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Analysis of Quarter-Car Suspension Dynamics in Sports Cars with PID Control

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ABSTRACT

The analysis and modeling of a sports cars suspension using a PID controller aims to minimize vibrations, effectively shifting the system from underdamped to nearly critically damped, thus enhancing comfort. This research emphasizes the significance of the mass-spring-damper model in suspension analysis, particularly highlighting the role of PID control in reducing vibrations for improved user comfort in sports cars. The comprehension of mechanical system dynamics is facilitated by utilizing the mass-spring-damper system as a mathematical tool, drawing parallels between suspensions and such systems, where energy is stored, similar to the shock absorption in a suspension. Ride comfort, a pivotal aspect of vehicle performance, often faces disruption due to road-induced vibrations, which suspensions manage to mitigate discomfort and disorder. Through quarter-car models, this research delves deep into suspension dynamics, contributing to optimizing suspensions for elevated ride comfort, stability, and enhanced handling.

1. Introduction

The mass-spring-damper system is utilized as a mathematical representation to analyze and portray the dynamic properties of various mechanical systems. The system consists of three main elements: a moving mass, a spring that provides restorative force, and a damper that absorbs energy. The model is applied in a variety of fields, including in the study of vehicle suspensions, buildings, industrial mechanical systems, and even in the physical sciences [1]. The mass of this system represents the mass of the observed moving object or system. Springs, usually represented as elastic elements, store energy during processing and generate restoring force when a mass is moved. In contrast, dampers or absorbers are in charge of absorbing and dampening the vibrations and energy generated by the movement of the mass [2]. In the automotive world, how a sports cars suspension operates is fundamentally represented by this model.

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Evaluating vehicle performance hinges significantly on ride comfort, a key aspect that researchers consistently aim to enhance across all types of vehicles. The presence of uneven road surfaces induces forced vibrations, negatively impacting the overall ride comfort and, in some cases, leading to unpredictable motion disturbances [3]. High-performance cars boast an excellent suspension system. In the event of a shock, such as encountering a bumpy road, the cars maintain stability, shielding passengers from feeling the impact. The suspension system plays a crucial role in minimizing passenger discomfort by carefully selecting springs and dampers to mitigate vehicle movements such as pitch and roll. Its function goes beyond ensuring a smoother ride; it also involves isolating passengers and safeguarding the vehicle and cargo from damage and fatigue caused by vibrations [4]. The suspension system additionally enhances vehicle stability across various driving situations, including turns, braking, and high-speed travel. A vehicle equipped with an upgraded suspension system reduces the effects felt by the driver when maneuvering through uneven or rough terrain [5].

Newton method is one way of analyzing vibration that uses the equilibrium equation of motion according to Newton Law II. By providing an excitation force in the form of road waves that are assumed to be sinusoidal waves and only a combination of pitching and rolling vibrations occurs, the equation of motion can be derived. The aspects of vehicle comfort and stability are considered in the design of a vehicle [6]. When a car is in motion, the vehicle suspension system has a significant impact on comfort and stability. If conditions deteriorate, this can lead to a reduction in the vehicle responsiveness to the controls exerted by the driver, resulting in a potential deterioration in the overall quality of the driving experience [7].

On the flip side, the suspension system is crucial to ensure optimal contact between the wheels and the road, enhancing steering stability, and enabling effective handling to maintain control of the vehicle. The effectiveness of the automotive suspension system is paramount, given that the forces involved in the interaction between the vehicle and the road rely on the contact area of the tires. Comprising numerous parts and components, this system establishes a link between the vehicle body and its wheel assemblies, facilitating the required suspension movement [8]. The optimal comfort level for suspension vibrations is typically achieved when the vibration frequency falls within the range of 1 Hz to 1.5 Hz. Nevertheless, when the frequency exceeds this threshold and falls within the range of 1.5 Hz to 2 Hz, passenger comfort diminishes. In response to this issue, many researchers have concentrated on creating and evaluating active or semi-active suspension configurations that include adjustable stiffness and damping parameters. Examples of such systems include hydro-pneumatic suspension and magnetorheological (MR) suspension, both of which exhibit both elastic and damping characteristics. These technologies aim to enhance the ride experience by effectively managing vibrations within the specified frequency range [9].

PID control has found important applications in vehicle suspension systems to improve vehicle comfort, stability, and handling. In automobile suspensions, the use of PID control is usually associated with adaptive control systems that attempt to adapt the suspension characteristics to road conditions and driver needs [10]. The ride height can also be adjusted using an adaptive suspension system controlled by PID. This is often the case with air suspension systems that use PID to maintain a constant ride height or adjust the ride height depending on road conditions and speed.

In the course of this paper, the authors will conduct a comprehensive examination focused on the distinction between the weight of a sports car under no-load conditions and its loaded state. It is evident that this weight disparity significantly influences the specific vibration dynamics experienced within the vehicle. The utilization of PID control emerges as an invaluable tool for refining the suspension model of the sports car, thereby augmenting its capability to ensure stability when confronted with diverse road-induced vibrations or shocks. Through the implementation of PID control mechanisms, the sports car acquires the adeptness to execute strategic measures

proficiently, resulting in a significant decrease in instances of overshooting, where the system response surpasses the desired value [11]. Simultaneously, PID control addresses the issue of under-damping, marked by insufficient damping within the suspension system.

2. Sports Car Suspension Model

An examination is conducted on a sports car that incorporates independent front and rear suspension. In addition to providing exceptional ride comfort, the multilink suspension system of this sports car must be designed with characteristics closely matching the optimal specifications for agile handling. This involves achieving high lateral acceleration and minimizing the tendency for body roll [12]. Table 1 provides overview information on the key components of a vehicle suspension and steering system, as well as the suspension technologies and types used to improve vehicle comfort, stability, and control.

Table 1
Characteristics of vehicle mechanical system [12]

System	Steering	Steering gear type	Front-wheel or axle location	Rear wheel or axle location	Front / Rear Springs	Steering	Front / Rear shock absorber	Front / Rear stabilizer type
Type	Speed-dependent, electro-hydraulic	Rack-and-pinion	Independent Suspension	Independent Suspension	Coil spring	Speed-dependent, electro-hydraulic	Double-tube gas pressure / single-tube gas pressure	Tubular Torsion bar / Tubular Torsion bar

This paper investigates a sports car equipped with independent front and rear suspensions. In addition to providing superior ride quality, a multilink suspension system in a sports car should be designed to embody attributes closely aligned with the ideal specifications for responsive handling. The goal is to achieve high lateral acceleration and minimize body roll tendency. Fig. 1 illustrates a model quarter-car suspension.

Fig. 1. Sports car suspension [12]

Pitching, bouncing, or rolling over may be caused by uneven roads in vehicles. As shown in Fig. 2, a two DOF (Degrees of Freedom) vehicle model with pitch and bouncing motion can provide an initial suspension model. This model combines the elastic and damping properties of tires and suspension in a system that incorporates equivalent springs and dampers in the front and rear suspensions [13].

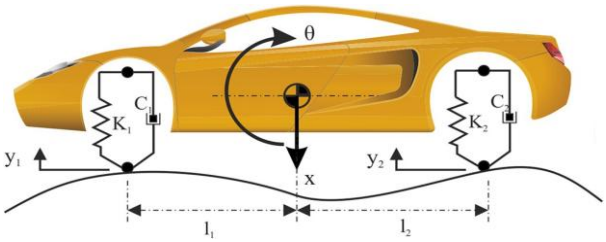

Fig. 2. A model with bounce & pitch motion [12]

Quarter car is an automotive engineering term that describes a simple car suspension model. It represents a quarter of the vehicle structure, mainly the wheels, suspension, and small parts of the vehicle frame or other structures. Quarter-car models study suspension characteristics and vehicle response to shocks and road irregularities [14]. This modeling technique enables a detailed study of the dynamics and behavior of suspension systems, helping to design and optimize vehicle suspensions to improve ride comfort, stability, and handling. It is a valuable tool for understanding how different suspension configurations and components affect overall vehicle performance [15].

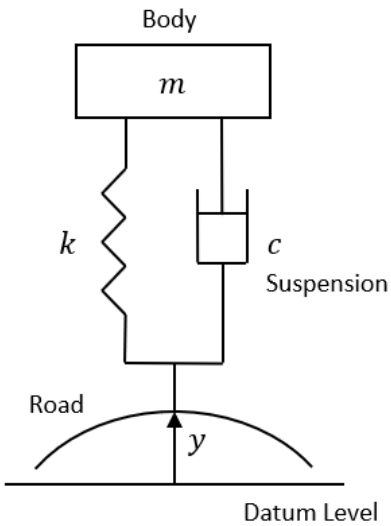

Fig. 3. Quarter car suspension model [7]

Fig. 3 represents a quarter-car suspension model is a simple way used in vehicle dynamics to understand the behavior of a single wheel and its suspension system. It is called a "quarter car" because it represents only a quarter of the vehicle as a whole, i.e. a single wheel and its suspension [16]. Meanwhile, Fig. 4 shows the free-body diagram, which is drawn assuming that $(\dot{y} > \dot{x})$ and that $(y > x)$. The illustration displays only the dynamic spring force since the force of gravity counteracts the static spring force.

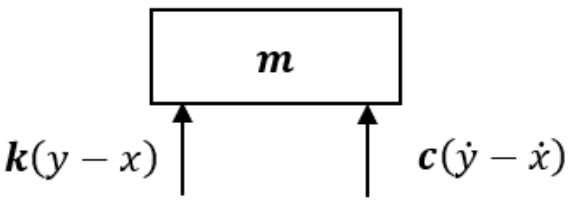

Fig. 4. Body diagram suspension model [7]

In the body diagram presented, it is identified that $\boldsymbol{m}$ refers to the without load and with load. In addition, $\boldsymbol{c}$ is the suspension damping coefficient which describes the level of damping or the ability of the suspension to absorb vibrations when the vehicle is moving. Moreover, $\boldsymbol{k}$ represents the suspension spring stiffness, determining the degree of rigidity or firmness in the suspension spring that influences the vehicle response and stability. The interaction among these three variables plays a crucial role in determining the quality and performance attributes of a vehicle suspension.

The amalgamation of factors, including parameters like spring stiffness, damping rates, and overall geometry, collectively influences how efficiently a suspension system absorbs road irregularities, ensuring optimal ride comfort and handling. Careful consideration and fine-tuning of these variables are essential in achieving a well-balanced suspension system. For instance, adjusting spring stiffness impacts the system ability to absorb shocks, while tweaking damping rates influences the control of body motions. This intricate relationship underscores the significance of a holistic approach in automotive engineering, where a nuanced understanding of these variables allows for the precise calibration of suspension systems, ultimately enhancing the overall driving experience. The equation (1) and (2) of motion are obtained from this free-body diagram.

$$m\ddot{x} = c(\dot{y} - \dot{x}) + k(y - x) \tag{1}$$

or

$$m\ddot{x} + c\dot{x} + kx + c\dot{y} + ky \tag{2}$$

The aim is for this to be expressed within the Laplace domain. The Laplace transform is used on each term to accomplish this. The Laplace transform of $\ddot{x}$ is $s^2X(s) - sx(0) - \dot{x}(0)$, assuming zero initial conditions. Similarly, assuming zero initial conditions for $y(0)$ and $\dot{y}(0)$. The Laplace transform of $\dot{x}$ is $sX(s) - x(0)$ and $\mathrm{sY(s) - y(0)}$. Upon the application of the Laplace transform to the differential equation (1) and (2), the following is obtained as represented by equation (3):

$$ms^2X(s) - msx(0) - m\dot{x}(0) = csY(s) - csx(0) + kY(s) - kx(0) \tag{3}$$

The initial condition can be expressed as $x(0)$ as $X(0)$ and $\dot{x}(0)$ as $\dot{X}(0)$. The transfer function is found by applying the Laplace transform to the equation (3), with the initial conditions set to zero:

$$ms^2X(s) + csX(s) + kX(s) = csY(s) + kY(s) + msX(0) + \dot{X}(0) \tag{4}$$

Now, rearranging the terms to isolate $Y(s)$ on one side and $X(s)$on the other. Then, the equation will be obtained as in equation (5):

$$Y(s) = \frac{cs+k}{ms^2+cs+k}X(s) + \frac{msX(0)\dot{X}(0)}{ms^2+cs+k} \tag{5}$$

The transfer function is derived through careful data acquisition and analysis, resulting in a precise expression of the intricate connection between input variables and the corresponding output parameters within the operational framework of the system. This yields the derived transfer function as obtained in equation (6).

$$\frac{Y(s)}{X(s)} = \frac{cs+k}{ms^2+cs+k} \tag{6}$$

The highlighted transfer function proves to be efficiently simulatable through the utilization of MATLAB software, providing a platform for visualizing and analyzing the system. The systematic integration of the information derived from Table 2 into the transfer function allows for the incorporation of specific details about the vehicle and suspension system. This comprehensive approach lays the foundation for a thorough examination of the system behavior, performance, and potential dynamics. By leveraging MATLAB simulation capabilities, researchers can gain valuable insights into how the system responds to various inputs, aiding in the optimization and fine-tuning of the vehicle suspension for enhanced overall performance.

Table 2
Design information on the suspension system [12]

Parameter	Value
Without load (m)	1595 kg
With load (m)	1890 kg
Suspension Damping Coefficient (c)	3000 N.s/m
Suspension Spring Stiffness (k)	29300 N/m

3. Methodology

This research uses PID control to stabilize the vibration generated by the sports car suspension. PID control will help stabilize the vibration generated by the sports car because that is the function of PID control. PID control is easy to implement, stable in most cases, and adaptable to many types of systems. With proper PID control, the cars suspension system can be enhanced to improve safety and the response to maneuvers is exhibited by the vehicle, optimizing the balance between comfort and handling.

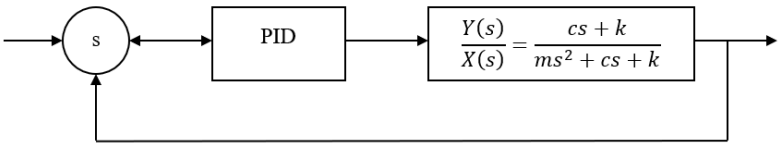

Fig. 5. System block diagram

Fig. 5 depicts a block diagram of the system incorporating a PID controller, which is anticipated to result in a smoother and more stable system response. PID (Proportional Integral Derivative) control is a widely utilized technique in automatic control systems. It involves three essential components to regulate a system by adjusting to the error, which is the difference between the setpoint value (the desired value) and the actual value of a variable in the system [17]. Not only the condition without load but also the state with load must be analyzed to understand the comparison between the two. PID aids in sustaining the desired performance in various automation systems by effectively optimizing the response to changes, minimizing errors, and ensuring system stability. The PID controller constants employed can be observed in Table 3 below.

Table 3
Parameters of PID Constant (without load)

Parameters	Without load	With load
Proportional	19	30
Integral	201	308
Derivative	11	20

PID parameter constants are the result of repetition to obtain a PID constant value that produces a response step that matches the desired target, namely eliminating overshoot and reducing settling time. Every application comes with distinct control requirements, necessitating the adjustment of PID parameters by the characteristics of the controlled system and the specific control objectives.

4. Result and Discussion

PID can be used to control the damping of automobile suspensions. Adaptively controlled using a PID algorithm, the damper can adjust its stiffness and damping depending on road conditions, speed, and suspension forces. This maintains the comfort and stability of the vehicle [18]. PID helps maintain desired performance in various automation systems by efficiently optimizing responses to change, reducing errors, and maintaining system stability [19]. In Fig. 6, the system without PID control, which exhibits a very high overshoot, is depicted. To improve the suspension system, the utilization of PID control with predefined parameters can be considered.

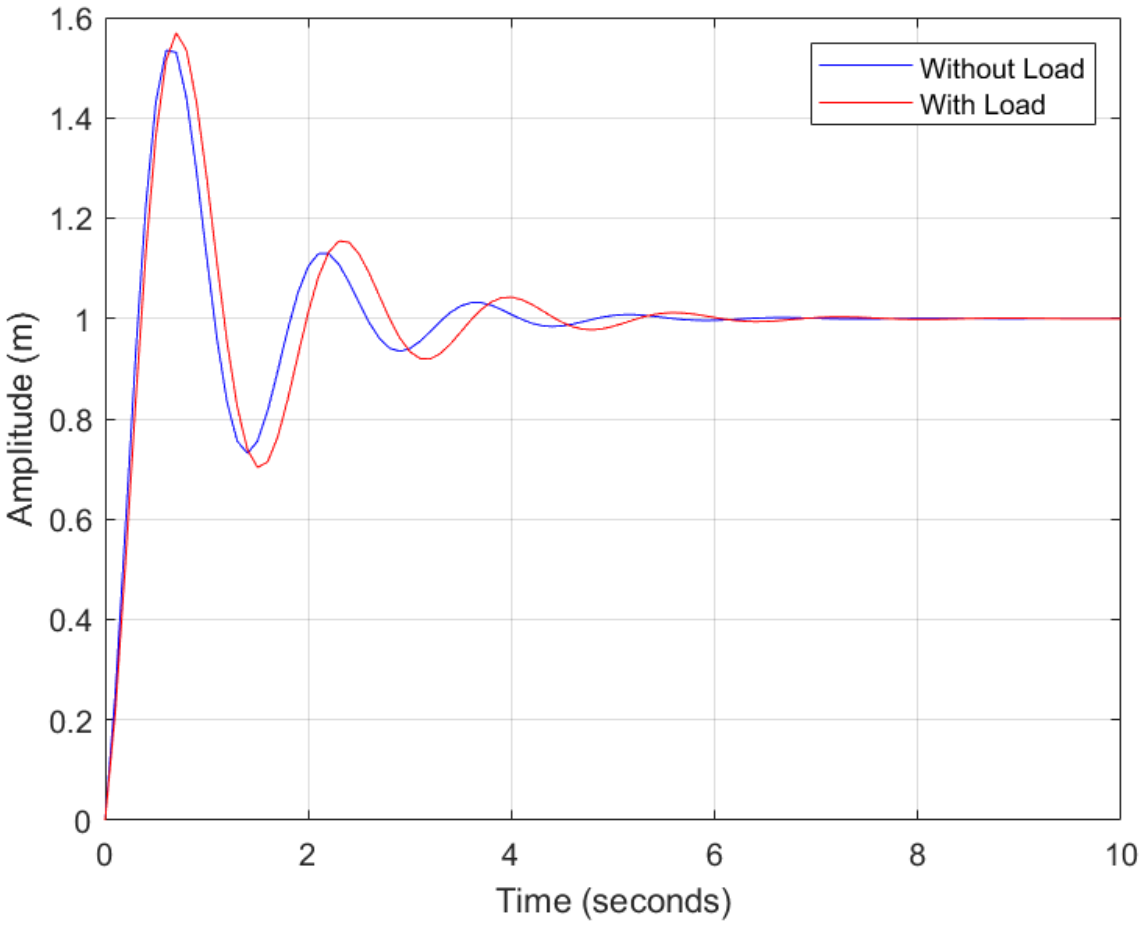

Fig. 6. Step response (without load & with load)

Table 4 shows the Result of Step response (without load & with load).

Table 4
Result of Step response (without load & with load)

Struct	Without load	With load
RiseTime	0.2469	0.2706
SettlingTime	3.8769	4.9038
SettlingMin	0.7325	0.7022
SettlingMax	1.5446	1.5687
Overshoot	54.4562	56.8748
Undershoot	0	0
Peak	1.5446	1.5687
PeakTime	0.6366	0.6963

Overshoot is a phenomenon when the system response exceeds the setpoint value or target value temporarily before reaching a stable value. It occurs when the controller, especially the proportional and derivative components, produces an overly strong or imprecise response to changes in the input [20]. Reducing overshoot is important in many control applications, especially where system precision and stability are required [21]. The cyclist will still feel the vibration before 1 second and will stabilize in the second. To reduce the overshoot of the system, it can be adjusted using PID control.

4.1 Without Load

Fig. 7 shows that the system, lacking PID control, exhibits substantial overshoot, stabilizing after 6 seconds. This results from an overly strong control output or the system tendency to overreact to changes, exceeding the target before returning to the set point. High overshoot, indicative of aggressive system response, can be attributed to factors like excessive control gain or significant system inertia [22].

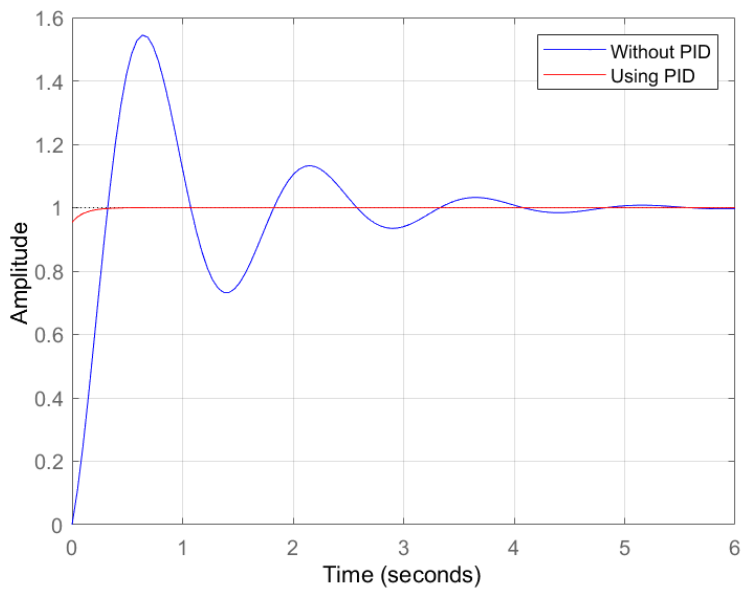

Fig. 7. Step response without load

Table 5
Result of Step response without load

Struct	Without PID	Using PID
RiseTime	0.2469	0.2456
SettlingTime	3.8769	0.4303
SettlingMin	0.7325	0.9956
SettlingMax	1.5446	1.0003
Overshoot	54.4562	0.0344
Undershoot	0	0
Peak	1.5446	1.0003
PeakTime	0.6366	0.8299

From the data result that has been given in Table 5, it can be concluded that there is a significant decrease in overshoot, from 54.4562 to 0.0344. This is evidence of the influence that PID Control has in reducing the overshoot level in sports cars. In addition, it was also noted that the Settling Time of the sports car suspension system also decreased, decreasing from 3.8769 to 0.4303. The implementation of PID control significantly improves system performance in the time domain with faster, more stable, and more accurate responses. This demonstrates the effectiveness of PID control in improving the overall control for sports car suspensions.

PID control is crafted to tackle issues found in control systems, notably the balance between rapid stabilization and overshoot reduction. Properly adjusted, it swiftly responds without excessive overshoot. Consequently, it achieves a fine equilibrium between swift response and tolerable overshoot, enhancing the overall system response. To determine the stability and pole-zeros trajectory of the system, a Root Locus approach is used. The Root Locus graph can be seen in Fig. 8.

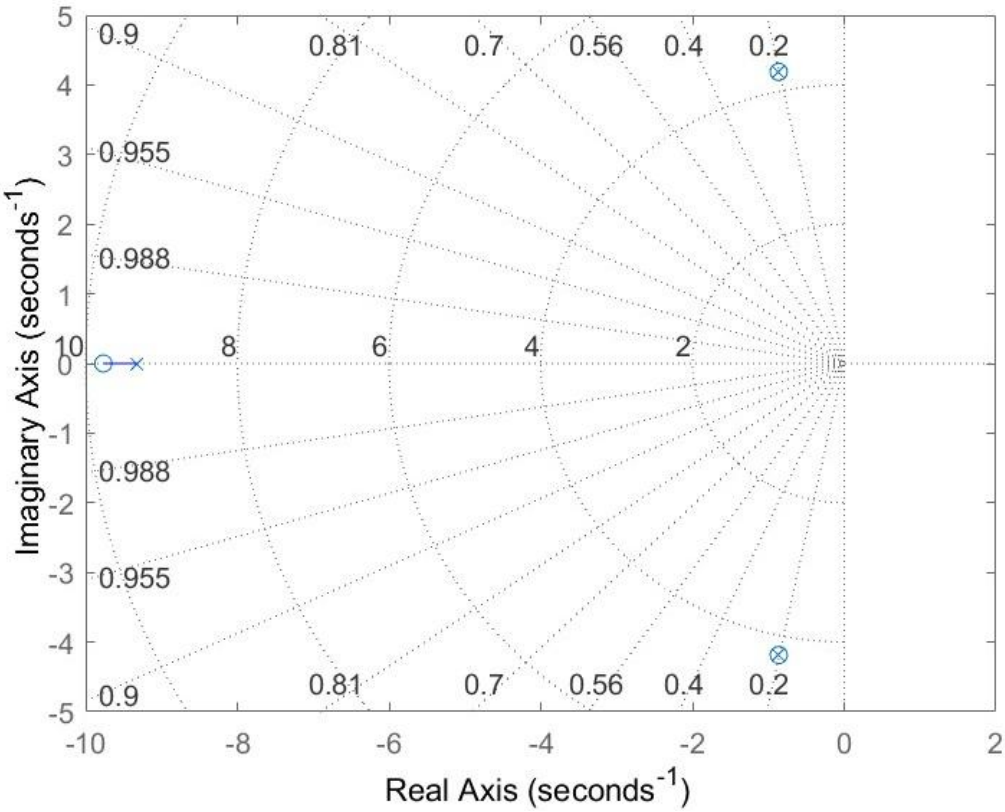

Fig. 8. Root Locus without load

Table 6 shows the Result of Root Locus without load.

Table 6
Result of Root Locus without load

Pole	-0.864 + 4.19i
Damping	0.202
Overshoot (%)	52.3
Frequency (Hz)	4.27

The root locus plot is a powerful tool for analyzing the dynamic behavior of a system by illustrating how the roots of its characteristic equation change with variations in the Proportional (P), Integral (I), and Derivative (D) parameters of a PID controller. The plot, as indicated in Fig. 8 Root Locus, visually represents the positions of these roots for different values of the control parameters. In this case, the plot indicates the presence of a zero on the real line and an imaginary twin pole in the system, offering valuable insights into its stability and response characteristics. Root locus analysis assists engineers in refining PID controller parameters to attain the desired system performance, ensuring stability and optimized control across various operating conditions. Analyzing the root locus plot shows that all system poles are situated on the left side of the s-plane, a critical indication of system stability. This finding underscores a significant improvement in the frequency response and dynamic performance of the sports cars suspension system following the implementation of PID control. The enhancement is particularly notable in the crossover frequency, signifying a more rapid response. This outcome highlights the effectiveness of the PID controller in achieving improved stability and dynamic characteristics for the sports cars suspension system, contributing to enhanced overall performance and responsiveness.

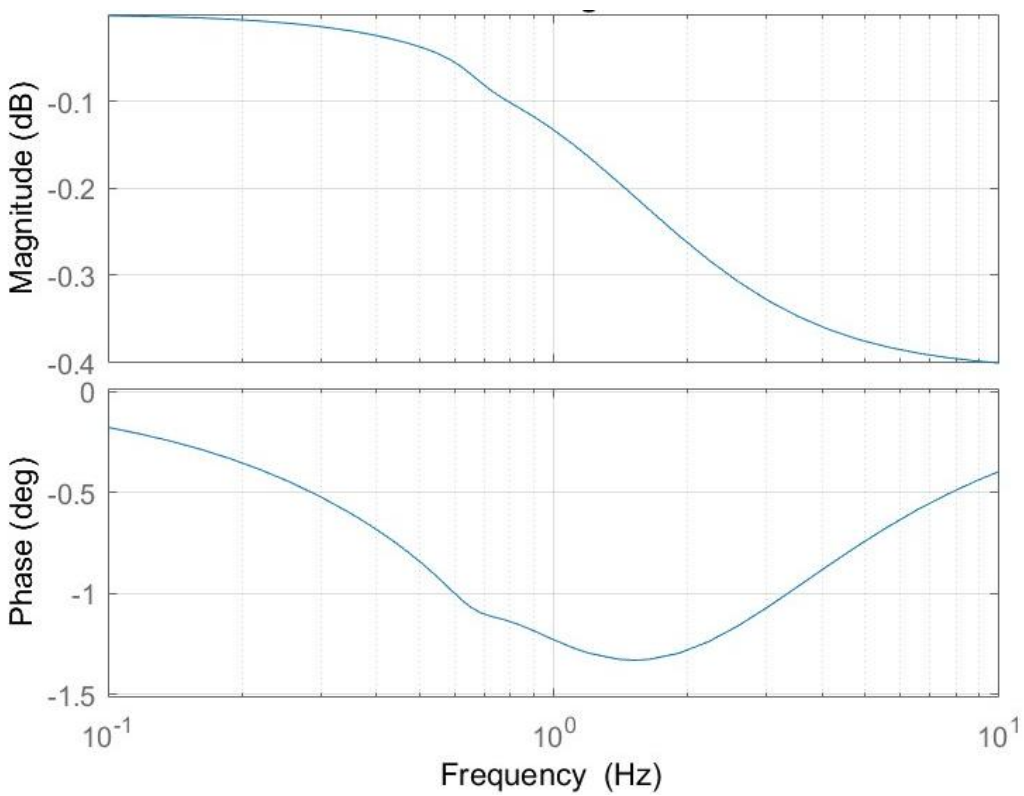

Fig. 9. Bode diagram without load

Bode diagrams serve as valuable tools for evaluating system stability and frequency response, enabling predictions about system behavior across various frequencies and guiding the design of stable controls. The graphical representation of magnitude and phase in these diagrams assists engineers in understanding how a system responds to different inputs. In particular, the comparison presented in Fig. 9, contrasting the Bode diagram with and without the controller, enhances the analysis. This comparison is instrumental in assessing the impact of the controller on stability and frequency response characteristics, allowing engineers to make informed adjustments and refine control parameters for the development of optimized and reliable control systems.

In addition to the root locus analysis, the original Bode diagram presents a noteworthy characteristic – the presence of an overshoot in the system. However, the controlled Bode diagram reveals a notable improvement, showcasing the absence of overshoot. This transformation is particularly significant for the sports cars suspension system as it indicates a smoother and more controlled driving experience. The absence of overshoot enhances the stability of the wheels, preventing excessive movement and ensuring that the suspension system operates within prescribed limits when encountering uneven surfaces or road obstacles. This outcome underscores the positive impact of the PID controller in mitigating overshoot and improving the overall performance and stability of the sports cars suspension system.

4.2 With Load

Fig. 10 reveals that without PID control, the system exhibits considerable overshoot, taking 6 seconds to stabilize. This overshoot causes discomfort to users due to persistent vibrations, exceeding the desired value before settling. Excessive overshoot can be annoying and uncomfortable in real-world applications, emphasizing the importance of well-tuned regulators like PID controllers to minimize it. Implementing effective control strategies is vital for stability and enhancing user comfort by reducing oscillations and disruptive vibrations in system response.

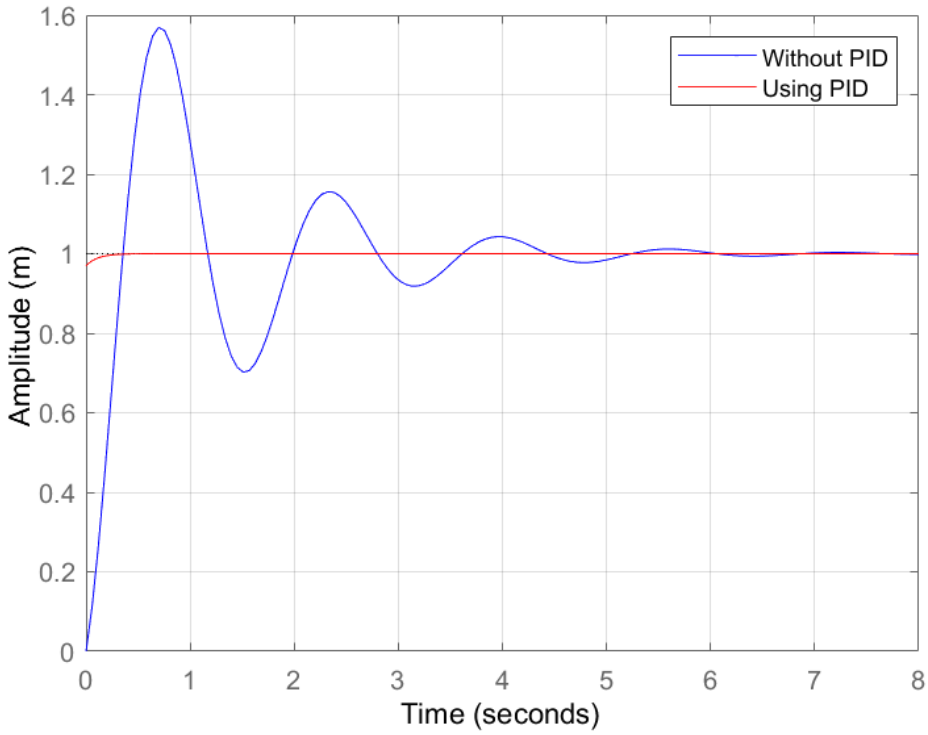

Fig. 10. Step response with load

Table 7
Result of Step response with load

Struct	Without PID	Using PID
RiseTime	0.2706	0.2387
SettlingTime	4.9038	0.4290
SettlingMin	0.7022	0.9970
SettlingMax	1.5687	1.0001
Overshoot	56.8748	0.0134
Undershoot	0	0
Peak	1.5687	1.0001
PeakTime	0.6963	0.9287

From the given data result, the overshoot value of the sports cars suspension system decreased from 56.8748 to 0.0134. By using PID control, the system will be much more stable than before which has a very high overshoot value. This is not good for driving comfort. In addition, the SettlingTime value decreases from 4.9038 to 0.4290, this is due to the influence of PID control. If the sports cars is equipped with a PID controller, the result is very good, there is almost no overshoot so the user does not experience excessive vibration when driving on rugged terrain. The application of the PID controller improves the system significantly. In this case, the near absence of overshoot indicates that the system response is now more controllable and able to respond to changes. This effect is especially noticeable in sports cars, making driving quieter and more comfortable on steep terrain that usually creates unpleasant vibrations for the user.

PID controllers efficiently minimize overshoot while preserving system stability, showcasing their precision in fine-tuning system responses for a smoother driving experience, particularly in challenging terrains. The Root Locus approach, illustrated in Fig. 11, analyzes system stability and the trajectory of poles and zeros, aiding in informed decisions for optimal PID controller parameters. This insight enables the refinement of the PID control strategy for enhanced control over overshoot and overall system response. PID controllers efficiently minimize overshoot while preserving system stability, showcasing their precision in fine-tuning system responses for a smoother driving experience, particularly in challenging terrains. The Root Locus approach, illustrated in Fig. 11, analyzes system stability and the trajectory of poles and zeros, aiding in informed decisions for optimal PID controller parameters. This insight enables the refinement of the PID control strategy for enhanced control over overshoot and overall system response.

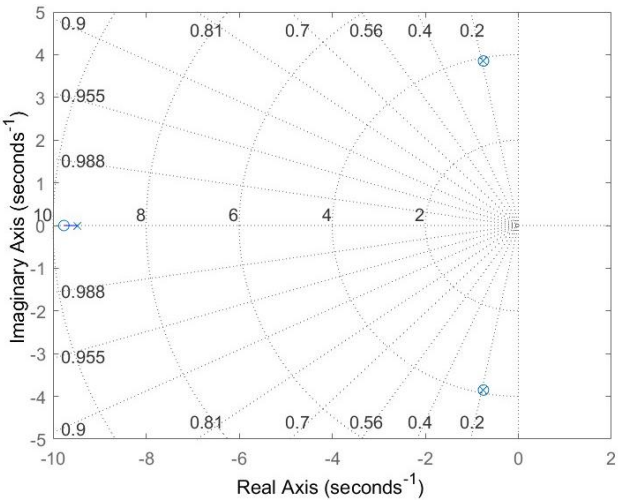

Fig. 11. Root Locus with load

Table 8
Result of Root Locus with load

Pole	-0.75 + 3.85i
Damping	0.191
Overshoot (%)	54.2
Frequency (Hz)	3.92

The root locus plot displays dots denoting the root positions of the closed control system transfer function for different control parameter values. Connecting lines illustrate how the characteristic root moves as control parameters change. With PID controllers, alterations in P, I, and D can shift the characteristic root, consequently impacting the system response and stability. From the given result, it can be concluded that the PID control effectively improves the frequency response and dynamic performance of the sports car suspension system. This can be seen from the crossover frequency which indicates a faster response.

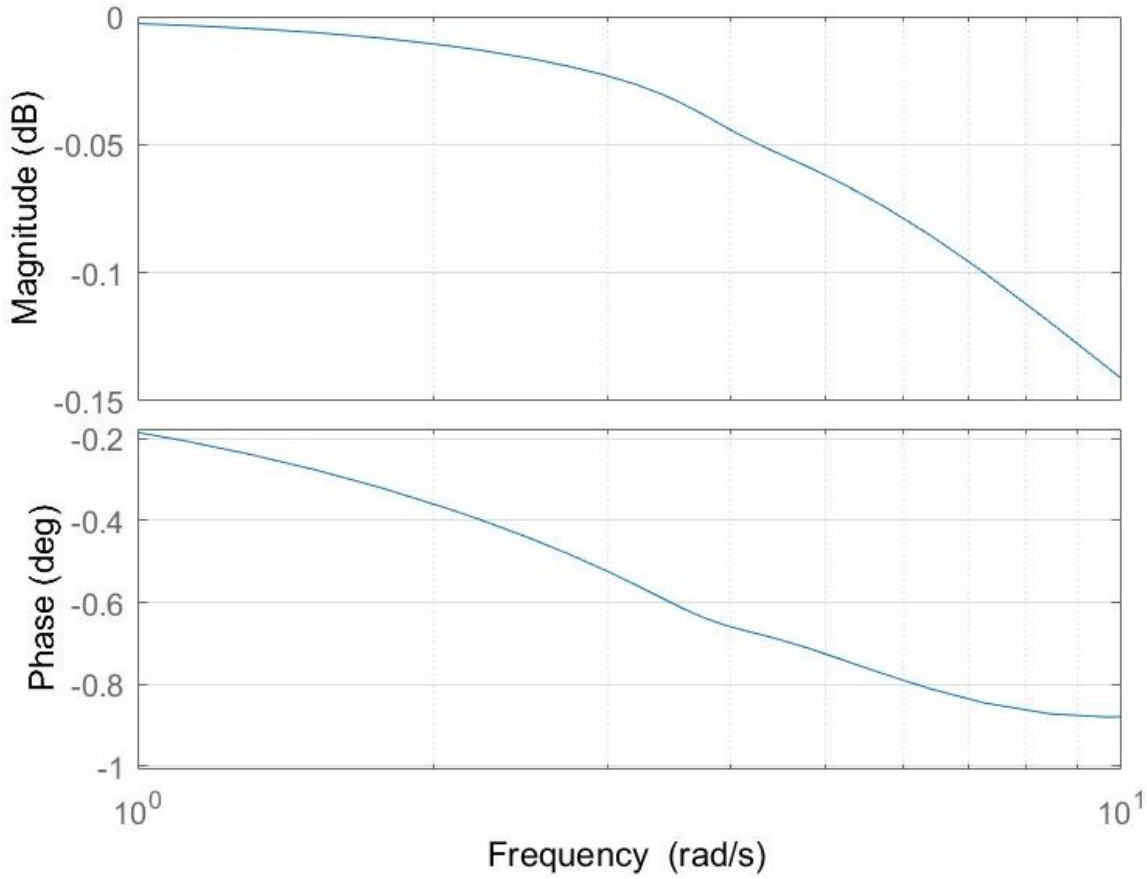

Fig. 12. Bode diagram with load

Bode diagrams in Fig. 12 are visualization tools used in system analysis to show the frequency response of the system. Bode diagrams can be used to analyze the frequency response of suspension systems to external disturbances and various road conditions. The use of bode diagrams in vehicle suspensions is often associated with the dynamic characteristics of the suspension which is part of a mechanical system that responds to external force.

5. Conclusion

Vehicle suspension is a system consisting of various components designed to provide comfort, stability, and control when driving a vehicle on various roads. Suspension is an important part of a vehicle that affects overall performance, driver and passenger comfort, and safety. The suspension on the sports car is unstable and the system is not well damped because there is still a very large overshoot. However, PID control on sports cars makes the system change from Underdamped, when given control the system approaches Critically Damped. To get the desired result, it is necessary to determine the constant value of PID parameters according to the existing transfer function. By using MATLAB, it is very easy to get the value of the PID constant by the existing transfer function. with the existing transfer function. Giving PID control to the system will not only change the response step but also change the Bode diagram. The purpose of a car suspension is to dampen vibrations due to uneven roads, maintain vehicle stability, improve driver and passenger comfort, and provide good and responsive capabilities when maneuvering and driving in various road conditions. The implementation of PID control significantly assist sports cars in minimizing overshooting.

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