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Bipolar Neutrosophic Dombi Shapley Weighted Average Aggregation Operator and Its Application in Decision-Making Problem

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ABSTRACT

A bipolar neutrosophic set (BNS) is a novel set theory characterized by positive and negative membership degrees. It is designed to handle bipolar, inconsistent, and uncertain data. BNS is used widely to solve complex decision-making problems. Besides uncertainty data, aggregation operators (AOs) also important in decision-making problems. The Dombi operations, comprising of Dombi T-norm and Dombi T-conorm (DTT), exhibit significant general parameter flexibility. According to the literature, the Dombi operation rules have addressed many intricate decision-making problems in diverse settings. However, the current study needs to examine the optimal method for obtaining the weight of criteria during the aggregation process. The importance of the standards is defined in this study using a Shapley fuzzy measure. Considering this, the article presents the bipolar neutrosophic Dombi Shapley weighted average aggregation (BNDSWAA) inspired by the Dombi operation. The paper examines the various properties of BNDSWAA. A decision-making method is also created based on the suggested operator. The proposed process under BNS is explained using an algorithm for selecting green suppliers. Finally, testing for parameter influence is conducted to identify any parameter changes. The finding shows the proposed method is effective in solving decision-making problems.

1. Introduction

For numerous individuals, arriving at optimal decisions in their daily lives can often prove to be a complex task. Solving decision problems of this nature presents challenges due to the inherent complexities associated with managing uncertain data, diverse cluster weights, and varying criteria. When choosing a location to establish a new business, an essential consideration would be an in-

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depth assessment that includes property prices, the availability of skilled labour, access to businesses and distribution channels, and proximity to potential customers. Because objects are complex and human thought is fuzzy, decision-making (DM) often involves values that are uncertain, inconsistent, and insufficient in terms of attributes. Zadeh [1] proposed the fuzzy set (FS) theory in order to process fuzzy information. Following this, Gargov *et al.,* [2] and Atanossov [3] extended intuitionistic fuzzy sets (IFS) to interval-valued fuzzy sets (IVFS), which include membership and non-membership degrees. However, FS is incapable of handling the discontinuous and inconsistent data that exists in real-world problems. Smarandache [4] introduced the concept of a "neutrosophic set" (NS) to characterize fragmentary and inconsistent information. Wang *et al.,* [5] subsequently presented the concept of single-valued neutrosophic sets (SVNS). The SVNS has been utilized in various fields and it appears better suited for handling uncertain and contradictory data. Subsequently, Wang *et al.,* [6] extended SVNS to an interval form denoted as INS. Important in set theory, the bipolar neutrosophic set (BNS) was introduced by Deli *et al.,* [7]. A bipolar neutrosophic set (BNS) is a new set theory characterized by positive and negative membership degree to address bipolar, inconsistent, and uncertain data. BNS is a valuable instrument for modelling and coping with informational uncertainties caused by inconsistent and unreliable data.

Aggregation Operators (AOs) are significant research topics that have attracted the interest of modern scholars. The function of AO is to combine multiple DM evaluation data into a single overall evaluation data. The arithmetic mean and the geometric mean are AOs that are frequently employed in data analysis [8]. In recent years, weighted AOs have emerged as an important multi-criteria decision-making (MCDM) technique. They permit decision-makers to assign weights to each criterion and to account for interactions. The MCDM-weighted aggregation operators consider the interdependence among criteria and employ various weighting strategies to address the requirements of diverse decision-making situations. These operators have been widely utilised in diverse domains, such as energy and environment. Ilham *et al*., [9] proposed MCDM technique to evaluate potential renewable energy in Malaysia. One of the notable benefits of weighted AOs lies in their capacity to effectively manage imprecise or uncertain data and integrate it into the DM process. The act of combining individual preferences or subjective evaluations can alter the perceived importance of expert knowledge and expertise.

Another notable operator in the realm of DM is the Dombi operator, as proposed by Dombi [10]. The distinguishing feature of this approach lies in the utilization of DTT operations, which serve to augment the operator that exclusively focuses on algebraic product and sum. The Dombi operator is widely recognized for its ability to offer significant flexibility in terms of operational parameters. Moreover, Liu *et al*., [11] extended the application of Dombi operations to IFSs and proposed a set of intuitionistic fuzzy (IF) Dombi Bonferroni mean operators for MAGDM problems. The authors placed significant emphasis on the capacity of the system to consolidate neutrosophic numbers into a singular value. After Liu *et al.,* [12] investigated the Dombi T-norm operational norms in the context of linguistic neutrosophic, a 2-tuple linguistic neutrosophic power Heronian mean (2-TLNPHM) operator was subsequently developed. Some Dombi Heronian means (HM) fused operators with IVIFNs were recently proposed by Wu *et al.,* [13], while Mahmood *et al.,* [14] proposed a BNS based on Dombi operations. In a recent paper, Jana *et al.,* [15] use the single-valued trapezoidal neutrosophic number and Dombi aggregation to provide the MCDM approach for picking the best contractor for a road building company.

Based on the above analysis, Dombi operators have solved various complex decision-making problems in multiple environments. However, the current study needs to consider the best way to obtaining the weight of criteria in the aggregating process. It is critical to evaluate the significance of standards as well as the relationships between bars and their combinations at the same time. As a result, in this study, we combined Shapley fuzzy measure (SFM) as a weighting method with Dombi AO. It is also an effective tool for determining weights when uncertainty and imprecision are significant factors compared to traditional weightage methods. SFM has been widely used in MCDM to assess the interaction between criteria and decision-makers importance [16]. Nie *et al.,* [17] built on the concept of SFM to create a Pythagorean fuzzy partitioned normalised weighted Bonferroni mean. Awang *et al.,* [18] recently developed a normalised weighted Bonferroni mean that takes SFM into account in an INS environment. Hashim *et al.,* [19] developed methods to solve investment decisions by combining the HM operator with SFM under interval neutrosophic vague sets (INVS). As a result, no literature to our knowledge integrates bipolar neutrosophic with Dombi operational rule and applies it to Shapley weighted aggregation.

The remaining sections of the paper are structured as follows. The second section provides a summary of BNSs, Dombi operations, and SFM. In section 3, goes over detail the bipolar neutrosophic Dombi Shapley weighted arithmetic average (BNDSWAA) and its properties. Section 4 describes a BNDSWAA decision-making strategy. Section 5 provides a numerical example. In Section 6, we examine the parametric analysis and the effects of varying the parameter values. The report concludes with a conclusion and recommendations for future research.

2. Preliminaries

This section reviews the fundamental concept of BNS, BNSs, Dombi operations, and Shapley fuzzy measures (SFM). These concepts are utilized in the creation of the BNDSWAA operator.

2.1 Bipolar Neutrosophic Set and Its Operation
2.1.1 Definition 1

[7] Bipolar Neutrosophic Set. Let X be a nonempty set. A bipolar neutrosophic set *A* is defined as follows:

$$A=\left\{\left\langle x,T^{+}\left(x\right),I^{+}\left(x\right),F^{+}\left(x\right),T^{-}\left(x\right),I^{-}(x),F^{-}\left(x\right)\right\rangle : x\in X\right\},\tag{1}$$

where $T^{+},I^{+},F^{+}:X\rightarrow[1,0]$ and $T^{-},I^{-},F^{-}:X\rightarrow[-1,0]$.

Here, the $T^{+}\left(x\right),I^{+}\left(x\right),F^{+}\left(x\right)$ represents positive membership degree, while $T^{-}\left(x\right),I^{-}\left(x\right),F^{-}\left(x\right)$ represents negative membership degree of an element $x\in X$.

2.1.2 Definition 2

[7] Bipolar Neutrosophic Sets. Let $a_1=\left\langle t_1^{+},i_1^{+},f_1^{+},t_1^{-},i_1^{-},f_1^{-}\right\rangle$ and $a_2=\left\langle t_2^{+},i_2^{+},f_2^{+},t_2^{-},i_2^{-},f_2^{-}\right\rangle$ be two BNSs and $\lambda>0$. The following are the fundamental operations for these numbers:

i. $a_1+a_2=\left\langle t_1^{+}+t_2^{+}-t_1^{+}t_2^{+},i_1^{+}i_2^{+},f_1^{+}f_2^{+},-t_1^{-}t_2^{-},-\left(-i_1^{+}-i_2^{+}-i_1^{+}i_2^{+}\right),-\left(-f_1^{+}-f_2^{+}-f_1^{+}f_2^{+}\right)\right\rangle$

ii. $a_1\cdot a_2=\left\langle t_1^{+}t_2^{+},i_1^{+}+i_2^{+}-i_1^{+}i_2^{+},f_1^{+}+f_2^{+}-f_1^{+}f_2^{+},-\left(-t_1^{-}-t_2^{-}-t_1^{-}t_2^{-}\right),-i_1^{-}i_2^{-},-f_1^{-}f_2^{-}\right\rangle$

iii. $\lambda a_1=\left\langle 1-\left(1-t_1^{+}\right)^{\lambda},\left(i_1^{+}\right)^{\lambda},\left(f_1^{+}\right)^{\lambda},-\left(-t_1^{-}\right)^{\lambda},-\left(-i_1^{-}\right)^{\lambda},-\left(1-\left(1-\left(-f_1^{-}\right)\right)^{\lambda}\right)\right\rangle$

iv. $a_1^{\lambda} = \left\langle \left(t_1^+\right)^{\lambda}, 1-\left(1-i_1^+\right)^{\lambda}, 1-\left(1-f_1^+\right)^{\lambda}, -\left(1-\left(1-\left(-t_1^-\right)\right)^{\lambda}\right), -\left(-i_1^-\right)^{\lambda}, -\left(-f_1^-\right)^{\lambda} \right\rangle$

2.1.3 Definition 3

[7] Assuming that $a_1 = \left\langle t_1^+, i_1^+, f_1^+, t_1^-, i_1^-, f_1^- \right\rangle$ is a BNS, the score function is defined as follows:

$$s\left(a_1\right) = \frac{t_1^+ + 1 - i_1^+ + 1 - f_1^+ + 1 + t_1^- - i_1^- - f_1^-}{6} \tag{2}$$

2.2 Dombi Operation
2.2.1 Definition 4

[10] Dombi T-norm and T-conorm (DTT). Let λ be a positive real number and $x, y \in [0,1]$. Then, DTT among x and y are depicted as follows:

$$T_{D,\gamma}\left(x,y\right) = \frac{1}{1+\left(\left(\frac{1-x}{x}\right)^{\gamma} + \left(\frac{1-y}{y}\right)^{\gamma}\right)^{\frac{1}{\gamma}}} \tag{3}$$

$$T^*_{D,\gamma}\left(x,y\right) = 1 - \frac{1}{1+\left(\left(\frac{x}{1-x}\right)^{\gamma} + \left(\frac{y}{1-y}\right)^{\gamma}\right)^{\frac{1}{\gamma}}} \tag{4}$$

On the basis of the Dombi T-norm and T-conorm, the operational rules of BNS can be determined.

2.2.2 Definition 5

[14] Dombi operations of bipolar neutrosophic sets. Let $a_1 = \left\langle t_1^+, i_1^+, f_1^+, t_1^-, i_1^-, f_1^- \right\rangle$ and $a_2 = \left\langle t_2^+, i_2^+, f_2^+, t_2^-, i_2^-, f_2^- \right\rangle$ be two BNSs and γ, δ be two positive real numbers. Consequently, Dombi sums and product operation are presented below:

i. $a_1 \oplus_D a_2 =$

$$< 1 - \frac{1}{1+\left(\left(\frac{t_1^+}{1-t_1^+}\right)^{\gamma} + \left(\frac{t_2^+}{1-t_2^+}\right)^{\gamma}\right)^{\frac{1}{\gamma}}}, \frac{1}{1+\left(\left(\frac{1-i_1^+}{i_1^+}\right)^{\gamma} + \left(\frac{1-i_2^+}{i_2^+}\right)^{\gamma}\right)^{\frac{1}{\gamma}}}, \frac{1}{1+\left(\left(\frac{1-f_1^+}{f_1^+}\right)^{\gamma} + \left(\frac{1-f_2^+}{f_2^+}\right)^{\gamma}\right)^{\frac{1}{\gamma}}},$$

$$\frac{-1}{1+\left(\left(\frac{1-t_1^-}{-t_1^-}\right)^{\gamma} + \left(\frac{1-t_2^-}{-t_2^-}\right)^{\gamma}\right)^{\frac{1}{\gamma}}}, \frac{1}{1+\left(\left(\frac{-i_1^-}{1+i_1^-}\right)^{\gamma} + \left(\frac{-i_2^-}{1+i_2^-}\right)^{\gamma}\right)^{\frac{1}{\gamma}}} - 1, \frac{1}{1+\left(\left(\frac{-f_1^-}{1+f_1^-}\right)^{\gamma} + \left(\frac{-f_2^-}{1+f_2^-}\right)^{\gamma}\right)^{\frac{1}{\gamma}}} - 1 >$$

ii. $a_1 \otimes_D a_2 =$

$$< \frac{1}{1+\left(\left(\frac{1-t_1^+}{t_1^+}\right)^{\gamma}+\left(\frac{1-t_2^+}{t_2^+}\right)^{\gamma}\right)^{\frac{1}{\gamma}}}, 1-\frac{1}{1+\left(\left(\frac{i_1^+}{1-i_1^+}\right)^{\gamma}+\left(\frac{i_2^+}{1-i_2^+}\right)^{\gamma}\right)^{\frac{1}{\gamma}}}, 1-\frac{1}{1+\left(\left(\frac{f_1^+}{1-f_1^+}\right)^{\gamma}+\left(\frac{f_2^+}{1-f_2^+}\right)^{\gamma}\right)^{\frac{1}{\gamma}}},$$

$$\frac{1}{1+\left(\left(\frac{-t_1^-}{1+t_1^-}\right)^{\gamma}+\left(\frac{-t_2^-}{1+t_2^-}\right)^{\gamma}\right)^{\frac{1}{\gamma}}}-1, \frac{-1}{1+\left(\left(\frac{1+i_1^-}{-i_1^-}\right)^{\gamma}+\left(\frac{1+i_2^-}{-i_2^-}\right)^{\gamma}\right)^{\frac{1}{\gamma}}}, \frac{-1}{1+\left(\left(\frac{1+f_1^-}{-f_1^-}\right)^{\gamma}+\left(\frac{1+f_2^-}{-f_2^-}\right)^{\gamma}\right)^{\frac{1}{\gamma}}} >$$

iii. $\delta a_1 =$

$$< 1-\frac{1}{1+\left(\delta\left(\frac{t_1^+}{1-t_1^+}\right)^{\gamma}\right)^{\frac{1}{\gamma}}}, \frac{1}{1+\left(\delta\left(\frac{1-i_1^+}{i_1^+}\right)^{\gamma}\right)^{\frac{1}{\gamma}}}, \frac{1}{1+\left(\delta\left(\frac{1-f_1^+}{f_1^+}\right)^{\gamma}\right)^{\frac{1}{\gamma}}}, \frac{-1}{1+\left(\delta\left(\frac{1+t_1^-}{-t_1^-}\right)^{\gamma}\right)^{\frac{1}{\gamma}}},$$

$$\frac{1}{1+\left(\delta\left(\frac{-i_1^-}{1+i_1^-}\right)^{\gamma}\right)^{\frac{1}{\gamma}}}-1, \frac{1}{1+\left(\delta\left(\frac{-f_1^-}{1+f_1^-}\right)^{\gamma}\right)^{\frac{1}{\gamma}}}-1 >$$

iv. $a_1^{\delta} =$

$$< \frac{1}{1+\left(\delta\left(\frac{1-t_1^+}{t_1^+}\right)^{\gamma}\right)^{\frac{1}{\gamma}}}, 1-\frac{1}{1+\left(\delta\left(\frac{i_1^+}{1-i_1^+}\right)^{\gamma}\right)^{\frac{1}{\gamma}}}, 1-\frac{1}{1+\left(\delta\left(\frac{f_1^+}{1-f_1^+}\right)^{\gamma}\right)^{\frac{1}{\gamma}}}, \frac{1}{1+\left(\delta\left(\frac{-t_1^-}{1+t_1^-}\right)^{\gamma}\right)^{\frac{1}{\gamma}}}-1,$$

$$\frac{-1}{1+\left(\delta\left(\frac{1+i_1^-}{-i_1^-}\right)^{\gamma}\right)^{\frac{1}{\gamma}}}, \frac{-1}{1+\left(\delta\left(\frac{1+f_1^-}{-f_1^-}\right)^{\gamma}\right)^{\frac{1}{\gamma}}} >$$

2.3 Shapley Fuzzy Measure

The related SFM used in this study are as follows:

2.3.1 Definition 6

[20] $\lambda-$fuzzy measures can also be expressed as follows:

$$A_{\lambda}(A)=\begin{cases}\frac{1}{\lambda}\left(\prod_{K_i\in A}\left(1+\lambda\mu_{\lambda}\left(\{K_i\}\right)\right)-1\right), & \text{if } \lambda\neq 0\\ \sum_{K_i\in A}\mu_{\lambda}\left(\{K_i\}\right), & \text{if } \lambda=0\end{cases} \tag{5}$$

In addition, the value of λ can be calculated as follows:

$$\left(\prod_{K_i \in A}\left(1+\lambda\mu_\lambda\left(\{K_i\}\right)\right)\right)=\lambda+1 \tag{6}$$

The $\lambda-$FM is based on the SFM theory and Eq. (5) is developed by Shapley is shown as follows:

$$\varpi_i\left(\mu,P\right)=\sum_{W\subseteq P/i}\frac{\left(P-|W|-1\right)!|W|!}{P!}\left(\mu\left(W\cup i\right)-\mu\left(W\right)\right),\forall i\in P \tag{7}$$

where μ is a fuzzy measure on P. Here $|P|$ and $|W|$ represent the cardinality of the set P and W respectively. Besides, $\varpi_i\left(\mu,P\right)$ represent the weight vectors on set P as $\varpi_i\left(\mu,P\right)>1$ and $\varpi_i\left(\mu,P\right)=1$.

3. BND-Shapley Aggregation operators

In this section, we have introduced the bipolar neutrosophic Dombi Shapley weighted arithmetic average aggregation (BNDSWAA) and have examined various properties associated with aggregation operators (AOs). The definition of BNDSWAA is provided as follows:

3.1.Definition 7

Assume $a_i=\left\langle t_i^+,i_i^+,f_i^+,t_i^-,i_i^-,f_i^-\right\rangle$ $\left(i=1,2,...,n\right)$ be a collection of BNSs. A mapping BNDSWAA: $A^r\rightarrow A$ is called BNDSWAA operator if it satisfies $BNDSWAA\left(a_1,a_2,...,a_n\right)=\overset{n}{\underset{i=1}{\oplus}}\left(\varpi_i\left(\mu,P\right)a_i\right)$where $\varpi_1\left(\mu,P\right),\varpi_2\left(\mu,P\right),...,\varpi_i\left(\mu,P\right)$ is the weight vector of a_n represent SFM in relation to the related fuzzy measure μ on P satisfying $\varpi_i\left(\mu,P\right)\in[0,1]$ and $\sum_i^n\varpi_i\left(\mu,P\right)=1$.

3.1.1 Theorem 1

Let $a_i=\left\langle t_i^+,i_i^+,f_i^+,t_i^-,i_i^-,f_i^-\right\rangle$ $\left(i=1,2,...,n\right)$ be a collection of BNSs and the value aggregated by the Definition 7 in form of BNS and

$$\begin{aligned}&BNDSWAA\left(a_1,a_2,...,a_n\right)=\\&<1-\frac{1}{1+\left(\sum_{i=1}^n\varpi_i\left(\mu,P\right)\left(\frac{t_i^+}{1-t_i^+}\right)^\gamma\right)^{\frac{1}{\gamma}}},\frac{1}{1+\left(\sum_{i=1}^n\varpi_i\left(\mu,P\right)\left(\frac{1-i_i^+}{i_i^+}\right)^\gamma\right)^{\frac{1}{\gamma}}},\frac{1}{1+\left(\sum_{i=1}^n\varpi_i\left(\mu,P\right)\left(\frac{1-f_i^+}{f_i^+}\right)^\gamma\right)^{\frac{1}{\gamma}}},\\&\frac{-1}{1+\left(\sum_{i=1}^n\varpi_i\left(\mu,P\right)\left(\frac{1+t_i^-}{-t_i^-}\right)^\gamma\right)^{\frac{1}{\gamma}}},\frac{1}{1+\left(\sum_{i=1}^n\varpi_i\left(\mu,P\right)\left(\frac{-i_i^-}{1+i_i^-}\right)^\gamma\right)^{\frac{1}{\gamma}}}-1,\frac{1}{1+\left(\sum_{i=1}^n\varpi_i\left(\mu,P\right)\left(\frac{-f_i^-}{1+f_i^-}\right)^\gamma\right)^{\frac{1}{\gamma}}}-1>\end{aligned} \tag{8}$$

Proof: The following result was obtained by using $n=2$ from BNSs for Dombi operations Definition 7:

$$
\begin{aligned}
& BNDWAA\left(a_1,a_2\right) \\
& =<1-\frac{1}{1+\left(\varpi_1\left(\mu,P\right)\left(\frac{t_1^+}{1-t_1^+}\right)^{\gamma}+\varpi_2\left(\mu,P\right)\left(\frac{t_2^+}{1-t_2^+}\right)^{\gamma}\right)^{\frac{1}{\gamma}}},\frac{1}{1+\left(\varpi_1\left(\mu,P\right)\left(\frac{1-i_1^+}{i_1^+}\right)^{\gamma}+\varpi_2\left(\mu,P\right)\left(\frac{1-i_2^+}{i_2^+}\right)^{\gamma}\right)^{\frac{1}{\gamma}}}, \\
& \frac{1}{1+\left(\varpi_1\left(\mu,P\right)\left(\frac{1-f_1^+}{f_1^+}\right)^{\gamma}+\varpi_2\left(\mu,P\right)\left(\frac{1-f_2^+}{f_1^+}\right)^{\gamma}\right)^{\frac{1}{\gamma}}},\frac{-1}{1+\left(\varpi_1\left(\mu,P\right)\left(\frac{1+t_1^-}{-t_1^-}\right)^{\gamma}+\varpi_2\left(\mu,P\right)\left(\frac{1+t_2^-}{-t_2^-}\right)^{\gamma}\right)^{\frac{1}{\gamma}}}, \\
& \frac{1}{1+\left(\varpi_1\left(\mu,P\right)\left(\frac{-i_1^-}{1+i_1^-}\right)^{\gamma}+\varpi_2\left(\mu,P\right)\left(\frac{-i_2^-}{1+i_2^-}\right)^{\gamma}\right)^{\frac{1}{\gamma}}}-1,\frac{1}{1+\left(\varpi_1\left(\mu,P\right)\left(\frac{-f_1^-}{1+f_1^-}\right)^{\gamma}+\varpi_2\left(\mu,P\right)\left(\frac{-f_2^-}{1+f_2^-}\right)^{\gamma}\right)^{\frac{1}{\gamma}}}-1> \\
& =<1-\frac{1}{1+\left(\sum_{i=1}^{2}\varpi_i\left(\mu,P\right)\left(\frac{t_i^+}{1-t_i^+}\right)^{\gamma}\right)^{\frac{1}{\gamma}}},\frac{1}{1+\left(\sum_{i=1}^{2}\varpi_i\left(\mu,P\right)\left(\frac{1-i_i^+}{i_i^+}\right)^{\gamma}\right)^{\frac{1}{\gamma}}},\frac{1}{1+\left(\sum_{i=1}^{2}\varpi_i\left(\mu,P\right)\left(\frac{1-f_i^+}{f_i^+}\right)^{\gamma}\right)^{\frac{1}{\gamma}}}, \\
& \frac{-1}{1+\left(\sum_{i=1}^{2}\varpi_i\left(\mu,P\right)\left(\frac{1+t_i^-}{-t_i^-}\right)^{\gamma}\right)^{\frac{1}{\gamma}}},\frac{1}{1+\left(\sum_{i=1}^{2}\varpi_i\left(\mu,P\right)\left(\frac{-i_i^-}{1+i_i^-}\right)^{\gamma}\right)^{\frac{1}{\gamma}}}-1,\frac{1}{1+\left(\sum_{i=1}^{2}\varpi_i\left(\mu,P\right)\left(\frac{-f_i^-}{1+f_i^-}\right)^{\gamma}\right)^{\frac{1}{\gamma}}}-1>
\end{aligned}
$$

If $n=s$ is depends on Eq. (8), then we obtained the following equation:

$$
\begin{aligned}
& BNDSWAA\left(a_1,a_2,\ldots,a_n\right)=\overset{s}{\underset{i=1}{\oplus}}\left(\varpi_i\left(\mu,P\right)a_i\right) \\
& =<1-\frac{1}{1+\left(\sum_{i=1}^{s}\varpi_i\left(\mu,P\right)\left(\frac{t_i^+}{1-t_i^+}\right)^{\gamma}\right)^{\frac{1}{\gamma}}},\frac{1}{1+\left(\sum_{i=1}^{s}\varpi_i\left(\mu,P\right)\left(\frac{1-i_i^+}{i_i^+}\right)^{\gamma}\right)^{\frac{1}{\gamma}}},\frac{1}{1+\left(\sum_{i=1}^{s}\varpi_i\left(\mu,P\right)\left(\frac{1-f_i^+}{f_i^+}\right)^{\gamma}\right)^{\frac{1}{\gamma}}}, \\
& \frac{-1}{1+\left(\sum_{i=1}^{s}\varpi_i\left(\mu,P\right)\left(\frac{1+t_i^-}{-t_i^-}\right)^{\gamma}\right)^{\frac{1}{\gamma}}},\frac{1}{1+\left(\sum_{i=1}^{s}\varpi_i\left(\mu,P\right)\left(\frac{-i_i^-}{1+i_i^-}\right)^{\gamma}\right)^{\frac{1}{\gamma}}}-1,\frac{1}{1+\left(\sum_{i=1}^{s}\varpi_i\left(\mu,P\right)\left(\frac{-f_i^-}{1+f_i^-}\right)^{\gamma}\right)^{\frac{1}{\gamma}}}-1>
\end{aligned}
$$

If $n=s+1$, then there is following result:

As a result, Theorem 1 is valid for $n=s+1$. Thus, the Eq. (8) is true for all n.

The BNDSWAA operator meets the following requirements such as idempotency, monotonicity and boundedness.

The proof of idempotency, monotonicity and boundedness are omitted.

4. The MCDM Approach based on BNDSWAA Operator

In this section, we discuss the comprehensive MCDM method that extended based on the proposed BNDSWAA operator. Let $A=\{A_1,A_2,...,A_n\}$ be a set of alternatives and $C=\{C_1,C_2,...,C_n\}$ be a set of criteria. Assume weight of vector denoted as $\varpi_1(\mu,P),\varpi_2(\mu,P),...,\varpi_i(\mu,P)$ obtained using SFM where $\varpi_i(\mu,P)\in[0,1]$ and $\sum_i^n \varpi_i(\mu,P)=1$. Suppose that $a=(a_i)_{n\times m}$ is BNS decision matrix where $a_i=\langle t_i^+,i_i^+,f_i^+,t_i^-,i_i^-,f_i^-\rangle=a$,$(i=1,2,3,...,n)$. The MCDM algorithm is based on the proposed bipolar neutrosophic Dombi Shapley aggregation operators, which are displayed below.

Algorithm

Step 1 Determine the Shapley fuzzy measure (SFM) of each criterion.

Firstly, the FM based on criteria is calculated by using Eq. (5) and Eq. (6). The SFM that are obtained are followed by using Eq. (7).

Step 2 Determine the aggregate BNSs for each alternative using BNDSWAA operator as stated in Eq. (8).

Step 3 The score values of each alternative can be determined by utilizing Eq. (2).

Step 4 Select the best alternative by ranking them all.

Step 5 End.

5. Numerical Example

The consideration of decision-making issues is adapted from Wu and Cao [21] work. Green production is receiving increasing attention in numerous industries due to global concerns about the environment. The car company desires to locate the finest green supplier for one of the most crucial components in the car manufacturing process. There are four alternatives that are presented and used to be considered in the following pre-evaluation. Three criteria are taken into account: Product quality (C_1), technological prowess (C_2), and pollution control (C_3). The weight of criteria $\{C_1,C_2,C_3\}$ is calculated using SFM. An expert has compiled the criterion values for the four viable options in a BNS environment as follows:

$$\begin{array}{cc} & \begin{array}{ccc} C_1 & C_2 & C_3 \end{array} \\ \begin{array}{c} A_1 \\ A_2 \\ A_3 \\ A_4 \end{array} & \begin{pmatrix} \langle 0.4,0.5,0.3,-0.6,-0.4,-0.5\rangle & \langle 0.6,0.1,0.2,-0.4,-0.3,-0.2\rangle & \langle 0.8,0.6,0.5,-0.3,-0.2,-0.1\rangle \\ \langle 0.6,0.4,0.2,-0.4,-0.5,-0.7\rangle & \langle 0.6,0.2,0.3,-0.5,-0.2,-0.3\rangle & \langle 0.7,0.4,0.5,-0.1,-0.3,-0.4\rangle \\ \langle 0.7,0.2,0.4,-0.2,-0.6,-0.4\rangle & \langle 0.9,0.3,0.6,-0.2,-0.2,-0.5\rangle & \langle 0.6,0.1,0.5,-0.2,-0.4,-0.6\rangle \\ \langle 0.8,0.6,0.5,-0.5,-0.3,-0.6\rangle & \langle 0.6,0.4,0.3,-0.1,-0.3,-0.4\rangle & \langle 0.9,0.6,0.4,-0.5,-0.3,-0.6\rangle \end{pmatrix} \end{array}$$

Step 1: Determine the Shapley fuzzy measure.

$\mu(C_1)=0.55;\quad \mu(C_2)=0.85;\quad \mu(C_3)=0.75$

The λ-value is then determined by applying Eq. (6), as demonstrated below:

$\lambda+1=(1+0.55\lambda)(1+0.85\lambda)(1+0.75\lambda)$

$\lambda=-0.9795$

Then, Eq. (5) is applied to get FM of each criterion, which is calculated as follows:

$$\mu(C_1,C_2)=\frac{1}{-0.9795}\left(\left(1-0.9795(0.55)\right)\left(1-0.9795(0.85)\right)-1\right)=0.9421$$
$$\mu(C_1,C_3)=\frac{1}{-0.9795}\left(\left(1-0.9795(0.55)\right)\left(1-0.9795(0.75)\right)-1\right)=0.8960$$
$$\mu(C_2,C_3)=\frac{1}{-0.9795}\left(\left(1-0.9795(0.85)\right)\left(1-0.9795(0.75)\right)-1\right)=0.9756$$
$$\mu(C_1,C_{2,}C_3)=\frac{1}{-0.9795}\left(\left(1-0.9795(0.55)\right)\left(1-0.9795(0.85)\right)\left(1-0.9795(0.75)\right)-1\right)=1$$

Then, Eq. (7) is applied to get measure SFM of each criterion, which is calculated as follows:

$$\varpi_1=\frac{(3-0-1)!0!}{3!}\left(\mu(C_1)-\mu(\phi)\right)+\frac{(3-1-1)!1!}{3!}\left(\mu(C_1,C_2)-\mu(C_2)\right)+\frac{(3-1-1)!1!}{3!}\left(\mu(C_1,C_3)-\mu(C_3)\right)+$$
$$\frac{(3-2-1)!2!}{3!}\left(\mu(C_1,C_2,C_3)-\mu(C_2,C_3)\right)=0.23$$

Using similar step, we have $\varpi_2=0.42$ and $\varpi_3=0.35$ respectively.

Step 2: Determine the aggregate BNS for all alternatives A_i where $i=1,2,\ldots,n$ using Eq. (8) and takes $\gamma=1$.

$$A_1=\langle 0.6859, 0.1907, 0.2804, -0.3846, -0.2962, -0.2721\rangle$$
$$A_2=\langle 0.6418, 0.2817, 0.3077, -0.2035, -0.3266, -0.4872\rangle$$
$$A_3=\langle 0.8288, 0.1653, 0.5063, -0.2000, -0.4059, -0.5234\rangle$$
$$A_4=\langle 0.8246, 0.4959, 0.3656, -0.1866, -0.3000, -0.5349\rangle$$

Step 3: Calculate the score function $S(A_i)$ where $i=1,2,\ldots,n$ of BNSs for each alternative. Based on Eq. (2), the BNDSWAA operator generates the following score values.

$S(A_1)=0.5664;\quad S(A_2)=0.6105;\quad S(A_3)=0.6478;\quad S(A_4)=0.6019$

Step 4: Sort all the options $A_i\ (i=1,2,...,n)$ in descending order and select the best one(s). The following is the ranking order when using the BNDSWAA operator: $A_3 \succ A_2 \succ A_4 \succ A_1$. Thus, the best alternative is A_3.

Step 5: End.

6. Influence of Parameter

In this example, we use different values of γ in step 2 of the above method to rank the options. This is done to demonstrate how γ affects the decision-making process. The ranking results are displayed in Table 1.

Table 1
Ranking of alternatives based on different parameters in the BNDSWAA operator

γ	$S(A_1)$	$S(A_2)$	$S(A_3)$	$S(A_4)$	*BNDSWAA*
1	0.5664	0.6105	0.6478	0.6019	$A_3 \succ A_2 \succ A_4 \succ A_1$
2	0.5995	0.6428	0.6679	0.6207	$A_3 \succ A_2 \succ A_4 \succ A_1$
3	0.6197	0.6643	0.6815	0.6317	$A_3 \succ A_2 \succ A_4 \succ A_1$
4	0.6325	0.6785	0.6910	0.6388	$A_3 \succ A_2 \succ A_4 \succ A_1$
5	0.6412	0.6883	0.6978	0.6438	$A_3 \succ A_2 \succ A_4 \succ A_1$
6	0.6475	0.6952	0.7029	0.6473	$A_3 \succ A_2 \succ A_1 \succ A_4$
7	0.6522	0.7004	0.7068	0.6500	$A_3 \succ A_2 \succ A_1 \succ A_4$
8	0.6559	0.7044	0.7099	0.6520	$A_3 \succ A_2 \succ A_1 \succ A_4$
9	0.6588	0.7076	0.7124	0.6536	$A_3 \succ A_2 \succ A_1 \succ A_4$
10	0.6612	0.7102	0.7144	0.6549	$A_3 \succ A_2 \succ A_1 \succ A_4$

Table 1 shows that the changing value of γ using the BNDSWAA operator changes the ranking order slightly, but the best alternative remains consistent. For $1 \le \gamma \le 10$, the ranking orders are shown as $A_3 \succ A_2 \succ A_4 \succ A_1$ and $A_3 \succ A_2 \succ A_1 \succ A_4$. As a results, the most viable option is A_3.

7. Conclusion

The purpose of this paper is to combine a weighted aggregation operator with SFM under BNS based on the Dombi operational rule. This combination aims to develop a solution for MCDM problems. The purpose of conducting these studies was to ensure the dependability and versatility of the bipolar neutrosophic Dombi operators. The validity and adaptability of the proposed methods were further investigated through a parameter analysis. Furthermore, various properties of the BNDSWAA operator are being discussed.

The contribution of the paper is described below: Initially, BNSs were utilised to present evaluation-based information for decision-making. The relative importance of each criterion is then determined using SFM. Thirdly, we used the score function of the BNDSWAA operator to rank the alternatives and select the optimal option(s) according to the score values in the various operational parameters. A numerical example confirmed the adaptability and dependability of the recommended operator.

In future applications, BNDSWAA operators may be employed to aggregate diverse neutrosophic environments. Moreover, the suggested aggregations have the potential to be implemented across various domains, including but not limited to pattern recognition, data analysis, and many more. To assist process of aggregating data, the graphical user interface should be developed such as the study by Ab Wahab *et al.,* [22].

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