Free Convection of Viscoelastic Nanofluid Flow on a Horizontal Circular Cylinder with Constant Heat Flux

Rahimah Mahat[1,*], Sharidan Shafie[2], Imran Ullah[3]

[1] Universiti Kuala Lumpur Malaysian Institute of Industrial Technology, Persiaran Sinaran Ilmu Bandar Seri Alam, 81750 Masai, Johor Bahru, Johor, Malaysia
[2] Department of Mathematical Sciences, Faculty of Science, Universiti Teknologi Malaysia, 81310 Johor Bahru, Malaysia
[3] College of Civil Engineering, National University of Sciences and Technology, 44000, Islamabad, Pakistan

ARTICLE INFO	ABSTRACT
	The current research study is focusing on a mathematical model for free convection boundary flow on a horizontal circular cylinder in a viscoelastic nanofluid has been constructed in this paper with boundary conditions constant heat flux. The Tiwari and Das Nanofluid model have been chosen in this analysis to investigate more nanofluid effects. A dimensionless set of partial differential equations is formed by using suitable non-dimensional variables. The transformed boundary layer equations are solved numerically using a finite difference scheme namely the Keller-box method. The effects of a few chosen factors on flow and heat transmission are investigated. Numerical solutions were obtained for the reduced skin friction coefficient, Nusselt number and Sherwood number as well as the velocity and temperature profiles. The features of the flow and heat transfer characteristics for various values of the viscoelastic parameter and nanoparticles volume fraction were analysed and discussed.

1. Introduction

Natural convection, often referred to as free convection, is a process, or form of mass and heat transfer, in which fluid motion is produced only by density variations within the fluid that arise because of temperature gradients, rather than by any external source like a pump, fan, suction device. Natural convection heat transfer has been researched for a long time because it is a model issue that is typical for many technical applications, including cooling electronics, heat exchangers, and other thermal systems. Recently, extensive research has been conducted on natural heat transfer such as [1-7]. Because of that, the heat transfers of natural convection that are applied on a horizontal circular cylinder also recently has attracted the attention of many researchers such as [8-12] due to their importance in engineering and industrial technology.

A few decades ago, the demands for heat transmission in industry, including manufacturing, refrigeration, automobiles, air conditioners, aviation, and other high-energy equipment, increased

* Corresponding author.
E-mail address: rahimahm@unikl.edu.my

quickly. The standard heat transfer techniques can scarcely keep up with the demands as they increase. Numerous variables that restrict the use of traditional procedures have been found after some research. Because they are less efficient at transferring heat than metal, one of the main limitations of traditional heat transmission is their poor thermal properties [13]. Increasing the thermal conductivity of fluids by suspending conducting particles became a viable method to address this poor attribute. However, the suspended particles in the range of micro to millimetres encountered a variety of issues, such as erosion caused by abrasive action, particle settling due to gravity, and blockage of narrow flow passageways. Despite having greater conductivities, they are still not useful as heat transfer fluids [14]. Nanometer-sized particles were used to replace these barriers. Argonne National Laboratory research by Choi [15] led to the concept's initial realization. Many researchers, such as [16-20], have undertaken extensive research on nanofluids with various effects and geometries.

For the current research, the natural convection of a viscoelastic nanofluid past a horizontal circular cylinder with constant heat flux is theoretically investigated. The effects of Prandtl number, Pr and nanoparticles volume fraction, ϕ and how these affect the thermal characteristics of the system are of particular interest.

2. Methodology

Choose the x-axis and y-axis such that they are orthogonal to one another. Consider a horizontal circular cylinder with radius a that is immersed in a nanofluid at room temperature T_∞ and heated to a constant heat flux q_w. Beginning at the lowest stagnation point = 0, and in the directions that are normal to it, the orthogonal coordinates of and are measured along the cylinder's surface. The dimensional governing equations of steady free convection boundary layer flow are given below [21, 22], assuming that the boundary layer approximations are accurate. The system Eqs. (1) - (3) and Eqs (15) - (16), together with the boundary conditions (4) and (17), are solved numerically using an implicit finite-difference method known as the Keller-box method. The model has been solved in two types of equations where Eqs. (12) - (13) are in the form of PDE (full equation) and Eqs (15) - (16) are in the form of ODE (stagnation point) corresponding to boundary conditions (14) and (17), respectively. As the scheme is unconditionally stable, Newton's method is first introduced to linearize the nonlinear system of equations before a block tridiagonal elimination method is employed on the coefficient matrix of the finite difference equations. The step size applied is $\Delta\eta = 0.025$. In this situation, the convergence criterion is set at 10^{-6} which gives accuracy up to six decimal places. The model used in this work is the Tiwari and Das model [23], which is a single-phase model that employs the Brickman viscosity model. The dimensional governing equations of momentum equation and energy equation may be stated as follows under the presumptions and taking the nanofluid model into account.

$$\frac{\partial \bar{u}}{\partial \bar{x}} + \frac{\partial \bar{v}}{\partial \bar{y}} = 0, \tag{1}$$

$$\bar{u}\frac{\partial \bar{u}}{\partial \bar{x}} + \bar{v}\frac{\partial \bar{u}}{\partial \bar{y}} = \frac{\mu_{nf}}{\rho_{nf}}\frac{\partial^2 \bar{u}}{\partial \bar{y}^2} - \frac{k_o}{\rho_{nf}}\left[\frac{\partial}{\partial \bar{x}}\left(\bar{u}\frac{\partial^2 \bar{u}}{\partial \bar{y}^2}\right) + \bar{v}\frac{\partial^3 \bar{u}}{\partial \bar{y}^3} - \frac{\partial \bar{u}}{\partial \bar{y}}\frac{\partial^2 \bar{u}}{\partial \bar{x}\partial \bar{y}}\right] + g\beta_{nf}\left(T - T_\infty\right)\sin\left(\frac{\bar{x}}{a}\right), \tag{2}$$

$$\overline{u}\frac{\partial T}{\partial \overline{x}} + \overline{v}\frac{\partial T}{\partial \overline{y}} = \frac{k}{\rho C_p}\frac{\partial^2 T}{\partial \overline{y}^2}, \tag{3}$$

considering the boundary conditions

$$\overline{u} = 0,\ \overline{v} = 0,\ T = -\frac{q_w}{k_{nf}} \qquad \text{at} \qquad \overline{y} = 0,\ \overline{x} \geq 0,$$

$$\overline{u} = \overline{u}_e(\overline{x}),\ \frac{\partial \overline{u}}{\partial \overline{y}} = 0,\ T = T_\infty \qquad \text{at} \qquad \overline{y} \to \infty,\ \overline{x} \geq 0, \tag{4}$$

where k_{nf} is the nanofluid thermal conductivity, T is the fluid temperature and q_w the constant heat flux. Table 1 provides the numerical values for the thermophysical characteristics of the base fluid and nanoparticles.

Table 1
Thermophysical characteristics between base fluid and nanoparticles

Physical Properties	$\rho\left(\text{kg }m^{-3}\right)$	$C_p\left(\text{J kg}^{-1}\text{K}^{-1}\right)$	$k\left(\text{Wm}^{-1}\text{K}^{-1}\right)$	$\beta \times 10^5\left(\text{K}^{-1}\right)$
Base Fluid (CMC)	997.1	4179	0.613	21
Nanoparticle (Cu)	8933	385	401	1.67

The controlling non-dimensional variables were then introduced:

$$x = \overline{x}/a,\ y = Gr^{1/4}\left(\overline{y}/a\right),\ v = \frac{a}{v}Gr^{-1/2}\overline{v},$$

$$u = \frac{a}{v}Gr^{-1/4}\overline{u},\ \theta = Gr^{1/4}\left(\overline{T} - \overline{T}_\infty\right)/\left(q_w a/k\right), \tag{5}$$

where *Re* is Reynolds number. The dimensionless system below is produced by replacing Eq. (5) with Eq. (1) through Eq. (3).

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{6}$$

$$\left[\left(1-\phi\right) + \phi\frac{\rho_s}{\rho_f}\right]\left[u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right] = \frac{1}{\left(1+\phi\right)^{2.5}}\frac{\partial^2 u}{\partial y^2}$$

$$-K\left[\frac{\partial}{\partial x}\left(u\frac{\partial^2 u}{\partial y^2}\right) + v\frac{\partial^3 u}{\partial y^3} - \frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial x \partial y}\right] + \left[\left(1-\phi\right) + \phi\frac{\left(\rho\beta\right)_s}{\left(\rho\beta\right)_f}\right]\theta \sin(x), \tag{7}$$

$$\left[\left(1-\phi\right) + \phi\frac{\left(\rho C_p\right)_s}{\left(\rho C_p\right)_f}\right]\left[u\frac{\partial \theta}{\partial x} + v\frac{\partial \theta}{\partial y}\right] = \frac{\left(k_s + 2k_f\right) - 2\phi\left(k_f - k_s\right)}{\left(k_s + 2k_f\right) + \phi\left(k_f - k_s\right)}\frac{1}{\text{Pr}}\frac{\partial^2 \theta}{\partial y^2}, \tag{8}$$

given the modified boundary conditions,

$$u = 0, \qquad v = 0, \qquad \frac{\partial \theta}{\partial y} = -1, \qquad \text{at} \qquad y = 0, x \geq 0,$$

$$u = 0, \qquad \frac{\partial u}{\partial y} = 0, \quad \theta = 0, \qquad \text{as} \qquad y \to \infty, x \geq 0, \tag{9}$$

where $\Pr = \mu_f C_p / k_f$ is Prandtl number, $K = k_o U_\infty / \mu_f a$ is viscoelastic parameter.

3. Mathematical Solution

The following variables have been considered in order to solve Eq (6) through (8), subject to the boundary conditions (9).

$$\psi = x F(x, y), \qquad \theta = \theta(x, y), \tag{10}$$

are introduced where ψ is the stream function defined as

$$u = \frac{\partial \psi}{\partial y}, \qquad v = -\frac{\partial \psi}{\partial x}. \tag{11}$$

Eqs. (10) and (11) were substituted into Eq. (6) through Eq. (8) to achieve;

$$\left[(1-\phi) + \phi \frac{\rho_s}{\rho_f} \right] \left[\left(\frac{\partial F}{\partial y} \right)^2 + x \frac{\partial F}{\partial y} \left(\frac{\partial^2 F}{\partial x \partial y} \right) - x \frac{\partial F}{\partial x} \frac{\partial^2 F}{\partial y^2} - F \frac{\partial^2 F}{\partial y^2} \right]$$

$$= \frac{1}{(1+\phi)^{2.5}} \frac{\partial^3 F}{\partial y^3} + \left[(1-\phi) + \phi \frac{(\rho\beta)_s}{(\rho\beta)_f} \right] \theta \frac{\sin x}{x} \tag{12}$$

$$+ K \left[2 \frac{\partial F}{\partial y} \frac{\partial^3 F}{\partial y^3} - F \frac{\partial^4 F}{\partial y^4} - \left(\frac{\partial^2 F}{\partial y^2} \right)^2 + x \left(\frac{\partial^2 F}{\partial x \partial y} \frac{\partial^3 F}{\partial y^3} - \frac{\partial F}{\partial x} \frac{\partial^4 F}{\partial y^4} + \frac{\partial F}{\partial y} \frac{\partial^4 F}{\partial x \partial y^3} - \frac{\partial^2 F}{\partial y^2} \frac{\partial^3 F}{\partial x \partial y^2} \right) \right],$$

$$\left[(1-\phi) + \phi \frac{(\rho C_p)_s}{(\rho C_p)_f} \right] \left[x \frac{\partial F}{\partial y} \frac{\partial \theta}{\partial x} - x \frac{\partial F}{\partial x} \frac{\partial \theta}{\partial y} - F \frac{\partial \theta}{\partial y} \right] = \frac{k_{nf}}{k_f} \frac{1}{\Pr} \frac{\partial^2 \theta}{\partial y^2}, \tag{13}$$

of which the following boundary conditions apply

$$F = 0, \qquad \frac{\partial F}{\partial y} = 0, \qquad \frac{\partial \theta}{\partial y} = -1, \qquad \text{at} \qquad y = 0, x \geq 0,$$

$$\frac{\partial F}{\partial y} = 0, \qquad \frac{\partial^2 F}{\partial y^2} = 0, \quad \theta = 0, \quad \text{as} \qquad y \to \infty, x \geq 0, \tag{14}$$

When $x \approx 0$, the ordinary differential Eqs. (12) and (13) are as follows:

$$\frac{1}{(1+\phi)^{2.5}} f''' - \left[(1-\phi) + \phi \frac{\rho_s}{\rho_f} \right] \left[f'^2 - ff'' \right] + K\left(2ff''' - ff^{iv} - f'^2 \right)$$

$$+ \left[(1-\phi) + \phi \frac{(\rho\beta)_s}{(\rho\beta)_f} \right] \lambda\theta - Mf' = 0, \tag{15}$$

$$\frac{\left(k_s + 2k_f\right) - 2\phi\left(k_f - k_s\right)}{\left(k_s + 2k_f\right) + \phi\left(k_f - k_s\right)} \frac{1}{\mathrm{Pr}} \theta'' + \left[(1-\phi) + \phi \frac{(\rho C_p)_s}{(\rho C_p)_f} \right] f\theta' = 0, \tag{16}$$

considering the boundary conditions

$$\begin{aligned} f(0) &= 0, & f'(0) &= 0, & \theta'(0) &= -1, \\ f'(\infty) &= 0, & f''(\infty) &= 0, & \theta(\infty) &= 0, \end{aligned} \tag{17}$$

4. Results

Analysis is done on the fluid flow behaviour for viscoelastic nanofluid via a horizontal circular cylinder subject to continuous heat flux. By comparing the skin friction and heat transfer coefficients with the findings from [24] and [25], the validity of numerical solutions is determined. The results as displayed in Table 2 have produced good agreements.

Table 2
Numerical values of C_f for different values of the x at $K = 0$, $\mathrm{Pr} = 1$, $\phi = 0$

	C_f			θ_w		
x	Merkin and Pop [24]	Kasim *et al.,* [25]	Present	Merkin and Pop [24]	Kasim *et al.,* [25]	Present
0.0	0.000	0.000000	0.0000	1.996	1.996116	1.99351
0.2	0.274	0.279052	0.2786	1.999	1.997936	1.99482
0.6	0.795	0.795651	0.7908	2.014	2.012061	2.01303
1.0	1.241	1.243863	1.2455	2.043	2.041627	2.04159

Figures 1 and 2 show graphs of skin friction and heat transfer coefficients as well as the impacts of nanoparticle volume fraction acting on fluid. It has been found that increasing the volume percentage of nanoparticles increases the measurements of skin friction and heat transfer. This is predicted because an increase in the volume percentage of nanoparticles improves the fluid's thermal conductivity, which causes an increase in the coefficients of skin friction and heat transfer.

The impact of the Prandtl number on the skin friction coefficient and heat transfer coefficient in comparison to K = 1.0 and $\phi = 0.03$ is discussed in Figures 3 and 4. Figure 3 shows that, for a given value of Pr, the skin friction coefficient grows monotonically. Additionally, it should be remembered that when Pr increases, the skin friction coefficient lowers. Figure 4 illustrates how the skin friction coefficient decreased as the Prandtl number increased. Additionally, for a certain Pr, the heat transfer coefficient rises in a specific area before falling along the positive *x*-direction.
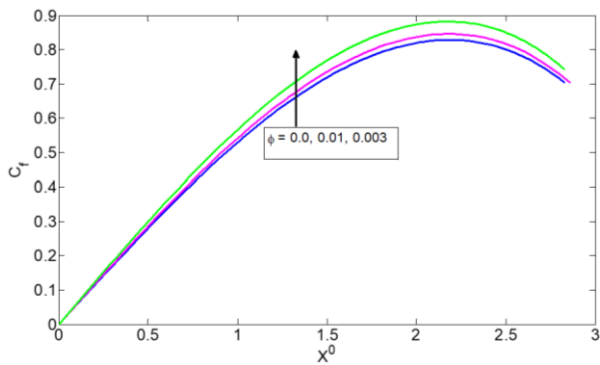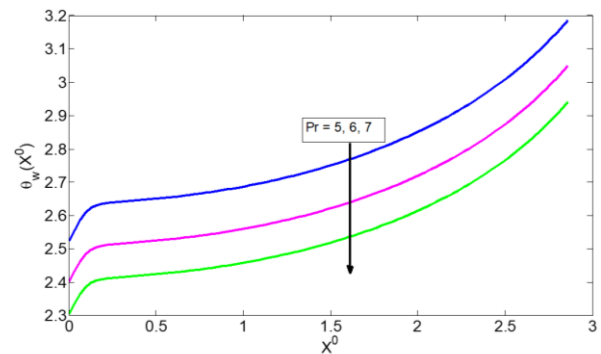
Fig. 1. Variation values of $\phi = 0$ for skin friction coefficient
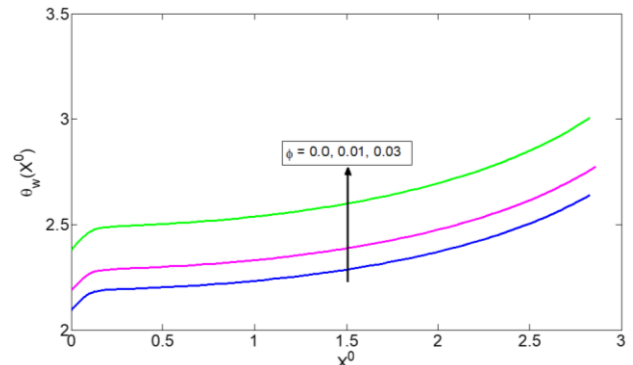
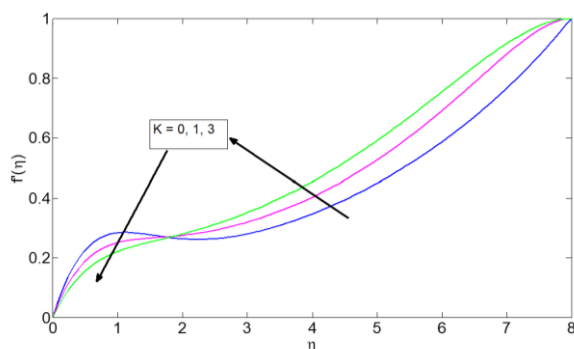**Fig. 2.** Variation values of $\phi = 0$ for heat transfer coefficient
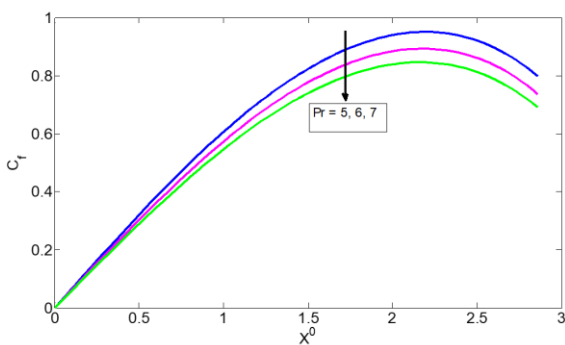
**Fig. 3.** Variation values of Pr for skin friction coefficient

Fig. 4. Variation values of Pr for heat transfer coefficient

The fluid where the graphs of velocity and temperature profiles are presented in Figures 5 and 6 is affected by the viscoelastic parameter, K. With an increase in the viscoelastic parameter, the profiles of velocity drop and grow while the temperature rises. According to the temperature profiles, a rise in the viscoelastic parameter's value causes a rise in the temperature distribution. The temperature then gradually drops to the surrounding air's temperature. This occurs because of the viscoelasticity's qualities, which combine both elastic and viscous traits. Due to the viscosity property, which states that a fluid with a higher viscosity opposes the motion, the velocity reduces as K increases. As K increases, the temperature profile accordingly rises.

Fig. 5. Variation values of K for velocity profiles.
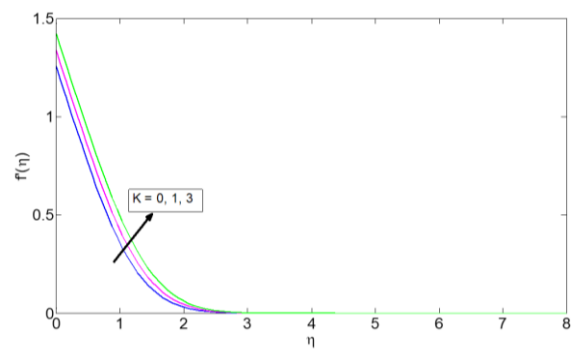
Fig. 6. Variation values of K for temperature profiles.

6

5. Conclusions

The conclusions drawn from this modern research are as follows: Velocity slows down and increases with augmentation in viscoelastic parameters. In the meantime, skin friction and heat transfer coefficients increase when nanoparticles volume increase. Besides that, the effects of the Prandtl number show the opposite results where skin friction and heat transfer coefficients decrease for the increasing values of Pr.

Acknowledgement

References
[1] Miroshnichenko, I. V., and M. A. Sheremet. "Turbulent natural convection heat transfer in rectangular enclosures using experimental and numerical approaches: A review." *Renewable and Sustainable Energy Reviews* 82 (2018): 40-59. https://doi.org/10.1016/j.rser.2017.09.005

[2] Teh, Yuan Ying, and Adnan Ashgar. "Three dimensional MHD hybrid nanofluid Flow with rotating stretching/shrinking sheet and Joule heating." *CFD Letters* 13, no. 8 (2021): 1-19. https://doi.org/10.37934/cfdl.13.8.119

[3] Mohamed, Muhammad Khairul Anuar, Anuar Mohd Ishak, Ioan Pop, Nurul Farahain Mohammad, and Siti Khuzaimah Soid. "Free Convection Boundary Layer Flow from a Vertical Truncated Cone in a Hybrid Nanofluid." *Malaysian Journal of Fundamental and Applied Sciences* 18, no. 2 (2022): 257-270. https://doi.org/10.11113/mjfas.v18n2.2410

[4] Rusdi, Nadia Diana Mohd, Siti Suzilliana Putri Mohamed Isa, Norihan Md Arifin, and Norfifah Bachok. "Thermal Radiation in Nanofluid Penetrable Flow Bounded with Partial Slip Condition." *CFD Letters* 13, no. 8 (2021): 32-44. https://doi.org/10.37934/cfdl.13.8.3244

[5] Murad, Nurul Musfirah, Noraihan Afiqah Rawi, Sharidan Shafie, Yeou Jiann Lim, and Rahimah Mahat. "Unsteady Falkner-Skan flow of Hybrid Nanofluid over a Nonlinear Moving Wedge." *Malaysian Journal of Fundamental and Applied Sciences* 18, no. 1 (2022): 116-123. https://doi.org/10.11113/mjfas.v18n1.2421

[6] Omar, Nur Fatihah Mod, Husna Izzati Osman, Ahmad Qushairi Mohamad, Rahimah Jusoh, and Zulkhibri Ismail. "Analytical Solution on Performance of Unsteady Casson Fluid with Thermal Radiation and Chemical Reaction." *Journal of Advanced Research in Numerical Heat Transfer* 11, no. 1 (2022): 36-41.

[7] Lam, Boon Chun, Yutaka Asako, Chungpyo Hong, Lit Ken Tan, and Nor Azwadi Che Sidik. "Validity of Performance Factors used in Recent Studies on Heat Transfer Enhancement by Surface Modification or Insert Devices: Constant Heat Flux Case." *Journal of Advanced Research in Numerical Heat Transfer* 11, no. 1 (2022): 1-4.

[8] Kanafiah, Siti Farah Haryatie Mohd, Abdul Rahman Mohd Kasim, Syazwani Mohd Zokri, and Mohd Rijal Ilias. "Combined Convective Transport of Brinkman-viscoelastic Fluid Across Horizontal Circular Cylinder with Convective Boundary Condition." *Journal of Advanced Research in Fluid Mechanics and Thermal Sciences* 89, no. 2 (2022): 15-24. https://doi.org/10.37934/arfmts.89.2.1524

[9] Mohamed, Muhammad Khairul Anuar, Mohd Zuki Salleh, Fadhilah Che Jamil, and Ong Huei. "Free convection boundary layer flow over a horizontal circular cylinder in Al2O3-Ag/water hybrid nanofluid with viscous dissipation." *Malaysian Journal of Fundamental and Applied Sciences* 17, no. 1 (2021): 20-25 https://doi.org/10.11113/mjfas.v17n1.1964

[10] Kanafiah, Siti Farah Haryatie Mohd, Abdul Rahman Mohd Kasim, Syazwani Mohd Zokri, and Nur Syamilah Arifin. "Numerical Investigation at Lower Stagnation Point Flow Over a Horizontal Circular Cylinder of Brinkman-Viscoelastic Fluid." *Journal of Advanced Research in Fluid Mechanics and Thermal Sciences* 87, no. 2 (2021): 56-65. https://doi.org/10.37934/arfmts.87.2.5665

[11] MAJDI, Hasan Shakir, Akeel Abdullah MOHAMMED, Amer Abdullah MOHAMMED, and Laith Jaafer HABEEB. "Effect of fibrous porous material on natural convection heat transfer from a horizontal circular cylinder located in a square enclosure." *Journal of Thermal Engineering* 7, no. 6 (2021): 1468-1478.https://doi.org/10.18186/thermal.990865

[12] Sultan, Ahmed Abd El-Rakik Ahmed. "Augmentation of Natural Convection Heat Transfer from Horizontal Circular Cylinder by Means of Helical-Coil-Turns.(Dept. M)." *MEJ. Mansoura Engineering Journal* 18, no. 3 (2021): 51-62. https://doi.org/10.21608/bfemu.2021.165563

[13] Gorla, Rama Subba Reddy, Ali J. Chamkha, and Eisa Al-Meshaiei. "Melting heat transfer in a nanofluid boundary layer on a stretching circular cylinder." *Journal of Naval Architecture and Marine Engineering* 9, no. 1 (2012): 1-10. https://doi.org/10.3329/jname.v9i1.7416

[14] Nazar, R., L. Tham, I. Pop, and D. B. Ingham. "Mixed convection boundary layer flow from a horizontal circular cylinder embedded in a porous medium filled with a nanofluid." *Transport in porous media* 86 (2011): 517-536. https://doi.org/ 10.1007/s11242-010-9637-1

[15] Choi, S. US, and Jeffrey A. Eastman. *Enhancing thermal conductivity of fluids with nanoparticles*. No. ANL/MSD/CP-84938; CONF-951135-29. Argonne National Lab.(ANL), Argonne, IL (United States), 1995.

[16] Harun, Muhammad Arif, Nor Azwadi Che Sidik, Yutaka Asako, and Tan Lit Ken. "Recent review on preparation method, mixing ratio, and heat transfer application using hybrid nanofluid." *Journal of Advanced Research in Fluid Mechanics and Thermal Sciences* 95, no. 1 (2022): 44-53. https://doi.org/ 10.37934/arfmts.95.1.4453

[17] Cherifa, Benygzer, Bouzit Mohamed, Mokhefi Abderrahim, and Khelif Fatima-Zohra. "Unsteady Natural Convection in a Porous Square Cavity Saturated by Nanofluid Using Buongiorno Model: Variable Permeability Effect on Homogeneous Porous Medium." *CFD Letters* 14, no. 7 (2022): 42-61. https://doi.org/ 0.37934/cfdl.14.7.4261

[18] Mahat, Rahimah, Sharidan Shafie, and Fatihhi Januddi. "Numerical analysis of mixed convection flow past a symmetric cylinder with viscous dissipation in viscoelastic nanofluid." *CFD letters* 13, no. 2 (2021): 12-28. https://doi.org/10.37934/cfdl.13.2.1228

[19] Mahat, Rahimah, Sharidan Shafie, and Noraihan Afiqah Rawi. "Numerical Investigation of Mixed Convection of Cu/Al2O3—Sodium CMC Nanofluids Past a Circular Cylinder." In *Advanced Transdisciplinary Engineering and Technology*, pp. 353-360. Cham: Springer International Publishing, 2022. https://doi.org/10.1007/978-3-031-01488-8_29

[20] Widodo, Basuki, Adhi Surya Nugraha, Dieky Adzkiya, and Mohd Zuki Salleh. "Magnetohydrodynamic Effects in Mixed Convection Copper-Water Nano Fluid Flow at Lower Stagnation Point on a Sliced Sphere." *CFD Letters* 13, no. 12 (2021): 21-31. https://doi.org/10.37934/cfdl.13.12.2131

[21] Khan, W. A., and I. Pop. "Boundary-layer flow of a nanofluid past a stretching sheet." *International journal of heat and mass transfer* 53, no. 11-12 (2010): 2477-2483. https://doi.org/10.1016/j.ijheatmasstransfer.2010.01.032

[22] Salleh, Mohd Zuki, and Roslinda Nazar. "Free convection boundary layer flow over a horizontal circular cylinder with Newtonian heating." *Sains Malaysiana* 39, no. 4 (2010): 671-676.

[23] Tiwari, Raj Kamal, and Manab Kumar Das. "Heat transfer augmentation in a two-sided lid-driven differentially heated square cavity utilizing nanofluids." *International Journal of heat and Mass transfer* 50, no. 9-10 (2007): 2002-2018. https://doi.org/10.1016/j.ijheatmasstransfer.2006.09.034

[24] Merkin, J. H., and Ioan Pop. "A note on the free convection boundary layer on a horizontal circular cylinder with constant heat flux." *Wärme-und Stoffübertragung* 22, no. 1-2 (1988): 79-81. https://doi.org/10.1007/BF01001575

[25] Kasim, A. R. M., Mohammad, N. F. and Shafie, S. "Effect of heat generation on free convection boundary layer flow of a viscoelastic fluid past a horizontal circular cylinder with constant surface heat flux", *AIP Conference Proceedings*, 286(2012), pp. 286–292.