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A Review on Picture Fuzzy Aggregation Operators

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ABSTRACT

Multiple Attribute Decision Making (MADM) is a fundamental concept in the scope of the decision sciences, serving as a structured method for evaluating and selecting the most appropriate alternative option from a pool of available alternatives. MADM methods have gained prominence over the years and are frequently applied in real-world scenarios. Nevertheless, decision-making in practical situations often involves information that is imprecise and uncertain, especially conflicting criteria or attributes. Therefore, Picture Fuzzy Sets (PFSs) and Aggregation Operators (AOs) have proven invaluable in effectively addressing decision challenges characterized by impression and uncertainty. During the past few years, various Picture Fuzzy Aggregation Operators (PFAOs) have been suggested and established but have not been thoroughly reviewed. The primary highlight of this research is to analyse as well as review the development and proposals surrounding PFAOs and their diverse applications within the decision-making paradigm. Regarding this, a review of 140 published articles from 2017 to 2022 appeared in 48 high-ranking journals cited from the "Scopus" and "Web of Science" databases. Other than that, all these articles have been classified by the nationalities of authors, publication year, published journal, research area, operators and methods. The findings of this study discovered that PFAOs have been increasingly applied for supporting decisions due to their frequent implications and applications in different managerial domains, either profit or non-profit organizations. This literature survey's significant contribution provides a platform for researchers to identify future dimensions of works as improvements for decision-making in picture fuzzy environments while also promoting future application of the approaches.

1. Introduction

Decision-making resembles an intellectual process in which decisions are made by identifying a decision, gathering information as well as evaluating alternative solutions [1]. Decision-making has gained popularity due to its frequent implications and applications in management settings, either in for-profit or non-profit organizations. In real-world problems, considering decision-making based on a single criterion or objective is usually impossible, which prevents the desired optimal decision outcome. Therefore, Multi-Criteria Decision-Making (MCDM) refers to a well-structured as well as multidimensional decision-support procedure established to handle decision problems in various

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domains and search for appealing alternatives by considering all important criteria [2]. In the context of MCDM, the criteria are also referred to as attributes. MCDM refers to a powerful decision-making tool used in various fields, for instance, engineering, business, and management [3,4].

Generally, MCDM problems may be categorized into two primary categories: Multi-Objective Decision Making (MODM) as well as Multi-Attribute Decision Making (MADM), depending on whether the problem involves designing or selecting alternatives. MODM methods deal with decision variables discovered via integer or continuous functions, with many alternatives. It aims to determine the best option fulfilling the decision maker's preferences as well as constraints. Linear programming and goal programming are both examples of MODM techniques. Conversely, MADM methods are typically discrete, involving a restricted set of predetermined alternatives. Moreover, MADM serves as an approach to addressing problems related to selection, screening, or determining final rankings among a finite number of options, considering the occurrence of multiple decision criteria or attributes that may often conflict with each other [5].

In recent years, MADM methods have gained significant popularity and find frequent applications in various real-life scenarios. However, the present hyper-competitive environment and the growing complexity with regard to decision problems make it challenging for a single decision-maker or expert to consider all related components comprehensively. As a result, many real-life problems are now approached by a group of decision-makers or experts, leading to Multiple Attribute Group Decision Making (MAGDM). It has become an intriguing and crucial aspect of contemporary decision science and research due to its ability to address complex decision challenges with collective insights and expertise [6]. In the context of MAGDM analysis, decision-makers are invited to partake in the evaluation process based on their individual skills, knowledge, experience as well as preferences within a decision space. This decision space comprises a finite set of alternatives, and the decision-makers provide evaluation information for each alternative across multiple attributes [7-9]. Moreover, the evaluation serves as the initial stage for ranking the alternatives or selecting the most suitable one. Currently, MAGDM is extensively employed to address a variety of social, economic, and management challenges, including safety assessments [10], supplier selection [11,12], bank recruitment [13], selecting an agricultural socialization service provider [14] as well as nanomaterial selection in biomedical problems [15], to name just a few.

One of the fundamental challenges in MADM as well as MAGDM is selecting the suitable Aggregation Operator (AO) to combine the performance of each attribute for various alternatives. An efficient AO should accurately represent the decision-making outcome, ensure that the evaluations of different attributes are considered comprehensively, and correctly reflect the impact of decisions [16]. Hence, the investigation of AO plays a crucial role in the MADM as well as MAGDM. Generally, AO serves as the fundamental form of information fusion [17].

Until this point, a diverse range of AOs has been proposed and applied in various domains. With the multitude of research findings in this field, it becomes highly imperative to provide a comprehensive review and consolidate valuable insights for both theoretical researchers and practitioners working in this domain. However, there is a scarcity of review studies on AO, particularly in picture fuzzy environments. Hence, this research focuses specifically on Picture Fuzzy Aggregation Operators (PFAOs) in MADM and MAGDM problems.

2. Picture Fuzzy Sets

In practical decision-making scenarios, numerous uncertain factors and imprecise information may arise, posing a challenge for decision-makers to accurately represent decision attribute values, particularly when using crisp numbers [18]. Consequently, the evaluation of decision-making information for alternatives becomes intricate and diverse [19]. To address this limitation, Zadeh [20] introduced the fuzzy set theory as a modelling tool for intricate systems influenced by human judgment. However, with its membership function signifying the degree of an element's belongingness to a set, the fuzzy set may not fully capture information in certain situations, particularly when decision-makers have conflicting opinions [21]. As an extension of fuzzy set theory, Atanassov [22] introduced the Intuitionistic Fuzzy Set (IFS) concept, which incorporates membership degree as well as non-membership degree to represent both agreement and disagreement in people's judgments. Nonetheless, this model still lacks the ability to account for various attitudes of decision-makers towards specific decision-making issues beyond just agreement or disagreement. The IFS do not accommodate exceptions for agreement as well as disagreement degrees or other possibilities, for instance, hesitancy or refusal degrees. In the voting system, the voters may be divided into four groups of those who vote yes, abstain, vote against, and refusal of the voting. For example, a group of students wants to visit two places: one is Pahang, and the other is Penang. Some students want to visit Pahang (voted yes), not Penang (voted against), but some students want to visit both places, Pahang and Penang (voted abstain or neutrality). Besides, a few students do not want to visit both places (voted refusal). Nevertheless, the IFS only cares about those who vote for or vote against and considers those who abstain and refuse to be equivalent. As a result, there is an inherent limitation in the IFS model [22].

The Picture Fuzzy Set (PFS) was established by Cuong and Kreinovich [23] as a direct extension of both FS as well as the IFS. PFS serves as an efficient modelling tool to capture the fuzziness in MADM and MAGDM problems when dealing with human opinions. It allows for a broader range of answers, including yes, no, abstain, as well as refusal, providing a more comprehensive representation of decision-makers' viewpoints. In PFS, there are four distinct degrees of an element representing the broader range of decision makers' answers, which are membership degree (α), hesitancy degree (β), non-membership degree (ϑ), and refusal degree ($\psi = 1 - \alpha - \beta - \vartheta$), respectively, satisfying $0 \leq \alpha + \beta + \vartheta \leq 1$. The pair of elements in PFS is known as Picture Fuzzy Number (PFN) or Picture Fuzzy Value (PFV).

3. PFOAs

At present, numerous researchers have widely investigated the theory and methods under the PFS environment. The research on PFSs mainly includes three aspects [24]. Firstly, the basic theoretical research of PFNs, such as the entropy measure [25-27], similarity measure [28-34], distance measure [35-42], correlation coefficients [43,44], and so on.

Another two aspects are to cope with quantified criteria assessment in MADM and MAGDM problems. One applies the extended traditional decision-making approaches relying on PFS, for instance, Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS) [11,45-47], VIKOR [26,27,48-53], ELECTRE [54-56], etc. Second includes the AOs methods for PFNs, for instance, picture fuzzy weighted AOs [57-63], Bonferroni Mean (BM) AOs [64], Heronian Mean (HM) AOs [21], Maclaurin Symmetric Mean (MSM) AOs [65], Muirhead Mean (MM) AOs [66], and Choquet Integral (CI) AOs [51]. Consequently, the approaches based on AOs are substantially rather suitable as well as convincing compared to extended traditional decision-making approaches, as they may present summary criterion values as well as rankings with regard to alternatives. Meanwhile, extended traditional decision-making approaches only produce output ranking of the alternatives [24,67]. As a result, PFAOs are easier to use and more intuitive compared to picture fuzzy-based traditional decision-making approaches.

4. Research Methodology

A thorough review of the literature concerning PFAOs and decision-making was conducted, utilizing papers referenced in the Web of Science and Scopus academic database. The literature search was performed using relevant keywords, for instance, PFAOs, PFS as well as picture fuzzy decision-making methods, focusing on papers published between 2017 and 2022. In total, 563 scholarly papers were extracted based on our search strategy. Of these records, 98 published papers were removed due to duplicates and redundant information, and 66 papers with miscellaneous problems irrelevant to picture fuzzy were removed. Moreover, since this study concentrates on PFAOs, the picture of fuzzy-based traditional decision-making methods was also removed. Following from here, we screened papers according to their titles and abstracts, and in total, 140 potentially related papers in the PFAOs field remained. The flowchart of the analysis and procedure is illustrated in Figure 1.

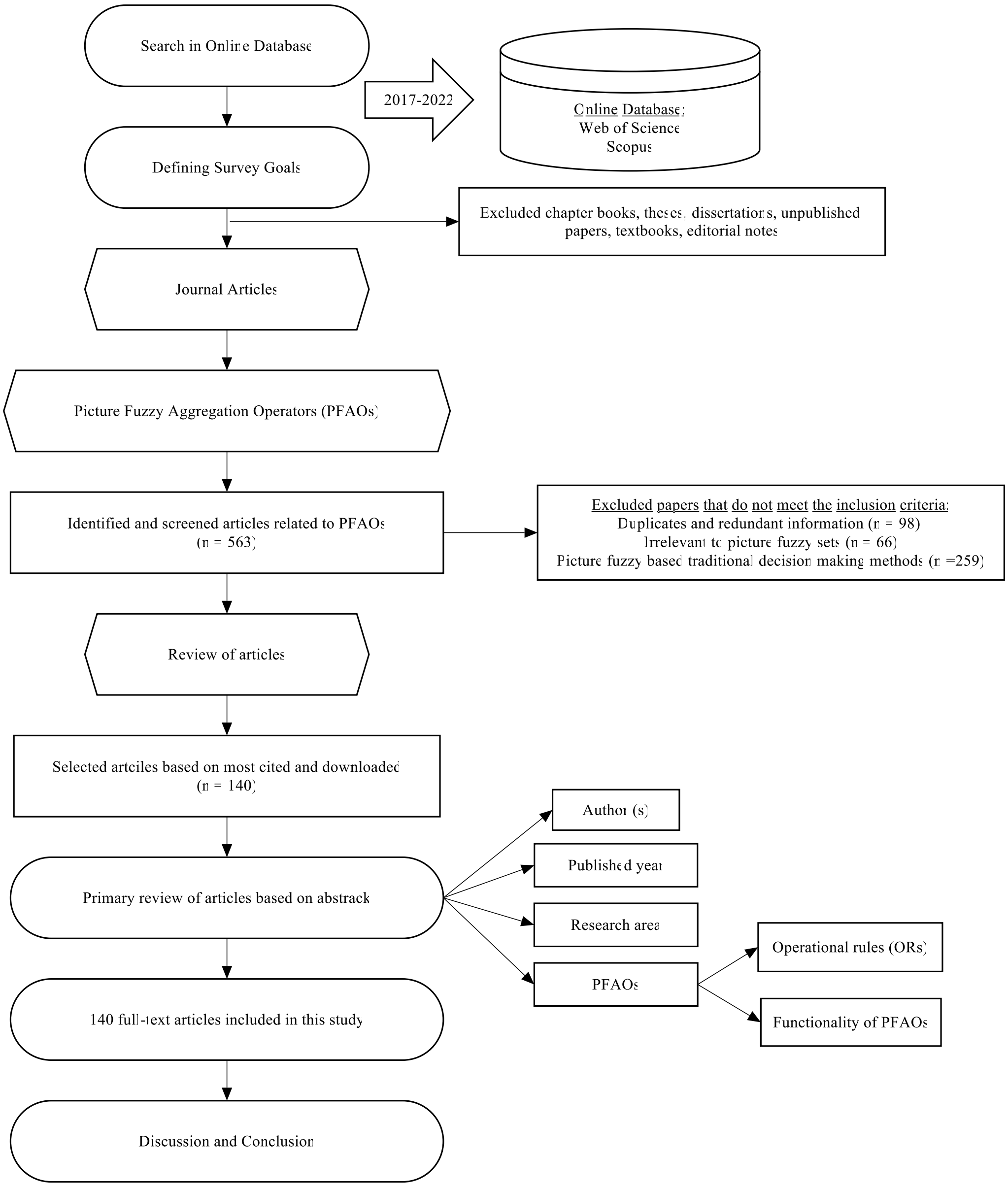

Fig. 1. Summary of analysis and procedure of study

5. Results

AOs have recently drawn great interest and have also taken the information fusion industry by storm. Normally, there are two parts to the AOs. The first part is the AOs with different Operational Rules (ORs), while the second part is the functionality of the AOs [24]. Thus, this research focuses on these two parts of PFAOs with the state of the research area and application in MADM or MAGDM. Subsequently, this study includes information pertaining to publication years, countries, as well as names of published journals. The research's findings are then summarized in several figures and tables.

5.1 Distribution Papers Based on PFAOs with Different ORs

The first part is the AOs with different ORs. So far, many MADM and MAGDM methods relying on AOs of PFNs have been discussed. Several well-known AOs mainly use the algebraic ORs, specifically obtained from the Archimedean t-norm and t-conorm, to carry the combination process dealing with aggregating criterion values [51]. Examples of PFAOs based on algebraic ORs are [60,67-70], which solve problems in electric vehicle charging station selection, circulation centre evaluation, manuscript evaluation, enterprise resource planning, and medical service efficiency, respectively. Following from there, several new ORs are obtained from new special cases of Archimedean t-norm as well as t-conorm, for example, Einstein ORs [68,69], Hamacher ORs [70,71], Frank ORs [72], Dombi ORs [60,73], and Schweizer-Sklar ORs [74] where all have some flexible parameters. In a word, each special case pertaining to t-norm and t-conorm has unique characteristics. The PFAOs with the basis of the ORs implemented in the methods are outlined in Table 1.

Table 1
A summary of some representative AOs and the basis of the implemented ORs in the existing MADM and MAGDM with PFNs

Reference	Year	Research Area	MADM or MAGDM	AOs of PFNs	Type of applied ORs	Application Type
Wei [108]	2017	Enterprise resource planning	MADM	Picture Fuzzy Weighted Average (PFWA), Picture Fuzzy Weighted Geometric (PFWG), Picture Fuzzy Ordered Weighted Average (PFOWA), Picture Fuzzy Ordered Weighted geometric (PFOWG), Picture Fuzzy Hybrid Average (PFHA), as well as Picture Fuzzy Hybrid Geometric (PFHG) operators	Algebraic	Illustrative study
Wang *et al.,* [61]	2017	Manuscript Evaluation	MADM	PPFWG, PFOWG, and PFHG operators	Algebraic	Illustrative study
Garg [59]	2017	Financial investment risk	MADM	PPFWA, PFOWA, and PFHA operators	Archimedean	Illustrative study
Wei [71]	2018	Enterprise resource planning	MADM	Picture Fuzzy Hamacher Weighted Averaging (PFHWA), Picture Fuzzy Ordered Weighted Averaging (PFHOWA), Picture Fuzzy Hamacher Hybrid Averaging	Hamacher	Illustrative study

				(PFHHA), Picture Fuzzy Hamacher Weighted Geometric (PFHWG), Picture Fuzzy Ordered Weighted Geometric (PFHOWG), and Picture Fuzzy Hamacher Hybrid Geometric (PFHHG) operators		
Wang *et al.,* [109]	2018	Service outsourcing supplier	MADM	Hesitant picture 2-tuple linguistic weighted averaging (ATS-HP2TLWA) and Hesitant picture 2-tuple linguistic weighted geometric (ATS-HP2TLWG) operators	Archimedean	Illustrative study
Liu and Zhang [110]	2018	Enterprise resource planning	MADM	Picture fuzzy linguistic weighted arithmetic averaging (A-PFLWAA) operator	Archimedean	Illustrative study
Zhang *et al.,* [88]	2018	Enterprise resource planning	MADM	Picture Fuzzy Dombi Heronian Mean (PFDHM), Picture Fuzzy Dombi Weighted Heronian Mean (PFDWHM), Picture Fuzzy Dombi Geometric Heronian Mean (PFDGHM), and Picture Fuzzy Dombi Weighted Geometric Heronian Mean (PFDWGHM) operators	Dombi	Illustrative study
Ju *et al.,* [111]	2019	Electric vehicle charging station selection	MAGDM	Picture Fuzzy Weighted Interaction Geometric (PFWIG) operator	Algebraic	Case study
Jana *et al.,* [73]	2019	Emerging technology commercialization	MADM	Picture Fuzzy Dombi Hybrid Weighted Geometric (PFDHWG), Picture Fuzzy Dombi Order Weighted Geometric (PFDOWG), Picture Fuzzy Dombi Weighted Geometric (PFDWG), Picture Fuzzy Dombi Hybrid Weighted Average (PFDHWA), Picture Fuzzy Dombi Order Weighted Average (PFDOWA), and Picture Fuzzy Dombi Weighted Average (PFDWA) operators	Dombi	Illustrative study
Qiyas *et al.,* [60]	2019	Emerging technology commercialization	MADM	Linguistic PFDWA (LPFDWA), linguistic PFDOWA (LPFDOWA), linguistic PFDHWA (LPFDHWA), linguistic PFDWG (LPFDWG), linguistic PFDOWG (LPFDOWG), linguistic PFDHWG (LPFDHWG) operators	Dombi	Illustrative study
Jana and Pal [70]	2019	Enterprise Performance Evaluation	MADM	PPFHWA and PFHWG operators	Hamacher	Illustrative study
Khan *et al.,* [69]	2019	Investment project	MADM	Picture Fuzzy Einstein Weighted Averaging (PFEWA) and Picture Fuzzy Einstein Ordered Weighted Averaging (PFEOWA) operators	Einstein	Illustrative study

Khan *et al.,* [112]	2019	Circulation centre evaluation	MADM	Logarithmic PFWA (Log-PFWA), Logarithmic PFWG (Log-PFWG), Logarithmic PFOWA (Log-PFOWA), and Logarithmic PFOWG (Log-PFOWG) operators	Algebraic	Illustrative study
Ashraf *et al.,* [47]	2019	Air quality	MAGDM	PPFWG, PFOWG, PFHWG, and Generalized Picture Fuzzy Weighted Geometric (GPFWG) operators	Algebraic	Illustrative study
Li *et al.,* [113]	2019	Emergency management centre (EMC)	MAGDM	Picture Fuzzy Hybrid Ordered Weighted Interaction Averaging (PFHOWIA), Picture Fuzzy Ordered Weighted Interaction Averaging (PFOWIA), as well as Picture Fuzzy Weighted Interaction Averaging (PFWIA) operators	Algebraic	Illustrative study
Zhang *et al.,* [68]	2020	Enterprise resource planning	MAGDM	PPFLWAA averaging operator	Archimedean	Illustrative study
Seikh and Mandal [72]	2021	Investment Project	MADM	Picture Fuzzy Frank Hybrid Geometric (PFFHG), Picture Fuzzy Frank Order Weighted Geometric (PFFOWG), Picture Fuzzy Frank Weighted Geometric (PFFWG), Picture Fuzzy Frank Hybrid Averaging (PFFHA), Picture Fuzzy Frank Order Weighted Averaging (PFFOWA), and Picture Fuzzy Frank Weighted Averaging (PFFWA) operators	Frank	Illustrative study
Kamacı *et al.,* [114]	2021	Investment Project	MADM	Dynamic Interval-Valued Picture Hesitant Fuzzy Einstein Weighted Average (DIVPH-FEWA) and Dynamic Interval-Valued Picture Hesitant Fuzzy Einstein Weighted Geometric (DIVPH-FEWC) operators	Einstein	Illustrative study
Rong *et al.,* [115]	2022	Emergency schemes assessment	MAGDM	Picture Fuzzy Archimedean Copula Prioritized Weighted Average (PFACPRWA) operator	Archimedean	Illustrative study
Tian *et al.,* [74]	2022	Investment Project	MADM	Picture Fuzzy Schweizer–Sklar Prioritized Weighted Average (PFSSPWA), Picture Fuzzy Schweizer–Sklar Prioritized Weighted Geometric (PFSSPWG) operators	Schweizer–Sklar	Illustrative study

5.2 Distribution Papers Based on the Functionality of the PFAOs

All the PFAOs mentioned above use additive AOs such as Weighted Average (WA) operator [75], Weighted Geometric (WG) operator [76], as well as Ordered Weighted Average (OWA) operator [77] that linear in nature and relying on the assumption that decision-makers' preferences are defined by an independence axiom [78,79]. The authors Abdullah *et al.,* [80] stated one of the major drawbacks of additive operators is that they are abortive in modelling the interaction among attributes during aggregation. In other words, these operators are improper for real-world decision-making phenomena since the decision-makers' preferences typically change dynamically, as well as numerous interactions are always present among various studied attributes. When aggregating the decision criteria for the computation of the global score, the employed AOs should take into account the interactive or interdependency characteristics of the decision criteria or the preferences of the decision-makers to produce reasonably more precise decision outcomes for these problems [81], [82]. Therefore, it needs to find some improvement in AO in dealing with the occurrences in which the decision data are related. Many aggregating operators for processing the interaction between attributes have been proposed, and some of the most popular basic as well as popular AOs are the BM operator, HM operator, MSM (MSM) operator, MM operator, as well as CI operator [83,84].

Basically, BM and HM are two robust aggregation technologies that may grasp the interrelationship between arguments. Ateş and Akay [64] extended the BM operator to PFSs and used them to develop a series of PFBM operators for solving enterprise resource planning problems. Alternatively, Yang and Li [63] extended the normalized weighted BM operator relying on Einstein operations to picture hesitant fuzzy environments. In addition, profiting from the idea of 2-tuple linguistics, G. Wei [85] utilized the BM as well as geometric BM operations to create several AOs for choosing the service outsourcing provider of communication industry. In determining the ability and proficiency, Mahmood *et al.,* [58] started a MAGDM technique relying on the Picture Hesitant Fuzzy Weighted Geometric Bonferroni Mean (PHFWBM) operator as well as the Picture Hesitant Fuzzy Geometric Bonferroni Mean (PHFGBM). To cope with the assessment of alternatives in MAGDM situations, P. Zhang *et al.*, [56] introduced picture fuzzy normalized weighted BM paired with decision-making approaches like MABAC. Meanwhile, Wei *et al.,* [62] proposed a series of Picture Fuzzy Heronian Mean (PFHM) operators in selecting enterprise resource planning systems. Luo and Xing [21] and Lin *et al.,* [86] demonstrated a new MADM method. Note that the partitioned weighted Heronian AO is modified under interactive PFNs to deal with correlated arguments for hotel selection issues and service quality ranking of nursing facilities, respectively. Wei *et al.,* [62]and Fan *et al.,* [87] proposed some picture fuzzy Heronian AOs under picture fuzzy environments in solving MADM problems. Meanwhile, Zhang *et al.,* [88] suggested new AOs based on Dombi operational rules considering the correlation between attributes to fuse PFNs, respectively. On the other hand, Li and Yang [89] recommended a combination of merits of both the Power Average (PA) operator as well as Improved Generalized Heronian Mean (IGHM) operator to eliminate the effect of abnormal data along with capturing the relationships among attributes in the MADM problem.

However, there are inherent flaws in HM and BM operators. First, the decision-makers may only assign the relationships between two attribute values and cannot process the interrelationships among three or multiple [17,21]. Furthermore, both the HM and BM operators necessitate the incorporation of p and q values, which signify the decision maker's risk attitude. These values can impact the interaction degrees of the variables and consequently influence the ranking outcomes of the alternatives [84]. For instance, when the values of p and q are less than 1, the ranking closely resembles that of the case without taking into consideration the criteria of interrelationship. On the other hand, if the p and q values are relatively larger, it indicates that the decision maker exhibits risk aversion. Conversely, if the decision maker's p and q values are smaller, they exhibit risk-seeking behaviour. To overcome the limitations in HM and BM operators, Maclaurin, in the year 1729 [90], introduced the MSM operator. The MSM operator represents a broader version of the Arithmetic Mean (AM), Geometric Mean (GM), BM as well as HM operators, offering enhanced adaptability by adjusting parameter values to capture interrelationships among any number of attributes. Its versatility allows for more robust information fusion, making it well-suited for addressing MADM scenarios involving various interaction patterns. Whether there are any interactions among all arguments, interactions between two arguments, or multiple integrated input arguments, the MSM operator can handle such situations effectively.

Furthermore, the MSM operator exhibits a monotonically decreasing trend concerning the parameter values for a given collection of arguments. This characteristic enables it to reflect the risk preferences of decision-makers in practical situations. By incorporating these properties, the MSM operator becomes a valuable tool in MADM tasks, offering greater flexibility and accuracy in capturing complex relationships among attributes and accommodating various risk attitudes during decision-making processes [91]. Qin *et al.,* [65] proposed a method that can minimize the negative effect of biased attribute values on the aggregation result. In contrast, Feng and Geng [92] proposed a convenient method for expressing cognitive information to prevent lost information. Here, both methods are based on MSM operators under the Archimedean operational rule. Apart from that, Chen and Ye [93] presented a MAGDM approach using Schweizer–Sklar operational rules under hesitant PFSs and solved enterprise informatization problems. Ullah [94] and Ashraf *et al.,* [95] proposed a novel MADM algorithm utilizing the PF MSM operators to evaluate the ERP systems' performance and address the Company Benefit Plan problem utilizing picture fuzzy information. Meanwhile, MSM proves to be a powerful aggregation method that accounts for interrelationships among multiple arguments, having some limitations. One such limitation is that MSM is unable to fully capture the interrelationships among all arguments in certain scenarios [83].

Muirhead [96] was the first person to employ the MM operator. The AM, BM, MSM as well as GM operators are all combined into the MM operator. It is an all-in-one AO that can be utilized to capture the relationships between any inputs assigned by a parameter vector of parameters that may be employed in various application situations. It is appropriate for circumstances in which all aggregated values do not depend on one another, in which any two values interact, and in which any number of values interact [97,98]. Wang *et al.,* [99] and Xu *et al.,* [83] developed a comprehensive MADAM operator such as picture fuzzy weighted MM as well as picture fuzzy weighted dual MM with PFNs for appraising financial investment risk and enterprise resource planning, respectively. Xian *et al.,* [66] established a MADM method with regard to the MM operator having the evaluation information defined in picture fuzzy linguistic values. Meanwhile, Qin *et al.,* [82] developed a set of novel operational rules as well as two power MM operators of PFNs in the framework with respect to Dempster-Shafer's theory. However, the MM operator may be challenging for decision-makers to identify an alterable vector with regard to parameters in the MM operator for aggregating attribute values [21].

The CI operator [100] is a robust tool in solving MADM as well as MAGDM problems by taking into account the interrelationships among decision criteria and utilizing a fuzzy measure [101] to express the weight of each criteria combination [102]. Many researchers used CI [103] as a sufficient substitute for the weighted arithmetic mean or OWA [104] operator to process the interacting criteria. For instance, for a picture fuzzy environment, using CI and point operator, Zhang *et al.,* [105] established a framework based on a point-CI operator to solve the problem in which diagnosis values are considered as PFNs for integrating the medical resources with respect to the whole society as well as enhancing the medical service system service efficacy. To globally mirror the interactions between criteria, Singh and Kumar [51] further defined the picture fuzzy Choquet averaging as well as picture fuzzy Choquet geometric mean operators. He suggested CI-based VIKOR for MCGDM problems having interdependent and non-commensurable criteria. By extending the power Shapley concept for fuzzy measures, Tian *et al.,* [106] suggested a new method to determine fuzzy measures of the CI, known as weighted picture fuzzy power Choquet ordered geometric operator as well as a weighted picture fuzzy power Shapley Choquet ordered geometric operator. Here, the CI can be more comprehensively processed and used as an efficient tool for modelling interaction phenomena in decision-making. It can also cope with the other AOs, such as BM, HM, MSM, and MM operators because it can consider redundant, complementary or independent characteristics among the criteria [107]. The summary of the available AOs that can consider the interrelationships among aggregated variables in picture fuzzy environments is outlined in Table 2.

Table 2
A summary of some AOs based on a picture fuzzy environment for processing the interaction between attributes

References	Year	Research Area	MADM or MAGDM	Operational rule	Whether combined with another decision-making approach/operator (No/Yes) / If Yes, name of decision-making method/operator	AOs of PFNs	Application Type
Wei [85]	2017	Communication industry	MADM	Algebraic	Bonferroni mean	Picture 2-Tuple Linguistic Weighted Bonferroni Mean (P2TLWBM), and Picture 2-Tuple Linguistic Weighted Geometric Bonferroni Mean (P2TLGBM) operators	Illustrative study
Wei [62]	2018	Enterprise resource planning	MADM	Algebraic	Heronian mean	Generalized Picture Fuzzy Heronian Mean (GPFHM) and Generalized Picture Fuzzy Weighted Heronian Mean (GPFWHM) operators	Illustrative study
Wang *et al.,* [116]	2018	Energy efficiency retrofit	MAGDM	Algebraic	Bonferroni mean + MABAC	Picture Fuzzy Normalized Weighted Bonferroni Distance (PFNWBD) operator	Case study
Zhang *et al.,* [88]	2018	Enterprise resource planning	MADM	Dombi	Heronian mean	PPFDHM, PFDWHM, PFDGHM and PFDWGHM operators	Illustrative study
Zhang *et al.,* [105]	2018	Medical service efficiency	MADM	Algebraic	Choquet Integral	Picture Fuzzy Point Choquet Averaging (PFPCA), Picture Fuzzy Point Choquet Geometric (PFPCG), Generalized Picture Fuzzy Point Choquet Averaging (GPFPCAF), and Generalized Picture Fuzzy Point Choquet Geometric (GPFPCGF) operators	Case study
Wei [62]		Enterprise resource planning	MADM	Algebraic	Heronian mean	GGPFHM operator and GPFWHM operator	Illustrative study
Luo and Xing [21]	2019	Hotel selection	MADM	Algebraic	Heronian mean	Picture fuzzy Interaction partitioned Heronian Mean (PFIPHA), Picture	Case study

						Fuzzy Weighted Interaction Partitioned Heronian Mean (PFWIPHA), Picture Fuzzy Interaction Partitioned Geometric Heronian Mean (PFIPGHA), and Picture Fuzzy Weighted Interaction Partitioned Geometric Heronian Mean (PFWIPGHA) operators	
Feng and Geng [92]	2019	Enterprise resource planning	MAD M	Archimede an	Maclaurin symmetric mean	Picture 2-Tuple Linguistic Maclaurin Symmetric Mean (2TLMSM), Picture 2-Tuple Linguistic Generalized Maclaurin Symmetric Mean (2TLGMSM), Picture 2-Tuple Linguistic Weighted MSM (2TLWMSM), and Picture 2-Tuple Linguistic Weighted Generalized MSM (2TLWGMSM) operators	Illustrative study
Wang *et al.,* [99]	2019	Financial investment risk	MAD M	Algebraic	Muirhead Mean	PPFWMM and PFWDMM operator	Illustrative study
Tian *et al.,* [106]	2019	Investment project	MAD M	Algebraic	Choquet Integral	Weighted Picture Fuzzy Power Choquet Ordered Geometric (WPFPCOG), as well as Weighted Picture Fuzzy Power Shapley Choquet Ordered Geometric (WPFPSCOG) operators	Illustrative study
Qin *et al.,* [65]	2020	Enterprise resource planning	MAD M	Archimede an	Maclaurin symmetric mean	Picture Fuzzy Archimedean Power Maclaurin Symmetric Mean (PFAPMSM) and Picture Fuzzy Archimedean Power Weighted Maclaurin Symmetric Mean (PFAPWMSM) operators	Illustrative study
Qin *et al.,* [82]	2020	Enterprise resource planning	MAD M	Algebraic	Muirhead mean + Power average + Dempster Shafer theory	Picture Fuzzy Muirhead Mean Dempster-Shafer Theory (PFPMMDST) and Picture Fuzzy Weighted Power Muirhead Mean Dempster-Shafer Theory (PFWPMMDST) operators	Illustrative study

Singh and Kumar [51]	2020	Supplier selection	MAGDM	Algebraic	Choquet Integral+VIKOR	Picture Fuzzy Choquet Integral-based VIKOR method	Illustrative study
Ateş and Akay [64]	2020	Enterprise resource planning	MADM	Algebraic	Bonferroni mean	PPFBM, Picture Fuzzy Normalized Weighted Bonferroni Mean (PFNWBM), as well as Picture Fuzzy Ordered Weighted Bonferroni Mean (PFOWB) operators	Illustrative study
Yang and Li [63]	2021	Enterprise resource planning	MADM	Einstein	Bonferroni mean	Picture Hesitant Fuzzy Normalized Weighted Bonferroni Mean (PHFNWBM) operator	Illustrative study
Li and Yang [89]	2021	Enterprise resource planning	MADM	Hamacher	Heronian mean + Power average	Picture Fuzzy Hamacher Weighted Power Improved Generalized Heronian Mean (PFHWPIGHM), as well as Picture Fuzzy Hamacher Weighted Geometric Power Improved Generalized Heronian Mean (PFHWGPIGHM) operators	Illustrative study
Mahmood *et al.,* [58]	2021	Company Performance Evaluation	MAGDM	Algebraic	Bonferroni mean	PPHFWBM and Picture Hesitant Fuzzy Weighted Geometric Bonferroni Mean (PHFWGBM) operators	Illustrative study
Ullah [94]	2021	Enterprise resource planning	MADM	Algebraic	Maclaurin symmetric mean	Picture Fuzzy Maclaurin Symmetric Mean (PFMSM), Picture Fuzzy Weighted Maclaurin Symmetric Mean (PFWMSM), Picture Fuzzy Dual Maclaurin Symmetric Mean (PFDMSM), and Picture Fuzzy Weighted dual Maclaurin Symmetric Mean (PFWDMSM) operators	Illustrative study
Xian *et al.,* [66]	2021	Bike design selection	MADM	Algebraic	Muirhead Mean	Picture Fuzzy Linguistic Weighted Muirhead Mean (PFLWMM) and Picture Fuzzy Linguistic Weighted Dual Muirhead Mean (PFLWDMM) operators	Illustrative study
Fan *et al.,* [87]	2022	Intelligent logistics performance evaluation	MADM	Algebraic	Heronian mean	Interval-Valued Picture Fuzzy Geometric Heronian Average Mean (IVPFGHM), Interval-Valued Picture Fuzzy	Case study

						Geometric Weighted Heronian Average Mean (IVPFGWHM), as well as Dynamic Interval-Valued Picture Fuzzy Geometric Weighted Heronian Average Mean (DIVPFGWHM) operators	
Lin *et al.,* [86]	2022	Quality services efficiency	MADM	Algebraic	Heronian mean	Picture Fuzzy Interactional Weighted Partitioned Heronian Mean (PFIWPHM) and Picture Fuzzy Interactional Weighted Partitioned Geometric Heronian Mean (PFIWPGHM) operators	Illustrative study
Chen and Ye [93]	2022	Enterprise informatization	MAGDM	Schweizer–Sklar	Maclaurin symmetric mean	The Hesitant Picture Fuzzy Schweizer–Sklar Weighted Maclaurin Symmetric Mean (HPFSSWMSM) and Hesitant Picture Fuzzy Schweizer–Sklar-Weighted Dual Maclaurin Symmetric Mean (HPFSSWDMSM) operators	Illustrative study
Ashraf *et al.,* [95]	2022	Company benefit plan	MADM	Algebraic	Maclaurin symmetric mean	Interval-Valued Picture Fuzzy Maclaurin Symmetric Mean Operator (IVPFMSM) operator	Illustrative study

5.3 Distribution Paper Based on Journal Name

Table 3 provides a comprehensive overview of the distribution of relevant papers employed for this review, focusing on PFAOs. The selected papers were sourced from 48 distinguished international scholarly journals, specifically focusing on MADM and MAGDM issues, extracted from Web of Science as well as Scopus databases. Among these journals, the Journal of Intelligent & Fuzzy Systems emerged as the top contributor, with a total of 25 papers related to PFAOs, indicating its prominent position in this research area. Following closely, the Journal of Soft Computing secured the second position with 15 papers, and the Journal of Symmetry ranked third with 13 papers. The Journal of the Institute of Electrical and Electronics Engineers Access secured the fourth position with ten papers. In addition, the International Journal of Fuzzy Systems and the Journal of Computers and Industrial Engineering occupied the fifth and sixth ranks with nine and five papers, correspondingly. The Journal of Computational and Applied Mathematics claimed the seventh rank, featuring four papers. Notably, the Journal of Cognitive Computing, Granular Computing, Informatica, International Journal of Intelligent Systems, Journal of Mathematics, and Mathematical Problems in Engineering shared the eighth rank, each contributing three papers to this field. The frequency of papers published in other notable journals is displayed in Table 3, offering a comprehensive insight into the diverse research dissemination in the realm of PFAOs.

Table 3
Distribution of papers based on the name of journals

Name of Journal	The number of papers published	Percentage (%)
Journal of Intelligent & Fuzzy Systems	25	17.86%
Soft Computing	15	10.71%
Symmetry	13	9.29%
Institute of Electrical and Electronics Engineers Access	10	7.14%
International Journal of Fuzzy Systems	9	6.43%
Computers & Industrial Engineering	5	3.57%
Computational and Applied Mathematics	4	2.86%
Cognitive Computing	3	2.14%
Granular Computing	3	2.14%
Informatica	3	2.14%
International Journal of Intelligent Systems	3	2.14%
Journal of Mathematics	3	2.14%
Mathematical Problems in Engineering	3	2.14%
Applied Soft Computing	2	1.43%
Artificial Intelligent Review	2	1.43%
Expert Systems with Applications	2	1.43%
Journal of Cleaner Production	2	1.43%
Mathematical Biosciences and Engineering	2	1.43%
Mathematics	2	1.43%
Technological and Economic Development of the Economy	1	0.71%
Mathematics and Statistics	1	0.71%
Mathematical Modelling of Engineering Problems	1	0.71%
RAIRO - Operations Research	1	0.71%
AIMS Mathematics	1	0.71%
Arabian Journal for Science and Engineering	1	0.71%
Bulletin of the Brazilian Mathematical Society	1	0.71%
Complexity	1	0.71%
Engineering Applications of Artificial Intelligence	1	0.71%
Entropy	1	0.71%
European Transport Research Review	1	0.71%
Expert Systems	1	0.71%
Frontiers in Public Health	1	0.71%
Funding Information	1	0.71%
International Journal of Computers Communications & Control	1	0.71%
International Journal of Environmental Research and Public Health	1	0.71%
International Journal of Intelligent Computing and Cybernetics	1	0.71%
International Journal of Knowledge-Based and Intelligent Engineering Systems	1	0.71%
International Journal of Machine Learning and Cybernetics	1	0.71%
International Journal of Environmental Research and Public Health	1	0.71%
Italian Journal of Pure and Applied Mathematics	1	0.71%
Journal of Function Spaces	1	0.71%
Mathematical Sciences	1	0.71%
PLOS ONE	1	0.71%
Reports in Mechanical Engineering	1	0.71%
Scientific Programming	1	0.71%
Separation and Purification Technology	1	0.71%
Sustainable Cities and Society	1	0.71%
Kybernetes	1	0.71%

5.4 Distribution Paper Based on Publication Year

Prominent growth in the number of papers published related to PFAOs was discovered from 2017 to 2022. From six articles in 2017, the yearly paper published increased to 15 papers in 2018. A further increment to 23 papers was observed in 2019 and remained at 23 in 2020. There is an approximately 5.67-fold increase in PFAO papers published during the 2017-2022 period. As the progress, there is an anticipated rise in the publication of papers pertaining to PFAOs in the coming years. Figure 2 illustrates the cumulative count of PFAO-related publications for each year, indicating a potential upward trend in research and scholarly contributions on this subject.

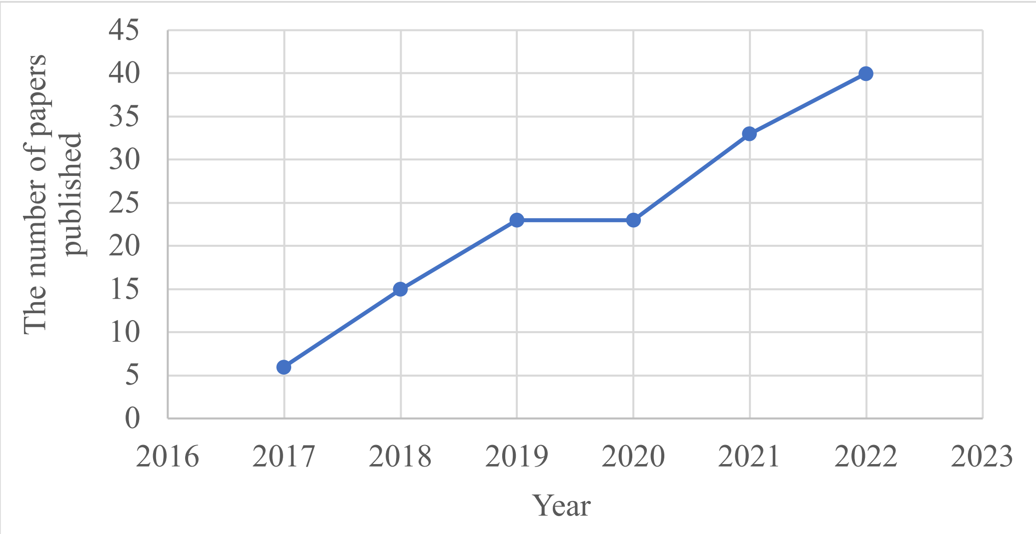

Fig. 2. Distribution papers based on year of publication (cumulative)

5.5 Distribution of Papers Based on the Nationality of Authors

Table 4 offers insights into the utilization of PFAOs across various application areas in 10 different countries. The data suggest that this technique is gaining popularity, particularly in developing and newly industrialized nations. Notably, China stands out as the leading contributor, with 71 papers (50.71%) on PFAOs. Pakistan, India, and Serbia have also made notable contributions, with 27, 20, and 6 publications, respectively, covering several application domains. The details concerning the nationality of the authors are presented in Table 4.

Table 4
Distribution of papers based on the author's nationality

Name of Country	The number of papers published	Percentage (%)
China	71	50.71%
Pakistan	27	19.29%
India	20	14.29%
Serbia	6	4.29%
Turkey	5	3.57%
Thailand	4	2.86%
Iran	3	2.14%
Malaysia	2	1.43%
Spain	1	0.71%
USA	1	0.71%

6. Conclusions

This comprehensive review study aimed to meticulously assess and categorize research papers that employed PFAOs across various application areas. The review encompassed papers published from 2017 to 2022, sourced from 48 renowned international journals indexed in Web of Science and Scopus. Analysing the distribution of PFAOs applications over the years, a noticeable trend emerged, with the application of these operators witnessing a steady and substantial increase annually. Remarkably, 85% of articles in this field were published within the last six years, indicating a surge of interest and attention towards this innovative methodology. Furthermore, articles from the most recent two years accounted for approximately 43% of the total, suggesting a promising trajectory for the future and continued advancements in this area. Examining the geographical distribution of research contributors, the study established that PFAOs had been applied by researchers from 10 different countries. China emerged as the frontrunner, boasting the highest number of applications across various fields, with an impressive count of 71 publications on PFAOs. Following closely, Pakistan demonstrated a significant presence with 27 publications, while India showcased a notable contribution with 20 papers in this realm. This review study sheds light on the growing significance of PFAOs in diverse domains and underlines the increasing interest and involvement of researchers worldwide. It opens up avenues for further exploration and potential advancements in the utilization and development of PFAOs in a myriad of applications in the near future.

In the real-world evaluation process, there are different formats of decision makers' opinion expression, for instance, yes, abstain, no, and refusal. The ordinary FS and IFS cannot be applied to the variety of decision-makers' preferences. To deal with this kind of situation and to develop a concept which is sufficiently close to human nature, in 2013, Cuong and Kreinovich [23] proposed PFSs to extend the FS and IFS theory by utilizing four membership functions, comprising the membership, hesitancy membership, non-membership and refusal degrees. AOs are used to consolidate individual evaluations or preferences into a collective or synthesized result. These operators serve a crucial role in the decision-making process by aggregating information from MCDM. Here, in context or picture fuzzy environments, PFAOs are among the popular MADM as well as MAGDM techniques, which have been improved and implemented by researchers to solve problems in the actual world. Recently, some former scholars controlled the PFAOs in a variety of study regions, for instance, enterprise resource planning [64], investment projects [74], quality services [85], financial performance evaluation [112], and so on. However, it was observed that there is a limitation in the application area in evaluating performance appraisal or similar issues that fall under Management Science problems.

In the realm of MADM and MAGDM, the significance of considering the interaction among attributes cannot be overlooked. Real-world decision-making scenarios often involve complex relationships and dependencies between various attributes, making it essential to incorporate these interactions into the decision-making process. However, upon conducting an in-depth review of the existing literature, it becomes evident that the research on PFAOs and their role in capturing attribute interactions in the picture fuzzy environment is limited. Meanwhile, some studies have touched upon this topic. Nevertheless, there remains a notable gap in comprehensive reviews and systematic analyses of the implementation of PFAOs in extension or hybridization, particularly concerning Einstein's ORs. The scarcity of such research hampers the development and practical application of PFAOs in solving real-world MAGDM problems. Without a thorough understanding of how PFAOs can effectively capture attribute interactions and be integrated into the picture fuzzy decision-making process, decision-makers may lack a robust tool to navigate the complexities of multi-attribute decision scenarios.

Thus, the significance of conducting further research in this area cannot be emphasized enough. A comprehensive examination of the role of PFAOs in capturing attribute interactions, particularly in conjunction with Einstein operational rules, can pave the way for more effective and accurate decision-making methodologies. By delving into practical applications of PFAOs in the picture fuzzy environment, researchers and practitioners can uncover valuable insights that may lead to innovative and enhanced solutions for addressing MAGDM challenges. In summary, it is essential to recognize the critical need for in-depth investigations and comprehensive reviews of PFAOs and their implementation in the context of attribute interactions in the picture fuzzy environment. Addressing this research gap can unlock the full potential of PFAOs and propel the advancement of robust decision-making techniques in the domain of MADM and MAGDM.

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