



## Hospital Bed Allocation using Three-Stage Weighted Optimization Method for Government Hospital in Pulau Pinang

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### ABSTRACT

Ineffective bed allocation across hospital departments leads to the imbalance between patients' needs and resource capacity. This study aims to simulate patients' arrivals, to measure the departments' importance and to design a mathematical model for allocating beds which will be realized by the proposed three-stage weighted optimization model. The stages consist of data simulation in ARENA software, weight evaluation based on bed occupancy rate patients' arrival rates, bed occupancy rate (BOR), patients' average length of stay (ALOS) and bed operation cost (BOC), and weighted optimization using goal programming (GP) model. The goals to be achieved in this study consists of minimizing the idle beds in departments and the hospital and minimizing the total BOC. The result of the study shows that obstetrics and orthopaedics wards obtained the biggest number of beds while dengue and paediatrics surgery wards obtained the least number. The statistical analysis made to the results shows that patients' arrival rate is the most influential factor in allocating the beds as its Pearson correlation value to the bed numbers is 0.789, which indicates strong correlation.

## 1. Introduction

Government hospitals are government-funded through the general taxation paid by the citizens [1]. Malaysian government hospital is relatively cheaper than private hospitals thus its services is critically important for Malaysian citizens – especially for the low income-earners. Ironically, the common issues that were addressed by patients were the long waiting time between patients' arrival to the hospital and admission to the ward [2]. This issue is highly connected to the availability of beds. Public hospitals are frequently associated with limited bed capacity. One of the reasons is the ineffective existing bed allocation system. Thus, the management team needs to improve the bed allocation system or expand the available capacity [3].

The study only covers regular wards and excludes the critical care wards such as intensive care unit (ICU). Another scope covered is the importance of each department which is measured based on the BOR, patients' ALOS, patients' arrival rates, and departments' BOC. The limitation for this

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study is the scarcity of the previous study regarding hospital bed allocation system in Malaysia. Besides that, there are limitations as to how much data from government hospitals can be disclosed to an outsider as it is closely related to the issues of confidentiality.

## 2. Literature Review

### 2.1 Hospital Bed Allocation

Hospital bed allocation is one of the processes in hospital bed management. According to Kao and Tung [4], managing hospital bed allocation involves determining the quantity of resources required to cater the yearly amount of patient and distributing the remaining resources based on marginal analysis while considering the monthly demand fluctuations. Length of stay (LOS) and bed occupancy rate (BOR) are the fundamental information that need to be secured before planning the bed allocation [3]. BOR is an indicator for the utilization level of a department. Higher BOR will increase the level of discharge and readmission, which means it offers less effective treatment thus the departments are critical for a change [5].

### 2.2 Mathematical Models and Method for Optimization

Various mathematical models can be applied when it comes to optimizing resources and minimizing costs. Staff allocation and scheduling was done using binary programming [6], integer programming [7] and hybrid binary integer GP [8]. GP was also applied for assigning inputs to utilize existing resources [9]. The operation cost related to the available resources can be minimized by using integer linear programming [10].

In addition, the weighted optimization method is among the most convenient and well-known to researchers. The importance of each objective was indicated by adding weights, thus converting different objective parts into single objective [11]. Instead of the decision variables, weighted optimization chose to optimize the objective based on their weight [12]. Computing the weight values for each objective is crucial in weighted optimization. The inability to quantitatively weight the objectives will hinder from measuring their importance [11].

### 2.3 Goal Programming Model

Among the reasons GP is widely used for decision making is it is an extension of the linear programming method that reduces complexity while dealing with more than one criterion [13]. The concept of goal appears more realistic to the real-world problems rather than optimization concept since corporations mainly set goals in terms of satisfaction criteria to be fulfilled [14]. The general mathematical equation for GP is as follows:

$$\min \sum_{i=1}^n |f_i(X) - g_i| \quad (1)$$

$$s. t. X \in F \quad (2)$$

where  $f_i(X)$  commonly denotes the linear function of the  $i$ -th goal and  $g_i$  represents its aspiration level [15].

GP models have been utilized widely for encountering hospital management issues. There are many studies that applied GP model for bed allocation worldwide such as in Iran by Mehrolhasani *et al.*, [16] and Ataollahi *et al.*, [17], in Indonesia by Sitepu *et al.*, [18], and in China by Li *et al.*, [19]. GP

also proven to solve other management problems such as the allocation of human resources [20], scheduling surgeries [21] and scheduling elective surgeries [22], and evaluating hospital service performance [14].

### 3. Methodology

#### 3.1 Data Collection and Analysis

This study was conducted using the data obtained from Seberang Jaya Hospital 2018 Annual Report that was accessible through Penang State Health Department website [23]. The sample included the distribution of inpatients admitted in the regular wards. The data collected at the hospital were analyzed by performing data cleaning, converting into suitable formats, and computing useful information.

#### 3.2 Three-Stage Optimization

This study proposed a three-stage optimization model similar to Luo et al., [3]. The previous study implemented analysis-simulation-optimization framework while this study applied simulation-weightage evaluation-optimization framework.

##### 3.2.1 Data simulation

Using the collected and analyzed information, a simulation model was built on ARENA software. To operate the simulation model, the percentage of admissions from each department, ALOS, number of resources in each department and the flow of patients' admissions is applied into the software. This stage is for understanding the flow of the patients' stay in the wards according to departments, from admission until discharge.

##### 3.2.2 Weightage evaluation

The weightage of each hospital department is measured according to their BOR, ALOS, patients' arrival rates and BOC. According to Offiong *et al.*, [24], weightage of elements in a set can be sum to one. Therefore, the weight of a department can be computed by equation suggested by Lal *et al.*, [25] as follows.

$$W_i = \frac{V_i}{\sum_{i=1}^n V_i} \quad (3)$$

where  $W_i$  is the weightage and  $V_i$  is the weight value for each department  $i$ . Thus, the weight values according to the four elements, can be calculated based on the equation (4).

$$W_i^C = \frac{C_i}{\sum_{i=1}^4 C_i} \quad (4)$$

$W_i^C$  is the weight value of a department  $i$  based on criteria  $C$ . Meanwhile,  $C_i$  is the value of criteria  $C$  for department  $i$ . when all the weight value of department  $i$  according to all criteria have been computed, the overall weight value of all departments can be obtained using the following equation.

### 3.2.3 Weighted optimization

There are two goals to be achieved in this study which are minimizing the idle beds and minimizing the operation cost. The model that is used to reach that purpose is a weighted GP model. GP model formulation consists of objective function, and the goal constraints. The optimization model referred for constructing the model in this study are Ataollahi *et al.*, [17] for the first goal and Sitepu *et al.*, [18] for the second goal. The following model is the combination of the two mentioned optimization models.

#### i. Model Formulations

The objective function for the model

$$\text{Min } Z = \sum(W_i^- d_{Yi}^- + W_i^+ d_{Yi}^+) + d_{Yh}^+ + (d_{Ch}^- + d_{Ch}^+) \quad (6)$$

Subject to

$$X_i + d_{Yi}^- - d_{Yi}^+ = Y_i \quad (7)$$

$$\sum X_i + d_{Yh}^- - d_{Yh}^+ = Y_h \quad (8)$$

$$\sum C_i X_i + d_{Ch}^- - d_{Ch}^+ = C_h \quad (9)$$

$$X_i, d_{Yh}^-, d_{Yh}^+, d_{Yi}^-, d_{Yi}^+, d_{Ch}^-, d_{Ch}^+ \geq 0, X_i \text{ integer} \quad (10)$$

#### ii. Model Description

The objective function is as stated in equations (6). Constraint (7) indicated the total number of beds in each department must be equal to their targeted number of beds. Constraint (8) is for restricting the total number of beds in all departments to the total number of beds exists in the hospital. Constraint (9) stated that the total BOC from each department must be within the budget allocated by the government. Lastly, Constraint (10) is the non-negative constraint of that denotes the decision variables of the model should not be less than zero and that the number of beds is an integer. The data collected at the beginning of the study are applied to the model formulations. Excel Solver is used to generate the solution. The solution can be interpreted using the report generated by Excel at the end of the optimization calculation.

### 3.3 Result Analysis

After generating the result through Excel Solver, some analyses are made. The analysis focuses on interpreting the relationship between the weightage and the deviation of the solution suggested. It also looked through the factors that have the most influence among the patients' arrival rates, BOR, ALOS and BOC. The analysis is performed using analytical tool in SPSS, one of the most common software for statistical analysis as it is relatively easier and practical to use [26].

## 4. Results

### 4.1 Three-stage Optimization

The optimization process involves 3 stages that have been explained earlier in the method part. This part discusses the implementation of the 3 stages and the result obtained.

#### 4.1.1 Data simulation

The selected information that was extracted from the annual report earlier are applied to the simulation in ARENA Software. The simulation is performed to simulate the arrival of patients and their admission flows into the ward. The simulation was performed in a 30-day period and involved 8766 entities of patients. Table 1 below shows the result of the simulation.

According to Table 1, male general medicine ward, obstetrics ward and female general medicine ward are the three wards with the highest number of untreated patients. Meanwhile, dengue ward, gynaecology ward and paediatrics surgery wards have the smallest number of untreated patients.

**Table 1**

The number of patients admitted and discharged

Departments	Admitted	Discharged	Difference
General Medicine (Male)	1295	1277	18
General Medicine (Female)	789	778	11
General Medicine (Dengue)	119	118	1
Gynaecology	499	498	1
Obstetrics	1526	1512	14
Orthopaedics	1030	1022	8
Paediatrics (Infant)	537	528	9
Paediatrics (Kids)	757	748	9
Surgery (Paediatrics)	583	581	2
Surgery (Male)	850	844	6
Surgery (Female)	781	776	5
Total	8766	8682	84

#### 4.1.2 Weight evaluation

The weightage of each department is evaluated based on four criteria which are the patients' arrival rates, BOR, ALOS and BOC. The weight value for each department is computed using the data collected, and equation (4) and (5). The weight values computed are as shown in Table 2.

Based on the weight values, Eq. (3) is implemented to compute the weightage of each department  $i$  as compared to other departments. The weightage can be separated into three categories to differentiate its priorities. The categories of priorities are high priority, medium priority and low priority. The weight of underachievement and overachievement for each department can be determined based on their category. To show the different ranges, the values can be categorized as stated in Table 3.

**Table 2**  
 Overall departments' weight value based on the 4 criteria

Department	Weight (Arrival)	Weight (BOR)	Weight (ALOS)	Weight (BOC)	Weight value	Weightage
General Medicine (Male)	0.1453	0.1254	0.1059	0.0983	0.4749	0.1128
General Medicine (Female)	0.0852	0.1246	0.1130	0.0983	0.4211	0.1000
General Medicine (Dengue)	0.0121	0.1104	0.0932	0.0983	0.3140	0.0745
Gynaecology	0.0578	0.0557	0.1048	0.1291	0.3474	0.0825
Obstetrics	0.1786	0.0771	0.1048	0.1291	0.4896	0.1162
Orthopaedics	0.1190	0.1259	0.1167	0.0626	0.4241	0.1007
Paediatrics (Infant)	0.0625	0.0846	0.1048	0.0983	0.3503	0.0832
Paediatrics (Kids)	0.0940	0.0959	0.1048	0.0983	0.3930	0.0933
Surgery (Paediatrics)	0.0632	0.0845	0.0599	0.0626	0.2701	0.0641
Surgery (Male)	0.0916	0.1159	0.0921	0.0626	0.3621	0.0860
Surgery (Female)	0.0908	0.1117	0.1003	0.0626	0.3653	0.0867
	1	1	1	1		1

**Table 3**  
 Ranges of weightage and their categories

Weightage Range	Category	Underachievement Weight	Overachievement Weight
Greater than 0.1	High priority	2	0
0.08 - 0.1	Medium priority	2	0.5
0.06 - 0.08	Low priority	1	0

#### 4.1.3 Weighted optimization

After the second stage of the optimization framework has been completed, now it is time for the implementation of the GP model. All the information that has been collected and computed is used in the model. The equations are then applied to Excel Solver to generate the optimal solution. The results generated are as shown in Table 4.

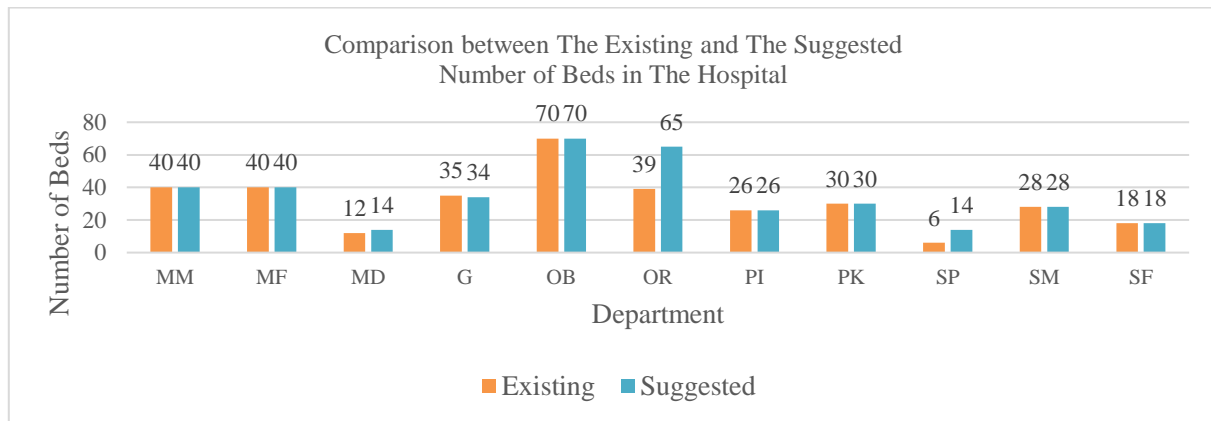
Based on Table 4, the obstetrics department secured the highest number of beds which is 70, followed by orthopedics department with 65 beds. Paediatrics surgery ward and dengue ward got the least number of beds which is 14. It is followed by female surgery ward with 18 beds.

**Table 4**  
 Number of beds suggested by excel solver

Department	Number of Beds
General Medicine (Male)	40
General Medicine (Female)	40
General Medicine (Dengue)	14
Gynaecology	34
Obstetrics	70
Orthopaedics	65
Paediatrics (Infant)	26
Paediatrics (Kids)	30
Surgery (Paediatrics)	14
Surgery (Male)	28
Surgery (Female)	18
Total	379

#### 4.2 Result Analysis

There are some differences between the existing and the suggested number of hospital beds. Figure 1 shows the comparison between the actual and the suggested number of beds for each department.



**Fig. 1.** Comparison between the actual and suggested number of beds

From the result generated, only four departments experience changes in the number of beds as compared to the initial number. However, it is not unexpected since previous studies such as Mehrolhasani *et al.*, [16] also experienced some unchanged numbers in terms of the bed reallocation. Based on the number of beds suggested by Excel Solver, the total number of hospital beds obtained was 379, which is 10.17% higher than the actual target of total 344 beds. However, with that number of beds, the total cost obtained was exactly equal to the budget allocated by the government, which is RM44695, hence the solution is optimal for the hospital since it does not exceed the allocated budget.

Then, the results are analyzed to find the relationship between the number of beds suggested and the weight of the four criteria, which are BOR, ALOS, patients' arrival rates and BOC. Table 5 shows the result of the analysis.

From the table, the Pearson Correlation value for the weight of arrival rates is the highest among the weight of the four criteria. The value 0.789 indicates a strong positive correlation between the weight of arrival rates and the bed numbers [27].

**Table 5**  
 SPSS correlations report regarding 4 criteria

Correlations		Beds	Arrival	BOR	ALOS	BOC
Pearson Correlation	Beds	1.000	.789	.053	.610	.325
	Arrival	.789	1.000	.140	.366	.144
	BOR	.053	.140	1.000	.260	-.569
	ALOS	.610	.366	.260	1.000	.367
	BOC	.325	.144	-.569	.367	1.000

#### 5. Conclusions

The goals of the model, which are to minimize idle beds and the total bed operation cost (BOC) are solved by designing a Goal Programming (GP) model and implemented on Excel Solver. The result of the statistical analysis shows that patients' arrival rates have the most influence on the bed

allocations in each department. It is believed that this model contributes to some of the departments, if not all, by suggesting bed allocations based on each department's needs for the resources that were measured according to their weightage. For future researcher, there are many more elements that can be explored regarding hospital bed allocations, either in Malaysia or other countries. More parameters such as the number of staff, types of beds and ward classes can be applied. Maximization objective function can be used for maximizing human resource utilization or occupying available spaces. Weight evaluation can be implemented using different method or different criteria. Lastly, other optimization software or mathematical application can also be used.

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