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Comprehensive Validation: Enhancing the Modified NERAH Model via Rigorous Sensitivity Analysis

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ABSTRACT

Marijuana is an illegal drug with several detrimental health impacts, remaining a significant threat to public health in developing nations. Transmission of marijuana occurs through interactions between smokers and non-smokers. The NERAH model, previously introduced in a study on marijuana usage within the target population, comprises five distinct stages: non-smokers (N_S), experimental smokers (E_S), recreational smokers (R_S), addicted smokers (A_S) and hospitalized individuals (H_S). This study modifies the Non-smokers, Experimental, Recreational, Addicted and Hospitalized (NERAH) smokers' model by introducing a new class called the prisoner's class, replacing the hospitalized class. Additionally, the Modified model's validation using the sensitivity analysis idea is the primary objective of this study. The reproduction number 〖(R〗_0) for marijuana use was obtained using the next-generation matrix method, aiding in assessing the rate at which marijuana smoking spreads within the population. The sensitivity analysis results reveal the factors significantly influencing marijuana smoking. Validation of the modified model incorporates various techniques, including the basic reproduction number and invariant region utilizing the sensitivity analysis concept. First-order non-linear ordinary differential equations are employed for modification of the NERAH model. Numerical simulations are conducted with the help of MATLAB using the Runge-Kutta method of order four. Finally, the results of the modified model are compared with those of the existing model, as part of the validation process for the proposed modification. Moreover, as the methodological and graphical sections demonstrated, the suggested updated models were effectively validated for enhanced performance.

1. Introduction

The background of the current study is pertaining to drug regulation throughout the general populace. The term drug denotes a medicinal substance utilised to accelerate chemical processes in

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the human body, such as illness treatment and cognitive enhancement [1]. Olaniyi *et al.,* [2] defines an illicit substance as any chemical able to inducing a biological alteration in human biological processes. This alteration is attributable to certain chemical constituents found in these drugs. A drug is described as a substance, either manufactured or natural, that induces physiological, chemical and behavioural changes in the body when administered [3].

In contemporary society, substance abuse has emerged as a significant global concern. Drug abuse is frequently covered in both print and electronic media. The emergence of addiction to drugs can adversely affect health, economic performance and societal structures. This condition is extremely damaging to individuals, society and the state, particularly affecting younger people. The authorities have implemented a range of initiatives using drug enforcement policies, focussing on the suppression of illicit drug trade and avoiding the spread of substance abuse within the community. Therefore, the involvement of various stakeholders is essential to combat drug issues and address the rising prevalence of drugs, ensuring a drug-free future for the nation [4].

There are several forms of drugs, but my research study focusses exclusively on marijuana. Marijuana is one of the illicit narcotics that is given the greatest abuse all over the world. As a result of changes in the nature of drug consumption, scientists and other academics are encountering new challenges in their studies and initiatives to track and investigate this public the outbreak of plague. The utilization of marijuana is often viewed as a warning sign for involvement with other illicit substances, being considered a potential "gateway" drug leading to more hazardous substances. Adolescents who engage in gateway drug use are at an increased likelihood of progressing to more harmful substances. One of the globally prevalent illegal substances is marijuana, also recognized by various names such as dried, Indian hemp, flame, vape, reefer, tough, grub, herb, cannabis and others [1].

The concept of a gateway in drug use suggests that individuals often initiate substance use with less harmful drugs, gradually progressing to more potent substances. This analogy likens the progression to ascending a staircase, where individuals may start with lighter narcotics like tobacco and then potentially advance to more potent drugs such as ice, heroin, beer and so forth [5]. Marijuana consumption poses a significant population health concern [6]. The most abused illicit drug in the United States is cannabis. As indicated by topical studies, 18.7 million Americans admitted to the use of marijuana, with 75% of cannabis consumers reporting consistent usage [7].

Marijuana use is becoming more common as more states legalize marijuana for recreational as well as medical purposes. According to national surveys, over 2 million individuals in the United States with existing cardiovascular conditions have either used or are currently using marijuana in its various forms, such as vaping and inhalation. Cannabinoid receptors are distributed across various tissues and cells, encompassing platelets, adipose tissue and myocytes. Information derived from observations suggests potential correlations between the consumption of marijuana and a broad spectrum of unfavourable cardiovascular risks. It is worth noting that the potency of marijuana is increasing and smoking marijuana shares similar cardiovascular health risks with tobacco smoking [8].

In the United States, marijuana and alcohol stand out as two widely favoured substances among young adults. According to statistics, 42 percent of individuals in the 19 to 30 age group and a total of 82 percent overall acknowledged the use of marijuana and alcohol in the preceding year. Both substances carry a variety of possible short-term and long-term risks and negative consequences. [9,10]. The latest studies have concentrated on examining concurrent use patterns, exploring the correlation between shifts in marijuana consumption and corresponding changes in alcohol consumption. The research investigates whether the use of these two substances is influenced by substitution effects, where the usage of one substitute for the other. Summarizing the research on

simultaneous alcohol and marijuana (SAM) smoking proves challenging due to variations in how SAM use is defined and the interchangeability of terminologies for simultaneous use and SAM use in the literature [11-14].

Marijuana can be used as medicine because it contains a complex blend of chemicals and cannabinoids. The positive impact ratio of marijuana has not been adequately characterized because there are more than 200 cannabinoids found in marijuana, some of which have been recognized and some of which are unknown. To extract cannabinoids, cannabis leaves are either vaporized or cooked. They are then smoked, typically in pipes or hand-rolled cigarettes (joints). Some users, however, combine it with food and brew it like tea [15]. High rates of marijuana use have a number of negative effects on society, including an increase in crime and the resulting loss of lives and property, an increase in the number of murders and accidents, a significant loss of efficient man hours, a decrease in the financial gain of individuals due to marijuana use and ongoing, high costs for substance abuse treatment and avoidance programs [16].

THC, the main ingredient in marijuana, quickly moves from the lungs to the circulation, where it is distributed to the brain and other organs, causing short-term effects. THC is absorbed more slowly when marijuana is taken orally, so effects usually start to take effect within 30 to 60 minutes. Marijuana can have immediate effects by excessively stimulating brain regions with lots of receptors. The long-term effects of marijuana usage affect the development of the brain. Marijuana use during adolescence can affect the development of vital connections between the many brain regions involved in thinking, memory and learning, as well as impair cognitive skills like thinking, memory and learning. The length of marijuana's effects and whether specific alterations may be permanent are still being studied by Meier *et al.,* [17].

An increased heart rate is caused by marijuana use and can last for up to three hours after smoking. Heart attacks could occur because of this rise in heart rate. People who are older or who already have cardiac problems may be in greater danger if they use marijuana [18]. Marijuana use during pregnancy can have negative impacts on parts of the foetus's developing brain. Compared to children who were not exposed, infants who were exposed to marijuana in the womb are more likely to struggle with attention, memory and problem-solving skills. Additionally, newborns exposed to marijuana during pregnancy are more likely to have cognitive and behavioural issues [19].

Regular marijuana usage can increase the amount of THC in breast milk, which could influence the nursing infant's growing brain. An in-depth understanding of the connection between marijuana use and pregnancy requires more study [20]. The cardiovascular effects of marijuana usage, including possible drug interactions, were summarized by DeFilippis *et al.,* [8]. They also emphasized the need for more studies to create more precise guidelines about the drug's safety for the cardiovascular system. When treating young patients who have cardiovascular issues, they advise healthcare professionals to actively screen and test for marijuana usage. There is a focus on screening to ensure proper management and treatment in clinical settings.

The study's author has presented a synthesis of peer-reviewed projects focusing on cannabis consumption in individuals aged more than 50 and older. This includes an examination of usage patterns, relation with marijuana smoking and the prevalence of cannabis use within this demographic. Notably, there has been a discernible rise in marijuana use among adults aged greater than or equal to 50. In contrast to smoking for fun, a greater proportion of individuals in this age group employ marijuana for medicinal purposes, with variations observed in how older adults utilize medical marijuana based on state regulations. Factors such as male gender and single marital status. Marijuana usage among older people is associated with emotional strain and the existence of several chronic illnesses. Interestingly, depending on ethnic background and level of schooling, these attributes are handled differently in the examined articles [21].

The study focused on young patients without any cardiovascular risk factors other than recent marijuana use and looked at the timing of marijuana usage in connection with various cardiovascular events. Atrial fibrillation, acute coronary syndromes and incidences of sudden death were among the consequences that were documented [22].

A drug use disorder, a medical disease characterized by the inability to stop taking a substance despite suffering detrimental effects on one's health and personal life, can occur because of marijuana usage. Addiction is a term that is often used to describe severe cases of substance use disorders. According to studies, between 9 and 30 percent of marijuana users may experience different types of marijuana use disorders [23]. The endocannabinoid system, which involves the distribution of cannabinoid receptors in different organs and cell types, facilitates the effects of marijuana. Cannabinoid receptors are present in platelets, adipose tissue, myocytes, the liver, the pancreas and skeletal muscle, in addition to the neurological systems of the peripheral and central regions, where they are found in large concentrations. Because of this, the body's many systems can be impacted by cannabinoids that come from outside the body [24,25].

The structure of nature has shown increased curiosity in mathematical models because of their applications in various therapeutic and healthcare domains [26-29]. Mathematical models employing ordinary differential equations have proven effective in elucidating the dynamic responses of biological phenomena. Most biological activities demonstrate memory or implications in their social behaviour; however, conventional integer-order mathematical frameworks overlook these implications. Fractional-order mechanisms, due to their ability to represent problems associated with memory and environmental factors, are often more appropriate than integer-order systems across different fields [30-32]. It seems that there seems to be significant interest recently in developing mathematical models of biological science utilising fractional differential equations (FDEs) [33-36].

Yusuf [15] formulated a probabilistic model to govern the dissemination of marijuana. Control measures, including awareness campaigns, rehabilitation efforts and educational initiatives, were implemented. To identify the most effective combination of preventive measures for reducing marijuana use's expense and frequency within a community, different approaches are strategically employed according to optimal control.

Ullah *et al.,* [37] mathematically articulated the attributes of illicit marijuana consumption within a general population. This encompassed individuals both engaging and refraining from substance use. To further classify marijuana users, four supplementary categories-experimental (E_S), recreational (R_S), addicted (A_S) and hospitalized (H_S) were introduced. This paper's main goal is to adapt and validate the NERAH model, which links marijuana use with the population's makeup. The following set of differential equations represents the NERAH model, as seen in Figure 1.

$$
\begin{aligned}
{dN_S}/{dt} &= £ - (cR_S + c_3A_S + \mu)N_S + c_2E_S + c_4R_S + c_7H_S,\\
{dE_S}/{dt} &= cR_SN_S - (c_1 + c_2 + \mu)E_S,\\
{dR_S}/{dt} &= c_3A_SN_S + c_1E_S - (c_4 + c_5 + \mu)R_S,\\
{dA_S}/{dt} &= c_5R_S - (c_6 + \mu)A_S,\\
{dH_S}/{dt} &= c_6A_S - (c_7 + \mu)H_S.
\end{aligned} \tag{1}
$$

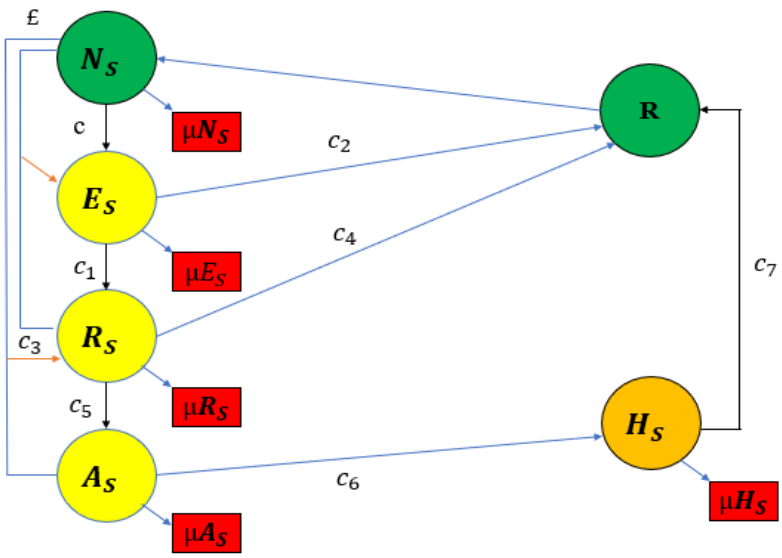

Fig. 1. A schematic diagram of the NERAH model [37]

Many researchers have studied marijuana consumption and created mathematical models to reduce and prevent its use in the general population; however, they have overlooked the crucial demographic of prisoners in their models, rendering initiatives to mitigate and prevent marijuana smoking significantly more difficult. The objective of the current study is to enhance the NERAH model by introducing a novel compartment and subsequently validate the updated framework to assess its improved performance.

2. Mathematical Model Formulation

Developing a model is an important first step in turning what we know about the world into a mathematical representation. This is done in two main steps: first, a conceptual actual-world issue is generated and then it is turned into mathematical models. First, we figure out what the main parts are (they're called state variables) and how information moves between them. Following the conservation principle, the theory model's versions show how much these state indicators have changed by putting up all the flows coming in and taking away all the proceeds going out across all components. To show this, you can use a schematic or flow chart with blocks that show state factors and lines that show flows connecting them. In the next step, the flows are carefully described and written as mathematical equations [38]. For a wide understanding of the mathematical modelling of real-world problems, see the articles [38-55]

Creating an idea for a noticed issue and subsequently evaluating it by forecasting the outcomes of numerous experiments according to particular conditions constitutes an essential academic process. Estimates are generally compared with experimental findings. distinctions necessitate the reformulation of the explanation, whereas coherence enables the reasoning to be acknowledged as an acceptable idea. The structure that outlines the key components of the occurrence, usually articulated mathematically, might be iteratively enhanced throughout the revision process to reconcile discrepancies with descriptive or experimental findings. The iterative procedure illustrated in Figure 2 is termed the mathematical modelling cycle of manufacturing.

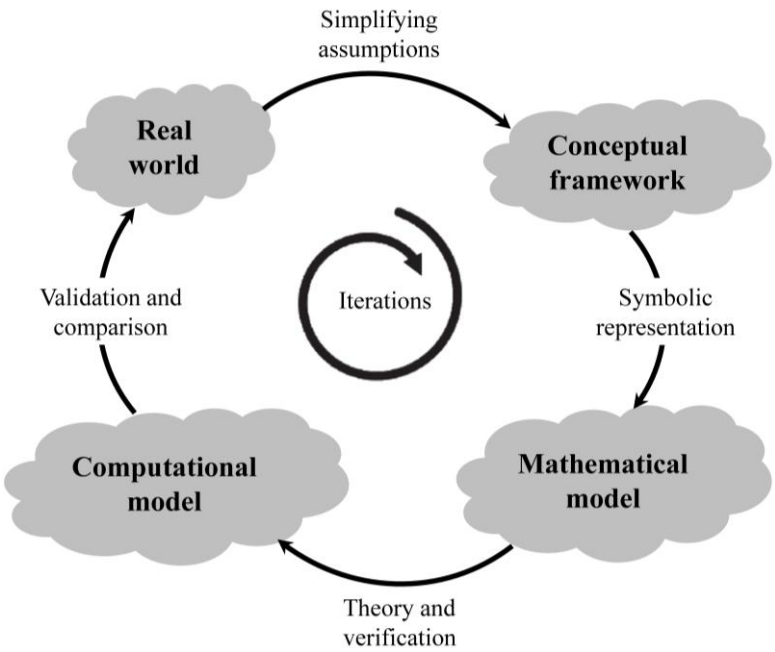

Fig. 2. The process of constructing, evaluating and validating models

Here, we are creating an updated version of the NERAH model (see Figure 3) from the viewpoint of a worldwide spread. There are two primary segments of the population: those who use marijuana and those who do not. There are four distinct categories of smokers, each of which represents a distinct stage of the addiction experience.

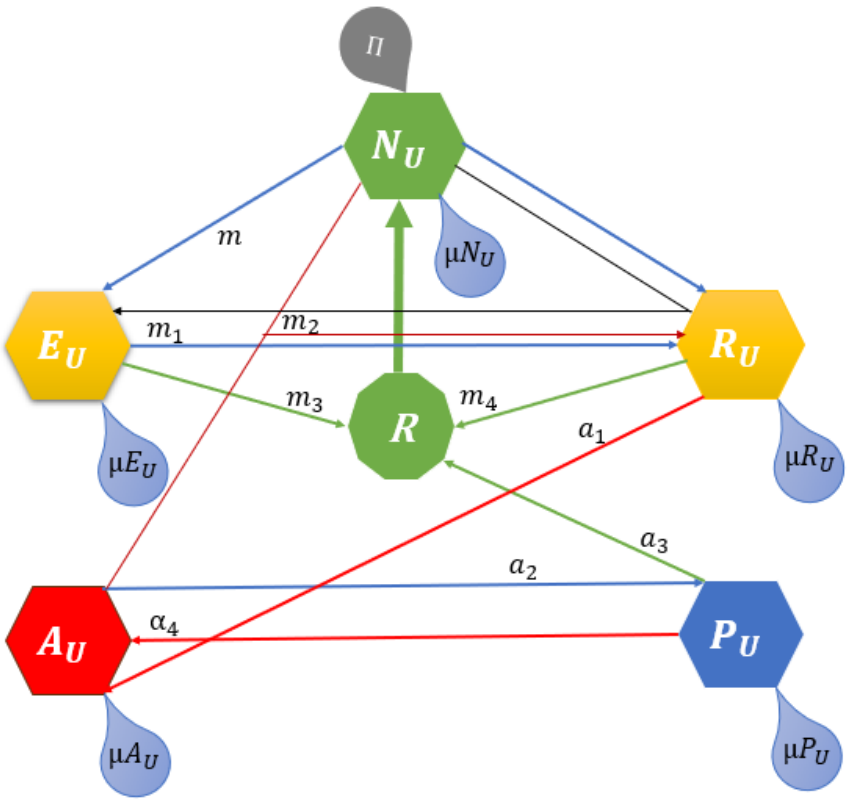

Fig. 3. A schematic diagram of the updated model

Table 1 below discusses these stages, including vulnerable individuals. Additionally, the whole population T_p(t) at a time 't' is shown in Eq. (2) [41].

$$T_p(t) = N_U(t) + E_U(t) + R_U(t) + A_U(t) + P_U(t). \tag{2}$$

The following set represents the general behaviours of marijuana users (refers to Eq. (3)) of first-order non-linear ordinary differential equations.

Table 1
Explanation of the state variables

State variables	Explanation
N_U	Non-smokers/Susceptible
E_U	Experimental smokers
R_U	Recreational smokers
A_U	Addict's
P_U	Prisoner's

$$\begin{aligned}
\dot{N_U} &= \Pi - (mR_U + m_2A_U + \mu)N_U + m_3E_U + m_4R_U + a_3(1-\alpha_4)P_U,\\
\dot{E_U} &= mR_UN_U - (m_1 + m_3 + \mu)E_U,\\
\dot{R_U} &= m_2A_UN_U + m_1E_U - (a_1 + m_4 + \mu)R_U,\\
\dot{A_U} &= a_1R_U + a_3\alpha_4P_U - (e + a_2 + \mu)A_U,\\
\dot{P_U} &= a_2A_U - (a_3 + \mu)P_U.
\end{aligned} \tag{3}$$

The detailed descriptions of the parameters are provided in Table 2.

Table 2
Details of the parameters

Symbols	Parameters descriptions	Parameters Values	References
Π	People's ratio of births	$0.0015875day^{-1}$	[56]
$\boldsymbol{m}$	Effects of experimental individuals in relation to non-smokers (vulnerable)	$0.0000511day^{-1}$	Assumed
$\boldsymbol{m_1}$	The proportion of people conducting experiments as opposed to recreational users	$0.002day^{-1}$	[37]
$\boldsymbol{m_2}$	The level of control addicts has over non-smokers (vulnerable)	$0.367day^{-1}$	Assumed
$\boldsymbol{m_3}$	the proportion of experimental participants who stopped smoking after getting advice	$0.001201day^{-1}$	[37]
$\boldsymbol{m_4}$	Chances that recreational cannabis users may stop due to their stressful environment	$0.0621day^{-1}$	Assumed
$\boldsymbol{a_1}$	What is the frequency with which recreational users become addicted once the transitional period ends?	$0.215day^{-1}$	Assumed
$\boldsymbol{a_2}$	The percentage of long-term users who serve time in jail	$0.025day^{-1}$	[41]
$\boldsymbol{a_3}$	The proportion of people who complete their period in jail	$0.045day^{-1}$	Assumed
μ	People's ratio of deaths	$0.066day^{-1}$	[41]
$\boldsymbol{e}$	The fatality rate that law enforcement witnesses	$0.000005day^{-1}$	Assumed
$\boldsymbol{\alpha_4}$	The proportion of prisoners who eventually go back to the addicted section	$0.03day^{-1}$	[41]

2.1 Invariant Region

We assumed that all the state variables and characteristics (parameters) of the proposed model are non-negative at time t = 0 because they are pertinent to the real-life society in which we live. This comprises the behaviours of the entire human population, which is depicted by the non-linear system below [41].

$$T_p = N_U + E_U + R_U + A_U + P_U. \tag{4}$$

Eq. (4) is resolved to acquire the next equation

$$\dot{T_p} = \Pi - \mu T_p - eA_U, \tag{5}$$

Eq. (5) provides us with

$$\dot{T_p} = \Pi \leq \mu T_p, \tag{6}$$

After resolving Eq. (6), we obtain

$$T_p \leq T_p(0)e^{-\mu t} + \frac{\Pi}{\mu}(1 - e^{-\mu t}) \Rightarrow \ T_p \leq \frac{\Pi}{\mu} \text{ when t} \dashrightarrow \infty \tag{7}$$

We put up the following assertion considering the earlier analysis.

2.2 Proposition

The suggested model's region is provided by

$$\S = \left[(N_U, E_U, R_U, A_U, P_U) \epsilon \ \mathbb{R}_+^5, T_p \leq \frac{\Pi}{\mu}\right] \text{[41].} \tag{8}$$

The domain is invariant positively, the proposed modified model is mathematically and epidemiologically well-formulated [56] and every pathway boundary points in the direction of transit. It indicates that there is a population limit. It is the initial stage in demonstrating the formulated model's validity.

3. Basic Reproduction Number

The reproduction number, denoted as R_0, is a fundamental epidemiological metric representing the average number of secondary addictions generated by a single addicted person in a completely non-smoking population. It serves as a crucial indicator of the potential for drug or disease spread and is calculated based on the rate of transmission and the duration of addictiveness. A value of R_0 greater than 1 indicates that drug potency is likely to sustain transmission, leading to an epidemic, while a value less than 1 suggests that the potency is likely to die out over time in the population. The commencement network throughput (reproduction number) is determined using a widely recognized method known as the "Next Generation Matrix Method" [57,58].

As:

$$\rho(\digamma V^{-1}) = \text{Ꞧ}_0 \text{ [56,59].} \tag{9}$$

Where the spectral range is given. Additionally, $\digamma = \mathcal{J}_{\text{ꝼ}}$ is the Jacobian of $'\text{ꝼ}'$.

$$\text{ꝼ} = \begin{pmatrix} \text{ꝼ}_1 \\ \text{ꝼ}_2 \\ \text{ꝼ}_3 \end{pmatrix} = \begin{pmatrix} mR_U N_U \\ m_2 A_U N_U \\ 0 \end{pmatrix}. \tag{10}$$

The individuals who become addicted are represented by the column in Eq. (10).

$$\digamma = \begin{pmatrix} \digamma_{11} & \digamma_{12} & \digamma_{13} \\ \digamma_{21} & \digamma_{22} & \digamma_{23} \\ \digamma_{31} & \digamma_{32} & \digamma_{33} \end{pmatrix} = \begin{pmatrix} 0 & mN_U & 0 \\ 0 & 0 & m_2 N_U \\ 0 & 0 & 0 \end{pmatrix}. \tag{11}$$

To make it simple, we interpret Eq. (11) as:

$$\digamma = \begin{pmatrix} 0 & \eta_1 & 0 \\ 0 & 0 & \eta_2 \\ 0 & 0 & 0 \end{pmatrix}_{(MFE)}. \tag{12}$$

Where $\eta_1 = mN_U$ and $\eta_2 = m_2 N_U$.
Similarly, $V = \mathcal{J}_\nu$ is the Jacobian of $'\nu'$ where

$$V = \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix} = \begin{pmatrix} -(m_1 + m_3 + \mu)E_U \\ m_1 E_U - (a_1 + m_4 + \mu)R_U \\ a_1 R_U - (a_2 + e + \mu)A_U \end{pmatrix}. \tag{13}$$

The individuals who join or leave the afflicted class are displayed in the matrix's 'V' column (Eq. (13)), apart from the individuals who are members of the vulnerable class.

$$V = \begin{pmatrix} V_{11} & V_{12} & V_{13} \\ V_{21} & V_{22} & V_{23} \\ V_{31} & V_{32} & V_{33} \end{pmatrix}, \tag{14}$$

$$V = \begin{pmatrix} -(m_1 + m_3 + \mu) & 0 & 0 \\ m_1 & -(a_1 + m_4 + \mu) & 0 \\ 0 & a_1 & -(a_2 + e + \mu) \end{pmatrix}_{(MFE)}. \tag{15}$$

To make it clear, we rewrite Eq. (15) as:

$$V = \begin{pmatrix} -\text{Ꞧ}_1 & 0 & 0 \\ m_1 & -\text{Ꞧ}_2 & 0 \\ 0 & a_1 & -\text{Ꞧ}_3 \end{pmatrix}_{(MFE)}. \tag{16}$$

Where $\text{Ꞧ}_1 = (m_1 + m_3 + \mu)$, $\text{Ꞧ}_2 = (a_1 + m_4 + \mu)$ and $\text{Ꞧ}_3 = (a_2 + e + \mu)$.

The dominant Eigenvalue of $(\mathrm{F}V^{-1})$ is:

$$\sqrt{\left(\frac{m*S*m_1}{\mathrm{H}_1*\mathrm{H}_2}\right)}, \tag{17}$$

Which is the basic representation number:

$$\mathrm{R}_0 = \sqrt{\frac{mm_1\Pi}{(m_1+m_3+\mu)(a_1+m_4+\mu)\mu}}. \tag{18}$$

3.1 Biological Implications of the Basic Reproduction Number R_0

In this case, Eq. (18) states that m denotes the proportion of recreational smokers who influence non-smokers (susceptible) and m_1 is the ratio of influenced (experimental) individuals who transition to the recreational smoker category.

The term $mm_1\Pi$ of R_0 indicates that certain people of the susceptible class may start smoking marijuana and so join the marijuana smoking group due to the high influence rate of marijuana users on vulnerable individuals. Consequently, $mm_1\Pi$ suggests that marijuana is being spread from smokers to non-smokers. The other parameters that are used in R_0, only specify the magnitude of R_0 [60].

4. Sensitivity Analysis of the Proposed Modified Model

Sensitivity analysis reveals the significance of each parameter on the spread of marijuana. Such knowledge is essential for complex and non-linear model simplification, analytic approach and research setup [61]. Since there are frequent mistakes in the collection of data and the assumption of model parameters, sensitivity analysis is frequently utilized to evaluate how robust numerical simulations are to parameter values. It is used to identify variables that intervention strategies should focus on because they have a significant impact on R_0. We can assess the proportional diversity in a variable when a parameter varies due to sensitive indices.

The normalized advance sensitivity indices of a variable with respect to a parameter refer to the ratio of the relative change in the variable to the corresponding difference in the parameter. Alternatively, these sensitivity indices can be established through partial derivatives when the variable is a differentiable function of the parameter [41].

4.1 Definition

With consideration to the parameter x, the generalized forward-sensitive indexes of R_0 are distinct from each other. These are defined as:

$$\mathbb{\Gamma}_{\mathrm{x}}^{\mathrm{R}_0} = \frac{\partial \mathrm{R}_0}{\partial \mathrm{x}} \times \frac{\mathrm{x}}{\mathrm{R}_0}. \tag{19}$$

The indexes for multiple correspondences are provided in Table 3.

Table 3
Sensitivity indexes of the parameters

Parameters	Parameters values	Sensitivity indexes
Π	0.0015875	Positive
m	0.0000511	Positive
m_1	0.002	Positive
m_3	0.001201	Negative
m_4	0.0621	Negative
a_1	0.215	Negative
μ	0.066	Negative

These sensitivity indices are graphically presented in Figure 4.

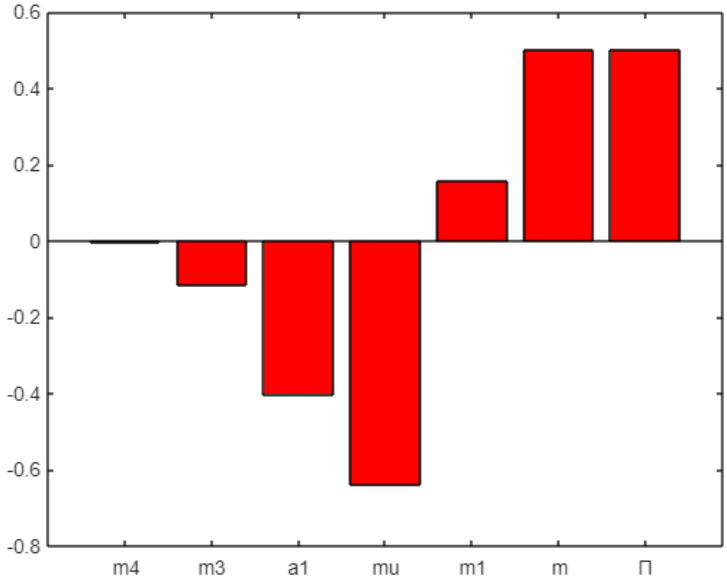

Fig. 4. The graphical representation of the sensitivity indices

5. Results and Discussion

The results of the approaches suggested are predicted in the following consequent figures. $N_U(0) = 1000$, $E_U(0) = 20$, $R_U(0) = 20$, $A_U(0) = 20$ and $P_U(0) = 10$ are the values we have assumed. We use the Runge-Kutta method of order four (RK4) with MATLAB's help to generate the proposed model's graphical results, as the RK4 method is a popular numerical solution for resolving ordinary differential equations (ODEs). RK4 is the numerical method most used to compute an estimation at each iteration of the programs.

The ten ODEs of the first-order problem with boundary conditions are solved using three different methods. Comparison statistics by Ahmad *et al.,* [62] show that the RK4 technique is superior in every circumstance and every field. Meanwhile, as RK4 requires less computing time to compute the truncation global error in the numerical solution, Islam [63] shows that RK5 and RK8 are less efficient than RK4.

Figure 5 shows that there will be approximately 206 individuals under control in 115 days. In contrast, Figure 6 illustrates that the addicted class of the modified model can achieve 468 recovered individuals only in 95 days. In a similar vein, Figure 7 illustrates that the hospitalized category within the previous model can be brought under control within a span of 112 days, resulting in the recovery of 18 individuals.

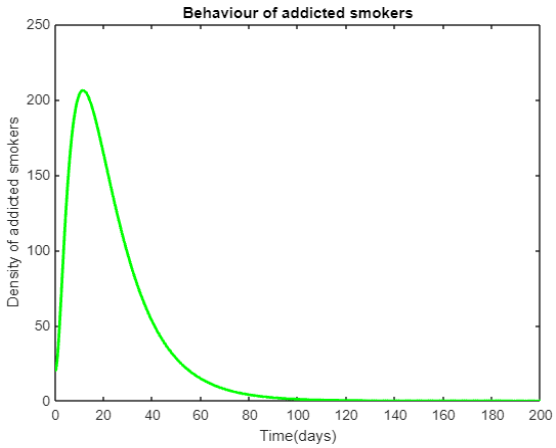

Fig. 5. Graphical outcomes of the addict's (the previous model)

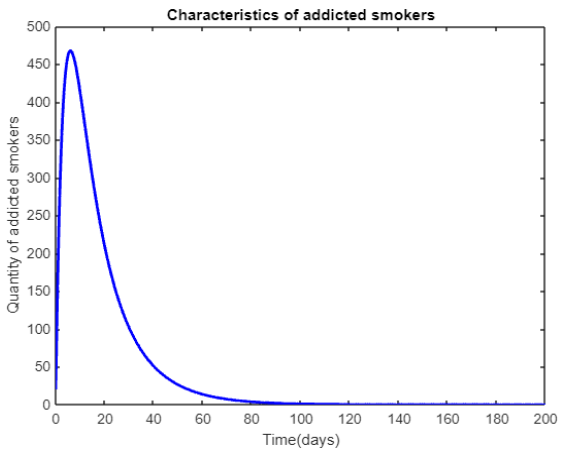

Fig. 6. Graphical outcomes of the addict's (the updated model)

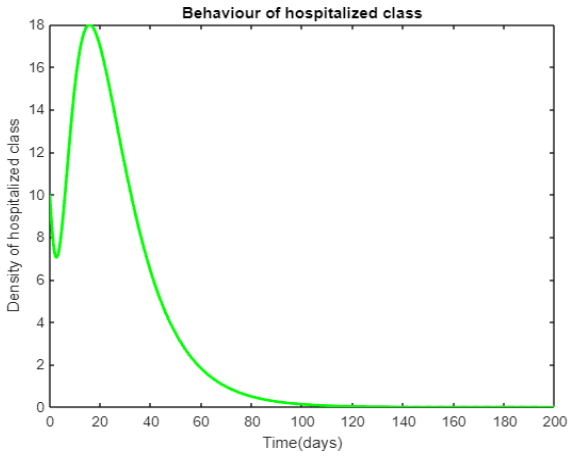

Fig. 7. Graphical outcomes of the hospitalized Individuals (the previous model)

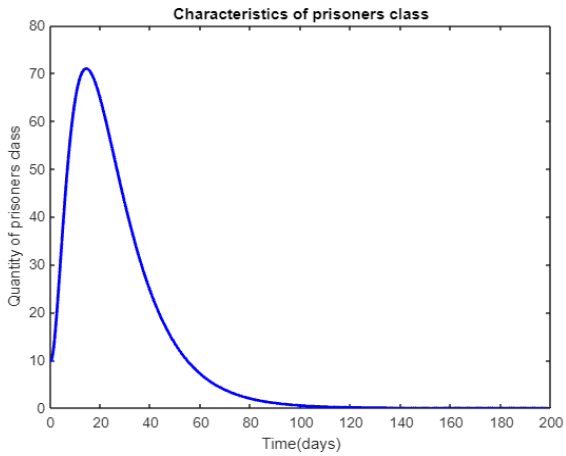

Fig. 8. Graphical results of the prisoner's (the updated model)

Concurrently, Figure 8 depicts that the prisoner's category within the modified model can achieve the recovery of 71 individuals over a period of 115 days. Furthermore, upon comparing the cumulative outcomes of the conventional model with the modified model, it is evident that the two classes in the previous model achieve a recovery rate of 1.96 individuals per day. In contrast, the updated model demonstrates a significantly higher recovery rate of 5.56 individuals per day. Consequently, the modified model exhibits a faster convergence with maximal recovery compared to the prior model.

6. Conclusion

The concluding remarks of this study encompass an enhanced mathematical model distinguishing between marijuana smokers and non-smokers. The emphasis lay on the elimination of addiction, alongside the introduction of a novel category termed the "prisoner's class." The study involved a meticulous comparison of outcomes derived from these classes within both the established (NERAH) model and the modified model, serving the purpose of validation. The comparative analysis revealed insights into the accuracy of each model in controlling marijuana smoking within the population. Notably, the investigation underscored the inherent constraint on the population, evident from the initial stage (the invariant part), where the human population was observed to be positively invariant.

The substantive significance of the initial marijuana transmission rate ($\mathbb{R}_0$) was emphasized, with a detailed biological analysis elucidating its importance. The suggested model underwent numerical simulation employing the Runge-Kutta method of order four. This study involved the modification of the NERAH model through the incorporation of first-order non-linear ordinary differential equations. Additionally, the proposed modified models were effectively validated for enhanced performance, as illustrated in both graphical representations and technical sections, substantiated by sensitivity analysis. Conclusively, based on the outcomes derived from the comprehensive validation processes, we affirm the validity of the refined model. This model proves to be a reliable tool for future endeavours aimed at preventing marijuana smoking in the human population. Subsequent research endeavours may consider employing established methodologies such as the 'Optimal Control Problem' and 'Threshold Conditions,' which can play a pivotal role in minimizing both the time and cost associated with controlling marijuana smoking effectively.

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