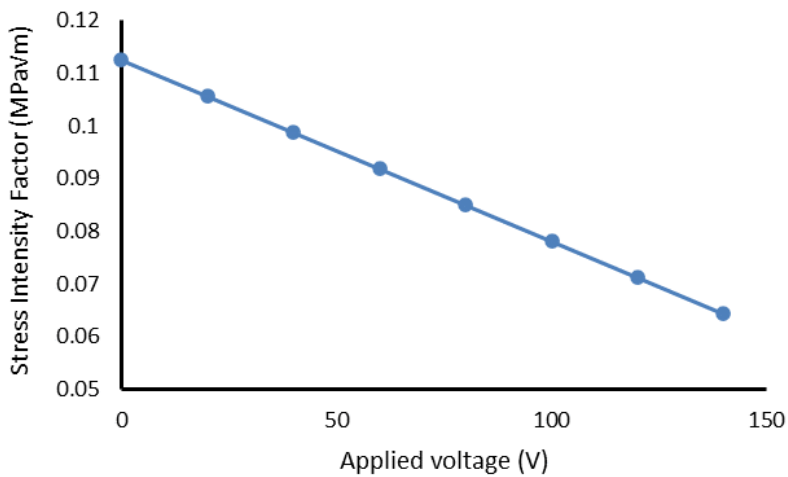


Fig. 7. SIF with various applied voltage

4.4 Effect of Adhesive Thickness on SIF

The impact of varying thicknesses of adhesive on the SIF is depicted in Figure 8. The figure reveals that even a slight increment in adhesive thickness leads to an elevation in the SIF value. This underscores the significance of making prudent choices regarding adhesive thickness. Greater thickness can increase adhesion strength, but it might hinder the patch's ability to transfer loads effectively, lessening its benefits. Conversely, opting for a smaller thickness shifts stress toward the patch, potentially increasing the risk of adhesive failure. Considering this, achieving an optimal adhesive thickness is pivotal for striking the right balance between these considerations.

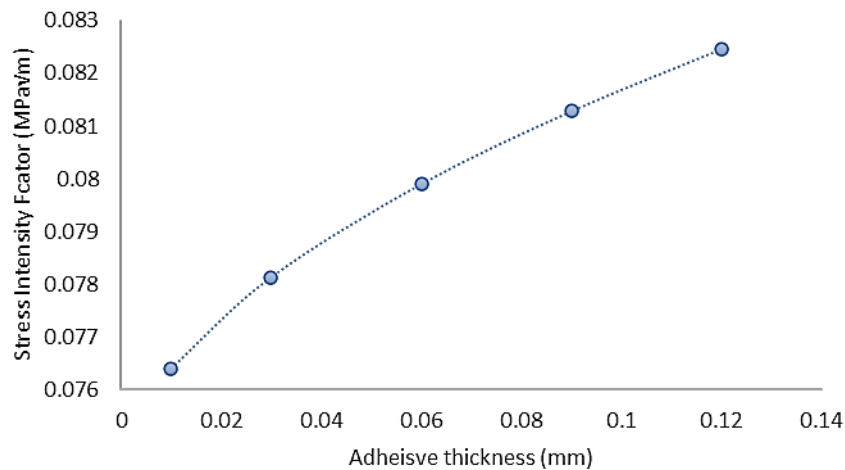


Fig. 8. Effect of adhesive thickness on SIF

4.5 Effect of Different Adhesive Materials on SIF

To identify the best adhesive materials, the repaired plate was further examined using various types of adhesives. There are multiple adhesives with various mechanical characteristics in this section. One of the elements that helps reduce the SIF is the mechanical characteristics of the adhesive. Table 3 displays the mechanical properties of the adhesive used in this analysis.

Table 3

Material properties of adhesives

Adhesive	Young's Modulus, E (GPa)	Poisson ratio	Shear Modulus, G (GPa)
Adekit A140	2.69	0.30	0.99
Araldite 2014	5.1	0.345	1.89
Epon 422J	3.49	0.29	1.10
Masterbond ESP 110	3	0.3	-
AV138/ HV998	4.59	0.47	1.56
FM73	2.55	0.32	0.96

Table 4 presents SIF values acquired through the utilization of different adhesive types. The outcomes from the table establish FM 73 as yielding the largest values of SIF for each crack length, surpassing the other adhesive options. On the contrary, Araldite 2014 yields the lowest SIF values, followed by AV138/HV998, Epon 422J, Masterbond ESP 110, and Adekit A140. This can be attributed to the higher shear modulus of Araldite 2014 compared to the other adhesives. A higher shear modulus indicates greater stiffness and resistance to deformation under applied stress. As a result, Araldite 2014 is better able to distribute loads efficiently from the host structure to the patch, leading to reduced stress concentrations at the crack tip and subsequently lower SIF values. Based on the analysis results, Araldite 2014 emerges as the most suitable adhesive choice due to consistently exhibiting the lowest SIF values across all crack lengths. Its application for bonding the PZT patch to the host structure can facilitate superior SIF reduction outcomes.

Table 4
 SIF for different Adhesive Materials

Adhesives under crack length of (mm)	Stress intensity factor (MPavm)					
	Araldite 2014	AV138/HV998	Epon 422J	Masterbond ESP 110	Adekit A140	FM73
5	0.078126	0.078358	0.07893	0.079329	0.079627	0.079807
7.5	0.098932	0.099205	0.099847	0.10031	0.10065	0.10086
10	0.11948	0.1198	0.12053	0.12107	0.12147	0.12172
12.5	0.14151	0.14189	0.14274	0.14337	0.14384	0.14413
15	0.1672	0.16764	0.16862	0.16934	0.16988	0.17021

4.6 Effect of Patch Thickness on SIF

The thickness of the patch holds a significant role in determining the repair efficiency of the mended structure. As evident from Figure 9, an observable trend is the escalating reduction in SIF as the thickness of the PZT actuator grows, particularly under passive conditions and lower voltages. However, this impact of increasing actuator thickness is not universally consistent and tends to lose effectiveness as the applied electric field intensifies. This shift becomes especially pronounced at higher electric field levels. This behavior could be attributed to the inverse relationship between the strain generated by the PZT actuator and its thickness. In this context, employing a thin actuator alongside a relatively higher voltage is preferred. This approach is conducive to achieving optimal SIF reduction outcomes, acknowledging that the strain produced by the actuator is inversely proportional to its thickness.

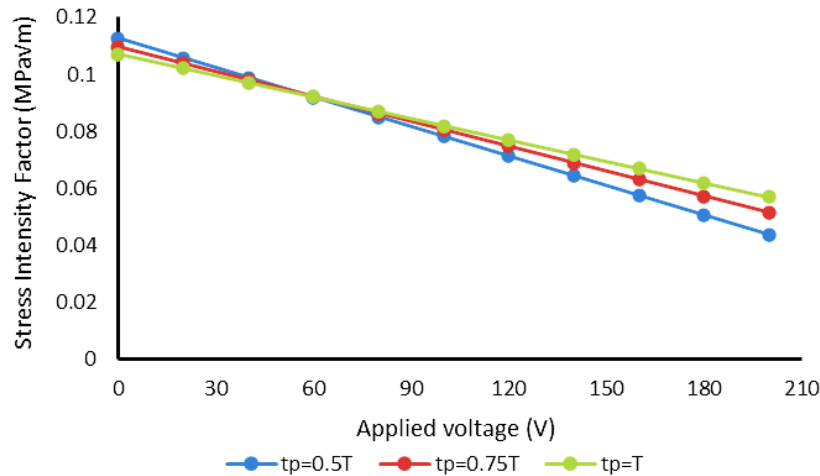


Fig. 9. Effect of patch thickness on SIF

4.7 Effect of Distance Between Crack Tip and PZT Actuator “S” on SIF

The effectiveness of the repair is influenced by the distance between the crack and the actuator, which controls how much stress is transferred to the crack surfaces. Consequently, due to the concentration of high stresses at the actuator's periphery within the integrated structure, it is expected that the edge of the PZT actuator is situated close to the fracture point. As demonstrated in Figure 10, augmenting the distance between point “S” and the fracture tip is mirrored by growth in the SIF. Consequently, the optimal placement strategy involves situating the PZT actuator near the crack tip. This positioning facilitates the attainment of minimal SIF values.

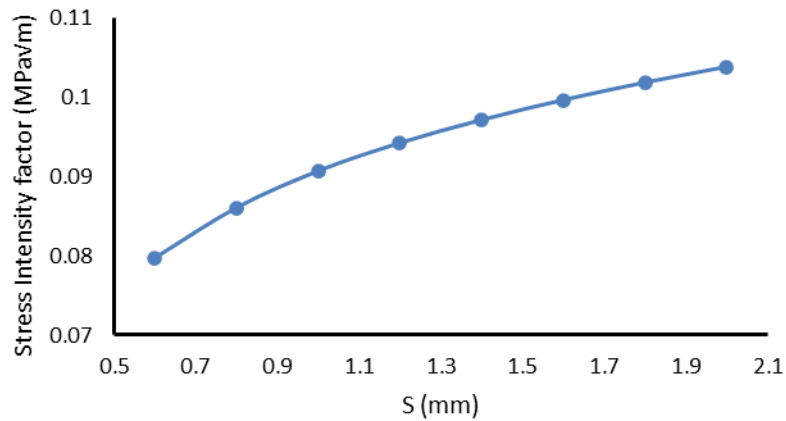


Fig. 10. Effect of distance between the crack tip and PZT actuator

4.8 Thermo-Mechanical Loading

The procedure of bonding through robust structural adhesives typically necessitates the adhesive's curing at temperatures surpassing ambient conditions. In the initial bonding stage, the plate is specifically heated to a designated temperature T_i during the curing procedure and after curing is performed, it is cooled down to ambient temperature. The strength of the bond between the PZT patch and the mended panel is notably impacted by the curing temperature. Nonetheless, this curing process concurrently leads to the emergence of thermal residual stresses, which in turn, impairs the overall effectiveness of the repair process. These stresses can be derived by

$$\sigma = -1/2 \alpha \cdot E \Delta T \tag{1}$$

To assess the potential impact of these stresses on repair outcomes, the stress intensity factor (SIF) is calculated under two scenarios: one accounting for thermal residual stress and the other without it. A mechanical load of 100MPa is imposed on the repaired plate, accompanied by thermal stress. The curing process takes place at a temperature of 120°C, while the ambient condition is represented by 20°C.

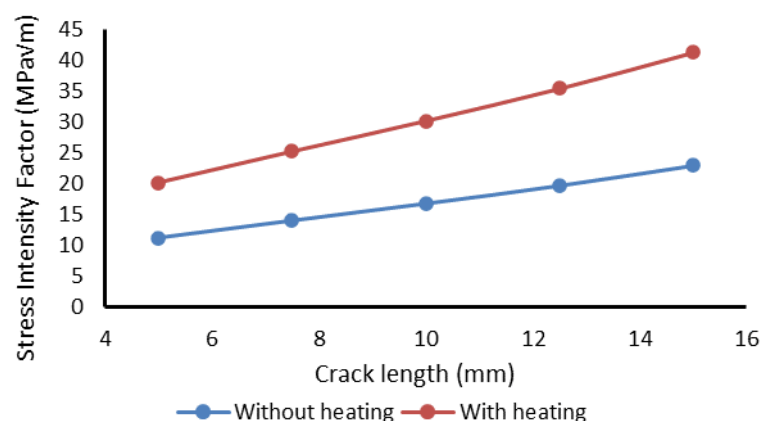


Fig. 11. Effect of thermal heating on SIF

The investigation reveals that the curing process involving adhesive bonding introduces stress residue at the crack tip. This residual stress notably expedites crack propagation, thereby diminishing the repair's efficacy. Consequently, when assessing the fatigue life of renewed aircraft structures, it

becomes imperative to account for the influence of thermal residual stresses. The graphical representation in Figure 11 illustrates that the introduction of thermal stress to the mechanical load leads to an approximate 57% increase in SIF across the entire crack length. The thermal residual stresses brought on by the cooling process do, in fact, significantly raise the values of the SIF. The PZT patch experiences compressive thermal stresses during cooling while the aluminum, whose thermal coefficient of expansion is larger than that of the PZT patch, is subjected to tensile thermal stresses. These stresses would result in a positive mean SIF, hastening the formation of cracks. This increment in SIF could potentially contribute to a reduction in the structure's overall fatigue life.

5. Conclusion

This study delved into the assessment of the PZT actuator's efficacy concerning mode-I SIF using finite element analysis. The results obtained from this investigation revealed a substantial 42.65% reduction in SIF at the crack tip of a cracked plate when compared to the unrepaired plate scenario. This reduction was achieved through the application of a negative field. Notably, the study shed light on the adverse influence of thermal stresses on the repair efficacy of mended plates. It was found that a 57% rise in SIF was observed due to the impact of thermal stresses. In this context, the utilization of adhesives capable of undergoing curing at room temperature was recommended as a strategy to mitigate the detrimental impact of thermal stress. Moreover, the study underscored the significance of optimizing certain factors for maximal SIF reduction. Employing a higher voltage in conjunction with a slender actuator thickness was found to be conducive to achieving the most substantial SIF reduction. Meanwhile, careful calibration of adhesive thickness emerged as crucial. Different adhesives were employed to achieve the least value of SIF, and Araldite 2014 performed well to attain the same. Overall, the findings underscored the commendable performance of PZT actuators in curtailing SIF, thereby fostering the encouragement for the application of intelligent materials in proactive efforts to reduce stress and repair.

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