Journal of Advanced Research in Applied Sciences and Engineering Technology

Journal homepage:
https://semarakilmu.com.my/journals/index.php/applied_sciences_eng_tech/index
ISSN: 2462-1943

SIR Fractional Order of Simulated Covid-19 Cases using Adams Bashforth-Moulton Method

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ARTICLE INFO

Article history:
Received 16 January 2023
Received in revised form 17 January 2024
Accepted 20 February 2024
Available online 26 March 2024

Keywords:

SIR model; Covid-19; Fractional calculus; Caputo sense; Adams Bashforth Moulton method

ABSTRACT

Fractional order derivative has been widely used in many different areas such as bioengineering, fluid mechanics, circuits systems, biomathematics, and biomedicine. This study introduces the system of the fractional differential equation on SIR (Susceptible-Infected-Recovered) model to simulate the COVID-19 in Malaysia. The fractional derivative is described in Caputo sense and solved by the Adams Bashforth Moulton method. The Runge-Kutta method is used to prove and validate the numerical results obtained. The graphical representations of the simulation with difference fractional order have been presented. The derivative order, α with values more than 0.5 are acceptable and reliable.

1. Introduction

The Coronavirus disease of 2019 (COVID-19) is a newly found coronavirus that causes an infectious disease [19]. COVID-19 was discovered around the end of 2019 in Wuhan, China. The virus continues to spread widely, and the cases are detected in countless countries worldwide, eventually becoming a pandemic. By 2020, COVID-19 has reached the United States, Europe, and Asia, brought by travellers from affected areas including Malaysia. The first outbreak of imported COVID-19 was reported on January 25, 2020, involving three Chinese tourists. Then, the outbreak became epidemic when the number of cases in Malaysia increased from 3 cases to 22 confirmed cases registered throughout the 23-day periods.

Despite minimal outbreak control measures, many researchers tried to understand the disease transmission and provide solutions for disease control. Mathematical modelling in epidemiology opens up a new era in understanding disease transmission and provides recommendations for disease control. A few mathematical models can describe the dynamic of the COVID-19. The Susceptible-Infected-Recovered (SIR) model is the most basic mathematical model to study epidemiology including COVID-19.

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https://doi.org/10.37934/araset.42.1.8292

In 1927, Kermack and McKendrick developed the model which studied a fixed population with only three compartments, which are susceptible (*S*), infected (*I*) and recovered (*R*). This model started with the number of people in the susceptible, infected, and recovered categories at initial time zero. This SIR model serves as a fundamental mathematical model for transmitting epidemic diseases. Later, various modifications of SIR model such as SIS, SIR, SIRS, SEIS, SEIR and SEIRS have been used to model the spread of disease. Several past studies focused on epidemic model have been done to describe the dynamic of COVID-19 in Malaysia and aimed to "flatten the epidemic curve". Ariffin *et al.,* [6]. The research interests include incorporating a partial time-varying force of infection into SIR model [11], an exposed population (*E*) as SEIR model [13], the influence of reinfection force and limited medical resources issues [16] and the effect of vaccination [18].

Research with different methods and approaches on SIR model has been conducted to understand its nature and transmission. Most researchers used Runge-Kutta's fourth and fifth-order and Euler's methods to solve the SIR model. A fractional derivative is a real or complex derivative of any arbitrary order in applied mathematics and mathematical analysis.

Fractional differential equations started to obtain extensive application in developing physical and biological simulation [20]. A fractional derivative extends the derivative and integral of integer order. It helps simulate multi-scale situations with a broader time or length scale. Fractional derivative is powerful fractional operator because of their non-local power-law form, making them suitable for explaining various physical phenomena memory and hereditary aspects.

The effect of SARS-CoV-2 infection on the dynamics of dengue and HIV is interpreted by utilizing the methods of fractional calculus. The model fitting applying the three fractional derivatives was done using real data from Argentina [21]. However, currently there is no study on fractional order derivative of SIR model for COVID-19 disease has been done in Malaysia.

It is important to highlight that the basic mathematical models of integer-order derivatives, as well as nonlinear models, do not fit into acceptable framework in numerous situations [20]. Since the fractional derivative can explain the model better, this study considers SIR fractional-order derivatives on COVID-19 infectious trend simulation in Malaysia based on the study by Ariffin *et al.,* [6]. The fractional derivatives used are in Caputo sense. Caputo derivative is one of the studied fractional derivatives, and Italian Caputo proposed it in 1967. Caputo fractional derivative is the most accessible fractional operator to deal with real-world problems because it allows initial and boundary conditions Atangana [5]. Caputo fractional derivatives and their applications for various epidemic model have been widely used such as SIR [1,2,12,13,15,17].

The Adams Bashforth Moulton approach is then applied to solve differential equations. Applying Adams Bashforth Moulton method to the fractional derivative allows computing the approximate solution. John Couch Adams developed the Adams Bashforth and Adams-Moulton methods to solve differential equation problems. The Adams Bashforth Moulton is obtained from the Adams-Bashforth and Adams Moulton methods. The Adams-Bashforth method is used as the predictor. In contrast, the Adams-Moulton method is used as the corrector in a multistep process for approximating the solution of a differential equation.

One of the main advantages of fractional derivatives are adaptability and non-locality. Since these derivatives are of fractional order, the method is able to simulate with more flexibility than the classical derivatives. Hence, this study focused on solving fractional derivatives by using Adam Bashforth method to simulate COVID-19 in Malaysia.

2. Methodology

2.1 SIR Formulation

A SIR models the theoretical number of people infected with an infectious disease over time in a closed population as presented in the compartment model in Figure 1. The name of this group of models comes from the fact that they involve coupled equations connecting the number of susceptible individuals (S), infected people (I), and recovered people (R).

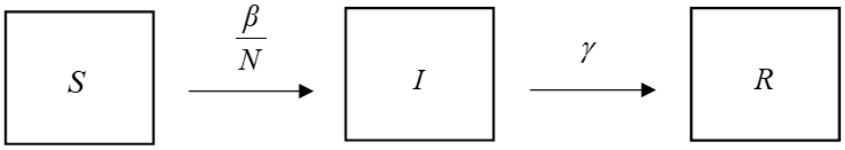

Fig. 1. SIR compartment diagram

The dynamics of the COVID-19 transmission are described using the following nonlinear ordinary differential equations (ODEs):

$$\frac{dS}{dt}=\frac{-\beta IS}{N},\frac{dI}{dt}=\frac{\beta IS}{N}-\gamma I,\frac{dR}{dt}=\gamma I \tag{1}$$

In the model (N,S,I,R) represent the total number of people in the area, the number of susceptible, the number of infected and the number of recovered individuals, respectively. All the variables are functions of time, t . The parameter β is a constant showing infectivity rate and γ is a constant showing recovery rate. Assume N is fixed a long time t , then

$$N=S+I+R \tag{2}$$

The SIR model Eq. (1) can be non-dimensionally by using dimensionless variables, which are

$$S^{*}=\frac{S}{N},\ I^{*}=\frac{I}{N},\text{and}\ R^{*}=\frac{R}{N}, \tag{3}$$

Hence, the dimensionless model of the COVID-19 transmission is then given as

$$\frac{dS^{*}}{dt}=-\beta I*S*,\ \frac{dI^{*}}{dt}=\beta I*S*-\gamma I*,\frac{dR^{*}}{dt}=\gamma I*, \tag{4}$$

where $S^{*},I^{*},$and R^{*} are dimensionless variable for the number of susceptible, the number of infectious individuals and the number of fully recovered individuals, respectively.

2.1.1 Apply fractional Caputo

The general initial value problem for the first order differential equation written as

$$Dy(t) = f(t, y(t)) \text{ and } y(0) = y_o \tag{5}$$

Then, the ordinary derivative model Eq. (4) is replaced by a fractional derivative of the following form (Diethelm *et al.,* [8]):

$$\begin{cases} D_*^{\alpha} y(t) = f(t, y(t)), 0 \le t \le T \\ y^{(k)}(0) = y_o^{(k)}, k = 0,1,...,m-1 \end{cases} \tag{6}$$

where $\alpha > 0$, T is a suitable positive number, and m is a positive integer. $D_*^{\alpha} y(t)$ is the α^{th} order (always fractional) derivative of y.

Consider y be a function with $(m-1)$ continuous integer-order derivatives, then the fractional Caputo derivative of order *a* is defined as [7]:

$$D_*^{\alpha}(y(t)) = \frac{1}{\Gamma(m-\alpha)} \int_0^t (t-\tau)^{m-\alpha-1} y^{(m)}(\Gamma) d\tau, \; m-1 < \alpha < m \in Z^+ \tag{7}$$

where $y^{(m)}$ denotes the derivatives of integer m^{th} order of *y* and in the above integral Γ represents the Gamma function, $\Gamma(y) = \int_0^{\infty} e^{-t} t^{y-1} dt, y \in \mathbb{C}$. Hence, the new system of fractional differential equations (FDEs) for the model:

$$D_t^{\alpha} S(t) = -\beta I^* S^*, \; D_t^{\alpha} I(t) = \beta I^* S^* - \gamma I^*, \; D_t^{\alpha} R(t) = \gamma I^* \tag{8}$$

subject to initial conditions $S^*(0)$, $I^*(0)$, $R^*(0)$.

2.1.2 Solve Fractional using Adams Bashforth-Moulton

Approximation and numerical approaches must be used since fractional differential equations (FDEs) are difficult to solve analytically [14]. Thus, the generalized Adams-Bashforth-Moulton method can be used to solve the system numerically.

The Eq. (6) is equivalent to Volterra integral equation:

$$y(t) = \sum_{k=0}^{m-1} y_o^{(k)} \frac{t^k}{k!} + \frac{1}{\Gamma(\alpha)} \int_0^t (t-u)^{\alpha-1} \quad f(u, y(u))\, du, t \le T \tag{9}$$

For solving the Eq. (6) and Eq. (9), the first study of the fractional Adams method on a uniform grid $\{t_j = jh : j = 0,1,....,N\}$ with some integer *N*, step length $h = \frac{T}{N}$, and $y \approx y(t_j)$ [9]. The following is the specified implicit formula

$$\begin{cases} y_{k+1}^{p} = \sum_{j=0}^{m-1} \frac{t_{k+1}^{j}}{j!} y_o^j + \frac{1}{\Gamma(\alpha)} \sum_{j=0}^{k} b_{j,k+1} f(t_j, y_j), \\ y_{k+1} = \sum_{j=0}^{m-1} \frac{t_{k+1}^{j}}{j!} y_o^j + \frac{1}{\Gamma(\alpha)} \left(\sum_{j=0}^{k} \alpha_{j,k+1} f(t_j, y_j) + \alpha_{k+1,k+1} f(t_{k+1}, y_{k+1}^{p}) \right), \end{cases} \tag{10}$$

where

$$\alpha_{j,k+1} = \frac{h^{\alpha}}{\alpha(\alpha+1)}, \begin{cases} \left(k^{\alpha+1} - (k-\alpha)(k+1)^{\alpha}\right), & \text{if } j=0, \\ \left((k-j+2)^{\alpha+1} + (k-j)^{\alpha+1} - 2(k-j+1)^{\alpha+1}\right), & \text{if } 1 \le j \le k, \\ 1, & \text{if } j=k+1 \end{cases} \tag{11}$$

and

$$b_{j,k+1} = \frac{h^{\alpha}}{\alpha}\left((k-j+2)^{\alpha} - (k-j)^{\alpha}\right), j = 0,1,2,...,k. \tag{12}$$

Therefore, the value $y_n^p(t_{n+1})$ is defined by the fractional Adams-Bashforth method

$$y_n^p(t_{n+1}) = \sum_{k=0}^{[\alpha]-1} \frac{t_{n+1}^{k}}{k!} y_o^{(k)} + \frac{1}{\Gamma(\alpha)} \sum_{j=0}^{n} b_{j,n+1} f\left(t_j, y_n(t_j)\right) \tag{13}$$

with the weights $\alpha_{j,n+1}$ and $b_{j,n+1}$ is defined according to Eq. (11) and Eq. (12), respectively.

2.1.3 Numerical analysis

By using the Adams-Bashforth-Moulton (ABMs) method, the numerical solutions of the system of fractional differential Eq. (8) are formulated as follow.

Considering the initial value problem

$$\begin{aligned} &D^{\alpha} S(t) = f\left(t, S(t)\right), S(0) = S_o, 0 < \alpha < 1, t > 0 \\ &D^{\alpha} I(t) = f\left(t, I(t)\right), I(0) = I_o \\ &D^{\alpha} R(t) = f\left(t, R(t)\right), R(0) = R_o \end{aligned} \tag{14}$$

and

$$\begin{aligned} S_{n+1}^{p} &= S_o + \frac{1}{\Gamma(\alpha)} \sum_{j=0}^{n} b_{j,n+1} \left(-\beta^* I_j^* S_j^*\right), \\ I_{n+1}^{p} &= I_o + \frac{1}{\Gamma(\alpha)} \sum_{j=0}^{n} b_{j,n+1} \left(\beta^* I_j^* S_j^* - \gamma I_j^*\right), \end{aligned} \tag{15}$$

$$R_{n+1}^{p} = Re_{o} + \frac{1}{\Gamma(\alpha)} \sum_{j=0}^{n} b_{j,n+1} \left(\gamma I_{j}^{*} \right),$$

where

$$\alpha_{j,n+1} = \begin{cases} \left(n^{\alpha+1} - (n-\alpha)(n+1)^{\alpha} \right), & \text{if } j = 0, \\ \left((n-j+2)^{\alpha+1} + (n-j)^{\alpha+1} - 2(n-j+2)^{\alpha+1} \right), & \text{if } 1 \le j \le n \\ 1, & \text{if } j = n+1 \end{cases} \tag{16}$$

Findings from the simulation conducted based on the above equations will be elaborated in the next section.

3. Results

This section shows the numerical results for SIR model in Eq. (15) associated with the susceptible, infected, and recovered populations. Parameters values infectivity rate, $\beta = 0.22$ and recovery rate, $\gamma = 0.09$ were collected from Ariffin *et al.,* [6].

3.1 Validation of the Study

The Runge-Kutta build-in method in Matlab@ software is performed to prove and validate the numerical results obtained from Adams Bashforth Moulton simulation method by comparing susceptible, infected, and recovered individuals against time t (days). The comparison is shown in Table 1. Both methods are in good agreement.

Table 1
Numerical table of solving SIR model with $\alpha = 1$

Time (days)	Population					
	Adams Bashforth Moulton			Runge-Kutta		
	Susceptible $S^{*}(t)$	Infectious $I^{*}(t)$	Recovered $R^{*}(t)$	Susceptible $S^{*}(t)$	Infectious $I^{*}(t)$	Recovered $R^{*}(t)$
0	1	0.001	0	1	0.001	0
5	0.998477	0.0018994	0.000623454	0.998452	0.00191409	0.000633659
10	0.995555	0.00362252	0.00182241	0.9955	0.00365497	0.00184508
15	0.990018	0.00687812	0.00410405	0.989899	0.00694749	0.00415305
20	0.979631	0.0129503	0.00841841	0.979386	0.0130931	0.00852129
25	0.960515	0.024005	0.0164795	0.960028	0.024284	0.0166879
30	0.926529	0.0432559	0.0312154	0.925626	0.0437572	0.0316165
35	0.869635	0.0742295	0.057136	0.868134	0.075017	0.0578494
40	0.783249	0.117824	0.0999271	0.781125	0.118821	0.101053
45	0.669372	0.167438	0.16419	0.666954	0.16835	0.165696
50	0.543342	0.208117	0.249541	0.541188	0.208635	0.251177
55	0.426638	0.225818	0.348544	0.425093	0.225951	0.349955
60	0.333298	0.217963	0.449739	0.332312	0.218	0.450688
65	0.265497	0.192431	0.543072	0.264837	0.192628	0.543534
70	0.218633	0.159495	0.622872	0.218096	0.159932	0.622972

3.2 SIR Simulation

The dynamic behaviour of Adams Bashforth Moulton SIR simulation is investigated based on the simulation curves.

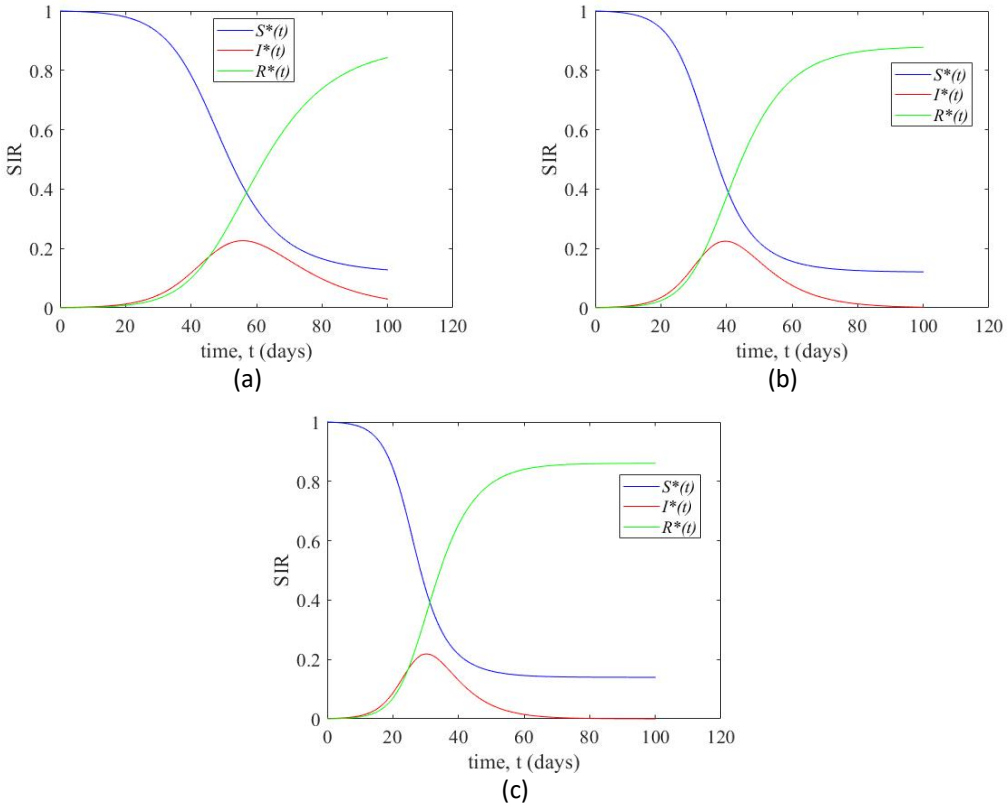

Fig. 2. SIR simulation with fractional derivative order (a) $\alpha = 1$, (b) $\alpha = 0.8$, and (c) $\alpha = 0.6$.

However, Figures 3(b) and 3(c) with α less than 0.5 show that the graph becomes insignificant compared to Ariffin [6] because Figure 2(a), the proportion of Susceptible, $S^*(t)$ decreases until it reaches asymptotic behaviour below 20 percent while Figure 3(c) shows asymptotic behaviour above 80 percent. Thus, the transition into a fractional model makes it particularly sensitive to the order of differentiation, α which is a slight shift may result in a significant difference in the outcome [4].

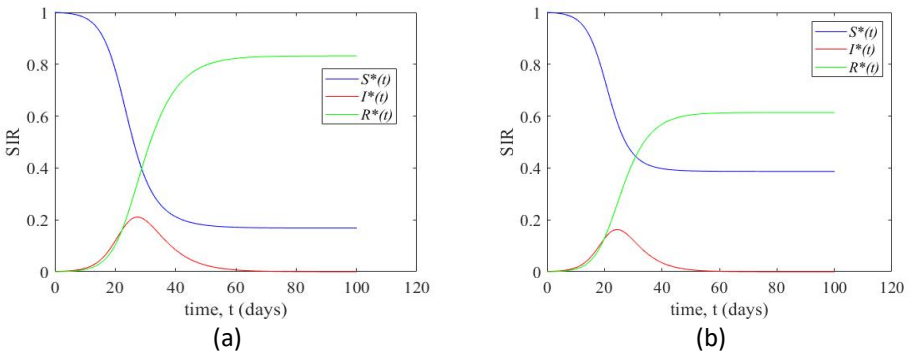

Fig. 3. SIR simulation with fractional derivative order (a) $\alpha = 0.5$, (b) $\alpha = 0.3$, and (c) $\alpha = 0.1$

Figure 4(a) indicates that the duration of time for Susceptible, $S^*(t)$ to reach asymptotic behaviour decreases as the fractional order, α decreases where $\alpha = 1$ is at 100 days, $\alpha = 0.8$ is at 93 days, and $\alpha = 0.6$ is at 79 days. However, Susceptible, $S^*(t)$ of $\alpha = 0.6$ starts to increase above Susceptible, $S^*(t)$ of $\alpha = 0.8$ after 65 days. Figure 4(b) illustrates that by decreasing the fractional order, α, Infected, $I^*(t)$ increases for the first 60 days and afterwards, it drops. The peak cases of Infected, $I^*(t)$ are different for the three different α where $\alpha = 0.6$ is at 30 days, $\alpha = 0.8$ is at 40 days, and $\alpha = 1$ is at 56 days. In contrast to Figure 4(a), Figure 4(c) shows that Recovered, $R^*(t)$ rises when the decrement of fractional order from $\alpha = 1$ to $\alpha = 0.6$. All three α reach more than 80 percent of the population in the number of Recovered, $R^*(t)$.

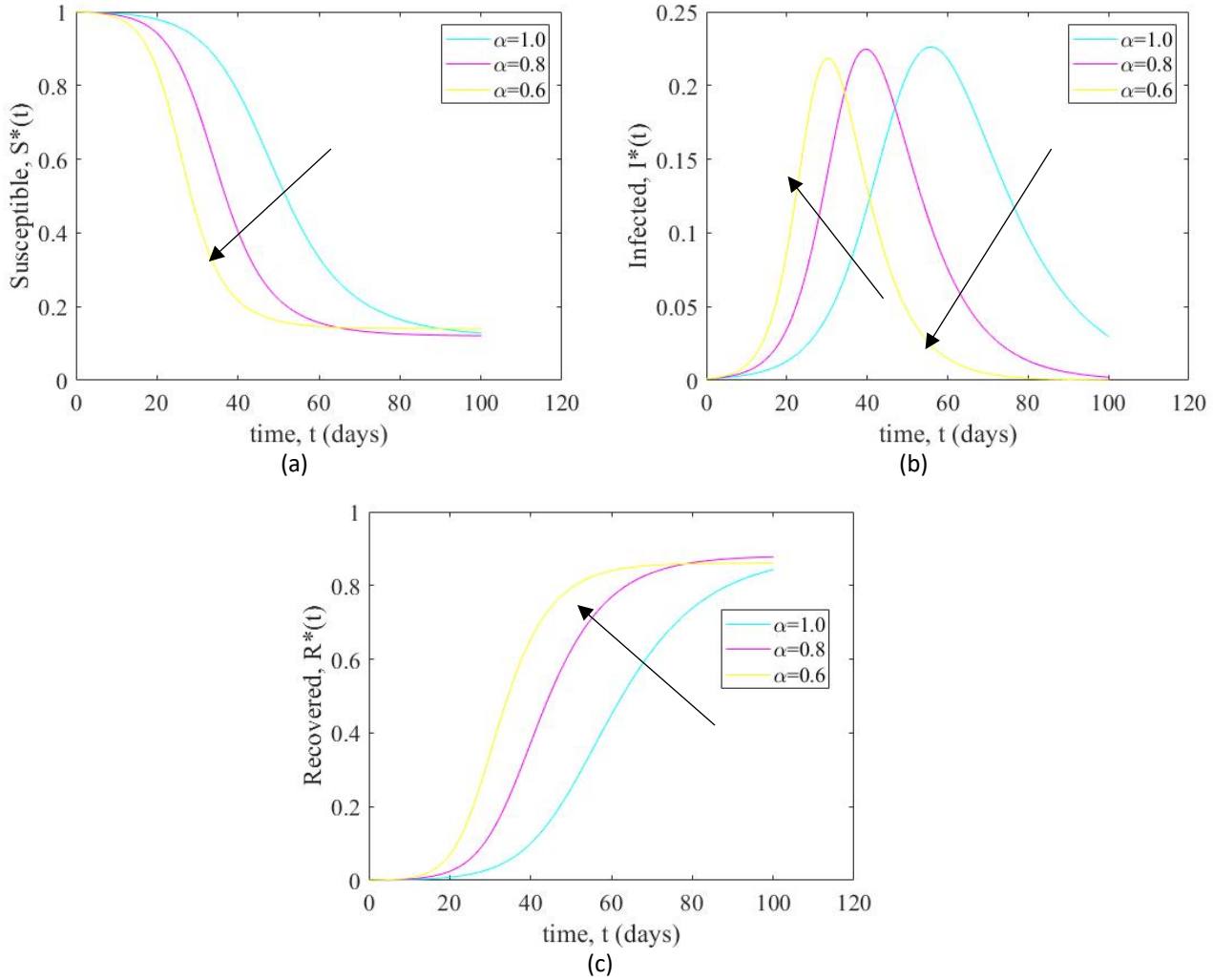

Fig. 4. Numerical solution for (a) Susceptible, $S^*(t)$,(b) Infected, $I^*(t)$, and (c) Recovered, $R^*(t)$ in a time, t (days) at $\alpha = 1, 0.8, 0.6$.

Figure 5(a) shows that the duration of time for Susceptible, $S^*(t)$ to reach asymptotic behaviour decreases as the fractional order, α decreases. However, the Susceptible, $S^*(t)$ asymptotic behaviour increases for $\alpha = 0.5$ is below 20 percent, $\alpha = 0.3$ is 40 percent, and $\alpha = 0.1$ is more than 80 percent when the fractional order, a decreases. Moreover, both Figure 5(b) and Figure 5(c) are opposite to Figure 5(a) as by lowering the fractional order from $\alpha = 0.5$ to $\alpha = 0.1$, the number

of Infected, $I^*(t)$ and Recovered, $R^*(t)$ decrease, respectively. Recovered is less than 20 percent for $\alpha = 0.1$. Nevertheless, it is below 62 percent for $\alpha = 0.3$, and $\alpha = 0.5$ showing at least 80 percent of the Recovered, $R^*(t)$ after 50 days.

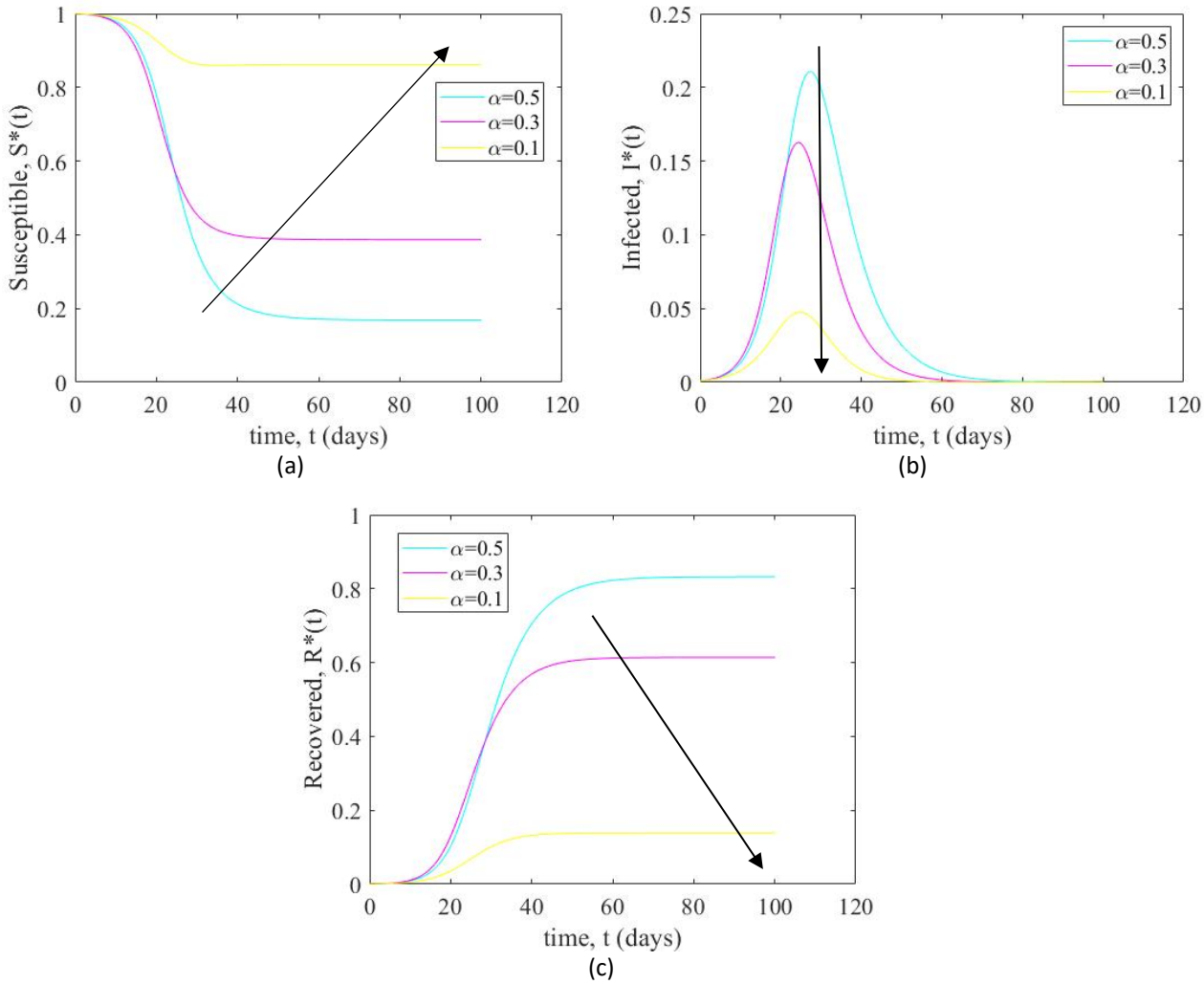

Fig. 5. Numerical solution for (a) Susceptible, $S^*(t)$,(b) Infected, $I^*(t)$, and (c) Recovered, $R^*(t)$ in a time, t (days) at $\alpha = 0.5, 0.3, 0.1$.

Figures 4(a) - 4(c) and Figures 5(a)-5(c) recognize that α values above 0.5 are reasonable and acceptable. The graphs demonstrate that reducing α values reduce the number of Infected, $I^*(t)$ while significantly increasing the number of Recovered, $R^*(t)$. On the other hand, the graphs with $\alpha = 0.5, 0.3$ and 0.1 are unacceptable. It is proven in Figures 5(b), where the graph demonstrates that when $\alpha = 0.1$ is applied, more than 80% of people are expected to become susceptible, and only 10% are recovered from the disease within 40 days to 100 days. Since the percentage of Susceptible, $S^*(t)$ is high and the percentage of Recovered, $R^*(t)$ is low in this circumstance, the graph is unreasonable compared to Figure 2(a).

4. Conclusions

In conclusion, simulated results showed that the Adams Bashforth Moulton method can solve the fractional SIR model. This method is valid and reliable since the result achieved is identical to the validation result obtained using the Runge-Kutta built-in method. Different sets of derivative order, α values are used to determine the most identical fractional derivative value of SIR simulation model for COVID-19 disease in Malaysia. Additionally, it can be shown from the simulation that values more than 0.5 are reliable and acceptable. All the populations reached the asymptotic behaviour in lesser time by decreasing the fractional order derivative values. Future work could be expanded to include more SIR parameters by using the current COVID-19 data.

Acknowledgement

The authors would like to express their gratitude to the College of Computing, Informatics & Media, Universiti Teknologi MARA (UiTM) Shah Alam Selangor Malaysia for their assistance, as well as Research Nexus UiTM (ReNeU), Office of Deputy Vice-Chancellor (Research & Innovation) UiTM for the publication incentive under Program *Pembiayaan Yuran Prosiding Berindeks* (PYPB).

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