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Analytical Solution of Unsteady MHD Casson Fluid with Thermal Radiation and Chemical Reaction in Porous Medium

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ABSTRACT

One of non-Newtonian fluid, Casson fluid, has a diversify application in the manufacturing and engineering sector due to its elasticity behaviour. This study provides the analytical solution of unsteady Casson fluid with the presence of thermal radiation and chemical reaction in a porous medium. Formulation of this fluid model is initiated with the partial differential equation (PDE) of the momentum and energy equations as well as the concentration equation. By applying the proper non-dimensional variables, these equations are then converted into dimensionless form. Subsequently, the derivation of the exact solutions for the concentration, temperature and velocity profiles is conducted with the operation of the Laplace transform method that satisfies both initial and boundary equations. Graphical illustrations, which portray the various effects on this study are generated. It has been revealed that as the thermal radiation, chemical species and porosity increases, so does the velocity profile. However, MHD and Casson parameter shown opposite behaviour in the velocity profiles.

1. Introduction

The complex rheological properties of non-Newtonian fluids possess the diverse nature from Newtonian fluids. Due to its scientific and technological applications in the biological sciences and processing industry, such as motion of biological fluid and lubricant's performance, the study of non-Newtonian becomes a popular research area at present [1]. To demonstrate the distinction between Newtonian and non-Newtonian fluids, there are several non-Newtonian fluid models have been presented such as Bingham plastic, power law, Walter-B, viscoplastic, Brinkman type, Oldroyd-B models and Casson fluid [2-9].

Casson fluid has captured the attention of researchers as it has extensive applications in practical and industrial applications, polymer industries, textile, MHD pumps and motors aerodynamic heating

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and purification of mineral oil [10]. Casson fluid model was introduced by Casson for the prediction of the flow behavior of pigment-oil suspensions [11]. As such, the properties of Casson fluid are widened studied to understand and distinguish from other non-Newtonian fluids. Pushpalatha *et al.,* [12] observed Numerical study of chemically reacting unsteady Casson fluid flow past a stretching surface with cross diffusion and thermal radiation and found that velocity and temperature profiles increase with an increase in radiation.

Swati [13] discussed heat transfer on Casson fluid flow over a stretching surface. She found that thermal radiation enhances the effective thermal diffusivity and the temperature rises. Kumar *et al.,* [14] studied effect of thermal radiation on MHD Casson fuid flow over an exponentially stretching curved sheet. It is seen that there is an enhancement in the field of temperature with the radiation, temperature-dependent thermal conductivity, and irregular heat parameters. Sulochana *et al.,* [15] studied Heat and mass transfer of magnetohydrodynamic Casson fluid flow over a wedge with thermal radiation and chemical reaction. They observed that an increase in skin friction values is noted due to an increment in the thermal Grashof number, whereas a decrease is observed due to the chemical reaction parameter. Mohan *et al.,* [16] discussed an unsteady heat transfer flow of Casson fluid through porous medium with aligned magnetic field and thermal radiation. They discovered that the velocity decreases with increases the values of Magnetic parameter, Prandtl number, aligned angle and Heat source parameter in case of cooling of the plate and opposite phenomenon is observed in case of heating of the plate and the velocity increases with increases the values of Porosity parameter and thermal radiation in case of cooling of the plate and opposite phenomenon is observed in case of heating of the plate. Most recently, Osman *et al.,* [17] explored the unsteady axial symmetric flows of incompressible and electrically conducting Casson fluids over a vertical cylinder with time variable temperature.

Bilal *et al.,* [18] investigated analytical treatment of radiative Casson fluid over an isothermal inclined Riga surface with aspects of chemically reactive species. It is found that the accelerating parameter, chemical reaction parameter, positive modified Hartmann number, and radiation values improve skin friction while heat absorption parameter retards friction. Moreover, an increment in chemical reaction and heat absorption parameters causes a reduction in momentum distribution [19]. Bejawada *et al.,* [20] presented radiation effect on MHD Casson fluid flow over an inclined non-linear surface with chemical reaction in a Forchheimer porous medium. The temperature profile reduces with augment of thermal radiation and Prandtl number. Concentration profile is decaying function for larger values of Schmidt number and chemical reaction. Saeed *et al.,* [21] conducted research for convective flow of a magnetohydrodynamic Casson fluid through a permeable stretching sheet with first-order chemical reaction. The results show that the augmented Darcy number, Casson and magnetic parameters have declined the velocity profile of the Casson fluid flow. Growth in Brownian motion augments the chaotic motion amongst the particles due to which the kinetic energy of the particles transforms to heat energy which consequently augmented the thermal profile, while reduced the concentration profile.

Inspired by the above research works, this research is aiming to establish an analytical solution of MHD unsteady Casson Fluid over an accelerated plate with thermal radiation and chemical reaction solution.

2. Methodology

2.1 Mathematical Formulation and Solution

Exact solutions for the flow of Casson fluid with presence of radiation and chemical reaction is performed in this research. An unsteady Casson fluid past an accelerated plate is considered in this paper, situated at the flow being confined to $x>0$, where x is the measure of coordinate in the normal direction to the surface. Initially, for time $t=0$, fluid and plate are both at stationary condition with constant temperature and concentration. At $t>0$, the plate is accelerated with velocity $u'=At$. At the same time, the plate temperature T' and concentration C' are raised to T_w' and C_w'.

The flow is governed by the following dimensional momentum, energy and concentration equations:

$$\rho\frac{\partial u'}{\partial t'}=\mu\left(1+\frac{1}{\gamma}\right)\frac{\partial^2 u'}{\partial x'^2}+\rho g\beta\left(T'-T_\infty'\right)-\sigma B_0^2u'-\frac{v}{K}u' \tag{1}$$

$$\rho c_p\frac{\partial T'}{\partial t'}=k\frac{\partial^2 T'}{\partial x'^2}-\frac{\partial q_r'}{\partial x'} \tag{2}$$

$$\rho c_p\frac{\partial C'}{\partial t'}=D\frac{\partial^2 T'}{\partial x'^2}-Kr'(C'-C_\infty') \tag{3}$$

Here, γ denotes Casson parameter, u' represents fluid in the x-direction, t refers to time variable T' is temperature of the fluid near the plate, while T_∞' represents temperature of the plate, ρ denotes fluid density, μ is dynamic viscosity, β refers to coefficient of the thermal expansion, $B_\circ$ is external magnetic field, represents kinematic viscosity, denotes thermal conductivity, refers to porosity, is specific heat at constant pressure and represents radiative heat flux, denotes chemical reaction along with initial and boundary conditions:

$$\begin{aligned}&u'\left(x',0\right)=0;\quad u'\left(0,t'\right)=At;\ u'\left(\infty,t'\right)=0;\\&T'\left(x',0\right)=T_\infty';\ T'\left(0,t'\right)=T_w';\ T'\left(\infty,t'\right)=T_\infty',\\&C'\left(x',0\right)=C_\infty';\ C'\left(0,t'\right)=C_w';\ C'\left(\infty,t'\right)=C_\infty',\end{aligned} \tag{4}$$

and by introducing dimensionless variable

$$u=\frac{u'}{(vA)^{\frac{1}{3}}};\ t=\frac{t'\ A^{\frac{2}{3}}}{v^{\frac{1}{3}}};\ x=\frac{x'\ A^{\frac{1}{3}}}{v^{\frac{2}{3}}};\ T=\frac{T'-T_\infty'}{T_w'-T_\infty'};\ C=\frac{C'-C_\infty'}{C_w'-C_\infty'} \tag{5}$$

The dimensionless form for Eqs. (1), (2) and (3),

$$\frac{\partial u}{\partial t}=A\frac{\partial^2 u}{\partial x^2}+GrT-Bu+GcC \tag{6}$$

$$\lambda\frac{\partial T}{\partial t}=\frac{\partial^2 T}{\partial x^2} \tag{7}$$

$$\frac{\partial C}{\partial t}=\frac{1}{Sc}\frac{\partial^2 C}{\partial x^2}-KrC \tag{8}$$

With its dimensionless initial and boundary condition:

$$\begin{aligned} &u(x,0)=0,\ u(0,t)=at,\ u(\infty,t)=0; \\ &T(x,0)=0,\ T(0,t)=1,\ T(\infty,t)=0; \\ &C(x,0)=0,\ C(0,t)=1,\ C(\infty,t)=0. \end{aligned} \tag{9}$$

Laplace transform method is applied for equations (6), (7) and (8),

$$\frac{d^2\overline{U}}{dx^2}-\frac{(s+B)}{A}\overline{U}=-P\overline{T}-Q\overline{C} \tag{10}$$

$$\frac{d^2\overline{T}}{dx^2}-\lambda s\overline{T}=0 \tag{11}$$

$$\frac{d^2\overline{C}}{dx^2}-\overline{C}(ScKr+Scs)=0 \tag{12}$$

where the parameters used in this research are:

$$A=1+\frac{1}{\gamma},\quad B=M+\frac{1}{K},\quad \lambda=\frac{\Pr}{1+N},\quad P=\frac{Gr}{A},\quad Q=\frac{Gc}{A},\quad M=\frac{\sigma {B_0}^2 v^{\frac{1}{3}}}{\rho A^{\frac{2}{3}}},$$

$$N=\frac{16\sigma T_\infty}{3k},\quad \Pr=\frac{\mu c_p}{k},\quad Gr=\frac{g\beta(T_w-T_\infty)}{A},\quad Gc=\frac{g\beta(C_w-C_\infty)}{A}.$$

Equations (10), (11) and (12) are then solved by using inverse Laplace transform technique:

$$T(x,t)=erfc\frac{x\sqrt{\lambda}}{2\sqrt{t}} \tag{13}$$

$$C(x,t)=\frac{1}{2}((\mathrm{e}^{x\sqrt{ScKr}}\,erfc\frac{x\sqrt{Sc}}{2\sqrt{t}}+\sqrt{Krt})+(\mathrm{e}^{-x\sqrt{ScKr}}\,erfc\frac{x\sqrt{Sc}}{2\sqrt{t}}-\sqrt{Krt})) \tag{14}$$

$$U(x,t)=U0(x,t)+U1(x,t)+U2(x,t)+U3(x,t)+U4(x,t) \tag{15}$$

where

$$U0(x,t)=(\frac{t}{2}+\frac{x}{4}\sqrt{\frac{1}{AB}})e^{x\sqrt{\frac{B}{A}}}erfc((\frac{x}{2\sqrt{At}})+\sqrt{Bt})+(\frac{t}{2}-\frac{x}{4}\sqrt{\frac{1}{AB}})e^{-x\sqrt{\frac{B}{A}}}erfc((\frac{x}{2\sqrt{At}})-\sqrt{Bt})$$

$$U1(x,t)=(-\frac{a1}{a2}erfc(\frac{x\sqrt{\lambda}}{2\sqrt{t}}))+(\frac{a1}{a2}\frac{e^{a2t}}{2})((e^{x\sqrt{\lambda a2}}erfc((\frac{x\sqrt{\lambda}}{2\sqrt{t}})+\sqrt{a2t}))+(e^{-x\sqrt{\lambda a2}}erfc((\frac{x\sqrt{\lambda}}{2\sqrt{t}})-\sqrt{a2t})$$

$$U2(x,t) = -\frac{a3}{2a4}(e^{x\sqrt{ScKr}} erfc(\frac{x\sqrt{Sc}}{2\sqrt{t}}) + \sqrt{Krt} + e^{-x\sqrt{ScKr}} erfc(\frac{x\sqrt{Sc}}{2\sqrt{t}}) - \sqrt{Krt}) +$$
$$\frac{a3}{a4}\frac{e^{a4t}}{2}(e^{x\sqrt{Sc(Kr+a4)}} erfc(\frac{x\sqrt{Sc}}{2\sqrt{t}}) + \sqrt{(Kr+a4)t} + e^{-x\sqrt{Sc(Kr+a4)}} erfc(\frac{x\sqrt{Sc}}{2\sqrt{t}}) - \sqrt{(Kr+a4)t})$$
$$U3(x,t) = \frac{a1}{2a2} e^{x\sqrt{\frac{B}{A}}} erfc[(\frac{x}{2\sqrt{At}}) + \sqrt{Bt} + e^{-x\sqrt{\frac{B}{A}}} erfc((\frac{x}{2\sqrt{At}}) - \sqrt{Bt}) -$$
$$(\frac{a1}{a2}\frac{e^{a2t}}{2})((e^{x\sqrt{\frac{1}{A}(B+a2)}} erfc((\frac{x}{2\sqrt{At}}) + \sqrt{Ba2t})) + (e^{-x\sqrt{\frac{1}{A}(B+a2)}} erfc((\frac{x}{2\sqrt{At}}) - \sqrt{Ba2t})$$
$$U4(x,t) = \frac{a3}{2a4} e^{x\sqrt{\frac{B}{A}}} erfc((\frac{x}{2\sqrt{At}}) + \sqrt{Bt}) + e^{-x\sqrt{\frac{B}{A}}} erfc((\frac{x}{2\sqrt{At}}) - \sqrt{Bt}) -$$
$$(\frac{a3}{a4}\frac{e^{a4t}}{2})e^{x\frac{(B+A4)}{A}} erfc((\frac{x}{2\sqrt{At}}) + \sqrt{(B+a4)t}) + e^{-x\frac{(B+A4)}{A}} erfc((\frac{x}{2\sqrt{At}}) - \sqrt{(B+a4)t})$$

with

$$a1 = \frac{aP}{a\lambda - 1}; \qquad a2 = \frac{B}{a\lambda - 1}; \qquad a3 = \frac{AQ}{ASc - 1}; \qquad a4 = \frac{AScKr - B}{ASc - 1}.$$

3. Results

3.1 Numerical Results and Discussions

Numerical analysis of temperature concentration and velocity profiles are done using MATHCAD software. Figure (1a) depicts the upshot of the radiation constraint on the over-temperature field. It is known that escalating the radiation parameter diminishes the thermal border thickness and temperature profile. Figure (1b) represents the effect of time towards temperature profile. Temperature increases progressively as time rises.

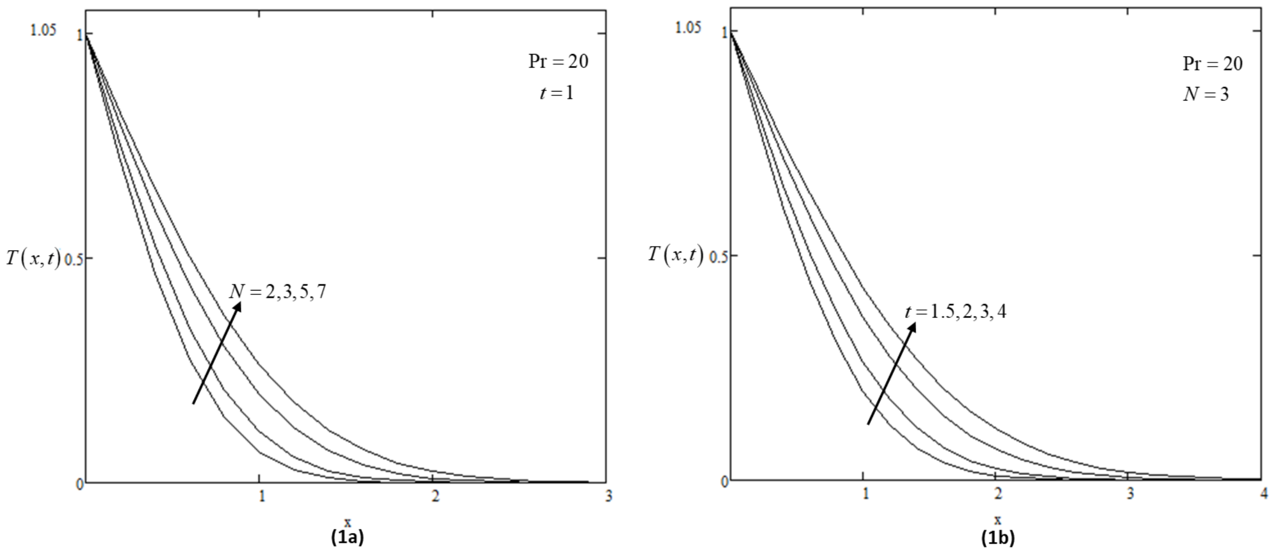

Fig. 1. Temperature profiles for various value of *N* and *t*

The impact of different values of the chemical reaction parameter on the concentration profile is illustrated in Figure (2a). When the chemical reaction parameter is increased, the concentration profile is found to be decayed. Figure (2b) presents a significant effect of the Schmidt number on the concentration profile. It is seen that the concentration profile decreases when the Schmidt level elevates. This is due to the inverse association between the Schmidt number and mass diffusivity. As the concentration distribution diminishes, a fluid flow regime with a higher Schmidt number incorporates lower mass diffusion values.

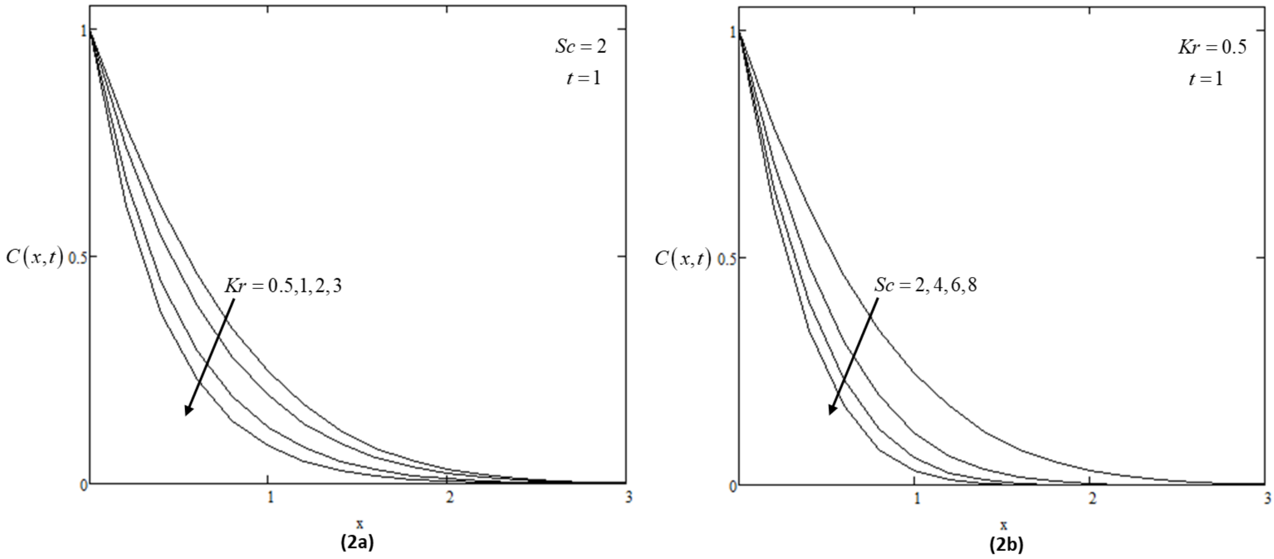

Fig. 2. Concentration profiles for various value of *Kr* and *Sc*

Figure 3 shows the effects of the magnetic parameter, *M* on velocity. The velocity increases with decreasing value of *M*. This is expected as the application of the Lorentz force resist and retard the flow in the velocity field. The effect of the Casson parameter experiences a decreasing trend on the velocity profile is portrayed in Figure 4. The characteristic of the Casson fluid determines that as the value decreases, the shear stress of the fluid gradually overcomes the yield stress, and subsequently the boundary layer thickness decrease. The fluid velocity profiles are shown in Figure 5 for different values of thermal radiation *N*. It is found that the velocity decreases with decreasing values of *N* in the presence of thermal radiation. This is predictable because lower radiation occurs when the temperature is lower and as an effect the velocity decreases. In order to visualise the velocity

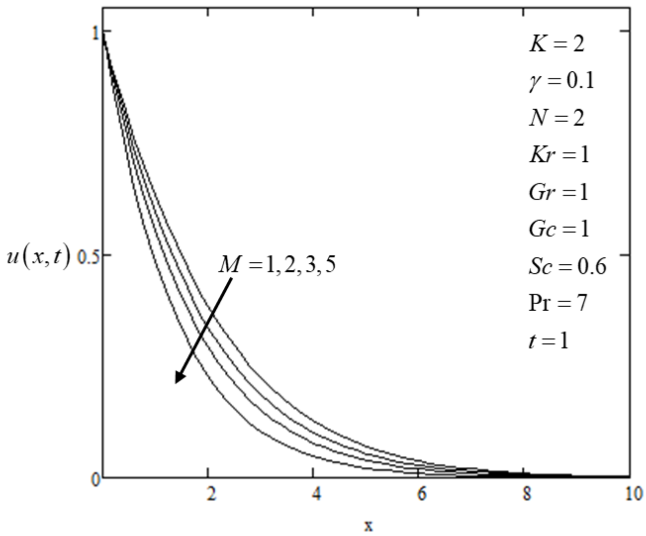

Fig. 3. Velocity profiles for various values of *M*

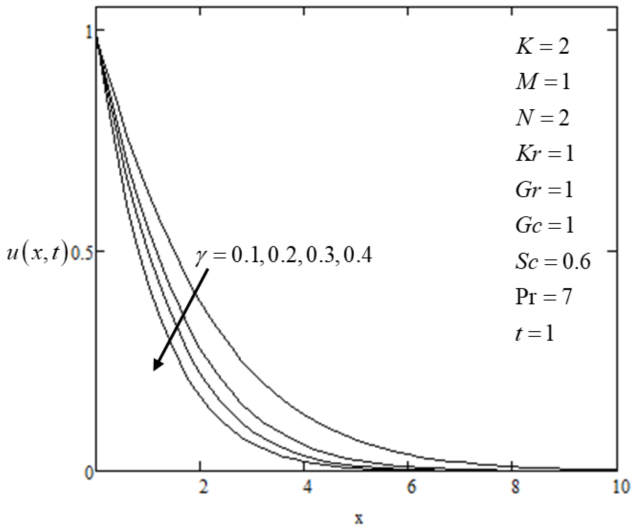

Fig. 4. Velocity profiles for various values of γ

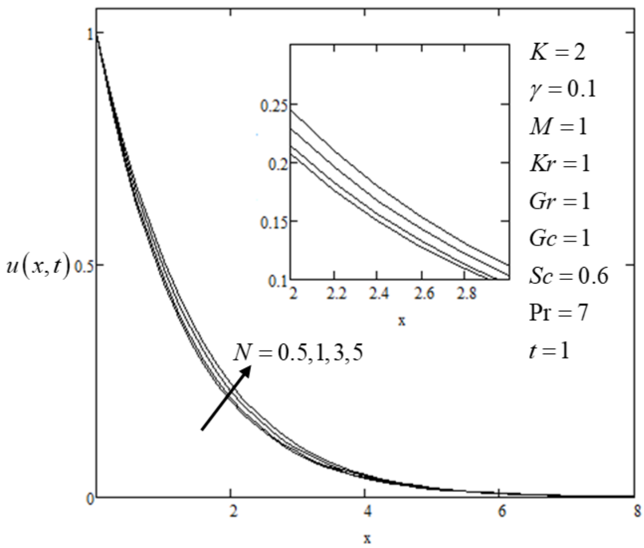

Fig. 5. Velocity profiles for various values of *N*

Figure 6 exhibits the effect of chemical reaction on velocity distribution. It is seen that velocity increases for big values of *Kr*. Thus, it is decided that the magnitude of coefficient of chemical reaction *Kr* plays a vital role on velocity distribution. The values of porosity parameters *K* are demonstrated in Figure 7 for different profiles of velocity, as the other flow parameters are kept fixed. Practically the velocity decreases with a decrease in *K*. This result may be explained by the fact of Darcy's law stated that the presence of a porous medium decreases the resistance to flow and hence enhances the fluid motion. Therefore, it is confirmed from the graph that porosity plays a vital role in the present analysis.

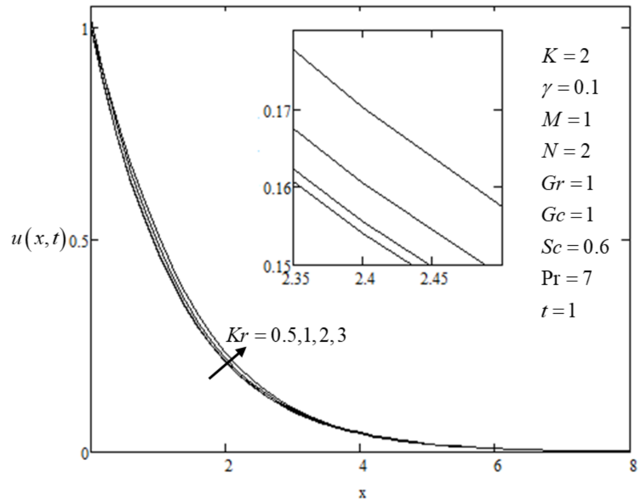

Fig. 6. Velocity profiles for various values of *Kr*

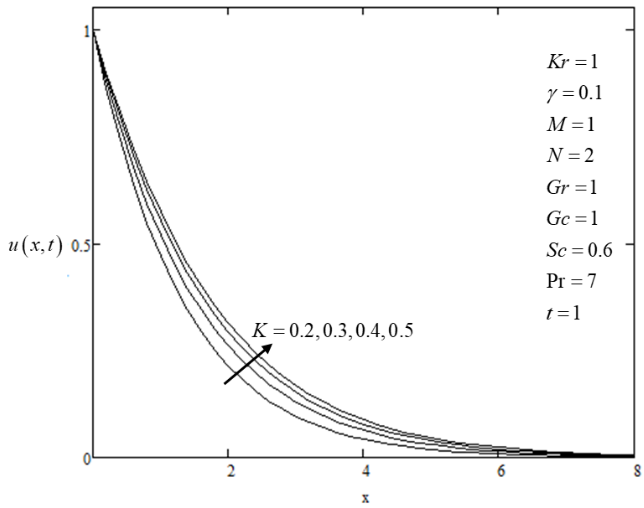

Fig. 7. Velocity profiles for various values of *K*

4. Conclusions

The analytical solution on performance of unsteady Casson fluid over with the inclusion of thermal radiation and chemical reaction is performed. The solution is derived by using Laplace transform technique. It has been observed that as the temperature rises, so does the thermal radiation and time. Besides, raising the chemical reaction parameter and Schmidt number reduce the concentration of the fluid. It is also concluded that increasing values of magnetic and Casson parameters tend to decreases the velocity field, whereas thermal radiation, chemical species and porous medium shown opposite results.

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