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Entanglement Detection: A Scoping Review

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ABSTRACT

Quantum entanglement is a critical physical process in quantum mechanics and quantum information theory. It is a required process in quantum computing, quantum teleportation, and quantum cryptography. Entanglement detection affects the performance of quantum information processing tasks. Entanglement detection has grown in popularity over the years, and various entanglement detection methods are available, though some have application and system scale limitations. This scoping review sought to identify various measurement methods for entanglement detection in both bipartite and multipartite entanglement systems. Secondary resource indexed literatures were selected based on specific keywords from literatures published between 2017 and 2021. The goal of this study is to present a proposed conceptual framework of entanglement detection based on previous work as a guidance and reference founded on one's specific requirements.

1. Introduction

Quantum entanglement is one of the most studied aspects of quantum mechanics [1], as well as one of the most important resources in quantum information processing, as interests in this resource has been growing and utilized in various quantum technologies namely quantum computation and quantum teleportation [2-10]. Entanglement occurs when two or more objects become entangled and remain correlated despite their distance [11]. In quantum computation, quantum entanglement is critical in demonstrating the advantages and capabilities of a quantum computer over a classical computer. Quantum entanglement has piqued the interest of a wide range of stakeholders in recent years. Though quantum entanglement has been known for decades, knowledge of the subject is extremely limited, especially in higher pure and mixed qubit state dimension systems [1,12].

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Entanglement detection, which is the process of determining whether a particular state is entangled or not, has proven to be difficult, especially in high-dimensional systems [12,13]. Entanglement detection is required to fully understand the structure of a state system, making it a hot topic in quantum technology [14]. The methods for detecting entanglement are considered sorted in a low qubit-dimension pure state system but not in a mixed state system [15]. Some well-known entanglement detection methods include extended and compounded entanglement witness, mutually unbiased bases (MUBs), mutually unbiased measurements (MUMs), general symmetric informationally complete measurements (GSIC-POVMs), randomness distillation, uncertainty relations, Bell inequality with semidefinite program (SDP) hierarchy, supervised machine learning-based approach, autoencoder neural network, tangle and entanglement measures based on fidelity, quantum Fisher information, separability criterion (positive partial transpose), tensor product decomposition of the coefficient vectors (ascending lexicographical order) and permutations of qubits, controlled SWAP (c-SWAP) test, genuine entanglement detection and genuine multipartite entanglement concurrence, additive entanglement of formation (EOF), k-partite entanglement and k-nonseparability, norms of the correlation tensors and multipartite concurrence.

This study aims to identify various existing entanglement detection methods used in previous research and provide a conceptual framework of entanglement detection as a guidance for future work based on one's specific requirements. This paper presents extended and combined entanglement detection methods that cover bipartite to multipartite quantum states and low to high-dimensional systems. The paper is organized as follows: The research methodology is detailed in Section 2. The results and entanglement detection methods are discussed in Section 3. Section 4 concludes the paper.

2. Methodology

This section discusses the method for retrieving literatures on entanglement detection. The study examined literatures on entanglement detection using a scoping review, which is a method that serves as an overview, revealing the fundamentals and theories underlying a research topic [16]. Peters *et al.,* [16] proposed a scoping review that includes five key phases:

i. formulation of research questions
ii. inclusion and exclusion criteria
iii. search strategy
iv. screening and selection
v. data extraction and analysis.

2.1 Formulation of Research Question

The research questions were developed based on the key context and concept of entanglement detection methods. The following research questions directed the review:

i. What are the existing entanglement detection methods in quantum information processing?
ii. What is the best alternate entanglement detection method for specific requirements?

These questions are consistent with the research goal of determining and providing a framework for the preferable entanglement detection method for specific conditions in quantum information processing.

2.2 Inclusion and Exclusion Criteria

The study established several inclusion and exclusion criteria (see Table 1). This scoping review examined indexed literatures and book series on entanglement detection published between 2017 and 2021. The five-year time frame was chosen due to the subject's maturity, which has seen increased interest in recent years as a result of numerous studies on the topic. As the focus of this review is entanglement detection, literatures on "entanglement quantification" and "entanglement classification" were excluded. Following that, only English-written literatures were considered to avoid conceptual misunderstandings and mistranslations on the subject.

Table 1
Inclusion and exclusion criteria

Inclusion criteria	Exclusion criteria
Literatures with the subject matter of "quantum entanglement" and "entanglement detection"	Literatures written in a language other than English
Literatures in indexed journals and book series	Literatures related to subject matter of "entanglement quantification" and "entanglement classification"
	Literatures published before the year 2017

2.3 Search Strategy

The literatures selected for the study were mainly drawn from Scopus and the Web of Science (WOS). Both databases are regarded as powerful search engines in research across a wide range of disciplines. Scopus is an abstract and citation database with over 30,000 journals from over 11,000 publishers worldwide, while WOS is an established, comprehensive, high-quality database platform with over 171 million reference records. As previously stated, the selected literatures span five years, from 2017 to 2021. An extensive search was conducted using the search field tags "TITLE-ABS-KEY" (title, abstract and keywords) in Scopus and "TS" (topic) in WOS using specific keywords, namely quantum detection, quantum entanglement and quantum measurement. Figure 1 depicts the search results on Scopus and WOS.

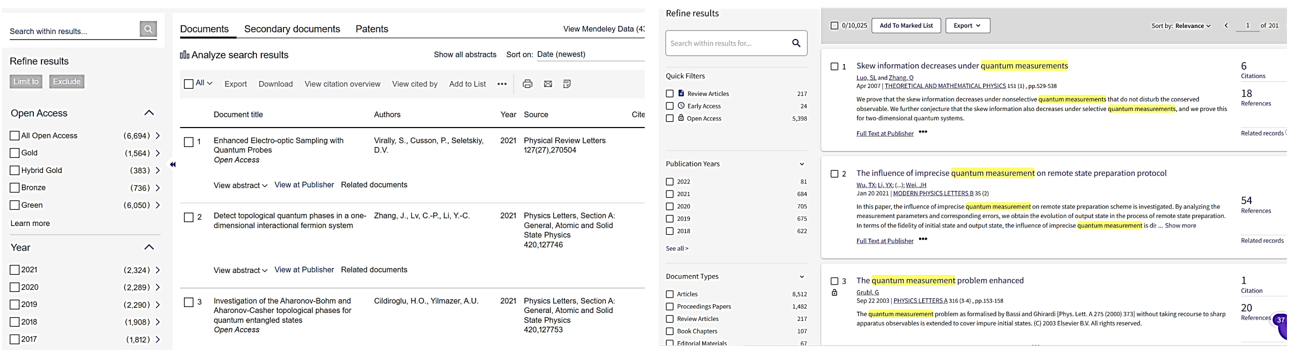

Fig. 1. Search results on Scopus and WOS

2.4 Screening and Selection

In the first stage of the review process, 1934 literatures were identified and extracted from the search results. 1847 literatures were excluded due to duplications by title reviews, and abstract reviews. In the second stage, the 87 remaining literatures were examined and validated to ensure that they met the inclusion and exclusion criteria, removing 33 publications in the process. In the third stage, the remaining 54 literatures were thoroughly reviewed again, resulting in 28 literatures being excluded by the inclusion and exclusion criteria. Finally, a total of 26 literatures are left for the study analysis. Figure 2 depicts the literature selection process.

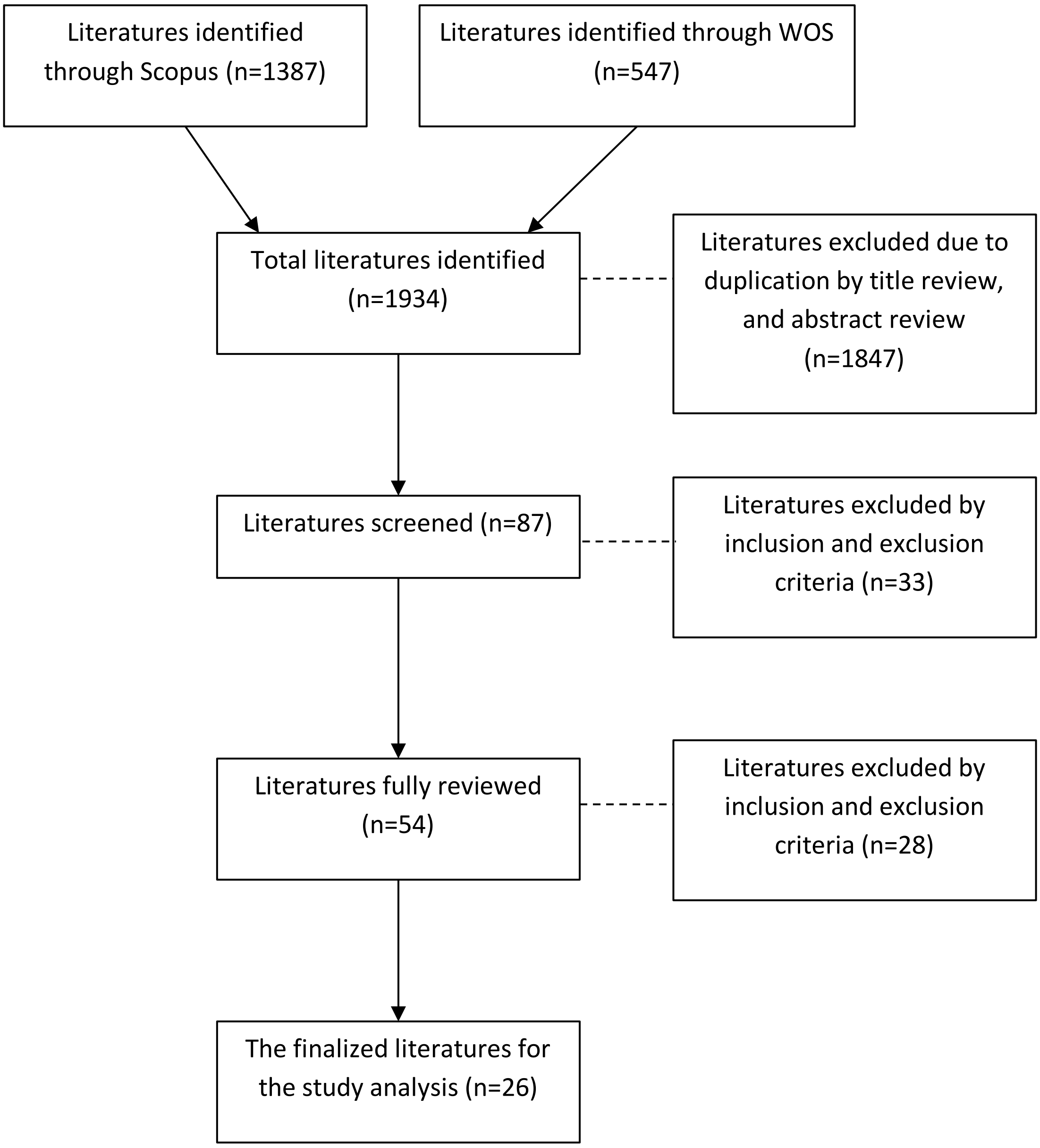

Fig. 2. Literatures selection process

2.5 Data Extraction and Analysis

The 26 selected literatures that correspond to the research questions were thoroughly examined. Through an in-depth reading of the literature, relevant and appropriate data were extracted in three steps. First, the abstract was read to see if there was any pertinent information. Following this, the conclusion and body of the literature were read.

The literatures were analysed to identify relevant themes and methods. The extracted data were tabulated in Microsoft Word. The literatures were grouped based on the year of publication (see Figure 3). One study was published in 2017, followed by four studies published in 2018, three studies were published in 2019, a total of eight studies were published in 2020 and lastly ten studies were published in 2021.

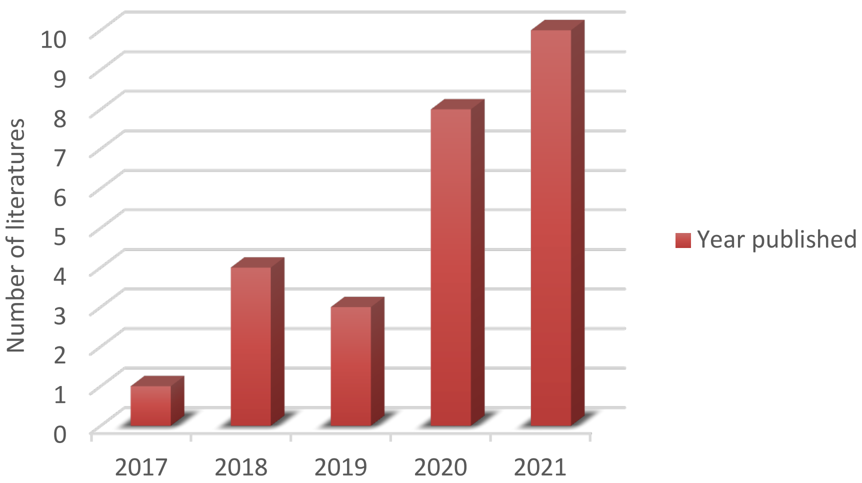

Fig. 3. The number of literatures based on the year of publication

The extracted data was then used to identify themes by observing methods used in the selected literatures. The identified themes are extended and compounded (additional variables or adaptation) entanglement witness; MUBs; MUMs; GSIC-POVMs; supervised machine learning-based approach; autoencoder neural network; randomness distillation; uncertainty relations; Bell inequality with semi-definite-program (SDP) hierarchy; tangle and entanglement measures based on fidelity; quantum Fisher information; separability criterion (positive partial transpose); tensor product decomposition of the coefficient vectors (ascending lexicographical order) and permutations of qubits; controlled-SWAP test; genuine entanglement detection and genuine multipartite entanglement concurrence; additive entanglement of formation (EOF); k-partite entanglement and k-nonseparability; and norms of the correlation tensors and multipartite concurrence.

3. Results

This section discusses the identified themes from the literature analysis (see Table 2). Based on the findings, a conceptual framework of entanglement detection was proposed for future work (see Figure 5).

3.1 Entanglement Detection Methods

Table 2 shows 27 entanglement detection methods used in previous studies. Some of these methods are extended and compounded (additional variables or adaptation) to facilitate the detection entanglement process.

Entanglement witness was used as a ground basis for extended and compounded entanglement detection methods in [13,17-21]. According to Baccari *et al.,* [1], the entanglement witness concept lacks generality on its own, hence the derivation of the concept to adapt to certain requirements is beneficial. Sen *et al.,* [14], Zhijin *et al.,* [22] and Sen *et al.,* [23] also used entanglement witness as a ground basis and employed a similar measurement-device-independent approach. The measurement-device-independent concept was introduced as a contrast to the device-dependent concept, providing entanglement detection with increased robustness and loss tolerance. Similarly, the device-independent concept was used by Baccari *et al.,* [1] but with a different measure setting, Bell inequality with SDP hierarchy.

The entanglement detection methods developed by Shen *et al.,* [24], Hiesmayr *et al.,* [25] and Lu Liu [26] are based on three related measurement concepts: MUB, MUM and GSIC-POVM. Due to its limitations, the concept of MUBs was then generalized to MUMs. Similarly, the symmetric informationally complete positive operator-valued measures (SIC-POVMs) were generalized to GSIC-POVMs. These measurements were effective as they are based on a few local measurements.

Other established entanglement detection methods include:

i. supervised machine learning-based approach
ii. autoencoder neural network
iii. randomness distillation
iv. uncertainty relations
v. tangle and entanglement measures based on fidelity
vi. quantum Fisher information
vii. separability criterion (positive partial transpose)
viii. tensor product decomposition of the coefficient vectors (ascending lexicographical order) and permutations of qubits
ix. c-SWAP test
x. genuine entanglement detection and genuine multipartite entanglement concurrence
xi. additive EOF
xii. k-partite entanglement and k-nonseparability
xiii. norms of the correlation tensors and multipartite concurrence.

The reviewed entanglement detection methods are divided into three clusters. Cluster 1 contains entanglement witnesses, Cluster 2 contains MUBs, MUMs, GSIC-POVMs, and Cluster 3 contains other methods. Figure 4 depicts the clusters of entanglement detection methods.

Table 2
Entanglement detection methods used in previous studies

Source	Methods	Qubit System		Quantum State			Remarks
		BP	MP	PU	MX	AR	
[1]	Bell inequality with SDP hierarchy		✓		✓		Up to 29 qubits
[12]	Supervised machine learning-based approach		✓	✓			Up to 8 qubits
[17]	Entanglement witness through lossy compression	✓		✓			
[13]	Optimal entanglement witness with separability criterion		✓	✓			4 qubits
[24]	MUMs, GSIC-POVMs	✓		✓			
[18]	Witness operator (for local model new genuinely multipartite entanglement)		✓	✓			
[14]	Measurement-device-independent entanglement witnesses (MDI-EW)	✓		✓			
[22]	Measurement-device-independent universal entanglement witness (MDI-UEW)	✓		✓	✓	✓	
[23]	Measurement-device-independent entanglement witness (MDI-EW)	✓	✓	✓	✓		
[27]	Autoencoder neural networks	✓				✓	Can be extended beyond 2 qubits
[19]	An optimal entanglement witness from random homodyne measurements	✓				✓	
[20]	Entanglement witness on graph state		✓	✓	✓		
[21]	Entanglement witnesses with local measurement settings		✓	✓			Up to 15 qubits
[25]	MUBs	✓		✓			
[26]	MUBs, MUMs, GSIC-POVMs		✓			✓	Qudit system and arbitrary high dimensional system
[28]	Randomness distillation	✓		✓			
[29]	Uncertainty relations		✓	✓	✓		Up to 6 qubits
[30]	Tangle and entanglement measures based on fidelity	✓	✓	✓	✓		
[31]	Quantum Fisher information		✓	✓			Arbitrary different local dimensions (N-qudit)
[32]	Separability criterion (positive partial transpose)		✓	✓			Up to 4 qubits
[33]	Tensor product decomposition of the coefficient vectors (ascending lexicographical order) and permutations of qubits		✓	✓			3 qubits and above
[34]	c-SWAP test	✓	✓	✓			2 qubits and above
[35]	Genuine entanglement detection and genuine multipartite entanglement concurrence		✓	✓	✓		
[36]	Additive EOF		✓	✓	✓		

[37]	k-partite entanglement and k-nonseparability	✓		✓	Up to 10 qubits Extendable to arbitrary high dimensional system
[38]	Norms of the correlation tensors and multipartite concurrence	✓	✓	✓	4 qubits and above N-partite qudit state

BP = Bipartite MP = Multipartite PU = Pure State MX = Mixed State AR = Arbitrary State

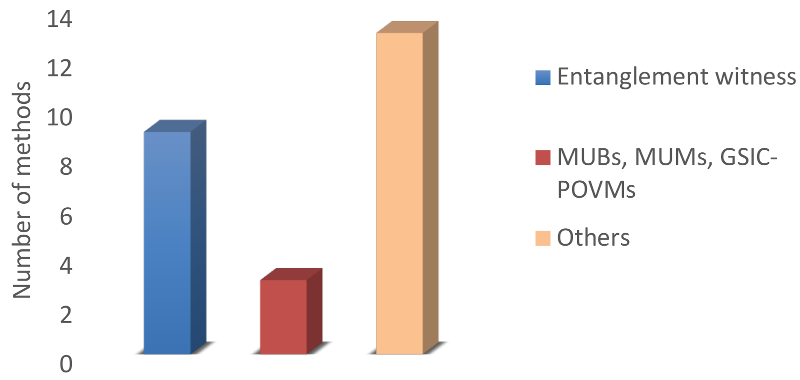

Fig. 4. Clusters of entanglement detection methods

3.2 The Proposed Framework of Entanglement Detection

The primary goal of this study is to propose a conceptual framework of entanglement detection in bipartite and multipartite systems based on previous work. Each method was evaluated individually to fully understand the concept of entanglement detection measurements. The framework was designed according to specific requirements identified in previous work, namely the quantum qubit system and quantum state. Due to its critical role in quantum information processing, the conceptual framework was built on the quantum qubit system ground, bipartite and multipartite qubit systems (see Figure 5).

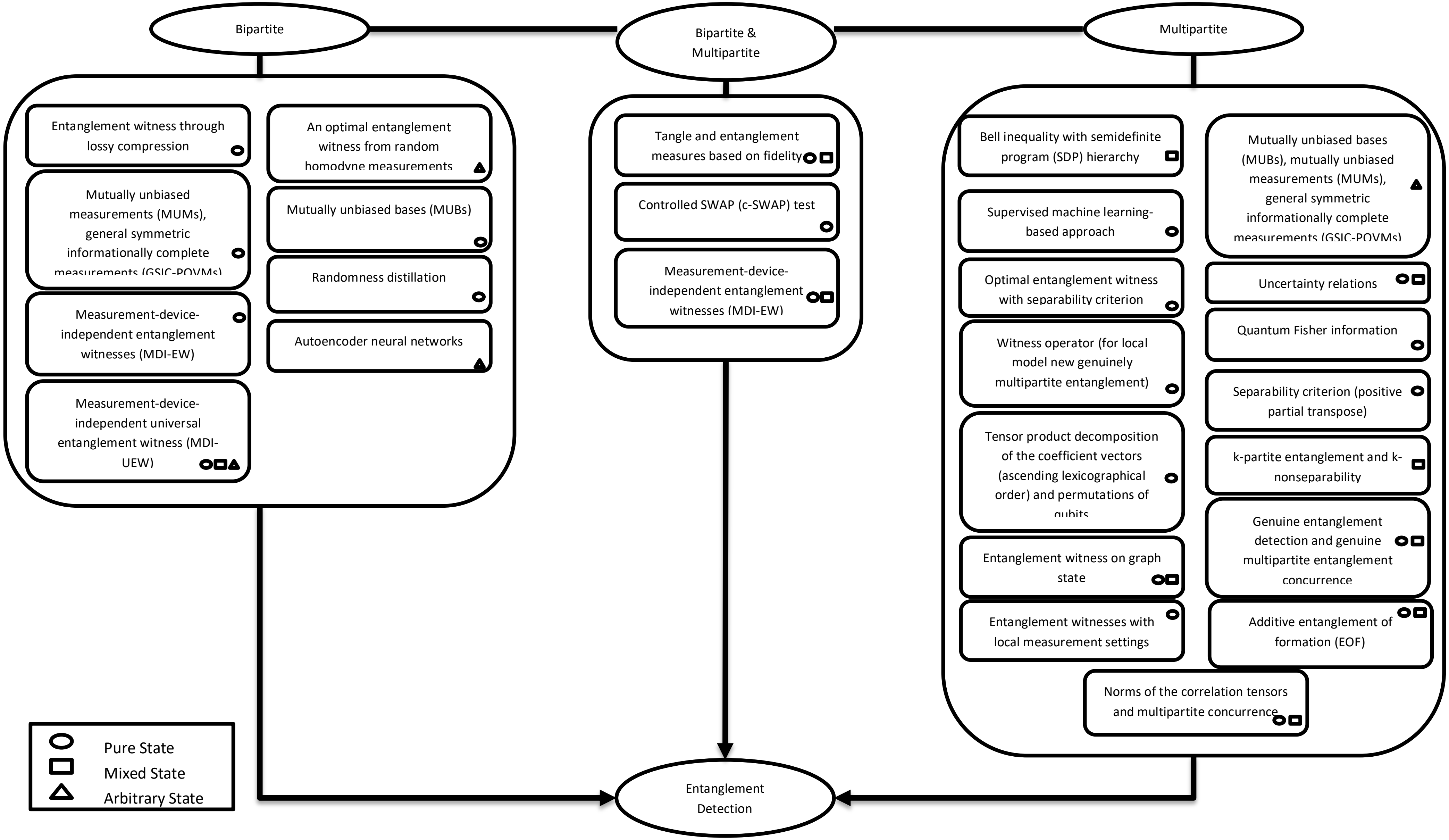

Fig. 5. Proposed conceptual framework of entanglement detection

4. Conclusions

Entanglement detection is a challenging task, especially in higher qubits and higher-dimensional systems. Some established methods from previous studies have been identified and presented. A proposed conceptual framework of entanglement detection in bipartite and multipartite qubit systems was developed and presented as a reference for future entanglement detection research based on one's specific requirements. Based on the proposed conceptual framework, more research on entanglement detection methods is required to produce an absolute universal entanglement detection measurement in higher qubits and higher-dimensional systems.

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