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Stability Analysis of Competition Model of iOS and Android

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ABSTRACT

iOS and Android are among the most used phone operating systems nowadays, and their existence and individual performance will heavily affect each other in the short and long term. In order to dominate the market or to engage in a healthy competition where both populations (iOS and Android) can coexist without harming one another, both populations must be aware of one another's performance and respond to it appropriately. Hence, this study aims to analyse the stability of the previous model competition between iOS and Android, which used the Lotka-Volterra model. The steady-state stability was analysed using the Jacobian matrix, and its phase portraits were plotted using the online tool. Besides that, parameter sensitivity analysis was carried out by varying different values for parameters presented in the equations to observe how the differences in parameters affected the dynamics of the relationship, which directly exhibits the effect of individual population performance and how it affects longevity and survival of the other populations in the system. In conclusion, this study has given us a better understanding of the dynamics of competition between iOS and Android in the current market.

1. Introduction

Smartphone sales have increased dramatically during the previous ten years. Most people's lives now include smartphones daily. These smartphone technologies allow users to do more than converse; they can also perform daily tasks like online banking, live map navigation for driving, online company promotion, supermarket shopping, and online restaurant ordering. These are accomplished at the tip of our fingers with only a few clicks. In essence, smartphones are little computers that fit in our hands. The smartphone's operating system has elevated to the position of being crucial. Now, two operating systems are dominating the market: iOS and Android. It will rely on several variables as to whether these two businesses will succeed in the future and whether they will capture the majority of the market stated by Goadrich and Rogers [1]. Which of these businesses, iOS or Android,

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has a better understanding of the consumer trends that fuel success in the mobile phone industry will be the most crucial.

In comparison to other African, Latin American, and Southeast Asian nations, only 3% of smartphone users in India are iPhone users, according to Statista [2]. Despite having 10 to 20% market shares in developing nations like Brazil and China, iOS's market share in its two biggest markets—the United States and Japan—is only about 60% due to its one-brand, one-operating-system strategy and the premium character of its products. Android currently holds the global market share lead for mobile operating systems. According to the Statista [2] research, Android held the top spot on the market share chart with 71.7 per cent as of March 2022. The iOS platform, on the other hand, is developing swiftly and is in a position to dominate a sizable market share. According to data from a study conducted in 2022 conducted by United Kingdom Competition and Markets Authority, Gilbert + Tobin Lawyers [3], users' purchasing decisions were impacted by the following elements: pricing, an easy-to-use operating system, a smartphone's design, camera, and battery life.

It is crucial to remember that the individual performances of iOS and Android may or may not impact one another Schreieck *et al.,* [4]. Hence, the dynamic interaction between the two operating systems can be used to ascertain this. This study aims to find the equilibrium points and investigate the stability analysis of the model competition of iOS and Android. In addition, parameter analysis sensitivity to the model was conducted. Hence, a better understanding of the relationship dynamics between iOS and Android. The system's behaviour may be evaluated by assessing the model's stability and whether intraspecific predation is helpful or harmful to iOS or Android in the market industry eventually. This is vital as it may help both companies to coexist, maximising profit and minimising loss due to miscalculations while maintaining a healthy dynamic relationship and harmless competition.

2. Competition of iOS and Android

A smartphone is a mobile phone with a computational power comparable to a personal computer. Today, smartphones are used worldwide by people of different ages and professions. Since 2008, smartphone operating systems have shown rapid change, and now the two primary operating systems that hold the market share are iOS and Android stated by Sheikh *et al.,* [5] and Remneland-Wikhamn *et al.,* [6]. It is interesting to model the relationship dynamics between these two major operating system. Moreover, it will be a great insight for students to see the application of the theoretical mathematics in economics for example article by Mateas [7].

Tseng *et al.,* [8] used an integrated model to anticipate sales volume to study the competitiveness between the currently available operating systems. Both the Delphi method and the co-diffusion model were employed. While, Bass Model and Modified Lotka-Volterra were utilised by Gündüç [9]. In his investigation, he concluded that extended Bass-type equations are better suited to modelling the rivalry between the iOS and Android operating systems. He also concluded that, the presence of one competitor aids the growth of the other's market share. Another study by Amelkin *et al.,* [10] employed a network model to describe the evolution of opinions of a group of people regarding their choice of smartphone operating systems, iOS or Android. Some researchers used the diffusion model to quantitatively analyse competing for smartphone operating systems' diffusion and co-diffusion effects. For example, Park and Ueda [11] did a comparative study on the diffusion of smartphones in Korea and Japan, and Lee and Lee [12] proposed the Gompertz model in examining the effects of smartphone platforms competition in OECD and BRICS countries.

This study focuses on the prey-predator equilibria in which the prey is represented by iOS, and Android OS represents the predator. The system's behaviour may be evaluated by assessing the model's stability and whether intraspecific predation is helpful or harmful to iOS or Android OS in the market industry. This is vital as it may help both companies to coexist, maximising profit and minimising loss due to miscalculations while maintaining a healthy dynamic relationship and harmless competition.

2.1 Modelling Competition of iOS and Android Using Lotka-Volterra Model

In 1910, Alfred J. Lotka proposed this Lotka-Volterra Model for the theory of autocatalytic chemical reactions. Later, this model was then extended to represent predator-prey interactions. However, now this Lotka-Volterra model has been applied in many applications. For example, Wu *et al.,* [13] proposed a model competitive relationship in the vehicle market in China, Windarto and Eridani [14] studied model on motorcycle manufacturers in Indonesia, and Xu [15] suggested a model on competitive intensity among major European container ports. Lotka-Volterra model is not only focused on two-species interaction but also 3 or 4 interactions. For example, Zhang and Huang [16] proposed a four-way interaction of relationship and competition among technological innovation, resource consumption, environmental quality, and industrial development quality centred around the industrial development of Shaanxi Province. A study by Nikolaieva and Bochko [17] has modelled the forecasting share market of the mobile operating system using a modified Lotka-Volterra equation, a system of an autonomous differential equation. They fitted the actual data of the share market, Statista and Statcounter, to the modified model. Also, they included the model of three interactions between mobile operating systems: iOS, Android and Microsoft. Wang *et al.,* [18] and Ngo [19] also modelled the competition of iOS-Android using modified Lotka-Volterra and stability analysis of the model was also done.

The model by Wang and Wang [20] was used for this study. It is decided due to various reasons. First, Wang and Wang [20] used a simple Lotka-Volterra model, which is much more reliable and easier to interpret. Moreover, Wang and Wang [20] has also fitted this model with the actual data. Therefore, the parameter values of the model have been determined. The representation of the competition between iOS and Android is modelled as below

$$\frac{dx}{dt} = x(a_1 - b_1 x - c_1 y) \tag{1}$$

$$\frac{dy}{dt} = y(a_2 - b_2 y - c_2 x) \tag{2}$$

This concept of the Lotka Volterra competition model in Eq. (1) and Eq. (2) refers to $x(t)$ and $y(t)$, which refer to the population of the prey (iOS) at time t and the population of the predator (Android) at time t, respectively. After thorough discussion and observation, the assumption behind this statement is that Android's growth strength is greater than that of iOS in the global data of market share as in Statista and Statcounter. The interaction of various species is referred to by the terms xy or yx. x^2 and y^2 represent the interactions that take place within the populations themselves. The first parameter, a_i, is the species' capacity for expansion. The second parameter, b_i, is the niche capacity limiting parameter for species i, while the third parameter, c_i, indicates the competition rate between the two species. The parameter values are as in Table 1. This work focuses on the stability of hypothetical prey-predator equilibria in which iOS serves as the prey, and Android OS serves as the predator. The latter is described in the following part.

Table 1
Parameter values of Eq. (1) and Eq. (2) from Wang and Wang [20]

Parameter	Value
a_1	0.424208569
a_2	0.393612619
b_1	0.0000001922
b_2	0.000001962
c_1	0.000001395
c_2	0.0000003267

3. Critical Points and Stability Analysis of the Model

3.1 Critical Points and its Stability

Let Eq. (1) and Eq. (2) set to be 0, which will lead to critical points, also known as steady-state points. Therefore, the first critical point obtained is (0,0). For when $x = 0$ in $a_2 - b_2 y - c_2 x = 0$, it will yield $y = \frac{a_2}{b_2}$. Hence, the second critical point obtained is $(0, \frac{a_2}{b_2})$. For when $y = 0$ in $a_1 - b_1 x - c_1 y = 0$, it will yield $x = \frac{a_1}{b_1}$. Hence, the third critical point obtained is $\left(\frac{a_1}{b_1}, 0\right)$. For the fourth critical point, a simultaneous calculation is needed. Hence, the last critical point is obtained, which is $\left(\frac{a_1}{b_1} - \frac{c_1}{b_1}\left[\frac{a_1c_2 - a_2b_1}{c_1c_2 - b_1b_2}\right], \frac{a_1c_2 - a_2b_1}{c_1c_2 - b_1b_2}\right)$.

To summarise, the steady states for this system are as follows

$$(0,0), \left(0, \frac{a_2}{b_2}\right), \left(\frac{a_1}{b_1}, 0\right), \left(\frac{a_1}{b_1} - \frac{c_1}{b_1}\left[\frac{a_1c_2 - a_2b_1}{c_1c_2 - b_1b_2}\right], \frac{a_1c_2 - a_2b_1}{c_1c_2 - b_1b_2}\right) \tag{3}$$

Next, by substituting the values as in Table 1 into Eq. (3), the actual critical points are (0,0), (0,200566.94), (2207120.547,0) and (3607960.285,801188.914610).

In order to find the phase portraits and their stability analysis, the eigenvalues must first need to be identified by the application of the Jacobian matrix. Hence, the Jacobian matrix for the system of Eq. (1) and Eq. (2) is as shown below

$$J = \begin{pmatrix} 0.4242085691 - 3.844\times10^{-7}x - 0.000001395y & -0.000001395x \\ -0.0000003267y & 0.3936126198 - 3.925\times10^{-6}y - 0.0000003267x \end{pmatrix}$$

Now that the system's Jacobian matrix is obtained, the eigenvalues $\lambda_1\ \lambda_2$ can be found.

(a) For the critical point (0,0),

$$J(0,0) = \begin{pmatrix} 0.4242085691 & 0 \\ 0 & 0.3936126198 \end{pmatrix}$$

Hence, its characteristic equation would be

$$\begin{vmatrix} 0.4242085691 - \lambda & 0 \\ 0 & 0.3936126198 - \lambda \end{vmatrix} = 0$$

Therefore, the eigenvalues are $\lambda_1 = 0.4242085691$, $\lambda_2 = 0.3936126198$ (real, same positive sign). It is concluded that the critical point (0,0) is an unstable node.

(b) For the critical point (0,2.005669400x10^5),

$$J(0, 2.005669400 \times 10^5) = \begin{pmatrix} 0.144417687 & 0 \\ -0.065525219 & -0.393612619 \end{pmatrix}$$

Therefore, the eigenvalues are $\lambda_1 = 0.144417687, \lambda_2 = -0.3936126198$ (real, opposite sign). The critical point (0,2.005669400x10^5) is an unstable saddle point.

(c) For the critical point $(2.207120547x10^6, 0)$ its Jacobian matrix and characteristic equation are as follows

$$J(2.207120547 \times 10^6, 0) = \begin{pmatrix} -0.424208569 & -3.078933163 \\ 0 & -0.327453662 \end{pmatrix}$$

Therefore, the eigenvalues are $\lambda_1 = -0.424208569, \lambda_2 = -0.327453662$ (real, same negative sign). The critical point $\left(2.207120547x10^6,\ 0\right)$ is stable.

(d) Last but not least, for the critical point $(-3.607960285x10^6,\ 8.01188914610x10^5)$,

$$J(-3.607960285 \times 10^6, 8.01188914610 \times 10^5) = \begin{pmatrix} 0.6934622549 & 5.033104598 \\ -0.2617484184 & -1.572333245 \end{pmatrix}$$

After simplifying the equation above, it is observed that this critical point produced complex values which exist in the form of $r_1, r_2 = \lambda \pm \mu i$. Hence, the eigenvalues, r_1 and r_2 for this point would be $-0.4394354951 \pm 0.1842548661i$, where its actual λ has the value of -0.4394354951. Since it is less than 0, it is concluded that the critical point $(-3.607960285x10^6,\ 8.01188914610x10^5)$ is asymptotically stable.

3.2 Phase Portraits of the Critical Points

The phase portraits of the determined critical points above were plotted using online tool Wolfram Alpha. Figure 1 shows phase diagrams according to types of critical points. In Figure 1(a), the arrows are relocating away from the critical point, as seen from the plane. This suggests that the point is unstable. The Jacobian matrix and characteristics equation can be manually calculated to confirm this. Based on our calculations, it is found that λ_1 and λ_2 are to be the values of a_1 and a_2 which are 0.4242085691 and 0.3936126198, respectively. Since they are both real numbers and have the same positive signs, this critical point is determined to be an unstable node. In Figure 1(b), the arrows are veering away from the crucial point, as seen in the graph above. More than that, their movements resemble the shape of a saddle. Hence, this critical point can be concluded as an unstable saddle. Similar to the previous critical point, this can also be manually calculated and it is found that the λ_1 and λ_2 are to be 0.144417687 and -0.3936126198 respectively. It is obvious that they have different signs and are real numbers. As a result, this provided strong evidence that the point is an unstable saddle point. Next, in Figure 1(c), the arrows swirling and going into the critical point can be observed at this critical point. This means that the point is a degenerate sink, and it can be concluded that it is indeed a stable point. By manual calculation, the λ_1 and λ_2 obtained are -0.424208569 and -0.327453662, respectively, showing that they have real values and are of the same negative sign. This increases confidence that Wolfram Alpha's results are accurate because they agree. Last but not least, in Figure 1(d), the phase plane of our fourth critical point is depicted in the image above. Several arrows going into and near the point can be observed. Hence, this shows that it is a stable point. To confirm this, the manual calculation shows that this point produced a complex value, which is $-0.439435495 \pm 0.1842548661i$. Since the value of λ is -0.4394354951, which is less than 0, it shows that the point is asymptotically stable.

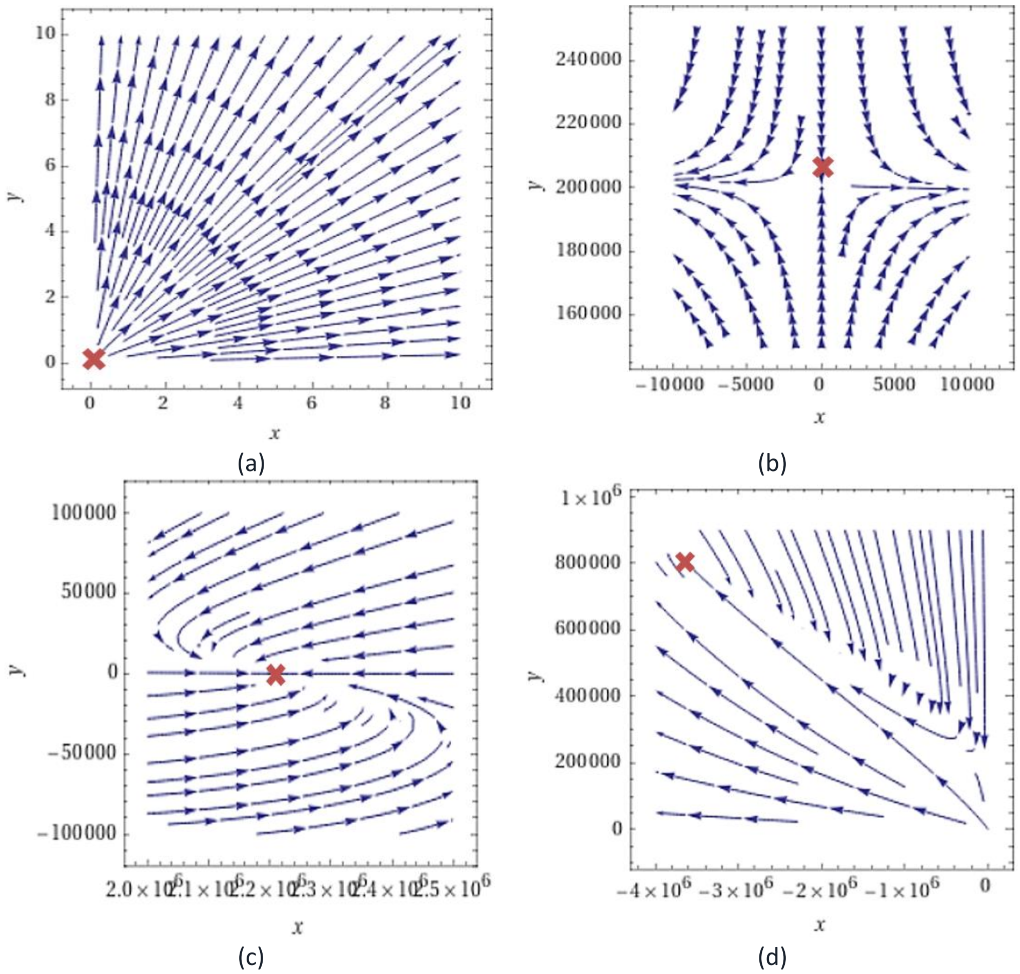

Fig. 1. a) Phase portrait at (0,0), (b) Phase portrait at (0, 2.005669400 x 10^5), (c) Phase portrait at (2.207120547 x 10^6,0), (d) Phase portrait at (-3.607960285 x 10^5, 8.011889146 x 10^5)

4. Parameter Sensitivity Analysis

In this section, the value and signs of the parameter c_i are altered. This is done to study how different types of relationships between Android and iOS. From the graphs shown below (Figure 2 to Figure 6), the quarter-year label that starts at Q0 represents quarter two for the year 2010, Q2 represents quarter four for the same year, Q4 represents the year 2011's quarter two and so on. It will eventually end in Q12, representing quarter two of 2013.

(a) c_1 and c_2 are both negatives (pure competition)

For this type of relationship, the parameters c_1 and c_2 were set to be the values of -0.000001395 and -0.0000003267, respectively. Figure 2 shows that $c_1 = -0.0000001395 < 0$ and $c_2 = -0.0000003267 < 0$, which implies a pure competition relationship between iOS and Android. This conclusion appears plausible given that both iOS and Android companies aim for the same market segment.

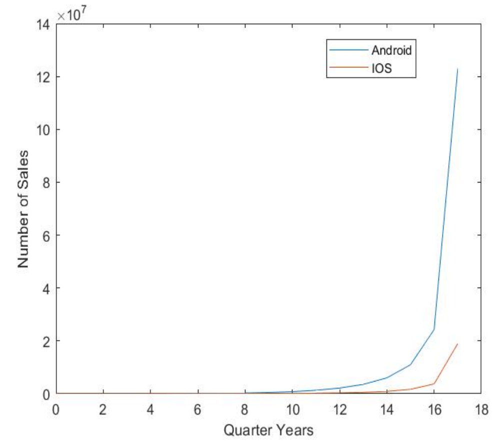

Fig. 2. Graph c_1 and c_2 are both negative

(b) c_1 is positive and c_2 is negative (predator-prey)

For this type of relationship, the parameters c_1 and c_2 were set to be the values of 0.000001395 and -0.0000003267, respectively. Figure 3 shows that $c_1 = 0.000001395 > 0$ and $c_1 = -0.0000003267 > 0$, which implies a predator-prey between iOS and Android. It is because the existence of iOS has a favourable influence on the growth of Android sales, while the growth of Android sales has a negative impact on the growth of iOS sales.

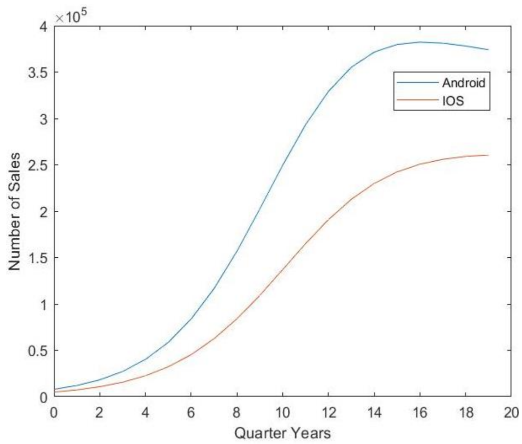

Fig. 3. Graph when c_1 is positive and c_2 is negative

(c) c_1 and c_2 are both positive (mutualism)

For this type of relationship, the parameters were set to 0.000001395 and 0.0000003267 for c_1 and c_2 respectively. Hence, the graph is obtained as in Figure 4. A mutualism relationship between iOS and Android is implied by the graph above, which shows $c_1 = 0.000001395 > 0$ and $c_2 = 0.0000003267 > 0$. This is feasible, considering iOS relies on Apple Company's production capability, and iOS orders are a significant source of income for Apple Company. The Apple Company's fairytale sales growth pushed them to expand their capacity. The same is true for Android, whose income is determined by its company's capabilities.

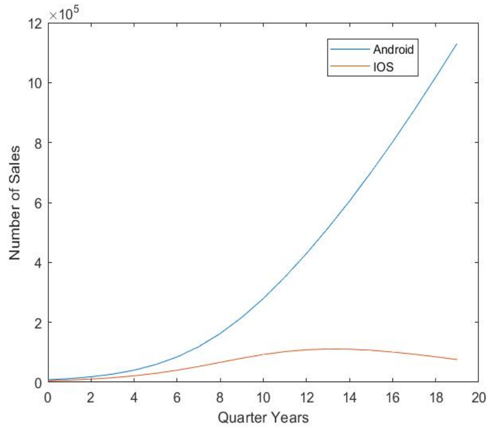

Fig. 4. Graph c_1 and c_2 are both positive

(d) c_1 is positive and c_2 is zero (amensalism)

For this type of relationship, the parameters were set to 0.000001395 and c_2 = 0 (see Figure 5). Other than that, when c_1 = 0.000001395 > 0 and c_2 = 0, which implies an amensalism relationship between iOS and Android. Android can generally be faster and smoother than iOS, then many will be interested in buying Android over iOS.

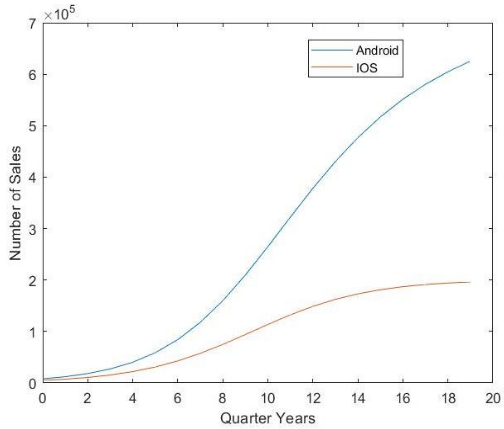

Fig. 5. Graph when c_1 is positive and c_2 is zero

(e) c_1 is negative and c_2 is zero (commensalism)

In Figure 6, for this type of relationship, the parameters were set to the values of -0.000001395 and c_2 = 0. This graph (Figure 6) shows when c_1 = -0.00000139 < 0 and c_2 = 0, which implies a commensal relationship between iOS and Android. This parasitic relationship occurs when one party takes the technology created by the other to create more sophisticated technology. This can be used as an example as Android imitates the iOS interface to make its system more sophisticated. Here it can be seen that one party becomes a parasite to another without affecting it.

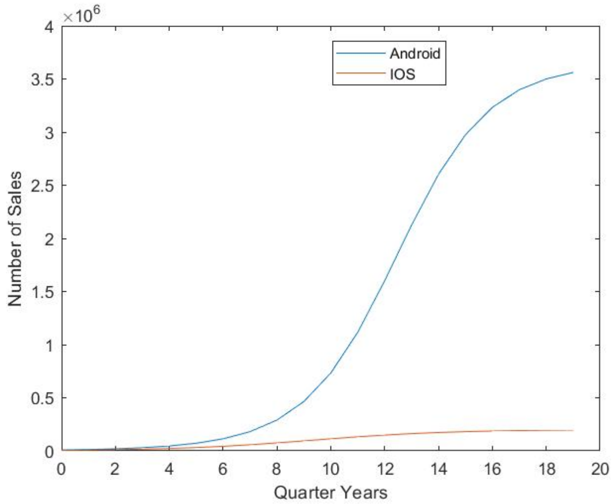

Fig. 6. Graph for when c_1 is negative and c_2 is zero

5. Conclusions

This study was conducted to study the dynamic relationship between Android and iOS using the Lotka-Volterra model, a work done by Wang and Wang [20] After thorough discussion and observation, it is shown that in this system, Android acts as a predator, and iOS acts as prey since Android's growth strength is greater than that of iOS. According to Wang and Wang [20], there are several reasons for this, including the smartphone's open platform, which permits all mobile phone manufacturers to use the Android system, which is well-liked by most users, and because Android is more customisable than iOS. Next, it is shown that iOS is still struggling to dominate the market if its performance is to be compared to Android's. In the case of this study, the production levels of Apple and Android significantly impact one another and the other products that their respective companies produce. Moreover, for this system, it is evident that iOS serves as prey for Android. These decisions must be made in accordance with and in response to the gathered data and figures. Wise and strategic plans need to be constructed and carried out by the top management level of the company in order to ensure the company's longevity in the market and be in a healthy relationship in the long run. Other than that, it is also crucial for them to act accordingly to dominate and hold a significant market share. In general, the Lotka Volterra model is a beneficial method for companies to evaluate their performance in the market and carry out strategic measures to help their company grow and generate more sales.

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