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General Fuzzy Switchboard Transformation Semigroup

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ABSTRACT

Commutative and switching processing are central to computation. However, some problems arise when classical versions do not reflect the true needs of current computer science, whereby they cannot predict the flow of the next input information in the specified output. The Switchboard State Machine is a controller that controls the direct flow of information from one state to another and also plays the main part in communication between the subsystems. If the simple system satisfies the two properties of the switching and commutative state machines, the system is therefore a finite switchboard state machine. The general fuzzy switchboard automation (GFSA) was introduced by incorporating the switchboard into the general fuzzy automation. This paper is intended to introduce the concept of a general fuzzy switchboard transformation semigroup (GFSTS) by combining the GFSA and the transformation semigroup. Some related definitions and properties are established. The example of certain products, such as GFSTS cascade products, has also been studied. General Fuzzy Switchboard Poly-Transformation Semigroup (GFSPS) is also introduced since it fulfils the switchboard state machine properties. Some of the definitions and properties associated with the GFSPS are defined. Applications for general fuzzy switchboard automata, such as washing machines, are also provided.

1. Introduction
1.1 Research Background

In the study of algebraic properties of the automaton theory, classical versions are often misunderstood to reflect the actual needs of modern computer science. When given input information is received from an integer sequence, it faces some problems in navigation or prediction of the flow of the next input information in a specified output. It is unable to formalize the switching and commutative processing, which is nowadays central to the computation when we need to re-

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evaluate the global transition of the finite state machine, as it was virtually unnecessary in the past. In other words, the algebraic approach still lacks its properties, which are the properties concerning the state machine of the switchboard. Therefore, the model of the switching mechanism as a control device must be understood. The reason for the use of GFSA is that there is a possibility of overtaking the transition to the same state on the same symbol from different current states. Several different membership values that happened at the same time are known as multi-memberships. GFA was introduced by Doorsfatemeh and Kremer [1] can also handle application problems that depended entirely on fuzzy automation as a modelling tool to assign membership values to active states of fuzzy automation, resolve multi-membership, and analyse the continuous operation of the fuzzy automaton.

According to Holcombe [2], algebra evolved in many different directions. In mathematics, abstract algebra deals mainly with algebraic structures. Most algebraic structures have multiple operations that require specific axioms to be satisfied. Semigroups, groups, rings, and fields are examples of algebraic structures. In the concepts of science and computation, the concept of change is that a system changes from state to state through internal processes or other external operations at different time scales. Therefore, the transformation of a finite set of states fulfils this concept [3]. The semigroup of transformations defined all types of different set transformations that can be combined over time. Algebraic products are considered to produce an automatic from existing automatons. Doostfatemeh and Kremer [1] introduced a new general definition of fuzzy automation to provide a better foundation of automation and basic equipment for the next applications. In addition, general fuzzy automata can also remove the burden of generating deterministic acceptors for calculating string membership values without developing a deterministic Moore machine. It can also be used for a wide variety of fuzzy grammar and language. Sato and Kuroki [4] introduced the notion of a finite switchboard state machine that is another extension of the finite state machine/finite automata. In 2002, they introduced the concept of fuzzy finite switchboard state machines and fuzzy switchboard transformation semigroups. Then, the idea of switching homomorphism is introduced. A finite switchboard state machine or finite switchboard automata brings together the concept of switching state machine and commutative state machine. The fundamental goal of this work is to introduce the concept of General Fuzzy Switchboard Transformation Semigroup (GFSTS) by the combination of GFSA and transformation semigroup. It is necessary to understand the general fuzzy switchboard automation.

1.2 Literature Review

Zadeh [5] was the first researcher to introduce the concept of a fuzzy set. In the late 1960s, fuzzy automatons were introduced by Santos [6] and Wee [7]. Since finite automata constitute a mathematical model of computation, a fuzzy finite automaton can be considered as an extension of 2 finite automatons, including notions such as “vagueness” and “imprecision”. Several researchers then introduced and studied fuzzy automation, such as (Li and Pedrycz [8]), (Malik *et al.,* [9]), and (Jin *et al.,* [10]). In principle, Li and Pedrycz [8] stated that a finite state automaton is a mathematical model that recognizes the formal language of classical computation and the former proposed fuzzy automata with membership values in the unit interval [0,1] with the max-min composition. In general, fuzzy finite state machines (FFSM) and fuzzy finite automation (FFA) have membership values of [0.1]. A finite state machine is a concept that binds a switching state machine and a commutative state machine. Sato and Kuroki [4] introduced the concept of a finite switchboard state machine, another extension of a finite state machine/finite automation. The main objective of this study was to propose an effective algebraic technique for the study of finite switchboard automata.

Understanding the importance of switching mechanisms modelling as electronic system control devices is necessary.

In general, Pedrycz and Gacek [11] mentioned that fuzzy automation offers a systematic way to generalize discrete applications where they can create capabilities that are rarely achieved by other tools. It offers a systematic approach to the integration of approximate reasoning into the system, as humans do [5]. Doostfatemeh and Kremer [1] therefore introduced a new general definition of fuzzy automation to establish a better basis for automation and the foundations for future applications. Furthermore, general fuzzy automata can also remove the burden of generating a deterministic acceptor to calculate membership values of the strings without developing a deterministic Moore automaton and it can also be used in large fuzzy grammars and languages [12]. In addition, this study takes into account the multi-member value of the General Fuzzy Switchboard Automata (GFSA), since it treats more than one membership value in a state. Horry [13] studied the general complex fuzzy transformation semigroups in automata and derived relationships between a max-min general complex fuzzy automaton and a general complex fuzzy transformation semigroup. Marapureddy [14] studied the connection between fuzzy theory and graph theory with an algebraic structure semigroup. Semigroups are basic algebraic structures in many branches of engineering like automata, formal languages, and finite state machines can be seen in review papers by several authors [15-20]. The concept of transformation semigroups has played an important role in the theory of finite automata [17,18]. East *et al.,* [21] describe general methods for enumerating sub semigroups of finite semigroups and techniques to improve the algorithmic efficiency of the calculations. In addition, Julatha and Rimcholakarn [22] investigate the characterizing semigroups by their generalized fuzzy ideals. It shows that semigroups are algebraic structures that show a very close relationship between self-adjoint operators.

The concept of switchboard properties is studied in general fuzzy automata, namely, the General Fuzzy Switchboard Automata (GFSA). A semigroup is important because it occurs in so many places. Since semigroups are important algebraic structures in automation theory, it is necessary to study their properties related to GFSA by expanding the algebraic properties of GFSA to the general fuzzy switching transition semigroup (GFSTS).

2. Methodology

2.1 Methods

General Fuzzy Automata (GFA) is used to resolve multi-membership because there are some problems defining the value of membership of an active state in the machine when an active state has a value of multi-membership. Doostfatemeh and Kremer provided an algorithm for resolving multi-membership. Some general definitions and introductions are presented, which will be later used in this study.

Definition 2.1: (Doorstfatemeh & Kremer [1]): A general fuzzy automaton (GFA) is an eight-tuple machine F = (Q, Σ, $\tilde{R}$, Z, $\tilde{\delta}^*$, ω, F_1 , F_2) where

i. Q is a finite set of states, Q = $\{q_1, q_2, \cdots , q_n \}$,
ii. Σ is a finite set of input symbols, Σ = $\{a_1, a_2, \cdots , a_n\}$,
iii. $\tilde{R}$ is the set of fuzzy start states, $\tilde{R} \subseteq P(Q)$,
iv. Z is a finite set of output symbols, Z = $\{b_1, b_2, \cdots , b_k\}$,
v. ω:$Q \rightarrow Z$ is the non-fuzzy output function,
vi. F_1:[0,1] × [0,1] → [0,1] is the membership assignment function,

vii. $\tilde{\delta}$: $(Q \times [0,1]) \times \Sigma \times Q)$ F_1 (μ,δ) → [0,1] is the augmented transition function,
viii. F_2:$[0,1]^*$→ [0,1] is a multi-membership resolution function.

The function has two parameters μ and δ that represent F_1 (μ, δ). μ is the membership value of a predecessor and δ is the weight of transition. According to the definition, the process of converting state q_i to q_j in the input ak is described as follows in Eq. (1).

$$\mu^{t_{i+1}}(q_j) = \tilde{\delta}(q_i, \mu^{t_i}(q_i), ak, q_j) = F_1(\mu^{t_i}(q_i), \delta(q_i, ak, q_j)) \tag{1}$$

It means that the membership value of the state q_j at time t + 1 is figured by function F_1 using the weight of the transition, δ, and the membership value of q_i at time t. Usually, the options for $F(\mu$, $\delta)$ are max{μ, δ}, min{μ, δ} and ($\frac{\mu+\delta}{2}$). The multi-membership resolution function, F_2 resolves the multi-membership active state and assigns a single truth value to them. Let Q_{act} $(t_i$) = {(q, μ t_i (q)): $\exists q$ ′ ∈ Q_{act} (t_{i-1}), $\exists a \in \sum$, $\delta(q$ ′ , a, $q)$ ∈ Δ},∀i ≥ 1. Since $Q_{act}(t_i)$ is a fuzzy set, then $q \in$ $Domain(Q_{act}$ $(t_i$)) and $T \subset Domain(Q_{act}$ $(t_i))$. Hereafter, simply denote as q ∈ (Q_{act} $(t_i$)) and $T \subset$ (Q_{act} $(t_i$)). The combination of the operations of functions F_1 and F_2 on a multi-membership state q_j indicates the multi-membership resolution algorithm.

2.2 Algorithm Construction to Check the Validity of Switchboard Automata

Algorithm (Doorstfatemeh & Kremer [1])

This algorithm is for multi-membership resolution. If there are various simultaneous transitions to the active state q_j at time t + 1, the following algorithm will assign a united membership value to it:

i. Each transition weight $\delta(q_i$, ak, $q_j)$ together with the membership value of state μ^t (qi), will be processed by the membership assignment function F_1, and will produce a membership value that is called as v_i , v_i = $\tilde{\delta}$ ((q_i, $\mu^t(q_i$)), ak, q_j) = F_1 (μ^t (q_i), $\delta(q_i$, ak, q_j)).
ii. These truth values are not necessarily equal. Hence, they need to be processed by the multi-membership resolution function F_2.
iii. The result produced by F_2 will be assigned as the instantaneous membership value of the active state q_j , μ^{t+1} (q_j) = $F_2 i$= $\frac{1}{n}$[F_1 (μ^t (q_i), $\delta(q_i$, ak, q_j))]. Where n is the number of simultaneous transitions to the active state qj at time t + 1. $\delta(q_i$, ak, $q_j)$ is the weight of a transition from q_i to $q_j$with input ak. μ^t (q_i) is the membership value of q_i at time t. μ^{t+1} (q_j) is the final membership value of q_j at time t + 1.

Let F = (Q, Σ, R, Z, $\tilde{\delta}^*$, ω, F_1, F_2) be a max-min bipolar general fuzzy automaton. When the number of input elements is 1, then F^* is switching and commutative. Follow the procedure below to check the validity of the switchboard automata.

Step 1: Enter the state transition $\tilde{\delta}^*{}_{a_1}, \tilde{\delta}^*{}_{a_2}, \cdots, \tilde{\delta}^*{}_{a_n}$.

Step 2: Set *i* be the initial value, i = 1 and n ≥ 2.

Step 3: for $i \leq n - 1$,

- Calculate $\tilde{\delta}^*{}_{a}\tilde{\delta}^*{}_{a_{i+1}}((q, \mu^{t_{i-1}}(q)), p)$ and $\tilde{\delta}^*{}_{a_{i+1}}\tilde{\delta}^*{}_{a}((q, \mu^{t_{i-1}}(q)), p)$ where $p, q \in Q$.
- If $\tilde{\delta}^*{}_{a}\tilde{\delta}^*{}_{a_{i+1}}\left((q, \mu^{t_{i-1}}(q)), p\right) \neq \tilde{\delta}^*{}_{a_{i+1}}\tilde{\delta}^*{}_{a}((q, \mu^{t_{i-1}}(q)), p)$, then $STOP$, it means that the output F^* is not commutative;
- If$\tilde{\delta}^*{}_{a}\tilde{\delta}^*{}_{a_{i+1}}\left((q, \mu^{t_{i-1}}(q)), p\right) = \tilde{\delta}^*{}_{a_{i+1}}\tilde{\delta}^*{}_{a}((q, \mu^{t_{i-1}}(q)), p)$,recalculate $\tilde{\delta}^*{}_{a_i}\tilde{\delta}^*{}_{a_{i+2}}((q, \mu^{t_{i-1}}(q)), p$ and $\tilde{\delta}^*{}_{a_{i+2}}\tilde{\delta}^*{}_{a_i}((q, \mu^{t_{i-1}}(q)), p)$.
- If both are not equal then $STOP$, it means that the output F^* is not commutative, NO;
- Otherwise recalculate $\tilde{\delta}^*{}_{a_i}\tilde{\delta}^*{}_{a_{i+3}}((q, \mu^{t_{i-1}}(q)), p)$ and $\tilde{\delta}^*{}_{a_{i+3}}\tilde{\delta}^*{}_{a_i}((q, \mu^{t_{i-1}}(q)), p)$ and so on;
- If necessary, calculate until $\tilde{\delta}^*{}_{a_i}\tilde{\delta}^*{}_{a_n}((q, \mu^{t_{i-1}}(q)), p)$ and $\tilde{\delta}^*{}_{a_n}\tilde{\delta}^*{}_{a_i}((q, \mu^{t_{i-1}}(q)), p)$;
- If both are not equal, the output F^* is not commutative, NO;
- If both are equal, go to Step 4.

Step 4: $i = i + 1$ repeat Step 3.

Step 5: $i = n$, $STOP$, the output F^* is commutative, YES.

Step 6: for $i \leq n$, calculate $\tilde{\delta}^*{}_{a_i}((q, \mu^{t_{i-1}}(q)), p)$ and $\tilde{\delta}^*{}_{a_i}((p, \mu^{t_{i-1}}(p)), q)$, $\forall q, p \in Q$.

- If $\tilde{\delta}^*{}_{a_i}((q, \mu^{t_{i-1}}(q)), p) \neq \tilde{\delta}^*{}_{a_i}((p, \mu^{t_{i-1}}(p)), q)$ then $STOP$, the output F^* is not switching;
- If $\tilde{\delta}^*{}_{a_i}\left((q, \mu^{t_{i-1}}(q)), p\right) = \tilde{\delta}^*{}_{a_i}((p, \mu^{t_{i-1}}(p)), q)$,recalculate $\tilde{\delta}^*{}_{a_{i+1}}\left((q, \mu^{t_{i-1}}(q)), p\right)$ and $\tilde{\delta}^*{}_{a_{i+1}}((p, \mu^{t_{i-1}}(p)), q)$;
- If both are not equal, then $STOP$, it means that the output F^* is not switching, NO;
- Otherwise recalculate $\tilde{\delta}^*{}_{a_{i+2}}\left((q, \mu^{t_i}(q)), p\right)$and $\tilde{\delta}^*{}_{a_{i+2}}\left((q, \mu^{t_i}(q)), p\right)$ and so on;
- If both are not equal, then the output F^* is not switching, NO;
- If both are equal, go to Step 7.

Step 7: $i = i + 1$, repeat Step 6.

Step 8: $i = n$, $STOP$, the output is switching, YES.

The flowchart below shows the simple algorithm which generates the switchboard in General Fuzzy Automata based on the step from 1 until 8.

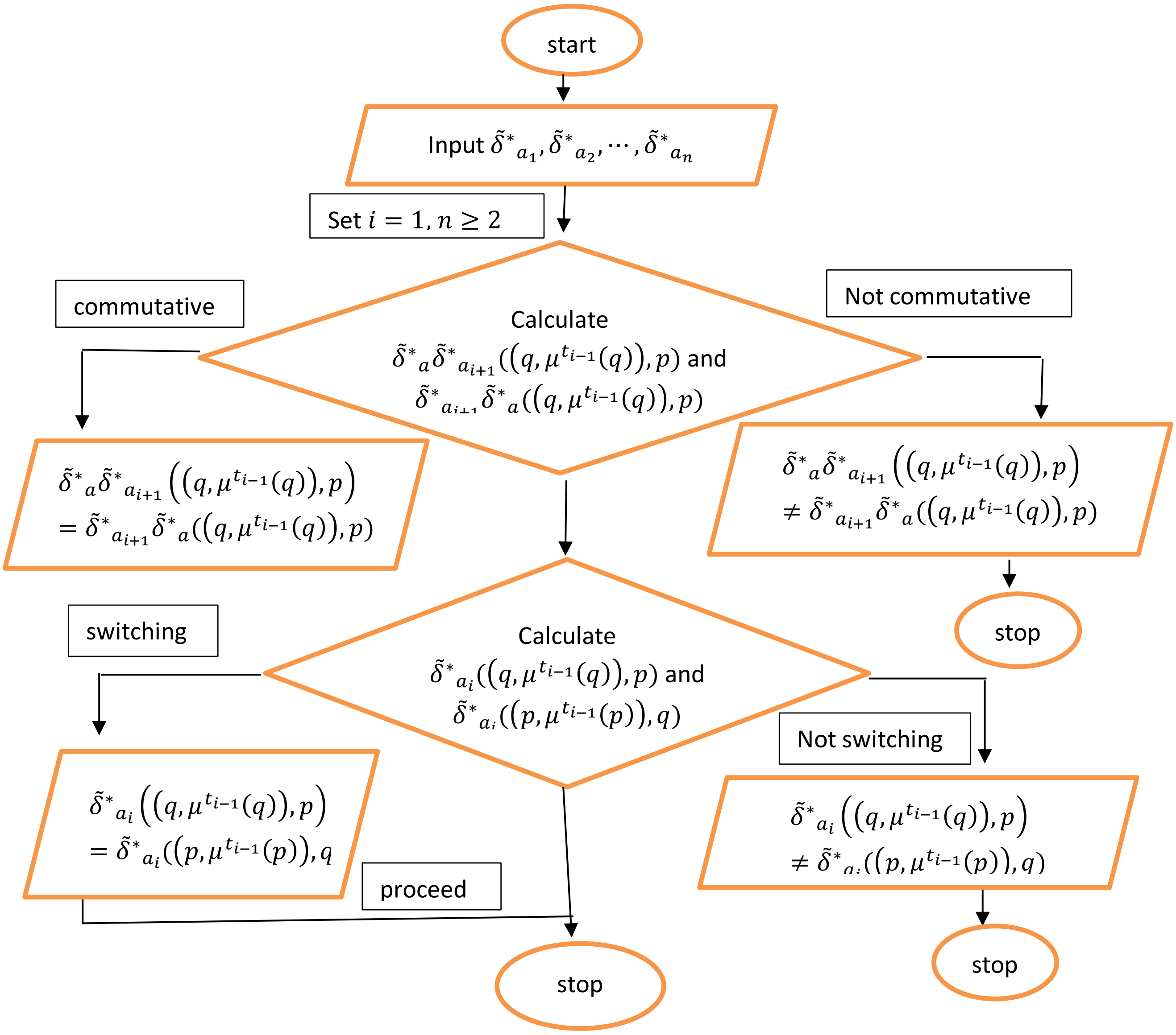

Fig. 1. Flowchart to check the validity of switchboard in General Fuzzy Automata

3. Results

3.1 General Fuzzy Switchboard Transformation Semigroup (GFSTS) and Product of GFSTS

Definition 3.1: A fuzzy transformation semigroup is a triple $TS(F) = (Q, S(F), \rho)$ where Q is a finite nonempty set, $S(F)$ is a finite semigroup of F, ρ is a fuzzy subset of $Q \times S(F) \times Q$. Such that:

i. $\rho(q, \mu^{t_i}(q), uv, p) = \vee_{r \in Q}\{\rho(q, \mu^{t_i}(q), u, r) \wedge \rho(r, \mu^{t_{i+1}}(r), v, p)$, for all $u, v \in S$, $q, p \in Q$, $i \geq 0$.
ii. If S contains the identity e, then $\rho(q, \mu^{t_i}(q), e, p) = 1$ if $q = p$ and $\rho(q, \mu^{t_i}(q), e, p) = 0$ if $q \neq p$, $\forall q, p \in Q$, $i \geq 0$. If the property holds, then $TS(F) = (Q, S(F), \rho)$ is called faithful.
iii. Let $u, v \in S(F)$, if $\rho(p, \mu^{t_i}(p), u) = \rho(p, \mu^{t_i}(p), v)$, $\forall p \in Q$, $i \geq 0$ then $u = v$.

Definition 3.2: Let $T = (Q, S(F), \rho)$ be a fuzzy transformation semigroup. Then:

i. T is commutative if it satisfied $\rho(q, \mu^{t_{i-1}}(q), uv, p) = \rho(q, \mu^{t_{i-1}}(q), vu, p), \forall q, p \in Q$, $\forall u, v \in S(F)$, $i \geq 1$.

ii. T is switching if it satisfied $\rho(q, \mu^{t_i}(q), uv, p) = \rho(p, \mu^{t_i}(p), uv, q)$, $\forall q, p \in Q$, $\forall u, v \in S(F)$, $i \geq 0$.

If T satisfied both conditions which are commutative and switching, thus it is called General Fuzzy Switchboard Transformation Semigroup (GFSTS).

Proposition 3.3: Let $F = (Q, \Sigma, R, Z, \delta, \omega, F1, F2)$ be general fuzzy switchboard automata. Then T is a general fuzzy switchboard transformation semigroup.

3.2 Cascade Product in General Fuzzy Switchboard Transformation Semigroup

Definition 3.4: Let $T_1 = (Q_1, S(\tilde{F})_1, \rho_1)$ and $T_2 = (Q_2, S(\tilde{F})_2, \rho_2)$ be GFSTS. Define the restricted cascade product $T_1 \varpi T_2 = (Q_1 \times Q_2, S(\tilde{F})_2, \rho^{\varpi})$ of T_1 and T_2 with respect to mapping $\varpi: S(\tilde{F})_2 \rightarrow S(\tilde{F})_1$ as, $\rho^{\varpi}((p_1, \mu^{t_i}(p_1), p_2, \mu^{t_i}(p_2)), s_2, (q_1, q_2) = \rho_1(p_1, \mu^{t_i}(p_1), \varpi(s_2), q_1) \wedge \rho_2(q_2, \mu^{t_i}(q_2), s_2, p_2)$, Where $\rho\ \varpi$: $(Q_1 \times Q_2) \times S(F)\ 2 \times (Q_1 \times Q_2) \rightarrow [0,1]$, $\forall (p_1, p_2), (q_1, q_2)) \in Q_1 \times Q_2$ and $s_2 \in S(\tilde{F})_2$ where $i \geq 0$.

Proposition 3.5: Let $T_1 = (Q_1, S(\tilde{F})_1, \rho_1)$ and $T_2 = (Q_2, S(\tilde{F})_2, \rho_2)$ be GFSTS. Then, there exists $\omega: Q_2 \times S(\tilde{F})_2 \rightarrow S(\tilde{F})_1 \forall \varpi: S(\tilde{F})_2 \rightarrow S(\tilde{F})_1 \forall$such that $T_1 \varpi T_2 \cong T_1\ \omega T_2$.

Proposition 3.6: Let $T_k = (Q_k, S(\tilde{F})_k, \rho_k)$ be GFSTS, $k = 1,2$ and $\varpi: S(\tilde{F})_2 \rightarrow S(\tilde{F})_1$ be a semigroup homomorphism. Then $T_1 \varpi T_2$ is a GFSTS if and only if both T_1 and T_2 are GFSTS.

Example: (Restricted cascade product of GFSTS) Let $T_1 = (Q_1, S(\tilde{F})_1, \rho_1)$ and $T_2 = (Q_2, S(\tilde{F})_2, \rho_2)$ be GFSTS's, where $Q_1 = \{p_1, p_2\}$,$Q_2 = \{q_1, q_2\}$, $S(\tilde{F})_1 = \{s_{1,} t_1, u\}$, $S(\tilde{F})_2 = \{s_{2,} t_2\}$ and ρ_1 and ρ_2 are defined as follows:

$\rho_1(p_1, \mu^{t_i}(p_1), s_{1,}, p_1) = 0.5$
$\rho_1(p_2, \mu^{t_i}(p_2), s_{1,}, p_1) = 0.3$
$\rho_1(p_1, \mu^{t_i}(p_1), t_{1,}, p_2)) = 0.2$
$\rho_1(p_2, \mu^{t_i}(p_2), t_{1,}, p_2) = 0.6$
$\rho_1(p_1, \mu^{t_i}(p_1), u_{1,}, p_2) = 0.4$
$\rho_2(p_2, \mu^{t_i}(p_2), u_{1,}, p_1) = 0.7$
$\rho_2(q_1, \mu^{t_i}(q_1), s_2, q_1) = 0.6$
$\rho_2(q_2, \mu^{t_i}(q_2), s_2, p_1) = 0.3$
$\rho_2(q_1, \mu^{t_i}(q_1), t_2, q_2) = 0.5$
$\rho_2(q_2, \mu^{t_i}(q_2), t_2, q_1) = 0.35$

Now the function $\varpi: S(\tilde{F})_1 \rightarrow S(\tilde{F})_2$ is defined as $\varpi(s_2) = s$, $\varpi(t_2) = t$. Next, define the partial function $\rho^{\varpi}: (Q_1 \times Q_2) \times S(\tilde{F})_2 \times (Q_1 \times Q_2) \rightarrow [0,1]$ as:

$\rho^{\varpi}(p_1, \mu^{t_i}(p_1), q_1, \mu^{t_i}, (q_1)), s_2, (p_1, q_1)) = \rho_1(p_1, \mu^{t_i}(p_1), \varpi(s_2), p_1) \wedge \rho_2(q_1, \mu^{t_i} q_1), s_2, q_1) = 0.5$

$\rho^{\varpi}(p_1, \mu^{t_i}(p_1), q_2, \mu^{t_i}, (q_2)), s_2, (p_1, q_1)) = \rho_1(p_1, \mu^{t_i}(p_1), \varpi(s_2), p_1) \wedge \rho_2(q_2, \mu^{t_i}(q_2), s_2, q_1) = 0.3$

ρ^{ϖ} (p_2, μ^{t_i} (p_2), q_1 , μ^{t_i}, (q_1)), s_2 , (p_1, q_1)) = ρ_1 (p_2, μ^{t_i} (p_2), $\varpi(s_2)$, p_1)$\wedge$ ρ_2 (q_1, μ^{t_i} (q_1), s_2 , q_1) =0.3

ρ^{ϖ} (p_2, μ^{t_i} (p_2), q_2 , μ^{t_i}, (q_2)), s_2 , (p_1, q_1)) = ρ_1 (p_2, μ^{t_i} (p_2),$\varpi(s_2$), p_1)$\wedge$ ρ_2 (q_2, μ^{t_i} (q_2), s_2 , q_1) = 0.3

ρ^{ϖ} (p_1, μ^{t_i} (p_1), q_1, μ^{t_i}, (q_1)), t_2 , (p_2, q_2)) = ρ_1 (p_1,μ^{t_i} (p_1),$\varpi(t_2$), p_2)$\wedge$ ρ_2 ($q_1\mu^{t_i}$ (q_1), t_2 , q_2) = 0.2

ρ^{ϖ} (p_1, μ^{t_i} (p_1), q_2, μ^{t_i}, (q_2)), t_2 , (p_2, q_1)) = ρ_1 (p_1,μ^{t_i} (p_1),$\varpi(t_2$), p_2)$\wedge$ ρ_2 (q_2, μ^{t_i} (q_2), t_2 , q_1) = 0.2

ρ^{ϖ} (p_2, μ^{t_i} (p_2), q_1, μ^{t_i}, (q_1)), t_2 , (p_2, q_2)) = ρ_1 (p_2, μ^{t_i} (p_2),$\varpi(t_2)$, p_2)$\wedge$ ρ_2 (q_1, μ^{t_i} (q_1), t_2 , q_2) = 0.5

ρ^{ϖ} (p_2, μ^{t_i} (p_2), q_2 , μ^{t_i}(q_2)), t_2 , (p_2, q_1)) = ρ_1 (p_2, μ^{t_i} (p_2),$\varpi(t_2$), p_2)$\wedge$ ρ_2 (q_2, μ^{t_i} (q_2), t_2 , q_1) = 0.35

And δ^{ϖ} is 0 elsewhere. It follows that $M_1\varpi M_2 \cong M_1\omega M_2$ is a restricted cascade product.

3.3 General Fuzzy Switchboard Polytransformation Semigroup

Definition 3.7: A poly-transformation semigroup is a triple T = (Q, $S(F)$, γ) where Q is a finite nonempty set, $S(F)$ is a finite semigroup of F, γ is a fuzzy subset of (Q × [0,1]) × $S(F)$ → P(Q × [0,1])\{∅}. Such that:

i. $\gamma(\gamma(p, \mu^{t_i}$ $(p), u), v)$ = $\gamma(p, \mu^{t_i}$ $(p), uv)\forall p \in Q$, u, $v \in S(F)$ and $\gamma(P, u)$ =∪ {$\gamma(p, \mu^{t_i}$ $(p), u)|p \in P$}, $P \subseteq Q$ and $i \geq 0$.
ii. If $S(F)$ contains the identity e, then $\gamma(p$, μ^{t_i} (p),$e)$ = {p} $\forall p \in Q$ and $i \geq 0$. If the property holds, then T = (Q, $S(F)$, γ) is called faithful.
iii. Let u, $v \in S(F)$, if $\gamma(p$, μ^{t_i} (p), $u)$ = $\gamma(p$, μ^{t_i} (p), $v)$, $\forall p \in Q$ then $u = v$.

Definition 3.8: An anti-poly-transformation semigroup is a triple T = (Q, $S(F)$, γ) where Q is a finite nonempty set, $S(F)$ is a finite semigroup of F, γ is a fuzzy subset of (Q × [0,1]) × $S(F)$ → P(Q × [0,1])\{∅}. Such that:

i. $\gamma(\gamma(p, \mu^{t_i}$ $(p), u), v)$ = $\gamma(p, \mu^{t_i}$ $(p), vu)\forall p \in Q$, u, $v \in S(F)$ and $\gamma(P, u)$ =∪ {$\gamma(p, \mu^{t_i}$ $(p), u)|p \in P$}, $P \subseteq Q$ and $i \geq 0$.
ii. If $S(F)$ contains the identity e, then $\gamma(p$, μ^{t_i} (p),$e)$ = {p} $\forall p \in Q$ and $i \geq 0$. If the property holds, then T = (Q, $S(F)$, γ) is called faithful
iii. Let u, $v \in S(F)$, if $\gamma(p$, μ^{t_i} (p), $u)$ = $\gamma(p$, μ^{t_i} (p), $v)$, $\forall p \in Q$ then $u = v$.

To summarize, if the definition of poly-transformation semigroup is equal to the anti-poly-transformation semigroup, then it is commutative properties.

Definition 3.9: Let T = (Q, $S(F)$, γ) be a poly-transformation semigroup. Then:

i. T is commutative if it satisfied $\gamma(p, \mu^{t_{i-1}}(p), uv) = \gamma(p, \mu^{t_{i-1}}(p), vu)$, $\forall p \in Q$, $\forall u, v \in S(F)$, $i \geq 1$.
ii. T is switching if it satisfied $\rho(q, \mu^{t_i}(q), uv, p) = \rho(p, \mu^{t_i}(p), uv, q)$, $\forall q, p \in Q$, $\forall u, v \in S(F)$, $i \geq 0$.

If T satisfied both conditions which are commutative and switching, thus it is called as General Finite Switchboard Poly-transformation Semigroup (GFSPS).

4. Application of General Fuzzy Switchboard Automata

In this section, the application of the Fuzzy Switchboard Automatic (GFSA) to washing machines is studied. The switchboard properties are applied to the machine because the switchboard can function as a control device and communicate between a subsystem and another subsystem. Therefore, it is necessary to include the idea of the switchboard properties to run the machine. Two conditions have to be fulfilled which are commutative and switching properties in order to incorporate the switchboard property into the General Fuzzy Automata. This system must therefore be checked to see if it is GFSA or GFA. First of all, we must check the commutative properties of this system. If $\rho(q, \mu^{t_{i-1}}(q), uv, p) = \rho(q, \mu^{t_{i-1}}(q), vu, p)$, where $\forall q, p \in Q, u, v \in S, i \geq 1$, then it is called as commutative state machine.

4.1 Washing Machine

Figure 2 shows the GFSA, which describes the behaviour of the washing machine influenced by the weight of the clothes and selects the appropriate timer for the entire process. Once the state of the machine is equal to or greater than 0.5, it will be move to the next process (the state). Meanwhile, if the machine receives less than 0.5 of membership value, it means the timer should increase by 5 minutes. However, this condition only occurs in the rinse state. It should be taken into account that clothing has different materials, thickness and weight. Thus, the time for rinse is different. To incorporate the Switchboard property into the general Fuzzy Automata, two conditions must be met, which are the switching properties and the commutative properties.

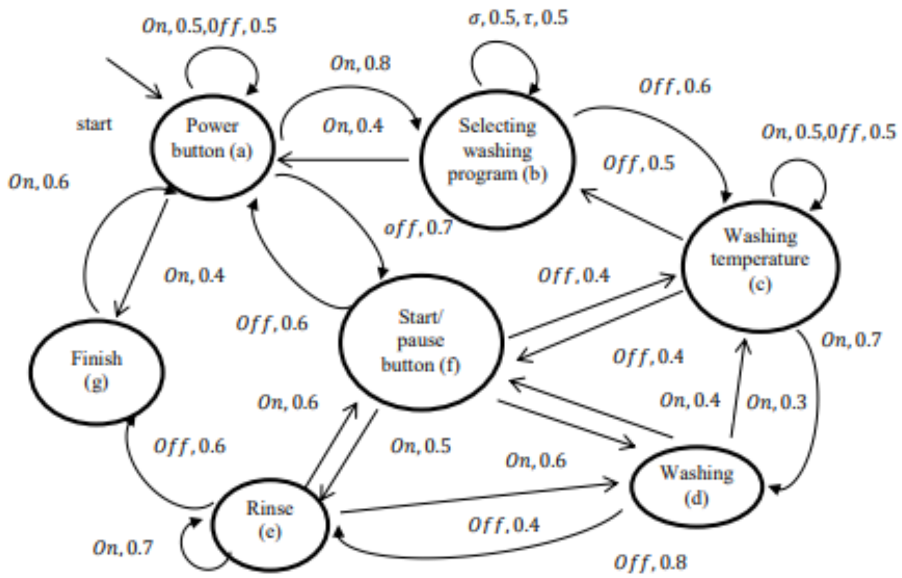

Fig. 2. The simple system of washing machine

Let a, b, c, d,e, f, $g \in Q$, and on, off, σ, $\tau \in \Sigma$. Denote that a, b, c, d,e, f, g are the states of the system, while on, off, σ, τ are the input symbols with the membership values. String x represents the situation that will occur from the state. For instance, from the initial state (power button) to f (start/pause button), if the power is off, it can be going to f. The meaning of the off input symbol is that the state is still in process, in the meantime, on means that the state is already finished and does not operate in that state. At f state, to continue for the next state, if the input symbol is on, it will go to the d state (washing). Then if something goes wrong with the machine, such as the water level is not suitable regarding the weight of the cloths and so on, it will go to the f state (on). However, suppose that there is nothing wrong in the state d, then from the d state it will go to the e state (rinse). Assume, at the e state, the membership value is still greater than 0.5, means that the timer for rinse must be increased by 5 minutes. Thus, at e state, the input symbol is on. The following calculation is for the GFSA in accordance with the situation given.

String x = on, off, on, off

$$\mu^{t_0}(a) = 1$$
$$\mu^{t_1}(b) = \tilde{\delta}((a, \mu^{t_0}(a), on, b) = F_1(\mu^{t_0}(a), \delta(a, on, b) = F_1(1{,}0.8) = 0.8$$
$$\mu^{t_2}(c) = \tilde{\delta}((b, \mu^{t_1}(b), off, c) = F_1(\mu^{t_1}(b), \delta(b, off, c) = F_1(0.8{,}0.6) = 0.6$$
$$\mu^{t_3}(d) = \tilde{\delta}((c, \mu^{t_2}(c), on, d) = F_1(\mu^{t_2}(c), \delta(c, on, d) = F_1(0.6{,}0.7) = 0.6$$
$$\mu^{t_4}(e) = \tilde{\delta}((d, \mu^{t_3}(d), off, e) = F_1(\mu^{t_3}(d), \delta(d, off, e) = F_1(0.6{,}0.8) = 0.6$$
$$\tilde{\delta}^*\left(\left(a, \mu^{t_0}(a)\right), on, b\right) = 0.8$$
$$\tilde{\delta}^*\left(\left(a, \mu^{t_0}(a)\right), onoff, c\right) = 0.8 \wedge 0.6 = 0.6$$
$$\tilde{\delta}^*\left(\left(a, \mu^{t_0}(a)\right), onoffon, d\right) = 0.8 \wedge 0.6 \wedge 0.6 = 0.6$$
$$\tilde{\delta}^*\left(\left(a, \mu^{t_0}(a)\right), onoffonoff, e\right) = 0.8 \wedge 0.6 \wedge 0.6 \wedge 0.6 = 0.6$$

If $\rho(q, \mu^{t_{i-1}}(q), uv, p) = \rho(q, \mu^{t_{i-1}}(q), vu, p)$ means that it is commutative state machine, where $\forall q, p \in Q, u, v \in S, i \geq 1$. Since the string $x = on, off, on, off$ then the other side must be vice versa. Next check the calculation of the string $x = off, on, off, on$

String $x = off, on, off, on$

$$\mu^{t_0}(a) = 1$$
$$\mu^{t_1}(f) = \tilde{\delta}((a, \mu^{t_0}(a), off, f) = F_1(\mu^{t_0}(a), \delta(a, off, f) = F_1(1{,}0.7) = 0.7$$
$$\mu^{t_2}(d) = \tilde{\delta}((f, \mu^{t_1}(f), on, d) = F_1(\mu^{t_1}(f), \delta(f, on, d) = F_1(0.7{,}0.6) = 0.6$$
$$\mu^{t_3}(e) = \tilde{\delta}((d, \mu^{t_2}(d), off, e) = F_1(0.6{,}0.8) = 0.6$$
$$\mu^{t_4}(e) = \tilde{\delta}((e, \mu^{t_3}(e), on, e) = F_1(0.6{,}0.7) = 0.6$$

$$\tilde{\delta}^*\left(\left(a, \mu^{t_0}(a)\right), off, f\right) = 0.7$$
$$\tilde{\delta}^*\left(\left(a, \mu^{t_0}(a)\right), offon, d\right) = 0.7 \wedge 0.6 = 0.6$$
$$\tilde{\delta}^*\left(\left(a, \mu^{t_0}(a)\right), offonoff, e\right) = 0.7 \wedge 0.6 \wedge 0.6 = 0.6$$
$$\tilde{\delta}^*\left(\left(a, \mu^{t_0}(a)\right), offonoffon, e\right) = 0.7 \wedge 0.6 \wedge 0.6 \wedge 0.6 = 0.6$$

The table below shows the operation of the fuzzy automation at the input string $(onoff)^2$and $(offon)^2$ for F_1 and F_2.

Table 1
Active states and their membership values of $onoffonoff$

Time	t_0	t_1	t_2	t_3	t_4
input	Λ	on	off	on	off
$Qact(ti)$	a	b	c	d	e
Membership value	1.0	0.8	0.6	0.6	0.6

Table 2
Active states and their membership values of $offonoffon$

Time	t_0	t_1	t_2	t_3	t_4
input	Λ	off	on	$offf$	on
$Qact(ti)$	a	f	d	e	e
Membership value	1.0	0.7	0.6	0.6	0.6

Since, $\rho(q,\ \mu^{ti-1}(q),\ onoffonoff,\ p) = \rho(q,\ \mu^{ti-1}(q),\ offonoffon,\ p)$, thus, the system is commutative. Then check the switching properties. If $\rho(q,\ \mu^{ti-1}(q),\ u,\ p) = \rho(p,\ \mu^{ti-1}(p),\ u,\ q)$, where $\forall q, p \in Q, u, v \in S, i \geq 1$, then the system is switching state machine. Here, from the above diagram, each state is switching state machine. For instance, $\rho(a,\ \mu^{ti-1}(a),\ on,\ b) = \rho(b,\ \mu^{ti-1}(b),\ on,\ a)$. Therefore, the system is GFSA since it fulfilled both conditions switching and commutative. If the system follows switchboard state machine, it means that the process can be vice versa depending on the situation. For instance, if the sudden failure happened which is e state is not working, it doesn't mean the whole system is breaking down. It is still can operate without e state. According to the Figure 2, from d state, it will go to the f state and go back to a and g state.

5. Conclusions

Algebraic properties are important for operations and throughout the system. In order to make the system or machine work correctly, the characteristics must be fulfilled and understandable. The theory is applied in real applications of general Fuzzy Switchboard Automata to make it more comprehensible and interesting. The importance of GFSA in real applications is to improve system function and automatically. In a nutshell, inspired by Doorsfatemeh and Kremer [1], the idea of Fuzzy General Automation and Switchboard in the concept of semi-group transformation for the study of algebraic automation is considered. Semigroups are important because they occur in many places. Since semigroups are important algebraic structures in automaton theory, they necessary to be studied. Transformation semigroups are important for the structure theory of finite state machines in automata theory. It defines all possible transition set transformations that can be joined in time and also as a collection of the functions from a set to itself. Since they have a huge number of sets of states, it is easier to explore the space of all possible finite computations by listing these semigroups. An example of the definition of a cascade product of a general fuzzy switchboard transformation group is presented since algebraic products are an effective way of studying the theory of state machines and automation. In addition, some of the related properties of General Fuzzy Switchboard Transformation Semigroup are examined. The definition of poly-transformation semigroup is equal to anti-poly-transformation semigroup meaning that it is satisfied commutative properties. Then, General Fuzzy Switchboard Poly-transformation Semigroup is introduced. In future research, it would be great to study the different products of the general fuzzy switchboard transformation semigroup. If there is a sudden failure, it can communicate between the subsystems and decide as a human being. Before that, check the system to follow the properties of the switchboard automation.

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