



## Reduce PAPR In Filtered-CPOFDM using Modified SCS Method System

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### ABSTRACT

Filtered Cyclic Prefix Orthogonal Frequency Division Multiplexing (F-CPOFDM) is a 5G multicarrier waveform that enables high data rates and spectrum efficiency improvements. The primary drawback of F-CPOFDM is that it has a high peak to average power ratio (PAPR), which is a characteristic of all multicarrier modulation techniques. This paper shows the study of the application on Group Codeword Shift (GCS) approach to reduce the peak to average power ratio (PAPR) in a Filtered-CPOFDM system in this article. Additionally, this paper also compared the results of peak to average power ratio (PAPR) reduction with low complexity in Filtered-CPOFDM using a Group Codeword Shift (GCS) approach, Selective Codeword Shift (SCS), and Conventional CPOFDM. The simulation results indicate that the Group Codeword Shift (GCS) approach reduces peak PAPR by 45.95 percent when compared to conventional Filtered-CPOFDM, 26.13 percent when compared to the Selective Codeword Shift (SCS) method and 22.53 percent when compared to the Median Codeword Shift (MCS) method.

## 1. Introduction

Various research and procedures have been offered to overcome high PAPR values by introducing a few tactics that may be grouped into three primary categories [1-5]. Selective Mapping (SLM), Partial Transmit Sequence (PTS), Selective Codeword Shift (SCS), Interleaving, Tone Reservation (TR), Tone Injection (TI), and Active Constellation Extension are some of the Signal Scrambling Techniques (ACE). Clipping and filtering, Compadding, Peak Windowing, and Envelop Scaling are the other Signal Distortion Techniques. Finally, there are two types of signal coding techniques: Block coding and Turbo coding. Previous study has shown that PAPR reduction is possible, but it comes at a cost, like high computational complexity, degraded bit error rate (BER) performance, side information, loss data rates, bandwidth, loss spectral efficiency, and distortion. There are two types of Block Coding techniques: Arithmetic coding and Huffman coding. Clipping and filtering is the basic strategy for lowering PAPR, and it is based on a clipping level that meets the signal to quantization noise ratio

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(SQNR) [7]. In comparison to SLM and Clipping and Filtering techniques, PTS approach reduces PAPR value significantly [8].

Multiple phase rotations are performed to the constellation points in the Selected Mapping (SLM) approach, where the one that minimises time signal peak is utilized, and selective mapping involves generating a wide collection of vectors with the smallest resultant PAPR is picked. This approach selects the lowest PAPR transmit signal from a group of suitably varied signals that all represent the identical data [4,9]. SLM has benefits such as no distortion and an independent number of carriers, but it also has drawbacks like side information and poor BER efficiency [10-12]. Due of this drawback, PAPR has been reduced using a mix of SLM and clipping techniques [5]. The PAPR value is improved by roughly 3.4dB using a modified SLM with M-QAM approach [8]. The PAPR value is reduced using the SLM approach, but the system's data rate and computing complexity are sacrificed. Few parameters must be considered when deciding which approach may lower a high PAPR value, including BER depreciation at the receiver, data rate loss, computational complexity, power increase in transmitted signal, PAPR reduction capabilities, and bandwidth expansion [13-15].

When compared to the original signal and conventional SLM, the Selective Codeword Shift (SCS) approach has demonstrated a considerable improvement in PAPR reduction; yet this technique is only applicable for modulation greater than 4 QAM or more than 2 bits per symbol. In terms of the IFFT block employed, this approach has a lower computational complexity than SLM, and there is no multiplication of phase factor involved in the transmission operation [16,17]. The codeword is circulant shifting in the SCS approach, and the time to complete this circulant shifting is greater since the codeword travels a lengthy journey. As a result, the PAPR and BER values in the SCS approach will not be reduced as much.

This paper present Group Codeword Shifting (GCS) is introduced, in which the codeword is separated into two groups (group A and group B) to create a reduced route for codeword shifting. The simulation results indicate that the Group Codeword Shift (GCS) approach reduces peak PAPR by 45.95 percent when compared to conventional Filtered-CPOFDM, 26.13 percent when compared to the Selective Codeword Shift (SCS) approach and 22.53 percent when compared to the Median Codeword Shift (MCS) approach. From the result, it shows the GCS approach outperform out SCS and MCS approach because GSC approach has a lowest value of PAPR and BER compare than others.

## **2. Methodology**

### **2.1 Filtered-Cyclic Prefix Orthogonal Frequency Division Multiplexing (F-CPOFDM)**

Filtered Cyclic Prefix Orthogonal Frequency Division Multiplexing (F-CPOFDM) is an additional multicarrier modulation approach that had be considered as a prospective signal for the implementation of 5G. F-CPOFDM potentially just keeps the characteristics of OFDM adopted in 4G, but also demonstrates great flexibility in spectrum use depending on a range of application scenarios, backward and forward stability, and improved OOB [18-20]. During OFDM decoding, the data is passed to the transmitter filter, which forms the transmission F-CPOFDM output. The receiver filter, which is identical to the transmitter filter, is initially applied to the F-CPOFDM receiver signal. The data collected from nearby transmissions is filtered out by the filter of the receiver. For this outcome, the receiver filter reduces the effects of other signals, ensuring that the F-OFDM data is sent without distraction by numerous additional signals [21,22].

The primary target of introducing the filter to the transmitter is to restrict the excessive OOB value of the OFDM system to enable parallel transmission and decrease delay [21,22]. With an outcome, the system's spectrum efficiency enhances, enabling it to fulfil the standards for 5G innovation. Since the filter length surpasses the cyclic prefix (CP) duration at the transmitter and the

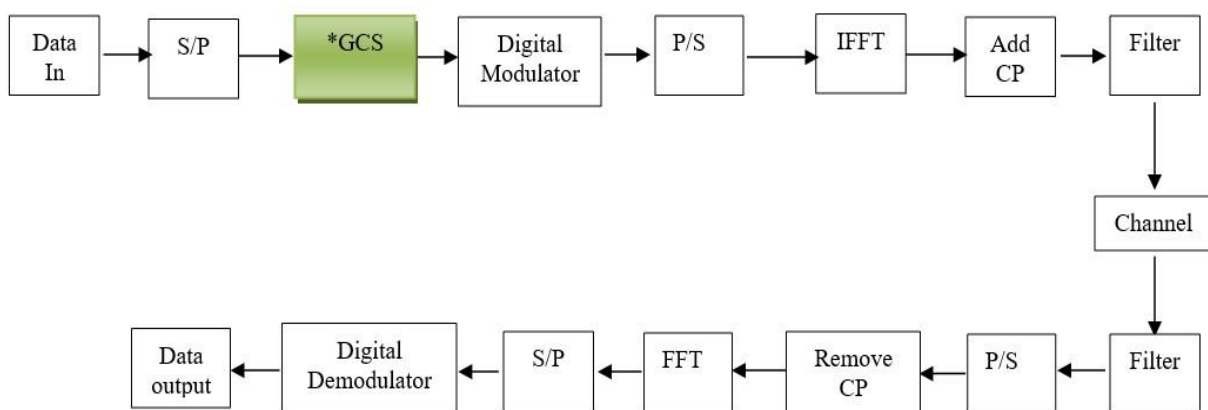
power distribution across the samples rises, the large PAPR might be seen as a possible barrier for the FCPOFDM system. The signal's average power is dropped as a consequence of this operation, and the difference between the F-CPOFDM signal's peak output and average output is extended. As an outcome, the PAPR of the F-CPOFDM system is better than that of the OFDM system [21,22].

In F-CPOFDM, filter layout is important to fulfil frequency localization of the signal and more flexibility between time and frequency localization. This results from the time-domain divergence caused by the desired frequency-domain localization. The filter architecture that can provide a good equilibrium between the temporal and frequency localization of the filter. In particular situations, the soft truncation of a filter is applied using a time-domain window with smooth transitions [23-25]. The filter used for the F-CPOFDM must contain some of the features mentioned in the list, such as a flat passband for the sub-carriers. In an effort to limit guard band consumption, the filter also needs a rapid transition. Thirdly, there must be enough stop band attenuation [25,26]. The sinc filter, commonly referred to as an ideal low pass filter, demonstrates each of the qualities listed above. This rectangular-function filter eliminates most frequency elements over a cutoff frequency. The sinc filter impulse response is obtained from the inverse Fourier transform (IFT) of the filter's frequency response [25,26].

The window that must be used to edit this impulse reaction transforms the infinite impulse reaction into a constrained reaction and creates a smooth negligible transition at both ends. There are several waves in the decreased sinc function's output. To stop these waves, extend the tone-offset low pass filter on both ends. Tone offset refers to the extra number of subcarriers produced to the output [26,27]. In attempt to retrieve the desired signal on the receiver section, a fitting filter that connects a known signal with the unknown signal is employed in the receiver section. The received signal combines the desired signal with AWGN. The purpose of the fitting filter is to distinguish the desired signal from the noisy received signal. In this circumstance, when the noise power is larger than the signal power, the SNR value is raised. At the receiver section of F-CPOFDM, a fitting filter is used to increase the SNR value [26,27].

## 2.2 Filtered-CPOFDM (F-CPOFDM) Based on GCS

The construction of F-CPOFDM signals for N subcarriers starts with the serial-to-parallel transformation of input data into information symbols. The information symbol will then be transferred into the constellation point using a 64-QAM modulating procedure. Lastly, IFFT will turn the modulated symbol into an F-CPOFDM signal. The entire procedure is depicted in Figure 1. In the yellow circular box following the serial-to-parallel conversion, the GCS method is encoded.



**Fig. 1.** Block diagram of Filtered-CPOFDM GCS

Throughout this research, the Group Codeword Shifting approach is utilized to construct new codewords via manipulating codeword architectures, followed by a permutation process (circulant shift) to produce a scrambled data sequence for enhanced PAPR reduction. This Group Codeword Shifting approach focuses on the layout of the codeword and the architecture of the bits to improve PAPR; by altering these two variables, a new codeword with a smaller PAPR is generated.

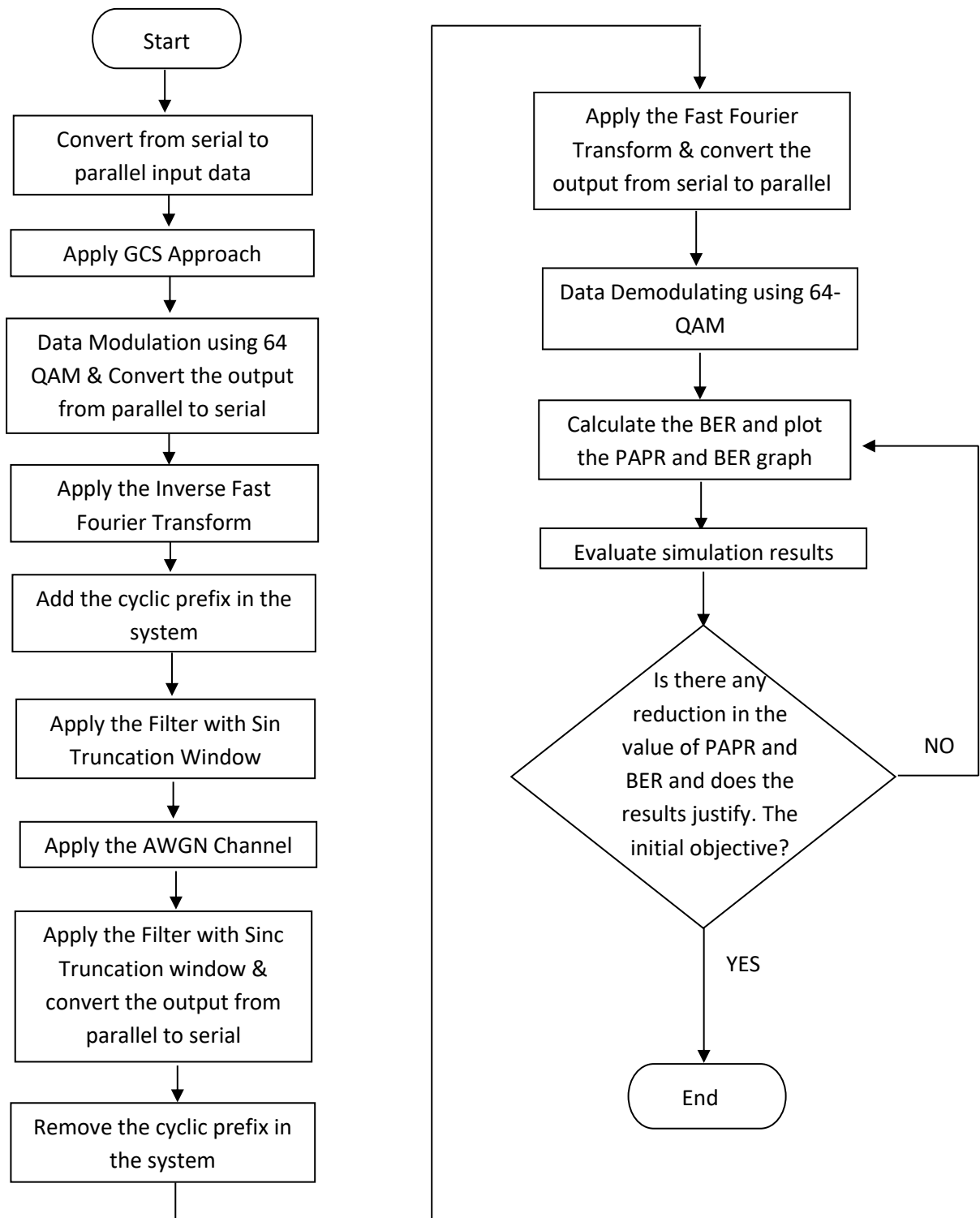


Fig. 2. Flowchart of the simulation process

As illustrated in Figure 3,  $R$  indicates the binary series codeword with  $r$  cumulative input bits and is denoted as  $R = [R_1, R_2, \dots, R_r]$ . The serial-to-parallel converter will split the codeword series across  $z$  sub-blocks, indicated by  $R = [R_1, R_2, \dots, R_z]$ , with  $y$  bits per symbol per sub-block, where  $z = r/y$ . Thus, the description for every sub-codeword block's is  $R_1 = [R_1, R_2, R_3, \dots, R_y]$ ,  $R_2 = [R_{y+1}, R_{y+2}, R_{y+3}, \dots, R_{2y}]$ , and so on till  $R_z$ .

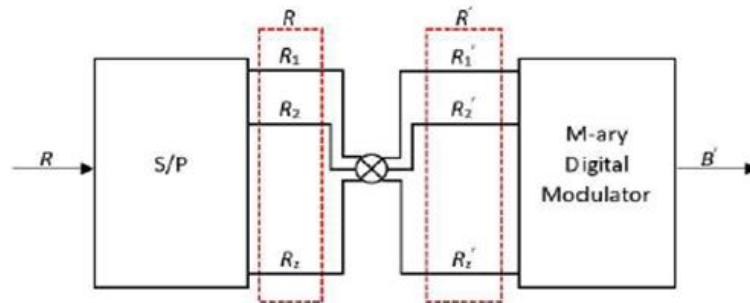


Fig. 3. GCS sub-block

As indicated in Figure 4, the initial phase of the group codeword shifting approach is to adjust the architecture of the codeword by separating it into two portions, A and B. The new codeword is developed in the second stage of development by executing the circulant shift across section A and part B individually.

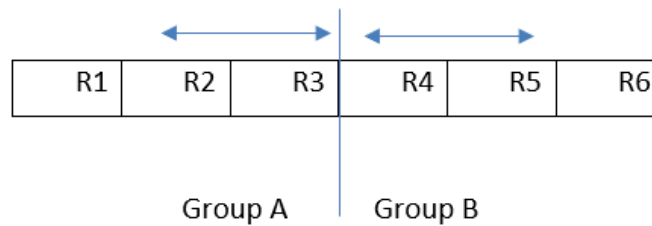


Fig. 4. Group Codeword Shifting structure

Table 1 illustrates the location of bits following number of shift operations in order to enhance explanation. The beginning location of the codeword bits is denoted by Codeword  $R_{1,0}$ . Part A's new bit location will be denoted as Codeword  $R_{1,1}$  as a result of the shifting action between A and B.  $R' = [R_1', R_2', \dots, R_z']$  expresses the updated option codeword series. Ultimately, the alternate Filtered-CPOFDM signal with the lower PAPR value will be picked for transmission.

**Table 1**  
 Bit arrangement of codeword for Group  
 Codeword Shifting Method

Sub-block codeword bits, $R_{z,\delta}$	Position of bits
Codeword, $R_{1,0}$	R1, R2, R3, R4, R5, R6
Codeword shift 1, $R_{1,0}$	R4, R2, R3, R1, R5, R6
Codeword shift 2, $R_{1,2}$	R1, R4, R3, R2, R5, R6
Codeword shift 3, $R_{1,3}$	R1, R2, R4, R3, R5, R6
Codeword shift 4, $R_{1,4}$	R5, R2, R3, R4, R1, R6
Codeword shift 5, $R_{1,5}$	R1, R5, R3, R4, R2, R6
Codeword shift 6, $R_{1,6}$	R1, R2, R5, R4, R3, R6
Codeword shift 7, $R_{1,7}$	R6, R2, R3, R4, R5, R1
Codeword shift 8, $R_{1,8}$	R1, R6, R3, R4, R5, R2
Codeword shift 9, $R_{1,9}$	R1, R2, R6, R4, R5, R3

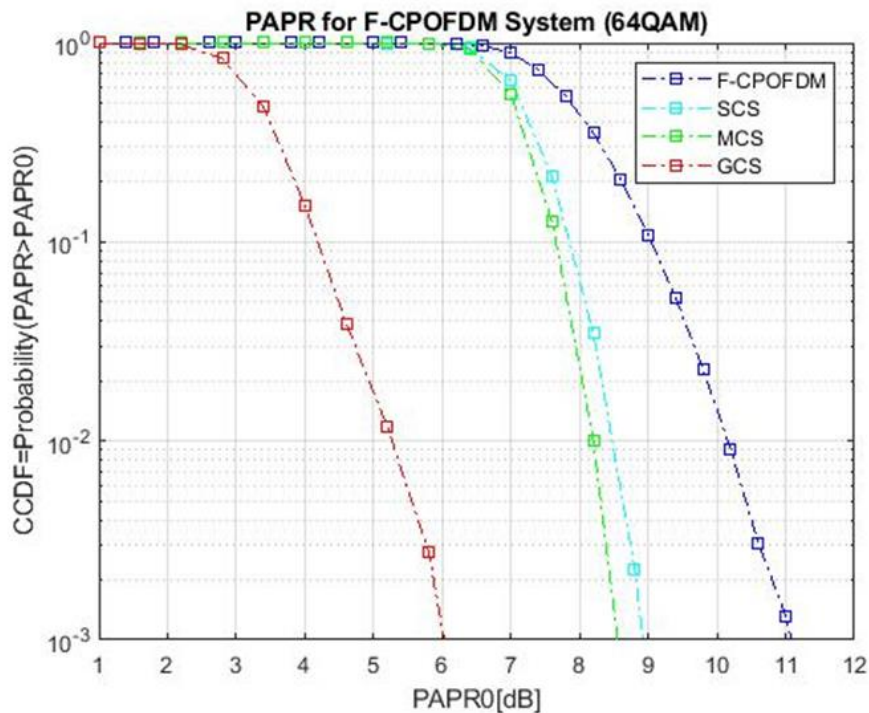
Computation will be utilized to analyse the PAPR performance of GCS. In the simulation,  $N = 128$  symbols of random input are created and mapped using 64-QAM modulation. The signal will be delivered through an AWGN channel. The cyclic prefix with a length of  $1/4$  is introduced to the Filtered-CPOFDM symbols to reduce intersymbol interference (ISI). Table 2 provides a comprehensive listing of all simulation-related parameters.

**Table 2**  
 Simulation parameters for 3<sup>rd</sup> Generation  
 Partnership Project Long Term Evolution (3GPP-LTE)  
 System [35]

Parameter	Value
Bandwidth (BM)	1.25 MHz
Sampling frequency	1.92 MHz
Sampling time	$5.208 \times 10^{-7}$ sec
IFFT size	128
Used subcarrier	76
Modulation technique	64QAM
Cyclic prefix length	$1/4$
Channel model	Rayleigh

### 3. Result and Discussion

In this section, the performance of the proposed GCS approach is compared with another PAPR and BER reduction approach. Figure 5 shows the PAPR effectiveness among MCS, SCS and GCS on Original Filtered-CPOFDM.



**Fig. 5.** PAPR performance for GCS, MCS, SCS and Original on Filtered-CPOFDM

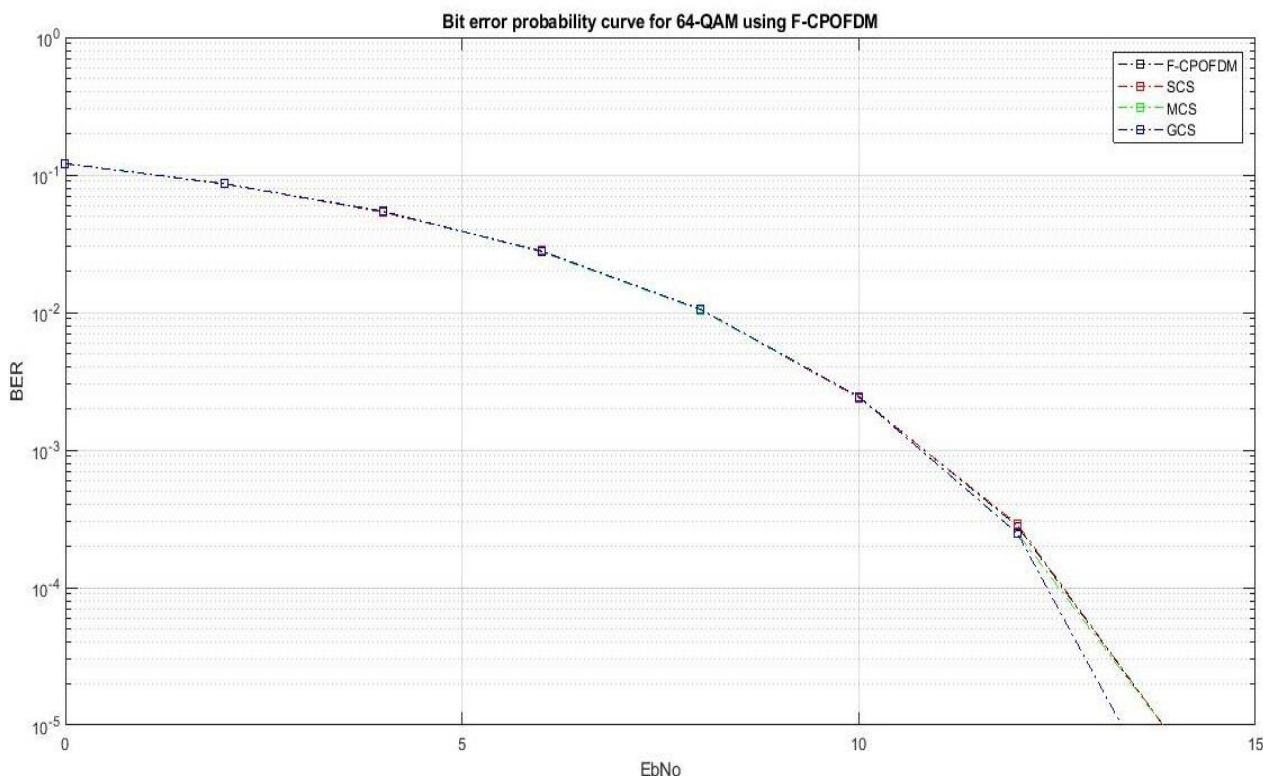
Table 3 demonstrates the PAPR assessment for Filtered-CPOFDM on MCS, SCS and GCS. Filtered-CPOFDM's original value is 11.1dB. SCS is 19.82 % and MCS is 23.42% better than Filtered-CPOFDM, while GCS is 26.13% better than SCS and 22.53% better than MCS at 6.0dB. By changing the way, the codewords are put together, the GCS PAPR effectiveness will be affected.

**Table 3**

PAPR analysis of GCS, MCS, SCS and Original Filtered-CPOFDM (PAPR =  $10^{-3}$ )

	PAPR	% of Improvement
Original	11.1	-
SCS	8.90	19.82
MCS	8.50	23.42
GCS	6.00	45.95

Figure 6 depicts the BER efficiency of MCS, SCS against GCS on Original Filtered-CPOFDM.



**Fig. 6.** BER performance for GCS, MCS, SCS and Original on Filtered-CPOFDM

Whereas the information in Table 4 illustrates the BER analysis for Filtered-CPOFDM on MCS, SCS and GCS. The original value of Filtered-CPOFDM is 14.1dB; the percentage of progress for MCS and SCS versus Filtered-CPOFDM is 0% at 14.1dB, however GCS manages to obtain a larger percentage of progress than MCS and SCS at 5.67 % at 13.3dB. By adjusting the codeword layout in light of its implications on GCS BER efficiency.

**Table 4**  
 BER analysis of GCS, MCS, SCS and Original Filtered-CPOFDM and Original CPOFDM (BER = 10<sup>-5</sup>)

	BER	% of Improvement
Original	14.1	-
SCS	14.1	-
MCS	14.1	-
GCS	13.3	5.67

#### 4. Conclusions

Throughout this study, the GCS show a better approach comparing with MCS, SCS and the original signal Filtered-CPOFDM for reducing PAPR and BER in the system, which is the primary drawback for all multicarrier modulation technique. The GCS has a lower computational complexity than SCS, MCS and the original Filtered-CPOFDM by lowering the system's IFFT block use and this contribute to 45.95% reduction in excessive PAPR has been observed and the minimize BER value. Nonetheless, this approach is only useful for modulations more than 4 QAM or greater than two bits per symbol. This GCS method's operations will be implemented on the transceiver. To evaluate the PAPR performance of the GCS system in other modulation approaches for other purposes, more study is required.



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