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# Mathematical Modeling on Integrated Vehicle Assignment and Rebalancing in Ride-hailing System with Uncertainty Using Fuzzy Linear Programming

Tubagus Robbi Megantara[1], Sudradjat Supian[1,*], Diah Chaerani[1]

[1,] Department of Mathematics, Faculty of Mathematics and Natural Sciences, Universitas Padjadjaran, Sumedang, Indonesia

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**ABSTRACT**

The general public frequently uses taxis as local transportation to get from one location to another. Ride-hailing is an innovation in taxi services that lets customers use their smartphones to find drivers, find prices, and submit requests. The two parts of ride-hailing are vehicle assignment and rebalancing. The task of the assignment is to allocate resources as efficiently as possible to fulfil demand. Demand and supply in the area are brought into balance through the rebalancing process. The rebalancing step is frequently carried out independently of the assignment process in ride-hailing pickup systems. However, it can be integrated into a single optimization process to improve system performance. The linear programming approach can integrate assignment and rebalancing. The batch assignment is an assignment algorithm in which each supply and demand is collected within a specific time window. The assignment is done after the vehicles and requests are collected. Fuzzy linear programming is used to deal with environmental uncertainty. Uncertain demand can reduce the reliability of ride-hailing pickup systems in addressing the problem of allocating empty vehicles to areas of high demand. The pickup time may change over time due to traffic conditions. Assignment integration and rebalancing modelling with fuzzy parameters are carried out to obtain a ride-hailing pickup system model that can handle demand uncertainty, pickup travel time, and travel delay times and increase the ride-hailing effectiveness of the pickup system. Numerical simulations were carried out on the assignment and rebalancing integration model without and with uncertainty parameters—numerical simulations based on publicly available taxi travel request data. The model selection for the ride-hailing system can be determined based on the numerical simulation results. This research can potentially advance and expand knowledge, especially in transportation.

## 1. Introduction

The development of science and technology has developed the public transportation system, resulting in the creation of internet-based public transportation that operates in real-time. Internet-based public transportation stems from telephone-based public transportation services whose service requests are made by telephone [1]. Assignment and dynamic pricing, part of the pickup

[*] *Corresponding author.*
*E-mail address: sudradjat@unpad.ac.id*

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system, are two crucial aspects of ride-hailing [2]. The transportation model's particular case of the assignment problem uses people to represent resources and jobs to represent goals [3]. The best way to allocate personnel to complete specific tasks is through the assignment. The most crucial element of the ride-hailing system is the assignment method, which motivates researchers to create an assignment model [2].

Assignments given to drivers to fulfill requests can be done using simple algorithms, such as the first-dispatch protocol in ride-hailing systems. The first-dispatch protocol assigns available vehicles to fulfill requests immediately to minimize travel time to the requested location. However, a sudden over-demand at a location can lead to a bad experience on the driver and passenger side. This happened because the available driver was immediately dispatched to fulfill the request. As a result, vehicle inventories are low and cause newly available vehicles to be shipped immediately to meet distant demand. This phenomenon was discussed by [4] under the name Wild Goose Chase (WGC). An alternative approach to minimize the occurrence of WGC is discussed by [2], namely batch assignment. Batch assignment works by accommodating requests in advance within a specific time window. Furthermore, assignments are made to vehicles to pick up requests that are accommodated during a predetermined time window. Unfulfilled requests will enter the next group of requests.

Surges in demand can cause a considerable increase in the waiting time required for passengers to get a ride. The average reduction in waiting time can be reduced by applying a batch-assignment approach [5]. Assignments generated by batch assignments are considered more efficient by aggregating requests and making group decisions [6]. However, collecting requests within a long window can result in longer waiting times [7]. Therefore, determining the time window must be considered to make the system more efficient.

In addition to applying batching to assignments, rebalancing strategies can play an essential role in minimizing the occurrence of WGC. The rebalancing problem is an assignment problem whose role is to assign the remaining vehicles to other areas based on estimated future demand [8]. The purpose of rebalancing is to minimize the possibility of an imbalance of resources and demand in the future. Rebalancing is also considered to improve system performance substantially, so it is often a component of ride-hailing pickup systems.

Alonso-Mora *et al.,* [9] proposed a rebalancing approach that, after the assignment process, allocates empty vehicles to satisfy unmet demand. A rebalancing model suggested by Mao *et al.,* [10] considers the price of reallocating vacant vehicles to areas with unmet demand. By considering the likelihood of reallocating vehicles based on unmet demand in a zone, Castagna *et al.,* [11] offer a rebalancing model that distributes empty vehicles to neighboring zones. A rebalancing mechanism that allocates accessible automobiles to fill insufficient demand in other zones is suggested by Guo *et al.,* [8].

The ride-hailing pickup systems' components (assignment and rebalancing) are frequently studied individually [12]. Assignment and rebalancing issues can be combined into a single optimization model that functions separately to create a ride-hailing pickup system [13]. It leads to more research in this area because the integration of assignment and rebalancing in one optimization model has not been commonly done [8]. In order to develop research on transportation, particularly the ride-hailing system, and enhance its performance of the ride-hailing system, assignments need to be integrated and rebalanced.

Assignment and rebalancing problems can be formulated into linear programming problems. Linear programming is an analytical planning technique using a mathematical model to find several alternative combinations of optimum solutions to the problems encountered [3]. Linear programming is generally concerned with allocating limited resources. In the real world, linear programming is often used to deal with problems of urban planning, currency arbitrage, investment, production planning, inventory control, oil blending and refining, and workforce planning.

Linear programming problems generally assume that the parameters are known precisely. However, situations in formulating real-life problems may be where the variables and parameters may not be known precisely. Linear programming problems with unknown decision variables or parameters play a significant role in several applications, such as transportation management [14]. Uncertainties in the parameters of linear programming problems can be handled by fuzzy approaches, known explicitly as Linear Fuzzy Programming (FLP) for single-objective functions and Multi-objective Linear Fuzzy Programming (FMOLP) for multiple-objective functions.

Fuzzy linear programming is a linear programming model combined with the concept of fuzzy logic. The concept of fuzzy logic is used to assist decision-making in determining optimal results by considering the uncertainty of the decision parameters or variables. Fuzzy linear programming can be divided into three groups based on the uncertainty in the model [15]: linear programming with fuzzy variables, linear programming with fuzzy parameters, and linear programming with fuzzy variables and fuzzy variables parameters.

Assignment problem with uncertainty parameters is discussed by [16]. Additionally, the rebalancing and assignment models are combined into one model. No one has considered the unknown travel time in the study on integrated vehicle assignment and rebalancing models that have been described, especially those that are solved using the fuzzy technique. The methodology described by [17] as [16] is then used to resolve the linear programming model with fuzzy parameters. The utilization of batch assignments minimizes post-assignment request imbalance.

Castagna *et al.,* [11] proposed an assignment model that maximizes the total reward using the reinforcement learning method. Using the expectation maximization approach, The rebalancing model focuses on maximizing the association of vehicle relocation probability. The proposed assignment and rebalancing are not integrated, and no parameters are assumed to be uncertain. The performance indicators are service level, waiting time, passenger distribution, and vehicle travel distance.

Mao, Liu, and Shen [10] proposed an assignment model that minimizes waiting time using a Reinforcement Learning approach. The rebalancing model focuses on minimizing the cost of relocating empty vehicles using a reinforcement learning approach. The proposed assignment and rebalancing are not integrated, and no parameters are assumed to be uncertain. The performance indicators used are waiting times and cancellation rates.

Alonso-Mora *et al.,* [9] proposed an assignment model that minimizes trip travel delays and penalties for unfulfilled requests. The rebalancing model used focuses on minimizing pickup times. The proposed assignment and rebalancing are not integrated, and no parameters are assumed to be uncertain. The performance indicators are fleet size, vehicle capacity, service level, waiting time, trip delays, and operational costs.

Guo, Caros, and Zhao [8] proposed an assignment model that minimizes the pickup distance. The rebalancing model used focuses on minimizing penalties for unfulfilled requests. The proposed assignment and rebalancing are integrated, and parameters are assumed to be uncertain. Uncertain parameters are overcome by using robust optimization methods.

All modeling is done without considering travel time and delay time uncertainty. It can be seen that there are no studies that use the fuzzy method as an optimization method. Generally consider the reward, cost, pickup distance, pickup time, and waiting time on the model destination. The model's performance is measured by comparing the primary reference and proposed models using service level, waiting time, cancellation rate, fleet size, vehicle capacity, service level, waiting time, trip delays, and operational costs.

Rebalancing can be done by dividing the area into several zones using clustering and assigning available vehicles to other zones considering demand [10,11]. In general, zone clustering can be done using two approaches: dynamic and static [8,10,11]. The primary difference between dynamic and static clustering is that the zone changes over time can only be obtained using dynamic clustering. Using a rebalancing strategy with a dynamic clustering approach provides a fairer workload without compromising performance related to waiting times and distribution of passengers per vehicle [11].

Vehicle utilization performance can be improved by implementing a ride-sharing scheme [9]. Tolerance limits for travel delays impact increasing service rates, waiting times, and vehicle mileage [9]. A strategy that considers the cancellation of orders by users increases the total waiting time of all passengers but decreases the total waiting time of "impatient passengers" [10]. Combining assignment and rebalancing components in a single optimization model that considers demand uncertainty performs better than the nominal model [8].

Assignment integration and rebalancing models have not been widely used in the articles studied. The model that considers the uncertain pickup time has not appeared in the article reviewed. This is because the research did not consider the uncertainty factor except for [8]. However, in their research, [8] only consider the uncertainty of the request, not the pickup time. Therefore, this study considers pickup times and uncertain requests. This study uses a linear programming approach with fuzzy parameters to solve optimization problems with uncertainty.

The main objective of this research is to form a linear programming model that can handle the uncertainty of parameters in the ride-hailing pickup system. The specific objectives of this research are:

i. Integrate the assignment and rebalancing model for the ride-hailing pickup system in a single model.
ii. Propose a linear programming model for assignment integration and rebalancing problems that can handle parameter uncertainty using a fuzzy approach.
iii. The proposed models with or without parameter uncertainty are compared through numerical experiments based on optimality criteria.

## 2. Methodology

### *2.1 Materials*

The object of this research is the assignment model discussed by [9] and the rebalancing model discussed by [13]. The assignment and rebalancing models are combined into a single linear programming model. The linear programming solution with fuzzy parameters discussed by [17] is then used to handle demand uncertainty and travel time in the ride-hailing system.

Manhattan, New York City taxi trip data in 2013 provided by [18] is popular public data used to simulate taxi demand. This data is published to assist researchers in researching traffic, taxi travel, and other transportation topics. This data is often used to demonstrate the model's reactivity and quality to real-world real-time taxi demand. Information regarding the origin and destination of passengers (time and distance traveled to take passengers to their destination) is included in the data. The time when the passenger requests a ride is represented by when the passenger is in the taxi. The taxi travel criteria used are as follows: (i) The origin and destination of the taxi are within the Manhattan area; (ii) The vehicle has a minimum capacity of zero and a maximum capacity of four. Vehicle capacity is checked by the number of passengers in the taxi plus the driver.

The shortest path between locations has been calculated using OSMnx and stored in the pivot data. OSMnx is a Python package for downloading OpenStreetMap geospatial data and modeling, projecting, visualizing, and analyzing real-world road networks [19]. OSMnx can provide complex road network mapping specific to the Manhattan, New York City location. Coordinates of origin and destination are needed to find the shortest path and the appropriate time and distance.

In addition, the shortest path can be calculated using several weights, such as travel time and distance. The travel time obtained from the path with the shortest time depends on the default vehicle speed of the OSMnx road network. In contrast, the travel time obtained from the path with the shortest distance depends on the constant speed set.

### *2.2 Mathematical Modeling*

The decision taken by the ride-hailing system to assign available vehicles to pick up passengers is determined by the optimal assignment of each alternative assignment decision. Each vehicle is assigned to pick up passengers according to the travel delay. The period between when a passenger calls for a ride and when the vehicle arrives at the passenger's location is the travel delay time. The time passengers must wait before getting in the car is sometimes called the travel delay time.

The problem of real-time ride-hailing pick-up systems that make pick-up decisions can be resolved with mathematical modelling. Mathematical modelling can be used to handle the ride-hailing pick-up system problem, which is modelled as an assignment problem. Decision variables, constraints, and objective functions are the essential elements of the assignment model.

#### *2.2.1 Integrated Vehicle Assignment and Rebalancing Model Formulation*

A model for the ride-hailing pick-up system is being developed that considers the remaining vehicles' rebalancing to reduce the supply-demand mismatch. The modelling is done by considering the assignment model stated by [9] and the rebalancing system discussed by [13]. The vehicle assignment and rebalance integration model uses the following set, parameters, and decision variables

**Sets:**

| | | |
|---|---|---|
| $\boldsymbol{G}$ | : | Set of regions |
| $\boldsymbol{\mathcal{R}}$ | : | Set of requests |
| $\boldsymbol{\mathcal{V}}$ | : | Set of vehicles |

**Parameters:**

| | | |
|---|---|---|
| $\rho$ | : | The level of excess inventory of vehicles |
| $\mathcal{H}$ | : | Time window of the rebalancing process |
| $M$ | : | Large constant as a penalty for ignoring a request |
| $t_{max}$ | : | Maximum tolerated travel delay time |
| $\lambda_g$ | : | Level of requests in region $g$ |
| $t_{i,j}$ | : | The time of delay passenger $i$ when the request is served by vehicle $j$ |
| $\tau_{j,g}$ | : | Travel time is taken by vehicle $j$ to reach the center of region $g$ |

**Decision Variables:**

| | | |
|---|---|---|
| $x_{i,j}$ | : | The decision variable that states whether vehicle $j$ is assigned to accept request $i$ or not (1/0) is denoted by equation (1) |
| $\chi_i$ | : | The decision variable that states whether request $i$ is rejected or not (1/0) is denoted by equation (2) |
| $y_{j,g}$ | : | The decision variable that states whether vehicle $j$ is assigned to go to the center of region $g$ or not (0/1) is expressed by equation (3) |

$$x_{i,j} \in \{0,1\} \tag{1}$$

$$\chi_i \in \{0,1\} \tag{2}$$

$$y_{j,g} \in \{0,1\} \tag{3}$$

The assignment model for the pick-up system needs to be considered: travel delay time, servicing request, and inventory control. The objective function that minimizes the total travel delay time and the total unserved requests is given by

$$\min \sum_{i\in\mathcal{R}} \sum_{j\in\mathcal{V}} t_{i,j} x_{i,j} + \sum_{i\in\mathcal{R}} M\chi_i \tag{4}$$

The objective function that maximizes the expected number of requests that will be served by the remaining vehicles is given by

$$\max \sum_{j\in V_r} \sum_{g\in G} y_{j,g} \lambda_g (\mathcal{H} - \tau_{j,g}). \tag{5}$$

Ride-hailing does not allow the vehicle to fulfill many requests simultaneously. Vehicles not assigned to fulfill the request will be directed to stay in place or to another place to rebalance the vehicle inventory. Constraints that ensure that each vehicle can only receive at most one request or are directed to balance the vehicle supply in an area are given by

$$\sum_{i\in\mathcal{R}} x_{i,j} + \sum_{g\in G} y_{j,g} \leq 1, \forall j \in \mathcal{V} \tag{6}$$

The assignment must ensure that only one vehicle can serve a request. Constraints that ensure that each request can only be received by at most one vehicle or ignored by each vehicle are given by

$$\chi_i + \sum_{j\in\mathcal{V}} x_{i,j} = 1, \forall i \in \mathcal{R}. \tag{7}$$

Constraints that guarantee that the travel delay time does not exceed a predetermined limit, given by

$$\sum_{j\in\mathcal{V}} t_{i,j} x_{i,j} \leq t_{max}, \forall i \in \mathcal{R}. \tag{8}$$

Vehicles assigned to balance inventory in an area must have a travel time that can be traveled during time $\mathcal{H}$, given by

$$y_{j,g}(\mathcal{H} - \tau_{j,g}) \geq 0, \forall j \in \mathcal{V} \text{ and } \forall g \in G. \tag{9}$$

The vehicle inventory balance must still take into account the expected future demand in the next batch to reduce the occurrence of excess inventory, given the following constraints

$$\sum_{j\in\mathcal{V}} y_{j,g}\left(\mathcal{H}-\tau_{j,g}\right) \leq \lambda_g \mathcal{H}^2 \rho, \forall g \in G \tag{10}$$

The integration model of assignment and balancing of vehicle inventory can be written as: objective functions (4) and (5); constraints (6), (7), (8), (9) and (10); decision variables (1), (2) and (3). The model is classified as linear programming with multi-objectives based on the function and decision variables.

### *2.2.2 Integrated Vehicle Assignment and Rebalancing Model Formulation with Fuzzy Parameters*

This study assumes that the parameters of travel time, travel delay time, and requests in each region are fuzzy. The pick-up has an uncertain time of travel delay caused by the uncertain pick-up travel time, which depends on unpredictable traffic conditions. In addition, the demand in each region is also uncertain because it can change over time. Linear programming with fuzzy parameters is solved by reformulating the model into parametric linear programming with crisp parameters.

The fuzzy parameters are travel delay time $\tilde{t}_{i,j}$, travel time $\tilde{\tau}_{j,g}$, and many requests $\tilde{\lambda}_g$ located at (4), (5), (8), (9), and (10). Assume that a trapezoidal fuzzy number can represent the fuzzy parameter. The crisp assignment and rebalancing integration model that can handle the fuzzy trapezoidal parameters $t_{i,j}, \tau_{j,g}, \lambda_g$ is as follows

$$\min\left(u+\sum_{i\in\mathcal{R}} M\chi_i\right) \tag{11}$$

$$\max v \tag{12}$$

Subject to:

$$\sum_{i\in\mathcal{R}} x_{i,j} + \sum_{g\in G} y_{j,g} \leq 1, \forall j \in \mathcal{V} \tag{13}$$

$$\chi_i + \sum_{j\in\mathcal{V}} x_{i,j} = 1, \forall i \in \mathcal{R} \tag{14}$$

$$(1-\alpha)\sum_{j\in\mathcal{V}} E_2^{t_{i,j}} x_{i,j} + \alpha\sum_{j\in\mathcal{V}} E_1^{t_{i,j}} x_{i,j} - t_{max} \leq 0, \forall i \in \mathcal{R} \tag{15}$$

$$(\alpha-1) y_{j,g}\left(\mathcal{H}-E_2^{\tau_{j,g}}\right) - \alpha y_{j,g}\left(\mathcal{H}-E_1^{\tau_{j,g}}\right) \leq 0, \forall j \in \mathcal{V}, \forall g \in G \tag{16}$$

$$(1-\alpha)\sum_{i\in\mathcal{R}}\sum_{j\in\mathcal{V}} E_2^{t_{i,j}} x_{i,j} + \alpha\sum_{i\in\mathcal{R}}\sum_{j\in\mathcal{V}} E_1^{t_{i,j}} x_{i,j} - u \leq 0 \tag{17}$$

$$(\alpha-1)\left[\sum_{j\in V_r}\sum_{g\in G} y_{j,g}E_2^{\lambda_g}\left(\mathcal{H}-E_2^{\tau_{j,g}}\right)\right] -\alpha\left[\sum_{j\in V_r}\sum_{g\in G} y_{j,g}E_1^{\lambda_g}\left(\mathcal{H}-E_1^{\tau_{j,g}}\right)\right]+v\leq 0 \quad (18)$$

$$(1-\alpha)\left[\sum_{j\in\mathcal{V}} y_{j,g}\left(\mathcal{H}-E_2^{\tau_{j,g}}\right)-E_2^{\lambda_g}\mathcal{H}^2\rho\right] +\alpha\left[\sum_{j\in\mathcal{V}} y_{j,g}\left(\mathcal{H}-E_1^{\tau_{j,g}}\right)-E_1^{\lambda_g}\mathcal{H}^2\rho\right]\leq 0, \forall g\in G \quad (19)$$

$$E_1^{t_{i,j}}=\frac{t_{1_{i,j}}+t_{2_{i,j}}}{2}, E_2^{t_{i,j}}=\frac{t_{3_{i,j}}+t_{4_{i,j}}}{2}, \forall i\in\mathcal{R}, \forall j\in\mathcal{V} \quad (20)$$

$$E_1^{\tau_{j,g}}=\frac{\tau_{1_{j,g}}+\tau_{2_{j,g}}}{2}, E_2^{\tau_{j,g}}=\frac{\tau_{3_{j,g}}+\tau_{4_{j,g}}}{2}, \forall j\in\mathcal{V}, \forall g\in G \quad (21)$$

$$E_1^{\lambda_g}=\frac{\lambda_{1_g}+\lambda_{2_g}}{2}, E_2^{\lambda_g}=\frac{\lambda_{3_g}+\lambda_{4_g}}{2}, \forall g\in G \quad (22)$$

$$x_{i,j}, \chi_i, y_{j,g}\in\{0,1\} \text{ and } u, v\geq 0 \quad (23)$$

The model is classified as linear programming with multi-objectives based on the function and decision variables.

## 3. Results
### *3.1 Case Study*

The case study used Manhattan, New York City, taxi trips in 2013. Numerical simulations were carried out on the proposed model to solve real-world vehicle inventory assignment and balancing problems based on data of Manhattan taxi trips on January 31, 2013, from 12.00 to 12.30, as shown in Figure 1, with 9817 requests. Based on historical sample data, 2000 vehicles were deployed at 11.59 pm. The addition or subtraction of vehicles during the simulation is assumed to be non-existent. Based on real-world demand data, vehicles will continue to pick up and drop off passengers. Requests are collected within a 30-second time window and assigned to the available vehicles in a batch. It is assumed that each passenger can only tolerate a maximum travel delay of 5 minutes. All requests not served within the maximum travel delay time will be removed from the queue.

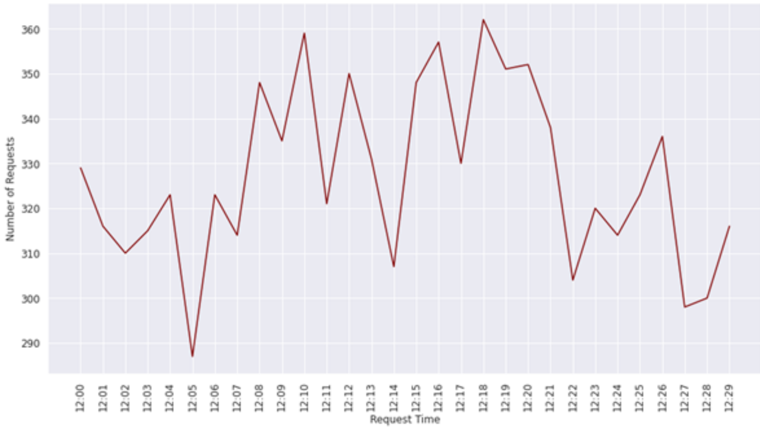


**Fig. 1.** Manhattan Taxi Trip Request

*3.2 Numerical Simulations*

Numerical simulation based on the case studies that have been described was carried out by applying the assignment model, the assignment and rebalancing integration model, and the assignment and rebalancing integration model with parameter uncertainty. The model reactively solves optimization problems that arise during the numerical simulation process based on case studies, namely 60 batches of ride-hailing pick-up problems each. The set time window is 30 seconds, meaning that the vehicle and request collection process and optimization will take place every 30 seconds.

Numerical simulation for the assignment and rebalancing integration model is carried out by setting the density of the rebalancing vehicle at 100%, i.e., ρ=1. Based on historical data, the decision maker wants to provide several vehicles to meet all possible requests for the next 30 seconds. The travel delay time based on the optimal results for the assignment and rebalancing integration model can be seen in Figure 2.

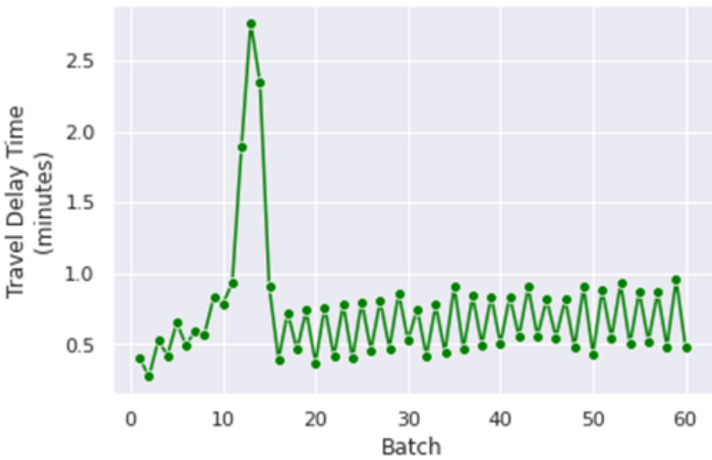


**Fig. 2.** The travel delay time of assignment integration and rebalancing models

The delay time for passengers is seen to be in the range of 0.5 to 1 minute, based on Figure 2. Travel delay times are generally far from the tolerance time of delay, which is 5 minutes. The average delay time is 0.72 minutes.

Numerical simulation for the assignment and rebalancing integration model with fuzzy parameters was carried out based on a confidence level of 0.5. That is, decision-makers have a tolerance of 50% of the feasibility of decisions due to uncertainty. The uncertain parameters, namely the delay in pick-up time, travel time, and requests, are represented by trapezoidal fuzzy numbers. Each uncertain parameter has its trapezoidal fuzzy number, which is represented by a quadruplet number (min,(min+mean)/2,(max+mean)/2, max). The travel delay time based on the optimal results for the assignment integration and rebalancing model with uncertainty can be seen in Figure 3.

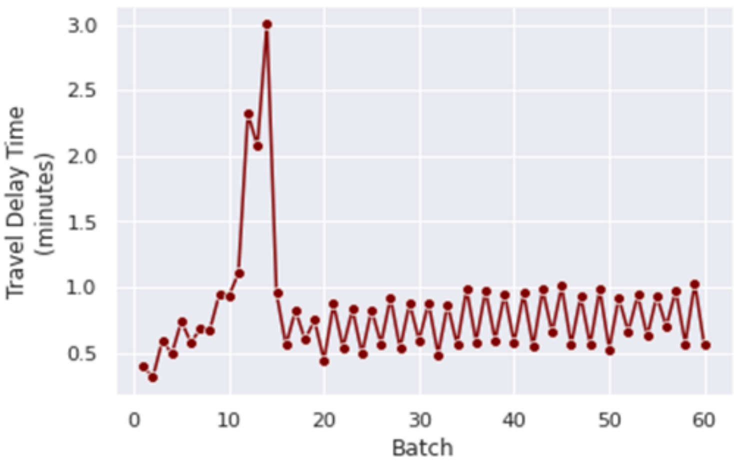


**Fig. 3.** Travel time delay of model integrated assignment and rebalancing with uncertainty parameters

The delay time for passengers is seen to be in the range of 0.5 to 1 minute, based on Figure 3. Travel delays are generally far from the delay tolerance time of 5 minutes. The average delay time is 0.82 minutes.

Travel time for the assignment and rebalancing integration model and the assignment and rebalancing integration model with uncertainty, respectively, has an average of 0.73 and 0.85 minutes. The optimal results for each model are presented in Table 1.

**Table 1**
Optimal results for each model

| Model | Service level (Percentage of requests served) | Request service |
|---|---|---|
| Integration of assignment and rebalancing | 56.98% | 5594 |
| Integration of assignment and rebalancing with uncertain parameters | 56.44% | 5541 |

Theoretically, the proposed model has reliability in overcoming the supply and demand imbalance, which is not considered by the assignment model proposed by some researchs. Traffic conditions are unpredictable, so the optimal results obtained by the assignment model and the assignment and rebalancing integration model may not be optimal in practice. This can happen when the post-optimized pick-up time does not change as calculated, thus increasing the travel delay time. Travel delays that do not match the information provided to passengers will cause inconvenience. The assignment and rebalancing integration model with fuzzy parameters can overcome this problem. Therefore, the assignment integration and rebalancing model can be used when the decision maker (ride-hailing company) wants a pick-up system that can handle the uncertainty of pick-up time, travel delay times, and requests.

## 4. Conclusions

In practice, the pick-up system has two often separate components: assignment and rebalancing. However, the two components are closely related, where rebalancing is carried out after the assignment is complete. Therefore, the assignment integration and rebalancing model is proposed in this study. Modeling is done by defining parameters, decision variables, constraints, and objective functions.

The assignment and rebalancing integration model have uncertain parameters of pick-up delay time, travel time, and demand which results in the quality of the ride-hailing pick-up system. In this study, the assignment and rebalancing integration model was then reformulated to handle the uncertainty of the parameters of pick-up delay time, travel time, and requests. The fuzzy approach is used to deal with the uncertainty of the parameters of the delay in pick-up time, travel time, and requests.

Numerical simulation results are given to compare the optimal results of each model proposed in this study. This research can be developed based on various existing obstacles. The relatively long computational time becomes a research obstacle that affects the selection of many data in numerical simulations. Data selection can reduce the representative nature of the model's reliability for dealing with real-world problems represented by numerical simulations. Long computation time can occur because this research uses basic methods to solve linear programming problems such as simplex. Heuristic methods can reduce computing time, such as the greedy algorithm [9].

Inaccurate forecasts of demand in each region can cause vehicle inventory balancing not to work optimally. The quality of demand forecasts in each region can be improved using the Spatio-temporal forecasting method, as [20] discussed.

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