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A Modified Wei-Yao-Liu Conjugate Gradient Method for Unconstrained Optimization and Motion Control of Robotic Motion Manipulators

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ABSTRACT

The WYL-type conjugate gradient (CG) iterative formulas have been widely studied by researchers because of their effective and robust algorithms as well as less memory requirements. However, recent modifications of the WYL formulas are very complicated for unconstrained optimization. Also, the performance of most of these methods are yet to be verified on real-life application problems. In this study, a new modification of Wei-Yao-Liu (WYL) CG formula was presented to compute the search direction of robotic motion control and unconstrained optimization problems (UOP). This study discussed the general convergence results of the new formula under suitable assumptions and showed that the scheme possesses the descent properties under inexact line search. Numerical results on some set of robotic motion control and UOP show that the new formula outperformed some existing formulas with the same characteristics. The new method was able to solved 100% of the test problems while WYL was able to solve about 90% and the other methods solved less than 50% of the test problems respectively.

1. Introduction

Optimization is the process of applying a set of defined condition to select the best elements from some set of available alternatives [1-4]. The conjugate gradient (CG) formula has been very instrumental in solving nonlinear optimization problems especially when the problem has large-dimension [5]. This is because of their good memory requirements and simplicity in their iteration formula [6-10]. The nonlinear CG algorithm is assigned to solve UOP of the form:

$$\min\{f(x) \mid x \in R^n\} \tag{1}$$

with the differentiable continuous function $f: R^n \to R$ and $\nabla(f(x)) = g(x)$ is the gradient of f [11,12]. The iterative scheme of the CG method is as follows:

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$$x_{k+1} = x_k + \alpha_k d_k, \qquad k \geq 1. \tag{2}$$

where x_{k+1} is the new iterative point obtained using a combination of the previous iterate x_k, the step size $\alpha_k > 0$, and the direction of search d_k. For the purpose of this study, the strong Wolfe (SWP) line search is considered which requires α_k to satisfy:

$$f(x_k + \alpha_k d_k) \leq f(x_k) + \delta \alpha_k g_k^T d_k \tag{3}$$

$$|g(x_k + \alpha_k d_k)^T d_k| \leq |\sigma g_k^T d_k| \tag{4}$$

where $0 \leq \delta < \sigma < 1$. A significant factor of any CG algorithm is the search direction which is usually computed via:

$$d_k = \begin{cases} -g_k, & \text{for} \quad k = 0 \\ -g_k + \beta_k d_{k-1}. & \text{for} \quad k \geq 1 \end{cases} \tag{5}$$

Here, g_k defines the gradient of f and β_k is a scalar denoting the coefficient of the CG formula. It is important to note that different formulas of β_k differentiate CG methods. Some of the earliest formulas for the CG methods are given by Hestenes-Steifel (HS), Polak-Ribiere (PR), Liu and Storey (LS) whose formulas are given as [13-15]:

$$\beta_k^{HS} = \frac{g_k^T(g_k - g_{k-1})}{d_{k-1}^T(g_k - g_{k-1})}, \beta_k^{PR} = \frac{g_k^T(g_k - g_{k-1})}{\|g_{k-1}\|^2}, \ \beta_k^{LS} = -\frac{g_k^T(g_k - g_{k-1})}{d_{k-1}^T g_{k-1}}. \tag{6}$$

These set of formulas are had been proved to possess efficient computational results. However, their major drawback is the inability to converge under certain line search procedure [16,17]. To overcome the above drawback, the Fletcher-Reeves (FR), Conjugate Descent (CD), and the latest by Dai-Yuan (DY) presented the following CG formulas [18-20]:

$$\beta_k^{FR} = \frac{g_k^T g_k}{\|g_{k-1}\|^2}, \beta_k^{CD} = -\frac{g_k^T g_k}{d_{k-1}^T g_{k-1}}, \beta_k^{DY} = \frac{g_k^T g_k}{d_{k-1}^T(g_k - g_{k-1})}. \tag{7}$$

It is obvious that the FR, CD, and DY formulas are modifications of the first section of CG formulas defined in (6). The convergence results of these formulas have been successfully studied under different line search procedures. However, the methods presented in (7) are influenced by jamming phenomena which affects their computational performance [16,21]. The convergence issues from the first set of CG methods defined in (6) and the poor numerical results from the second set (7) has led to numerous studies of the CG formulas [16,22-32].

This study is interested in solving unconstrained optimization problem of the form (1) using a new class of CG formula and further illustrate the real-life application of the new formula on robotic motion control problems. The motivation of this study is from the fact that only few studies have investigated the WYL CG formula for solving (1). Most of the WYL modifications are only applied to solve unconstrained optimization but their performance has not been evaluated on real-life problems and some of the formulas available are very complicated.

Therefore, this study aims to design a new WYL CG formula for both unconstrained optimization as well as robotic motion control problem. A novel modification of WYL CG method will be constructed for unconstrained optimization. The new formula will produce a new decent direction which is different from those available in the literature and the global convergence will be established under some mild assumptions. Computational efficiency of the new method would be compared with other existing algorithms. Lastly, the performance will further be evaluated by solving problem of robotic motion control.

In this study, we developed a new formula for the CG method for unconstrained optimization and motion control of robotic manipulators in section 2. The convergence analysis of the new formula would be discussed under suitable conditions in section 3. Results from numerical computation on UOP and robotic motion control problems are discussed in section 4 and finally conclusion in the last section.

2. Motivation and New CG Coefficient

Recently, Wei *et al.,* [33] constructed a new numerator for the PRP formula as follows [14]:

$$\beta_k^{WYL} = \frac{g_k^T\left(g_k - \frac{\|g_k\|}{\|g_{k-1}\|} g_{k-1}\right)}{\|g_{k-1}\|^2}. \tag{8}$$

This new modification not only converges globally but also possesses good numerical results under exact and strong Wolfe conditions [33,34]. Based on these nice properties possessed by the WYL method, and to address the drawbacks of the CG methods mentioned above, we defined a new formula for the CG method by considering the classical LS and WYL formulas as follows [15]:

$$\beta_k^{SFA} = -\frac{g_k^T\left(g_k - \frac{\|g_k\|}{\|g_{k-1}\|} g_{k-1}\right)}{d_{k-1}^T g_{k-1}}. \tag{9}$$

This new formula β_k^{SFA} where SFA denotes the researchers' names: Saleh, Fadhilah, and Aidya, replaced $\|g_{k-1}\|^2$ in the denominator of WYL method by $-d_{k-1}^T g_{k-1}$ of LS method.

Remarks 2.1
For the CG method, we have

$$g_1^T d_1 = -\|g_1\|^2 \tag{10}$$

provided $g_1 \neq 0$ [34]. The remarks would be very significant in the convergence analysis of the proposed formula.

Next is the algorithm of the new formula.

Algorithm 1.
Step 1: Starting with an initial guess x_k, set $k = 0$.
Step 2: Check if $\| g_k \| = 0$, terminate. Else, proceed to the next step
Step 3: Determine β_k based on (9).
Step 4: Calculate d_k via (5) and α_k using (3) and (4).
Step 5: Update new point using (2).
Step 6: Re-evaluate the stopping criteria and convergence test.

If $\|g_k\| \leq \varepsilon$ or $f(x_{k+1}) < f(x_k)$, terminate.
Else go back to Step 1 by replacing $k = k + 1$.

3. Convergence Analysis

This section will discuss the descent condition and global convergence of β_k^{SFA}. The following simplification of our proposed formula follows from remark (2.1).

$$\beta_k^{SFA} = -\frac{g_k^T\left(g_k - \frac{\|g_k\|}{\|g_{k-1}\|}g_{k-1}\right)}{d_{k-1}^T g_{k-1}} = \frac{g_k^T\left(g_k - \frac{\|g_k\|}{\|g_{k-1}\|}g_{k-1}\right)}{\|g_{k-1}\|^2} \tag{11}$$

since $g_1^T d_1 = -\|g_1\|^2$. This reduces our proposed method to the classical WYL method defined in (8).

3.1 Sufficient Descent Condition (SDC)

One of the fundamental properties that every CG method should satisfy is the sufficient descent condition discussed in this section.

For the SDC, we have

$$g_k^T d_k \leq -c\|g_k\|^2 \tag{12}$$

where $c \in (0,1)$ [34]. The SDC will guarantee that $f(x)$ will be reduce along d_k.

Theorem 3.1
Let $\{x_k\}$ be generated by algorithm 1 and α_k is determined by SWP line search (3) and (4), if $\sigma < \frac{1}{4}$, then the SDC defined in (12) holds for d_k.

Proof
We begin by proving the decent property of d_k. Multiplying (5) by g_k will produce

$$g_k^T d_k = -\|g_k\|^2 + \beta_k^{SFA} g_k^T d_{k-1} \tag{13}$$

Substituting (11) in (13) and dividing through by $\|g_k\|^2$ will give

$$\frac{g_k^T d_k}{\|g_k\|^2} = -1 + \left(1 - \frac{g_k^T g_{k-1}}{\|g_k\|\|g_{k-1}\|}\right)\frac{g_k^T d_{k-1}}{\|g_{k-1}\|^2} \tag{14}$$

Applying (4) on (14), we have

$$-1 + \left(1 - \frac{g_k^T g_{k-1}}{\|g_k\|\|g_{k-1}\|}\right)\sigma\frac{g_{k-1}^T d_{k-1}}{\|g_k - 1\|^2} \leq \frac{g_k^T d_k}{\|g_k\|^2} \leq -1 + \left(1 - \frac{g_k^T g_{k-1}}{\|g_k\|\|g_{k-1}\|}\right)\frac{-\sigma g_{k-1}^T d_{k-1}}{\|g_k - 1\|^2} \tag{15}$$

But, from remark 2.1, we know that $g_1^T d_1 = -\|g_1\|^2$ provided $g_1 \neq 0$. Next, if we assume that for $i = 1,2,\ldots,k-1$, all d_i's are descent directions, which, based on the definition of descent direction implies $g_i^T d_i < 0$. Then, it follows from Cauchy-Schwarz inequality that:

$$0 \leq 1 - \frac{g_k^T g_{k-1}}{\|g_k\|\|g_{k-1}\|} \leq 2 \tag{16}$$

Combining (15) and (16) will produce:

$$-1 + 2\sigma \frac{g_{k-1}^T d_{k-1}}{\|g_k-1\|^2} \leq \frac{g_k^T d_k}{\|g_k\|^2} \leq -1 - 2\sigma \frac{g_{k-1}^T d_{k-1}}{\|g_k-1\|^2} \tag{17}$$

By further considering the fact that $g_1^T d_1 = -\|g_1\|^2$ and continuous repetition of the process, we get:

$$-\sum_{j=0}^{k-1}(2\sigma)^j \leq \frac{g_k^T d_k}{\|g_k\|^2} \leq -2 + \sum_{j=0}^{k-1}(2\sigma)^j \tag{18}$$

But,

$$\sum_{j=0}^{k-1}(2\sigma)^j < \sum_{j=0}^{\infty}(2\sigma)^j = \frac{1}{1-2\sigma} \tag{19}$$

By considering (19) in (18), it becomes:

$$-\frac{1}{1-2\sigma} \leq \frac{g_k^T d_k}{\|g_k\|^2} \leq -2 + \frac{1}{1-2\sigma} \tag{20}$$

By restricting $\sigma \in \left(0, \frac{1}{4}\right)$, we get $g_k^T d_k < 0$ which, by induction, implies that $g_i^T d_i < 0$ holds $\forall k \in N$. With the above decent property, we can finally prove the SDC of d_k. By setting $c = 2 - \frac{1}{1-2\sigma}$ and considering the fact that $\sigma \in \left(0, \frac{1}{4}\right)$, then, we have $0 < c < 1$. Then, from (20), we have

$$c - 2 \leq \frac{g_k^T d_k}{\|g_k\|^2} \leq -c \tag{21}$$

and this implies that (12) holds for all d_k and thus, completes the proof.

3.2 Global Convergence Properties Under Inexact Line Search

To discuss the global convergence of the proposed method, the following assumption is very important.

Assumption (ASPT) 3.1

(i) The smooth function $f(x)$ is bounded below on the level set $\ell = \{x \epsilon R^n : f(x) \leq f(x_0)\}$ where x_0 is the initial point.

(ii) Also, $g(x)$ is Lipschitz continuous in N, which implies for some constant $L > 0$, we have $\|g(x) - g(y)\| \leq L\|x - y\|$ for any x, y in the neighborhood N.

Lemma 3.1

Suppose ASPT 3.1 is true for CG scheme of the form (2), where α_k satisfies conditions (3) and (4) and d_k is a decent direction. Then, we have

$$\sum_{k=1}^{\infty} \frac{(g_k^T d_k)^2}{\|d_k\|^2} < \infty \text{ or } \sum_{k=1}^{\infty} \frac{\|g_k\|^4}{\|d_k\|^2} < \infty. \tag{22}$$

This lemma is called the Zoutendijk condition whose proof follows from Zoutendijk [35].

Theorem 3.2
Supposed the sequence $\{x_k\}$ is generated by Algorithm 1 and ASPT 3.1 holds true where α_k satisfies (3) and (4). Then Lemma 3.1 holds for all $k \geq 0$.

Proof
The proof of this theorem is by contradiction. We assume Theorem 3.2 is not true. This implies $\exists c > 0$ satisfying $\|g_k\| \geq c$.

From (5), we have

$$d_{k+1} + g_{k+1} = \beta_{k+1} d_k \tag{23}$$

$$\|d_{k+1}\|^2 = -\|g_{k+1}\|^2 - 2g_{k+1}^T d_{k+1} + \beta_{k+1}^2 \|d_k\|^2 \tag{24}$$

$$\frac{\|d_{k+1}\|^2}{\|g_{k+1}\|^4} \leq \frac{\beta_{k+1}^2 \|d_k\|^2}{\|g_{k+1}\|^4} - \frac{2g_{k+1}^T d_{k+1}}{\|g_{k+1}\|^4} \tag{25}$$

Applying (9) in (25) will give:

$$\frac{\|d_{k+1}\|^2}{\|g_{k+1}\|^4} \leq \frac{4\|g_{k+1}\|^4 \|d_k\|^2}{\|g_k\|^4 \|g_{k+1}\|^4} - \frac{2g_{k+1}^T d_{k+1}}{\|g_{k+1}\|^4}. \tag{26}$$

And from (12), it follows that:

$$\frac{\|d_{k+1}\|^2}{\|g_{k+1}\|^4} \leq \frac{\|g_{k+1}\|^2}{\|g_k\|^2} \left(\frac{4\|g_{k+1}\|^4}{\|g_k\|^2} - \|g_{k+1}\|^2 (1 - 2c) \right)$$

$$\frac{\|d_{k+1}\|^2}{\|g_{k+1}\|^4} \leq \frac{\|g_{k+1}\|^4}{\|g_k\|^2} \left((1 - 2c) + \frac{4\|g_{k+1}\|^2}{\|g_k\|^2} \right)$$

since $\|g_k\| \geq c$, we know that $\|g_{k+1}\| \to \infty$ and $\|g_k\| \to \infty$. Therefore, it follows that

$$\sum_{k=0}^{\infty} \frac{\|d_{k+1}\|^2}{\|g_{k+1}\|^4} \approx 0$$

Implying that:

$$\sum_{k=0}^{\infty} \frac{\|g_{k+1}\|^4}{\|d_{k+1}\|^2} \geq \infty$$

which contradicts Lemma 3.1 and thus completes the proof.

Next, we consider the following property (*) presented by Gilbert and Nocedal [36] which is very important in the study of CG methods. The property is used to show that by generating a small α_k, the next direction of search automatically approaches the steepest direction.

Property (*)
Consider a CG algorithm generated using algorithm 1. Assume for all $k \geq 0$, we have
$$0 < \gamma \leq \|g_k\| \leq \overline{\gamma}\,, \tag{27}$$

for some positive constants γ and $\overline{\gamma}$. Then, we say the method possesses the property (*), if

$\exists\, b > 1, \lambda > 0,$ such that $|\beta_k| \leq b$ and $\|s_k\| \leq \lambda$,

then, $|\beta_k| \leq \frac{1}{2b}$, with $s_k = \alpha_k d_k$.

Using the next lemma, we will show that our proposed formula possesses the property (*).

Lemma 3.2
For any CG method generated by algorithm 1 where assumption 3.1 holds true, then, we say the property (*) holds for β_k^{SFA}.

Proof
Let $b = 2\left(\frac{\overline{\gamma}}{\gamma}\right)^2 > 1, \lambda = \frac{\gamma^4}{8L\,\overline{\gamma}^3}$. By (11) and (20), it follows that

$$\left|\beta_k^{SFA}\right| = \frac{\left|\|g_k\|^2 - \frac{\|g_k\|}{\|g_{k-1}\|}\left|g_k^T g_{k-1}\right|\right|}{g_{k-1}^T g_{k-1}} = \frac{\left|\|g_k\|\left(\|g_k\| - \frac{\left|g_k^T g_{k-1}\right|}{\|g_{k-1}\|}\right)\right|}{\|g_{k-1}\|^2} \leq \frac{\overline{\gamma}\left(\overline{\gamma} + \frac{\|g_k\|}{\gamma}\|g_{k-1}\|\right)}{\gamma^2} \leq \frac{\overline{\gamma}\left(\overline{\gamma} + \frac{\overline{\gamma}}{\gamma}\gamma\right)}{\gamma^2} = \frac{\overline{\gamma}(\overline{\gamma} + \overline{\gamma})}{\gamma^2} = 2\left(\frac{\overline{\gamma}}{\gamma}\right)^2 = b.$$

By assumption 3.1. If $\|s_k\| \leq \lambda$, it implies,

$$\left|\beta_k^{SFA}\right| \leq \frac{\left(\|g_k - g_{k-1}\| + \left\|g_{k-1} - \frac{\|g_k\|}{\|g_{k-1}\|}g_{k-1}\right\|\right)\|g_k\|}{\|g_{k-1}\|^2} \leq \frac{(L\lambda + \|g_{k-1}\| - \|g_k\|)\|g_k\|}{\|g_{k-1}\|^2} \leq \frac{(L\lambda + \|g_{k-1} - g_k\|)\|g_k\|}{\|g_{k-1}\|^2} \leq \frac{2L\lambda\|g_k\|}{\|g_{k-1}\|^2} \leq \frac{2L\lambda\overline{\gamma}}{\gamma^2} = \frac{1}{2b}$$

and this completes the proof.

Theorem 3.3
For any CG method generated using algorithm 1, where β_k^{SFA} follows from (9) and (11), and satisfy the conditions that follows:

i. $\beta_k > 0$
ii. d_k satisfy the SDC.
iii. The Zoutendijk condition holds.
iv. Property (*) holds.

If the boundedness and Lipschitz conditions holds true, then, the iterative points are globally convergent.

Proof
From (11), Theorem 3.1, Theorem 3.2, Lemma 3.2 and since WYL satisfy these conditions in Theorem 3.3 under SWP procedure, then, it follows that the proposed SFA formula also possesses all the conditions under the SWP line search, and thus, so the method is globally convergent.

4. Results

This section demonstrates the numerical performance of the new formula compared to other existing methods that includes LS method, HS method, FR method, and WYL method [13,15,18,33]. The analysis would be evaluated on a set of functions most of which follows from Andrei [37] as shown in Table 1. For every test problem, the study considered four different initial points with dimension ranging from $2 \leq n \leq 1000$. All formulas for these computations are written on MATLAB R2022a software and run on Ryzen5 Windows 11 Professional operating system with the stopping criteria set as $\|g_k\| \leq 10^{-6}$.

Table 1
List of Test Problems

No.	Functions	Dimensions	Initial Points
1	Extended Rosenbrock	2, 4, 80, 500	(2,2),(6,…,6),(12,…,12),(20,…,20)
2	Extended White & Holst	2, 4, 20, 100	(2,2),(4,…,4),(10,…,10),(30,…,30)
3	Extended Freudenstein & Roth	4, 10, 100, 500	(-10,…,-10),(-5,…,-5),(7,…,7),(19,…,19)
4	Extended Beale	10, 40, 100, 500	(-12,…,-12),(-4,…,-4),(4,…,4),(13,…,13)
5	Extended Powell	4, 40, 100, 500	(-10,…,-10),(-4,…,-4),(4,…,4),(10,…,10)
6	FLETCHCR	2, 4, 60, 150	(2,2),(4,…,4),(10,…,10),(16,…,16)
7	Diagonal 2	4, 8, 20, 200	(-5,…,-5),(5,…,5),(12,…,12),(26,…,26)
8	Extended Penalty	4, 10, 40, 400	(5,…,5),(7,…,7),(25,…,25),(35,…,35)
9	Hager	8, 20, 40, 80	(3,…,3),(7,…,7),(17,…,17),(27,…,27)
10	Extended Maratos	4, 10, 50, 100	(-4,…,-4),(3,…,3),(9,…,9),(17,…,17)
11	Generalized Quartic	2, 4, 6, 8	(2,2),(4,…,4),(8,…,8),(28,…,28)
12	Quadratic1	10, 100, 500, 1000	(5,…,5),(10,…,10),(15,…,15),(20,…,20)
13	Extended Quadratic Penalty	8, 20, 200, 1000	(2,…,2),(10,…,10),(22,…,22),(34,…,34)
14	NONDIA SHANO	2, 10, 100, 500	(3,3),(6,…,6),(15,…,15),(25,…,25)
15	ARWHEAD	2, 4, 40, 100	(-15,-15),(-2,…,-2),(2,…,2),(15,…,15)

The performance results are evaluated using a tool introduced by Dolan and Moré [38] under SWP line search. This tool plots the performance of every method as shown in Figure 1 based on iteration number and Figure 2 based on CPU time. The formula whose graph lies above other graph is considered as superior.

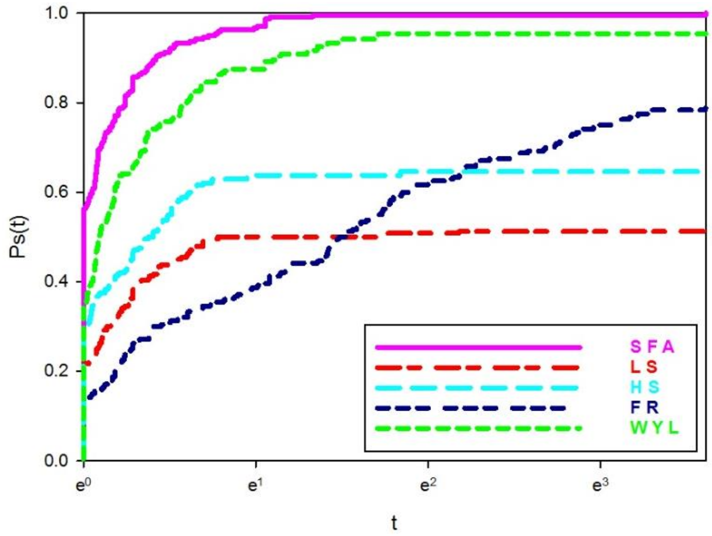

Fig. 1. Performance metric of iteration number

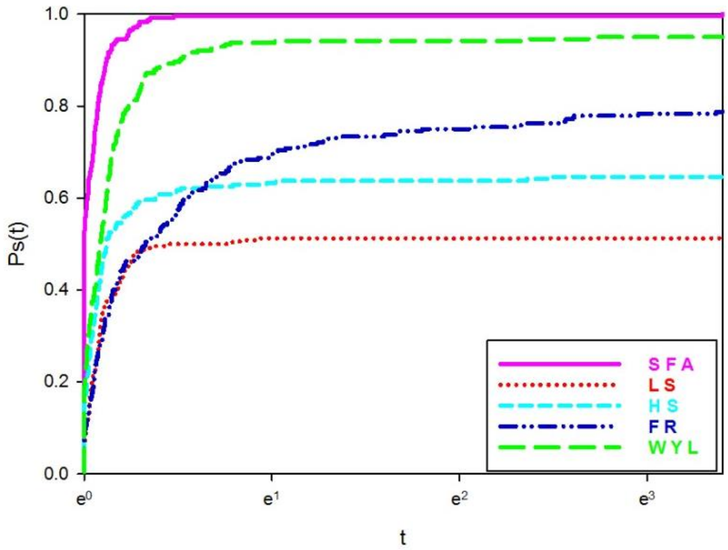

Fig. 2. Performance metric for CPU time

Based on this assertion, it can be clearly seen that the proposed SFA formula has the best performance both in terms of iteration numbers and CPU time as it was able to solve majority of the test problems considered for this study. The LS method is the least performer which implies that the method solved the least number of problems. The performance of HS method is similar to that of the FR methods on all the metrics. The FR method despite the poor performance but was able to solve a higher number of problems compared to the LS and HS methods. With this, we can conclude that the proposed method is promising.

5. Application to Robotic Motion Control

The CG method is one of the popular iterative algorithms used to solve real life application problems because of their efficiency and good memory requirement [39]. Recently, researchers have investigated the performance of different CG formula on image restoration, Portfolio selection, regression model, signal recovery, and motion control of robotic trajectories [6,17,22,24,26,39-45]. Therefore, in this study, we study the performance of the proposed CG method on problem for robotic motion control. Robot trajectories is related to the planning or process of finding a time series of successive joint angles. This involves the planning or process of determining a time sequence of successive joint angles that allow a robot to travel from a beginning configuration to a goal configuration to complete a task, such as picking up an object from a conveyor belt and placing it on a shelf. In robotics, trajectories also refer to a robot's configuration as a function of time. Meanwhile, the end effector refers to the part of the robot that interacts with its surroundings. An end effector is a robotics device that is placed to the end of a robot arm in place of the hand that would normally be there. On the other hand, a residual is a measurement that indicates how far a point is located away from the regression line in either the vertical or the horizontal axis. The difference between the observed value and the estimated value is the error.

For the discrete-time kinematics of robotic motion control, the equation is given as below,

$$f(\vartheta_k) = q_k, \tag{28}$$

where $\vartheta_k \in R^2$ denotes the joint angle vector, $q_k \in R^2$ defines the end effector position vector, and $f(\vartheta)$ is the kinematics function computed as follows:

$$f(\vartheta) = \begin{bmatrix} l_1 \cos(\vartheta_1) + l_2 \cos(\vartheta_1 + \vartheta_2) \\ l_1 \sin(\vartheta_1) + l_2 \sin(\vartheta_1 + \vartheta_2) \end{bmatrix}. \tag{29}$$

Here, l_1 and l_2 denotes the length of first rod and second rod respectively. Next is to define the robotic motion control equation based on nonlinear least square problem:

$$\min_{q_k \in R^2} \frac{1}{2} \left\| q_{dk} - q_k \right\|^2, \tag{30}$$

where q_{dk} represent the controlling the end-effector used in tracking the Lissajous curve which can be obtained via:

$$q_{dk} = \begin{bmatrix} 0.2\ \sin\left(\frac{\pi t_k}{5}\right) + 1.5 \\ 0.2\ \sin\left(\frac{2\pi t_k}{5} + \frac{\pi}{3}\right) + \frac{\sqrt{3}}{2} \end{bmatrix}. \tag{31}$$

where the initial point of the joint angle vector is selected in the range $\vartheta_0 = \left[0, \frac{\pi}{3}\right]^T$.

In this study, we consider the rod length $l_1 = l_2 = 1$ with the time duration ranging between 0 to 10 which is divided into 200 parts. Figure 3 to Figure 6 present the graph of experimental results obtained using MATLAB software. The results are described as follows:

i. Figure 3 illustrates the 3-dimensional graph of the synthetic robot trajectories obtained by the proposed algorithm.
ii. In Figure 4, we demonstrate the end-effector trajectories and desired path from the new algorithm.
iii. Lastly, Figure 5 and Figure 6 are the graph of trailing error for x and y axes respectively.

From Figure 3, it can be observed that the proposed algorithm was able to solve the robotic motion control problem and the end-effector trajectories and desired path has further demonstrated the efficiency of our algorithm. For residual error, the new modification has an error of 10^{-5} for x-

axis and residual error of 10^{-1} for y-axis. Thus, we can deduce that the new CG algorithm is efficient and promising on real-life application problems.

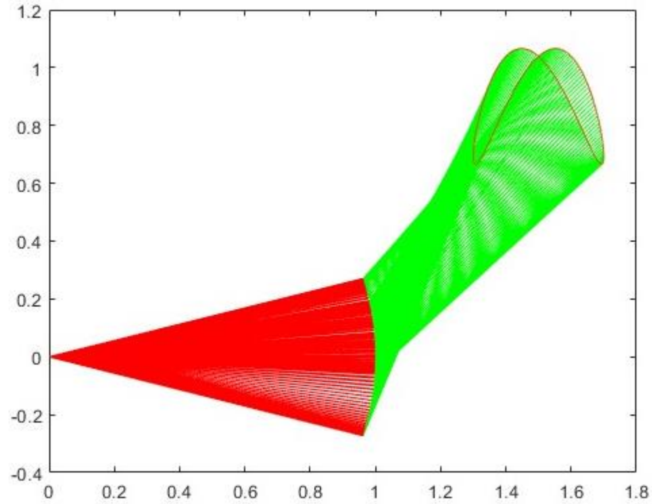

Fig. 3. Performance metric of iteration number

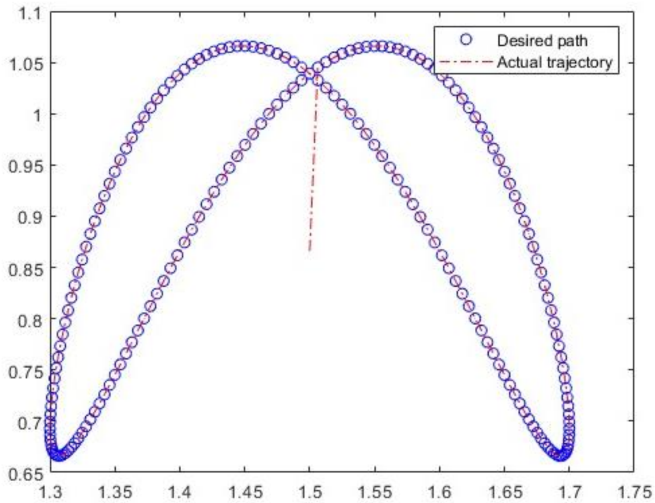

Fig. 4. Performance metric for CPU time

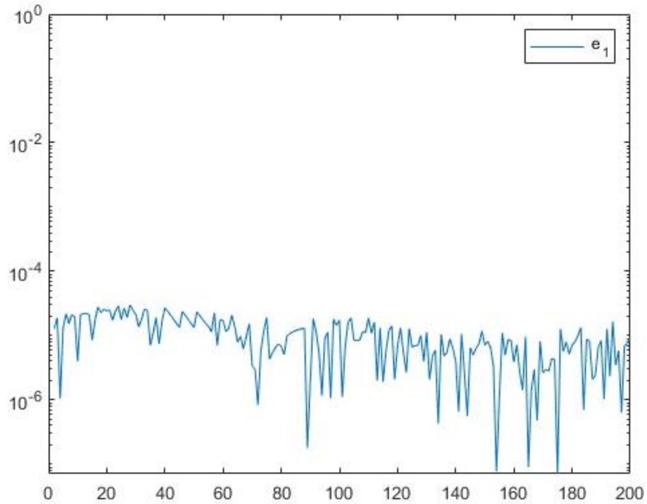

Fig. 5. Performance metric of iteration number

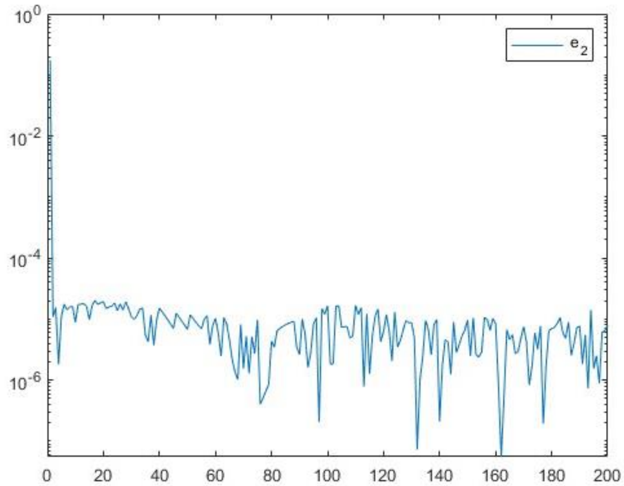

Fig. 6. Performance metric for CPU time

6. Conclusions

In this study, a new CG formula was developed for solving problem of motion control of robotic manipulator and unconstrained optimization. The proposed method was derived based on the classical LS and WYL formulas. Under suitable conditions, we established the global convergence of our new formula and sufficient descent condition. For the numerical performance, the study considered a number of test problems and robotic motion control problems for numerical test. Numerical results show that the new method was able to solved 100% of the test problems while WYL was able to solve about 90% and the other methods solved less than 50% of the test problems respectively. This illustrates that the new formula is promising and very efficient as it outperformed all the existing methods under both iteration number and CPU time.

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