



Natural Frequencies Optimisation of Hybrid Composite Laminates using Response Surface Method

Muhammad Hadri Mohd Rosaidi¹, Jamaluddin Mahmud^{1,*}, Siti Mariam Abdul Rahman¹, Nurul Hayati Abdul Halim¹, Raja Izamshah Raja Abdullah², Mohd Nor Azmi Ab Patar¹

¹ School of Mechanical Engineering, College of Engineering, Universiti Teknologi MARA, 40450 Shah Alam, Selangor, Malaysia

² Fakulti Kejuruteraan Pembuatan, Universiti Teknikal Malaysia Melaka, Hang Tuah Jaya, 76100 Durian Tunggal, Melaka, Malaysia

ARTICLE INFO

Article history:

Received 5 February 2023

Received in revised form 20 June 2023

Accepted 28 June 2023

Available online 15 July 2023

Keywords:

Natural frequency; hybrid composite laminates; carbon; glass; free vibration

ABSTRACT

Natural frequency is an important property in designing structures, as resonance can induce catastrophic failure. Nevertheless, the vibration behaviour of hybrid composite laminates due to the hybridisation has still not been fully understood. This study aims to analyse and optimise the natural frequency response of hybrid composite laminates under free vibration due to the effect of various lamination schemes, plate thicknesses and hybridisation volume fractions. Initial stage involved mesh convergence analysis and numerical validation. Design of Experiments approach was employed to set up the important parameters and effective case studies. The natural frequencies for each case study were determined and analysed using finite element analysis software. The final stage involved optimisation using Response Surface Method. The results from the 34 case studies showed that the range of natural frequency was between 116.53Hz and 5598.4Hz. It was found that both symmetric and anti-symmetric laminates with 0° fibre angle produced the highest natural frequency of 5598.4Hz. Considering other parameters, the thicker plate and higher volume of carbon produced higher natural frequency. In conclusion, this study has contributed significant knowledge such as better understanding the effect of the studied parameters on the natural frequencies of hybrid composite laminates.

1. Introduction

Hybridisation is one method for enhancing the mechanical characteristics of composite materials. It is possible to mix various fibres or matrices, or both, to create a new material that meets the specified specifications. Previous research by Adali and Verijenko [1] has presented the optimum stacking sequence of symmetric multi-layered graphite epoxy/ glass-epoxy hybrid composites undergoing free vibration. They selected high stiffness plates for exterior layers and low stiffness plates for inside layers. Another research [2] studied the vibration behaviour of aluminum-epoxy/glass-epoxy hybrid composite plates while taking into consideration the impact of thickness ratio, layer count and material type. It was reported that the natural frequency decreased when the

* Corresponding author.

E-mail address: jm@uitm.edu.my

<https://doi.org/10.37934/araset.31.2.197209>

layer thickness ratio and the number of stacked layers increased [3]. Among the primary parameters that significantly impact a structure's serviceability are its natural frequencies and mode shapes. When an external vibration coincides with one of the natural frequencies of laminated composites, a resonant phenomenon develops [4], resulting in catastrophic failure of the structure. Thus, a thorough examination would be necessary prior to the final laminate design. Findings from recent research indicate that laminated composites will create lower natural frequencies and could contribute significantly to the resonant phenomena [5]. Nonetheless, this resonant phenomenon is still poorly understood, particularly in the case of laminated composites. Thus, it is critical to define the underlying theory that underpins this phenomenon. According to the literature, it can be concluded that when designing hybrid laminated composites under dynamic loading conditions, natural frequency is an important property to be measured. Therefore, it is crucial to investigate the dynamic characteristics of carbon/glass hybrid laminated composite plates. This work is novel as there is no recorded study that used these parameters and method before. Moreover, the new data generated in this study could be useful for validation, comparison, or reference prior to designing carbon/glass hybrid laminated composite plates.

2. Literature Review

2.1 Natural Frequencies

The natural frequency of an object is the rate at which it vibrates when disturbed. An object that vibrates naturally may have one or more natural frequencies. Resonance occurs when the forced frequency equals to the natural frequency. These are the frequencies at which the structure will typically vibrate in response to external forces. These frequencies are dependent on the way mass and stiffness are distributed within the structure. When an object is subjected to a dynamic force, it vibrates. When a force is applied at the natural frequency of an object, it enters resonance, producing a larger amplitude vibration response [6].

2.2 Response Surface Method (RSM)

Response surface methodology is a collection of statistical and mathematical modelling and analysis techniques applicable to engineering problems and particularly beneficial for optimising chemical reactions, industrial operations, and experimental designs [7]. The primary purpose of this method is to optimise the response surface affected by various process parameters. The link between the controllable input parameters and the resulting response surfaces is quantified by response surface methodology [8]. RSM also provides the advantages of being able to see the interaction effects between independent factors [9].

Response Surface Methodology (RSM) employs numerous rounds of optimisation and can be completed in three simple steps. These techniques involve conducting tests to define the criteria that will be considered, followed by selecting the optimal ascent or descent route. The quadratic polynomial model is subsequently fitted and optimized [10, 11]. Displaying the yield as a surface plot is one of the most crucial inputs that RSM takes into account. It may provide several answers simultaneously by addressing the interactions between components, which is essential for process optimization and design. Given the limited understanding of theoretical correlations between dependent and independent parameters, which is mostly based on a second-order equation, a multiple regression model can be used to predict dependent parameters [12].

2.3 Previous Studies on Hybrid and Laminated Composite Material

The term "natural frequency" relates to the dynamic qualities of a structure, indicating the degree of vibration in relation to its mass and stiffness [13]. To avoid unexpected and catastrophic failure, it is critical to understand a structure's dynamic properties. Laminated composite structures offer a wide variety of dynamic characteristics. A study by Imran *et al.*, [14] discussed the initiation of delamination, which is a primary cause of failure of laminated composite structures, as influenced by dynamic characteristics and vibration properties. Other research and experiments have also been conducted; for example, Nayak [15] conducted physical testing and numerical simulations of woven fiberglass/epoxy laminated composite plates' free vibrations. Norman *et al.*, [16] used ANSYS software and analytical technique to study the free vibrations of a laminated composite beam caused by the effect of lamination schemes based on previous research of Khayal *et al.*, [17]. Shi *et al.*, [18] established an analytical approach for free vibrations analysis of cross-ply composite laminates with shallow shells by incorporating semi-analytic method into arbitrary classical and elastic boundary conditions. A study on the free vibrations of thick laminated composite plates has also been conducted by Xue *et al.*, [19].

Numerous previous works have focused on using numerical simulations to analyse the natural frequencies of hybrid laminated composites. An investigation about the effects of different fibre volume fraction ratios had also been conducted by Pingkular and Suresha [3] while several stacking sequences on hybrid laminates' natural frequency had been conducted by Madhu and Kumarasamy [20]. In addition, mode shape and natural frequency had also been studied for hybrid laminate composites such as Glass-epoxy, Carbon-epoxy and Graphite fibre reinforced polyimide materials by Suragimath [21], E-Glass fibre 60% and Kevlar 40% by Jadhav *et al.*, [22] and laminated composites hybrid with Shape Memory Alloy (SMA) materials by Yusof *et al.*, [23]. This study is novel as to date, no identical method has been published in literature for analysing the natural frequency of Carbon/Glass hybrid laminates with different lamination schemes, fibre angles and ply thickness.

2.4 Material Selection and Characteristic Study

Material selection is a crucial factor to consider when fabricating a mechanical component. Taking example in the wind turbine applications, the work of Sunil Kumar *et al.*, [24] proved that selecting the right light material could increase the propulsion velocity four times higher than the standard velocity. The study demonstrated that when selecting materials, durability and endurance strength must be considered. Composite materials need to have the best stiffness properties because of the high impact forces that act mostly on the surface of the material and the shear forces that change the shape of the material when it is tilted [25]. The common materials used in braking systems have been developed from the engineering materials to natural fibres such as banana peels, however these materials have less endurance strength due to certain limitations subjected to operating temperatures of 100°C. The selection of materials for braking systems is a crucial and important feature in the vehicle industry [26]. From other perspective, aluminum alloys are widely used in the aerospace and automotive sectors due to their high strength-to-weight ratio and strong mechanical qualities such as greater corrosion and wear resistance, as well as less thermal expansion when compared to other metals. In the study of Muniamuthu *et al.*, [27] Alumina particle (Al₂O₃) was selected to be reinforced into Aluminum matrix and the results show improved mechanical characteristics such as impact strength and hardness. Therefore, from this brief review, it can be strongly emphasised that material selection is very important in designing an economical, green yet reliable structure.

3. Results

There were four stages involved in this study

- i. Stage 1: Convergence Analysis and Numerical Validation
- ii. Stage 2: Setting up case studies using Design of Experiment (DOE)
- iii. Stage 3: Free Vibration Analysis
- iv. Stage 4: Optimisation using Response Surface Method (RSM)

3.1 Stage 1: Convergence Analysis and Numerical Validation

3.3.1 Convergence analysis

The aim of conducting convergence analysis is to determine the minimum precise mesh size required for the models. The smaller the mesh size, the more accurate the simulations will be. However, it will require more time to make a lot of mesh. As a result, a range of mapped meshes (10x10, 20x20, 30x30, 40x40, 50x50, 60x60 and 70x70) elements using shell 281 element type with dimensions of 150 mm long and 75mm wide had been made. The lamination schemes analysed were [0g/+45c/-45c/90g]_s and [0c/+45g/-45g/90c]_s.

Figure 1 shows that the mesh size effect on the simulation result was just by a minor percentage. It showed that when the mesh size was 60x60 onwards, the result would constantly be the same.

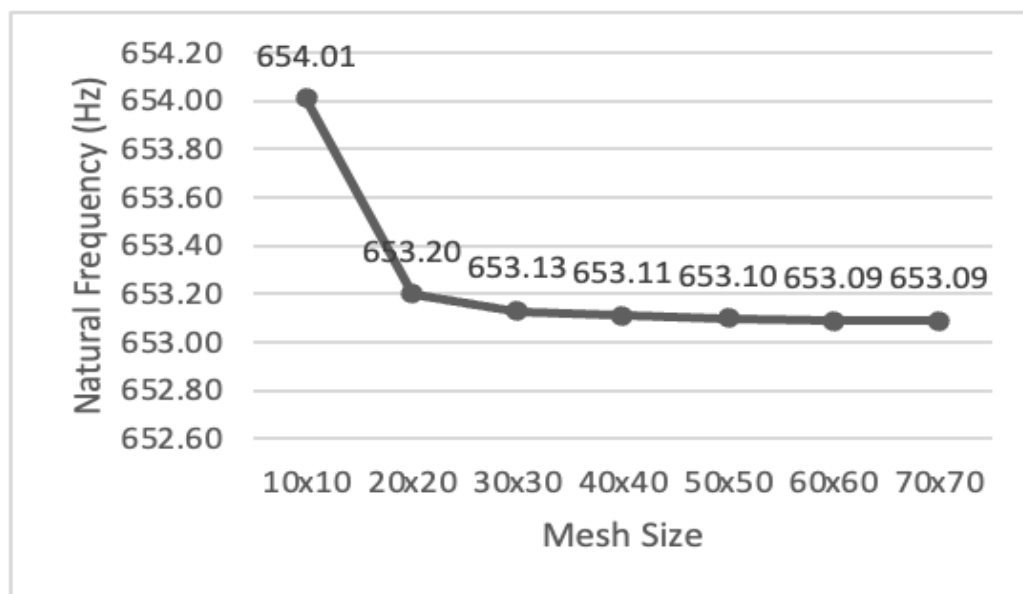


Fig. 1. Mesh convergence results for Hybrid Composite Plate [0g/+45c/-45c/90g]_s

3.1.2 Numerical validation

To confirm that the finite element simulation procedure conducted is correct, numerical validation was performed by comparing the current simulated results to the results presented in published literature. The works were related to free vibration analysis of a hybrid of the laminated composite plate. Therefore, the numerical validation was conducted based on the work of Pingulkar *et al.*, [3] and Norman *et al.*, [5]. The composite plate was made of hybrid carbon and glass fibre composites with dimensions of 150 mm long and 75mm wide as shown in Figure 2. The ply thickness for the glass fibre lamina was 0.15 mm and the ply thickness for carbon fibre lamina composite was

0.13 mm. The plate was meshed into 20 x 20 elements using shell 8-noded element type. The lamination schemes analysed were [0g/+45c/-45c/90g]s.

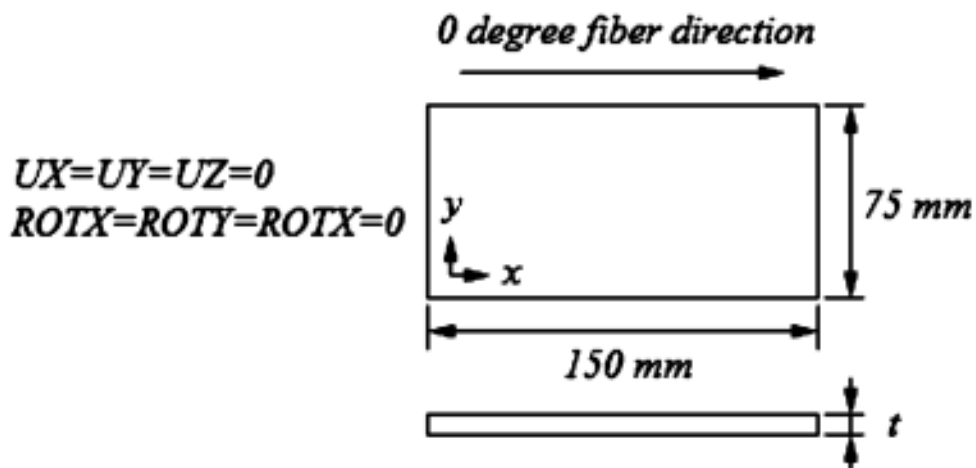


Fig. 2. Rectangular plate configuration with boundary conditions

In this study, both percentage error and percentage difference were calculated. The percentage error was calculated by comparing the current simulated results to the analytical results [3]. The percentage difference was calculated by comparing the current simulated results to the simulated results presented in a past study [5]. The percentage error and percentage difference and are presented in Table 1, where the maximum error produced was found to be 1.17%. Since the error was found to be less than 5%, it proves that the current finite element simulation procedure is correct. In terms of percentage difference, the difference may be due to the different parameters used in conducting the simulation. Another study [26] has also highlighted the same issue. In that finite element analysis, when comparing coarse mesh and elemental size of 0.5 mm assigned to a rotor surface, the elemental mesh sizes produce lower deviation compared to the coarse mesh.

Table 1

Comparison between previous analytical results and current simulation results for the natural frequency

Laminate	Mode	Analytical (Pingulkar) (Hz)	Simulation (Norman) (Hz)	Current Result (Hz)	Difference %	Error%
[0g/+45c/-45c/90g]s	1B	37.374	37.737	37.778	0.10	1.08
	1T	165.050	163.290	165.620	1.43	0.35
	2B	231.260	233.990	233.930	0.04	1.15
	2T	528.390	524.400	531.120	1.27	0.52
	1C	645.650	653.140	653.200	0.01	1.17

3.2 Stage 2: Setting up case studies using Design of Experiment (DOE)

Design Expert is a piece of software that assists in designing and analyzing multi-factor experiments [28]. Using this software, it helps to optimise the number of case studies that need to be carried out in composite processing [29]. It manages to process variables such as rotor speed as well as mixture variables such as the resin proportion in a plastic compound. Design Expert provides computer-generated D-optimal designs when traditional designs are not appropriate or where we want to improve an existing design, such as to meet a more flexible model.

There were two cases, which were symmetric and anti-symmetric in this study, and Box-Behnken Design was used to optimise the needed parameters. In each case, there were three numeric factors and for each factor, there were three levels as shown in Table 2. For the second level, the midpoint was chosen between the highest level and the lowest level. For symmetric lamination scheme the level was [0°/- 0°/ -0°/ 0°], [30°/- 30°/ -30°/ 30°] and [60°/-60°/ -60°/ 60°], for volume fraction 0 = fully carbon, 0.5 = hybrid and 1 = fully glass and for volume fractions highest was 0.002m, lowest was 0.0001m and the middle point was 0.00105m.

Table 2

Box-Behnken design

Parameters/ Factors	Units	Low	High
Lamination Scheme	Degree	0	60
Volume fraction	%	0	1
Thickness	M	0.0001	0.002

3.3 Stage 3: Free Vibration Analysis

A comprehensive analysis about the natural frequency of laminated composite plates and their hybrids was carried out. This analysis will enable in-depth knowledge of how the lamination schemes, plate thickness and hybridisation (volume fraction) influence the natural frequencies of composite laminates. The present work involves finite element modelling and simulation of composite laminates under free vibration to analyse the corresponding natural frequencies using ANSYS Mechanical APDL (v16.0, 2014 SAS IP, Inc.). In this study, symmetric and anti-symmetric laminated composite plates having 4 layers with the variations of angles of fibre orientation were studied. The boundary conditions are the same as shown earlier in Figure 2. The plate was meshed into 20 x 20 elements using shell 8-noded element type. The simulation related work was set into the modal analysis. Block Lanczos modal analysis was implemented, where the initial frequency was set to 0. The following procedure involved analysing the relationship between factors using a structured Design of Experiment (DOE) approach.

3.4 Stage 4: Optimisation using Response Surface Method (RSM)

Response surface methodology, or RSM, is a set of mathematical and statistical techniques that can be used to model and analyse problems in which a response of interest is influenced by several variables and the goal is to optimise this response [30]. Designing a series of experiments for adequate and reliable measurement of the response of interest.

The coefficients of the second polynomial model as in Equation 1 were derived from the responses collected from the simulation works using Design-Expert software. Regression analysis's fitted quadratic polynomial equation was utilised to produce 3D response surface and contour graphs.

$$y = \beta_0 + \sum_{i=1}^k \beta_i x_i + \sum \beta_{ii} + \sum_{i < j} \sum \beta_{ij} x_i x_j + \varepsilon \quad (1)$$

Following is the design process for response surface methodology

- i. Designing a series of experiments for adequate and reliable measurement of the response of interest.

- ii. Developing a mathematical model of the second order response surface with the best fittings.
- iii. Finding the optimal set of experimental parameters that produces a maximum or minimum value of the response.
- iv. Representing the direct and interactive effects of process parameters through two and three-dimensional plots.

4. Results and Discussion

4.1 Analysis of Natural Frequency using ANSYS

The hybrid composite laminates in this study were carbon/epoxy and glass/epoxy. Moreover, a composite laminate consisting of 4 layers with layup studied were symmetric $[\theta^\circ/-\theta^\circ/-\theta^\circ/\theta^\circ]$ and anti-symmetric $[\theta^\circ/-\theta^\circ/\theta^\circ/-\theta^\circ]$ with $\theta = 0^\circ, 30^\circ$ and 60° . Three different cases of hybridisation volume fraction were studied where the 1st case was 100% of carbon, the 2nd case was 100% glass and the 3rd was in sandwich form with 50% carbon and 50% glass at the middle. As stated in Tables 3 and 4, the volume fraction 0% show that the lamination was fully carbon, 1% was fully glass and 0.5% was hybrid lamination.

Table 3 and Table 4 show the results of natural frequency of composite laminates under free vibration due to the variables with various ply thicknesses and fibre angles for symmetric $[\theta^\circ/-\theta^\circ/-\theta^\circ/\theta^\circ]$ and anti-symmetric $[\theta^\circ/-\theta^\circ/\theta^\circ/-\theta^\circ]$ lamination schemes respectively. Based on the numerical validation conducted and the results presented in Table 1, the specification for other case studies are maintained. Based on the work of Norman *et al.*, [5], for clear comparison, the boundary condition was set as Free-Free-Free-Free (FFFF) around the plate. A commercially available finite element software (ANSYS Mechanical APDL, v16.0, 2014 SAS IP, Inc.) was used to determine the natural frequencies for each case.

Table 3
 Natural frequencies for symmetric laminate

Std	Run	Factor 1 Lamination Scheme (Degree)	Factor 2 Volume Fraction (%)	Factor 3 Thickness (m)	Response 1 Freq 1 (Hz)	Response 2 Freq 2 (Hz)	Response 3 Freq 3 (Hz)
9	1	30	0	0.00010	258.94	600.42	1282.9
2	2	60	0	0.00105	1301.6	4683.3	7405.9
16	3	30	0.5	0.00105	2105.2	4592.8	9761.3
8	4	60	0.5	0.00200	2171.8	6376.7	8472.4
4	5	60	1	0.00105	1124.5	3245.9	6551.4
1	6	0	0	0.00105	3750.7	4629	9226.4
5	7	0	0.5	0.00010	337.15	438.54	908.72
15	8	30	0.5	0.00105	2105.2	4592.8	9761.3
11	9	30	0	0.00200	4241.5	8425.5	16100
14	10	30	0.5	0.00105	2105.2	4592.8	9761.3
3	11	0	1	0.00105	1936.8	3093.5	8209.2
7	12	0	0.5	0.00200	5598.4	6729.3	9620.1
10	13	30	1	0.00010	149.39	362.59	875.37
12	14	30	1	0.00200	2767.4	5885.9	9608.6
13	15	30	0.5	0.00105	2105.2	4592.8	9761.3
6	16	60	0.5	0.00010	116.53	400.87	711.19
17	17	30	0.5	0.00105	2105.2	4592.8	9761.3

Table 4
 Natural frequencies for anti-symmetric laminate

	Factor 1	Factor 2	Factor 3	Response 1	Response 2	Response 3	
Std	Run	Lamination Scheme (Degree)	Volume Fraction (%)	Thickness (m)	Freq 1 (Hz)	Freq 2 (Hz)	Freq 3 (Hz)
11	1	30	0	0.00200	4315.6	8173.2	14999
13	2	30	0.5	0.00105	1887.3	3868.5	7867.9
12	3	30	1	0.00200	2782.6	5825.8	9519.7
5	4	0	0.5	0.00010	337.15	438.54	908.72
3	5	0	1	0.00105	1936.8	3093.5	8209.2
6	6	60	0.5	0.00010	114.36	432.32	699.55
4	7	60	1	0.00105	1124.6	3339.8	6557.2
15	8	30	0.5	0.00105	1887.3	3868.5	7867.9
14	9	30	0.5	0.00105	1887.3	3868.5	7867.9
2	10	60	0	0.00105	1316.3	5121.3	7496.9
1	11	0	0	0.00105	3750.7	4629	9226.4
10	12	30	1	0.00010	150.7	370.07	872.24
16	13	30	0.5	0.00105	1887.3	3868.5	7867.9
17	14	30	0.5	0.00105	1887.3	3868.5	7867.9
9	15	30	0	0.00010	266.12	644.39	1340.8
7	16	0	0.5	0.00200	5598.4	6729.3	9620.1
8	17	60	0.5	0.00200	2140.4	6425	8398.6

4.2 Analysis of Variance (ANOVA) using Full Factorial Design

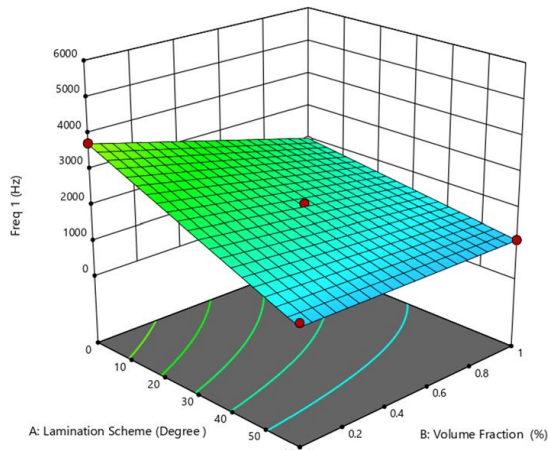
Analysis of variance (ANOVA) is a set of statistical models used to examine variations in group means and their related methods [31]. Table 5 shows the analysis of variance for the response of the composite laminates (symmetric) for natural frequency 1. The table showed that the model terms were significant because the p-value was less than 0.05. Also found that the ply thickness was the dominant factor influencing the response, followed by lamination scheme and the interaction between these two factors. While for model's determination coefficient (R^2) at 0.9947, indicating that it accurately represents the actual relationship between the control components. The adequate precision that larger than 4 indicates an adequate signal and desirable.

Table 5
 ANOVA for Reduced Cubic model for Symmetric Laminate

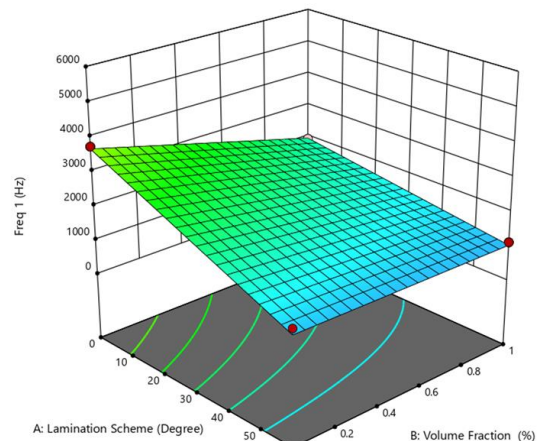
Source	Sum of Squares	Df	Mean Square	F-value	p-value	
Model	3.548E+07	6	5.913E+06	232.08	< 0.0001	significant
A-Lamination Scheme	5.966E+06	1	5.966E+06	234.16	< 0.0001	
B-Volume Fraction	1.597E+06	1	1.597E+06	62.69	< 0.0001	
C-Ply Thickness	2.421E+07	1	2.421E+07	950.24	< 0.0001	
AB	6.698E+05	1	6.698E+05	26.29	0.0004	
AC	2.570E+06	1	2.570E+06	100.85	< 0.0001	
BC	4.655E+05	1	4.655E+05	18.27	0.0016	
Residual	2.548E+05	10	25478.40			
Lack of Fit	2.548E+05	6	42464.00			
Pure Error	0.0000	4	0.0000			
Cor Total	3.573E+07	16				

R-squared	0.9947
Adj. R-squared	0.9916
Pred. R-squared	0.9757
Adeq. precision	59.6419

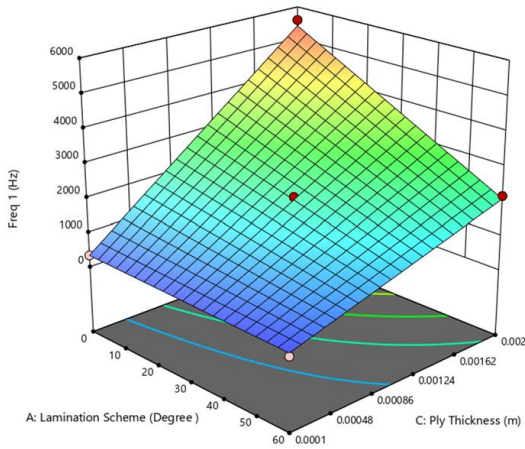
Graphs from a 3D surface model were used to analyse the influence of control parameters on responses, as illustrated in Figure 3. As can be seen, the contour and the highest value of natural frequency were the same for both conditions, symmetry and anti-symmetry. The highest natural frequency was the same for both, but the minimum natural frequency varied slightly. From Figure 3, it shows that fully carbon would have the highest natural frequency and the lowest would be the fully glass. Hybrid composite laminates (C-G-G-C) stayed in between fully carbon and fully glass. It could be concluded that fully carbon with 0° lamination scheme and thickest plate would have the highest natural frequency.



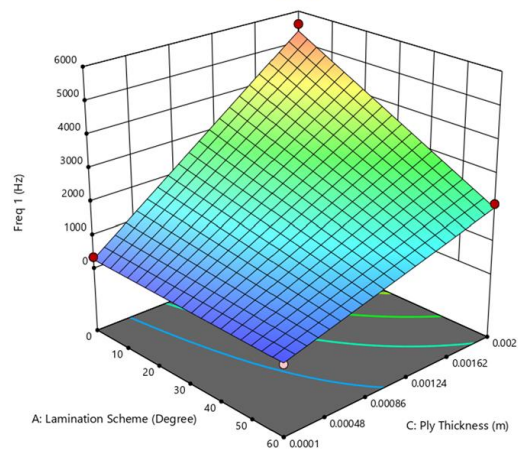
(a)



(d)



(b)



(e)

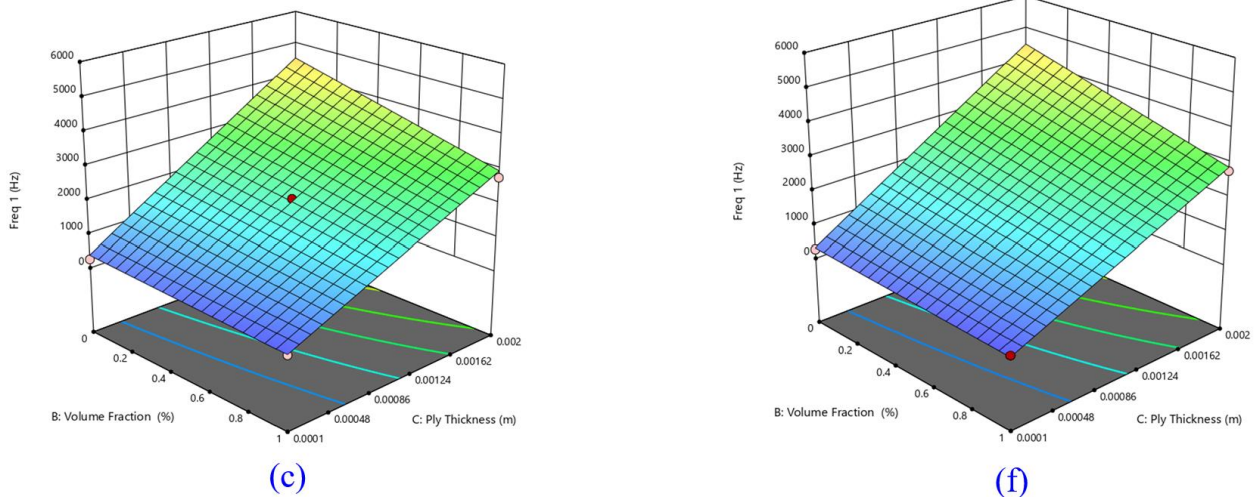
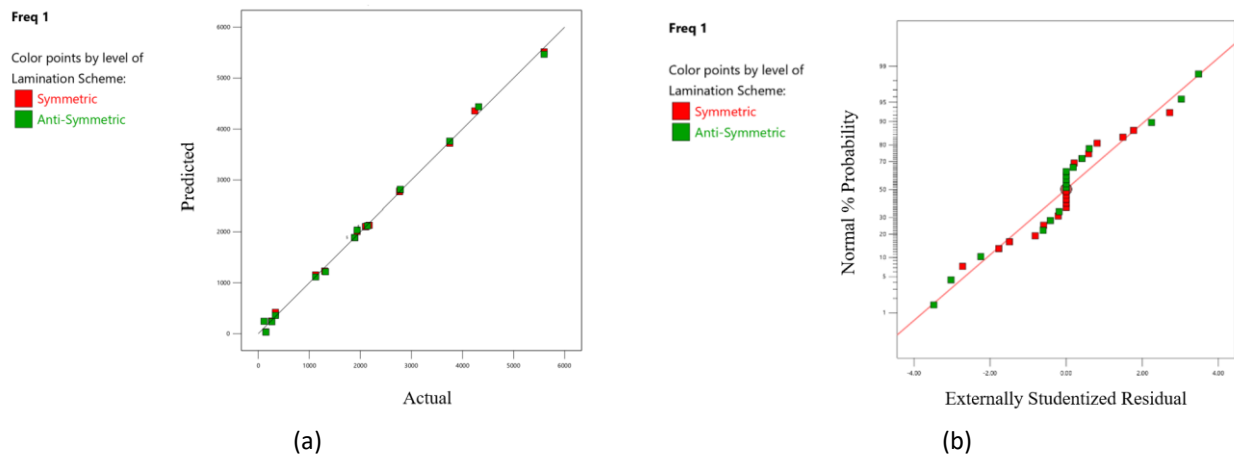
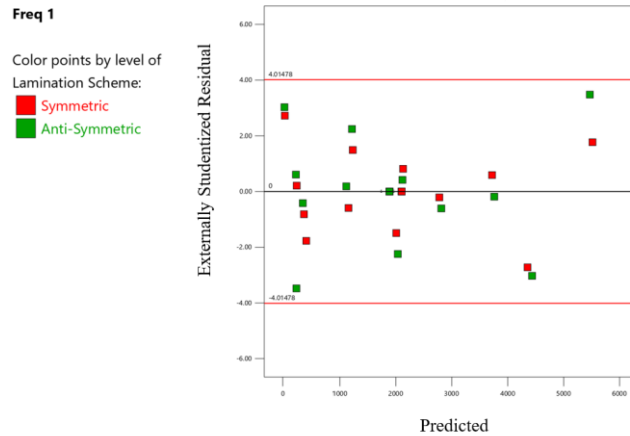


Fig. 3. 3D surface graph of (a) volume fraction versus lamination scheme for symmetric laminate (b) ply thickness versus lamination scheme for symmetric laminate (c) ply thickness versus ply thickness for symmetric laminate (d) volume fraction versus lamination scheme for anti-symmetric laminate (e) ply thickness versus lamination scheme for anti-symmetric laminate (f) ply thickness versus ply thickness for anti-symmetric laminate

4.3 Diagnostic Plots

Figure 4 illustrates diagnostic graphs of natural frequency for the symmetric and anti-symmetric hybrid composite laminates. By showing the normal data distribution, it is obvious that all plots had satisfied the general statistical assumptions. This is achieved after many iterations. Observing Figure 4a and comparing to the work of Narwade *et al.*, [31] it could be seen that the current results show better prediction (versus actual responses). Due to this achievement, the final data was used for the optimisation process using the Response Surface Methods.



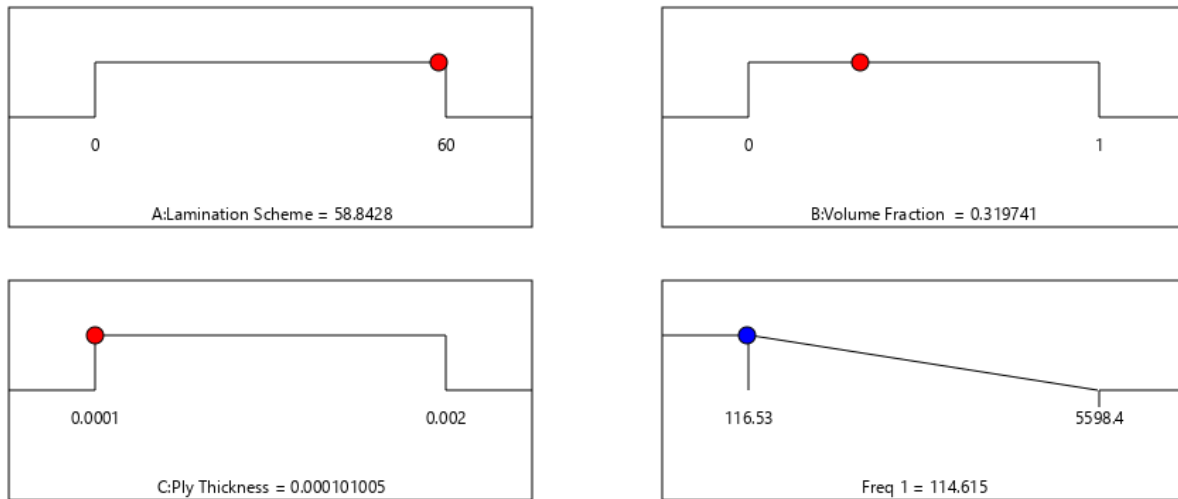


(c)

Fig. 4. Diagnostic graph of (a) Regression analysis of predicted vs actual, (b) Normal probability together with residual and (c) Studentized residual versus predicted values

4.4 Optimisation using Response Surface Methods

One of the potentials of Design-Expert software is that it can show optimum point in the form of Ramp chart as shown in Figure 5. In this study, RSM suggests the minimum value of frequency can be achieved at optimum parameter of lamination scheme (58.8°), volume fraction (32%) and ply thickness (0.0001 m). In general, the findings of the current study are in good agreement with the findings presented by Narwade *et al.*, [31] and Norman *et al.*, [32].



Desirability = 1.000
 Solution 4 out of 100

Fig. 5. The Ramp chart shows optimum point gained with Design-Expert software

5. Conclusions

Using finite element software, modal analysis was performed on hybrid composite laminates built of Carbon/Glass epoxy in this study. This study investigated and forecasted the natural frequency under various lamination schemes, volume fractions and ply thicknesses under the boundary conditions Free-Free-Free-Free (FFFF). Even though the laminates had the same geometry, mass and boundary conditions, the natural frequency generated varying results. This study demonstrated that all three parameters would influence the structure's natural frequency. In addition, finite element simulation is useful for vibration analysis because it can produce graphical findings relating to the fundamental frequency. This study's findings and collected data could be utilised by composite manufacturers in the design of structures with reduced vibration levels by merely considering the lamination scheme, volume fraction and ply thickness. Consequently, it can be concluded that this work provides a better understanding of the finite element techniques of hybrid composite laminates and their effect on natural frequency.

Acknowledgement

This work was supported by the Ministry of Higher Education (MOHE) Malaysia and Universiti Teknologi MARA, under the Fundamental Research Grant Scheme: Grant No. FRGS/1/2018/TK03/UITM//02/8 and 600-IRMI/FRGS 5/3 (165/2019).

References

- [1] Adali, S., and V. E. Verijenko. "Optimum stacking sequence design of symmetric hybrid laminates undergoing free vibrations." *Composite structures* 54, no. 2-3 (2001): 131-138. [https://doi.org/10.1016/S0263-8223\(01\)00080-0](https://doi.org/10.1016/S0263-8223(01)00080-0)
- [2] Chen, Chun-Sheng, Wei-Ren Chen, and Rean-Der Chien. "Stability of parametric vibrations of hybrid composite plates." *European Journal of Mechanics-A/Solids* 28, no. 2 (2009): 329-337. <https://doi.org/10.1016/j.euromechsol.2008.06.004>
- [3] Pingulkar, Pushparaj, and B. Suresha. "Free vibration analysis of laminated composite plates using finite element method." *Polymers and Polymer Composites* 24, no. 7 (2016): 529-538. <https://doi.org/10.1177/096739111602400712>
- [4] Suryadi, A., Yanuar, and Gunawan. "Numerical Analysis and Optimization Research on Backflow Effect of Nitrogen Compressor Piping System Based on Vibration Data". *Journal of Advanced Research in Applied Mechanics* 78 (1) (2021): 1-12. <https://doi.org/10.37934/aram.78.1.112>
- [5] Norman, Mohd Arif Mat, and Jamaluddin Mahmud. "Investigation of Natural Frequencies on Kevlar/Glass Hybrid Laminated Composite Plates Using Finite Element Simulation." *International Transaction Journal of Engineering, Management, & Applied Sciences & Technologies* 12, no. 9 (2021): 1-8. <https://doi.org/10.14456/ITJEMAST.2021.169>
- [6] Peng, Z. K., Zi-Qiang Lang, and S. A. Billings. "Resonances and resonant frequencies for a class of nonlinear systems." *Journal of Sound and Vibration* 300, no. 3-5 (2007): 993-1014. <https://doi.org/10.1016/j.jsv.2006.09.012>
- [7] Khoo, Miinyi, Vasanthi Sethu, Anurita Selvarajoo, and Senthil Kumar Arumugasamy. "Performance of Fenugreek and Okra for the Physico-Chemical Treatment of Palm Oil Mill Effluent – Modeling Using Response Surface Methodology". *Progress in Energy and Environment* 15 (2021): 8-30.
- [8] Anderson, Mark J., and Patrick J. Whitcomb. *RSM simplified: optimizing processes using response surface methods for design of experiments*. Productivity press, (2016). <https://doi.org/10.1201/9781315382326>
- [9] Sunarti, A. R., and A. L. Ahmad. "Performances of Automated Control System for Membrane Gas Absorption: Optimization Study." *Journal of Advanced Research in Applied Mechanics* 6 (2015): 1-20.
- [10] Durakovic, Benjamin. "Design of experiments application, concepts, examples: State of the art." *Periodicals of Engineering and Natural Sciences* 5, no. 3 (2017): 421-439. <https://doi.org/10.21533/pen.v5i3.145>
- [11] Khuri, André I., and Siuli Mukhopadhyay. "Response surface methodology." *Wiley Interdisciplinary Reviews: Computational Statistics* 2, no. 2 (2010): 128-149. <https://doi.org/10.1002/wics.73>
- [12] Gonzalez-Cortes, Tania, Rogelio Recio-Vega, Robert Clark Lantz, and Binh T. Chau. "DNA methylation of extracellular matrix remodeling genes in children exposed to arsenic." *Toxicology and applied pharmacology* 329 (2017): 140-147. <https://doi.org/10.1016/j.taap.2017.06.001>

- [13] Reddy, Mutra Raja Sekhara, Bathini Sidda Reddy, Vanguru Nageswara Reddy, and Surisetty Sreenivasulu. "Prediction of natural frequency of laminated composite plates using artificial neural networks." (2012): 329-337. <https://doi.org/10.4236/eng.2012.46043>
- [14] Imran, Muhammad, Rafiullah Khan, and Saeed Badshah. "Vibration Analysis of Cracked Composite Laminated Plate and Beam Structures". *Romanian Journal of Acoustics and Vibration* 15 (1), (2018): 3-13.
- [15] Nayak, P.' Vibration Analysis of Woven Fiber Glass / Epoxy Composite Plates Vibration Analysis of Woven Fiber Glass / Epoxy Composite Plates. Master Thesis, Technology in Civil Engineering, National Institute of Technology, Rourkela. (2008): 1-69.
- [16] Norman, M. Arif Mat, M. Amiruddin Zainuddin, and Jamaluddin Mahmud. "The effect of various fiber orientations and boundary conditions on natural frequencies of laminated composite beam." *International Journal of Engineering & Technology* 7, no. 3.11 (2018): 67-71. <https://doi.org/10.14419/ijet.v7i3.11.15932>
- [17] Khayal, O., Algarray, A., & Mahdi, "Free Vibration Analysis of Composite Laminated Beams." *International Journal of Engineering Research & Advanced Technology*, (2019): 3, 9-25.
- [18] Shi, Dongyan, Dongze He, Qingshan Wang, Chunlong Ma, and Haisheng Shu. "Wave based method for free vibration analysis of cross-ply composite laminated shallow shells with general boundary conditions." *Materials* 12, no. 23 (2019): 3808. <https://doi.org/10.3390/ma12233808>
- [19] Xue, Zechang, Qihong Li, Wenhao Huang, Yongxin Guo, and Jiufa Wang. "Vibration characteristics analysis of moderately thick laminated composite plates with arbitrary boundary conditions." *Materials* 12, no. 17 (2019): 2829. <https://doi.org/10.3390/ma12172829>
- [20] Madhu, S., and M. Kumaraswamy. "Experimental investigation and free vibration analysis of hybrid laminated composite beam using finite element method." *International Journal for Research in Applied Science and Engineering Technology (IJRASET)* 5, no. 6 (2017): 40-53. https://doi.org/10.1007/978-3-030-80618-7_24
- [21] Suragimath, Channabasavaradhy. "Modal analysis of composite beam using MATLAB." *International Journal of Engineering Science* (2019): 19551-19555.
- [22] Jadhav, A.K., Joshi, M.M., & Jadhav. S. M. "Vibration Analysis of Laminated Hybrid Composite Material for Racing Car Panels." *International Journal for Research in Applied Sciences & Engineering Technology*, (2019): 7(IX), 823-831. <https://doi.org/10.22214/ijraset.2019.9113>
- [23] Yusof, Zarina, Zainudin A Rasid, and Jamaluddin Mahmud. "The active strain energy tuning on the parametric resonance of composite plates using finite element method." *Journal of Mechanical Engineering (JMechE)* 15, no. 1 (2018): 104-119
- [24] Sunil Kumar, K., Sumathy Muniyandhu, and B. T. Tharanisrisakthi. "An investigation to estimate the maximum yielding capability of power for mini venturi wind turbine." *Ecological Engineering & Environmental Technology* 23 (2022): 72-78. <https://doi.org/10.12912/27197050/147150>
- [25] Muniyandhu, Sumathy, K. Sunil Kumar, K. Raja, and P. L. Rupesh. "Dynamic Characterization of Hybrid Composite Based on Flax/E-Glass Epoxy Composite Plates." *Materials Today: Proceedings* 59 (2022): 1786–1791. <https://doi.org/10.1016/j.matpr.2022.04.379>
- [26] Sunil kumar, K., and Deepika Bishnoi. "Pressure Exertion and Heat Dissipation Analysis on Uncoated and Ceramic (Al₂O₃, TiO₂ and ZrO₂) Coated Braking Pads. *Materials Today: Proceedings* 74 (2023): 774-787. <https://doi.org/10.1016/j.matpr.2022.11.170>
- [27] Muniyandhu, S., Raju, N.L., Sathishkumar, S. and Kumar, K.S., "Investigation on mechanical properties of al 7075-al₂o₃ metal matrix composite." *International Journal of Mechanical Engineering and Technology* 7, no. 6 (2016): 474–482. <https://doi.org/10.13140/RG.2.2.11112.06400>
- [28] Khaleel, Hawraa, Adnan A. Ateeq, and Amjed A. Ali. "The effect of temperature and inhibitor on corrosion of carbon steel in acid solution under static study." *Int. J. Appl. Eng. Res* 13, no. 6 (2018): 3638-3647.
- [29] Mohamed, Omar A., Syed H. Masood, and Jahar L. Bhowmik. "Optimization of fused deposition modeling process parameters: a review of current research and future prospects." *Advances in manufacturing* 3 (2015): 42-53. <https://doi.org/10.1007/s40436-014-0097-7>
- [30] Montgomery, D.C. (2012) Design and Analysis of Experiments. 8th Edition, John Wiley & Sons, New York.
- [31] Narwade, P. A., and Supriya R. Deshmukh. "An RSM approach to maximize fundamental frequency of e-glass epoxy composite panels." *Journal of Advanced Research in Applied Mechanics* 35, no. 1 (2017): 18-27.
- [32] Norman, Mohd Arif Mat, Kishen Sivakumar, Mohd Nor Azmi Ab Patar, and Jamaluddin Mahmud. "A Prediction Model for Natural Frequencies on Kevlar/Glass Hybrid Laminated Composite using Artificial Neural Networks (ANN)." *International Journal of Integrated Engineering* 14, no. 5 (2022): 207-214. <https://doi.org/10.30880/ijie.2022.14.05.023>