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Modularity Density Based-Edge Betweenness Method for Catchment Classification in Sabah

Siti Aisyah Tumiran[1,*], Farah Nurul Ain Mohd Suffian@Laurance[1] , Suzelawati Zenian[1], Kawi Bidin[1]

[1] Mathematics Visualization (MathViz) Research Group, Faculty of Science and Natural Resources, Universiti Malaysia Sabah, Jalan UMS, 88400 Kota Kinabalu, Sabah, Malaysia
[2] Faculty of Science and Natural Resources, Universiti Malaysia Sabah, Jalan UMS, 88400 Kota Kinabalu, Sabah, Malaysia

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ABSTRACT

One of the most significant studies in hydrology is the classification of catchments. There are various reasons why catchment classification studies have been conducted, but most importantly is to anticipate the prediction of ungauged basin (PUB), among others. There are several ways of classification, each with its own set of assumptions and basis, that have been used for catchment classification. Recently, complex network concepts, specifically community structure, have emerged as key classification methods, and are now attaining traction in catchment classification. As a result, in this study, community structure approach, specifically the Modularity Density based- Edge Betweenness (MDEB) method is implemented to split the network into communities. By using the proposed method, a network of 30 monthly streamflow stations across Sabah is considered for catchment classification. The impact of correlation threshold, which vary from 0 to 1, denoting the strength between a pair of catchments is also investigated. As a result, four threshold values are chosen (T = 0.5, 0.55, 0.6, and 0.65), and the communities established with threshold value, T=0.6 are interpreted. The relationship between catchment characteristics and flow characteristics are investigated as well as distance-correlation relationships for communities identified to understand the Sabah catchment's behaviors.

1. Introduction

Catchments are the areas of land from which water drains into a particular river, lake, or other water body. The classification of catchments is important for various purposes, including water resource management, flood prediction, and environmental planning. Catchment classification refers to the categorization of catchments based on certain characteristics or criteria. Grouping catchments according to their salient properties is especially helpful for identifying the appropriate complexity of models and for interpolating and extrapolating data (most importantly predictions in ungauged basins), among other things, as vast types of catchments exhibit varying levels of complexities. The fundamental concept of catchment classification is to categorize catchments according to their

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[*] *Corresponding author.*
E-mail address: sitiaisyah.tumiran@ums.edu.my

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prominent qualities (such as system, process, size, and data attributes). There are several strategies and techniques for classifying catchments. Measures of fluxes and storages, river/flow regimes, hydroclimatic variables, river morphology, hydrologic similarity indexes, hydrologic signatures, catchment topography, hydropedological variables, ecohydrologic and geomorphic variables, data-based mechanistic strategies, and data-driven methodologies as well as by hydrological responses and statistical regression analysis are some of these [1-6]. With the advancement of complex network science in recent years [7-10] the concept of community structure [9-12] has been applied for catchment classification [13-17]. The idea of community structure is extremely helpful in the specific situation of categorization. A network structure known as a "community structure" is one in which separate groupings, or "communities," are established by a collection of nodes (such as catchments), each of which is more densely linked than the others. Catchments have lately been classified using this idea. Walktrap, leading eigenvector, edge betweenness, modularity optimization, greedy algorithm and label propagation, are commonly used techniques for community structure in networks. Even though these techniques have been demonstrated to be helpful and successful for classifying catchments including a variety of systems, they frequently have drawbacks when used to classify actual dynamically developing systems. [19]. Recently, Tumiran and Sivakumar [18] has improved one of community structure methods specifically the edge betweenness algorithm called as the Modularity Density-based Edge Betweenness (MDEB) method. The MDEB method is improved from the edge betweenness (EB) method by using measures of the geodesic (shortest) path between links in the network. There are several signature steps that are still adopted from the EB method into the MDEB method. Due to the limitation contained in the EB method through the Modularity function, somehow the Modularity Density [18] is applied to combat scale resolution limit problem in the Modularity function. To achieve an accurate and dependable framework in catchment classification, it is crucial to find a consistent technique or procedure to classify catchments due to changes in the quantity of catchments (or the network size).

The advantage of the MDEB method against the EB method were validated through monthly streamflow data from two large networks of 639 stations from the United States (US) and 218 stations from Australia in understanding catchment behaviors [16]. The findings not only provide more support for the community structure concept's applicability and value in classifying catchments, but also demonstrate the value and potential of the MDEB method in the creation of a general framework for catchment classification. This is because network sizes are frequently uncertain due to the variability of available streamflow monitoring stations. However, the study only considered two regions and a very limited number of network sizes. In order to verify the competency of the MDEB method, one should extend the analysis by examining a variety of global areas and consider different various network size combinations (with different climatic and demographic factors).

Therefore, in this study, we address this by using the Modularity Density-based Edge Betweenness (MDEB) approach for catchment classification in a region with relatively small number of catchments with different climatic and demographic factors. Specifically, a monthly streamflow data of 30 catchments across Sabah is analyzed with different values of correlation thresholds to assess the classification's sensitivity to threshold. We also look into catchment and flow characteristics i.e., elevation, drainage area, flow mean and flow covariance (CV) for their behaviours assessments. Detail of relationship between distance with the communities' correlation is also presented.

The rest of this paper is organized as follows. The MDEB method is described in Methodology. The research region (Sabah) and the streamflow data taken into consideration are described in detail in the section of study area. The classification of streamflow across Sabah is presented in results and discussion. Final section presents recommendations for more study and makes some conclusions.

2. Methodology

Mathematically, a network is described as graph *G = {V, E},* where *E* consists of a set of *n* links and *V* consists of a set of *N* nodes $(V_1, V_2, \ldots, V_N)$. The connection of a pair of nodes in a network is assigned and does not have to be a simple binary relationship, somehow can be rely on a metric that quantifies the strength of the link.

Many complex networks feature nodes that cluster into distinct groups, with each group being more tightly linked than the rest of the network. These communities' features frequently stand apart from those of particular nodes and the network as a whole. These affiliations are referred to as "communities," and this network structure is referred to as a "community structure" [9]. Finding groupings in networks is very helpful as nodes in a community are mostly shared similar network characteristics and behaviors (such degree distribution, centrality, shortest path length, clustering and communicability).

2.1 Modularity Density-based Edge Betweenness Method

In this study, let an unweighted and undirected network is considered where a pair of nodes *i* to *j* is equivalently connected like a pair of nodes *j* to *i* [10]. In this study, the cross-correlation values are obtained based on streamflow values between a pair of streamflow stations (nodes), since the correlations between the streamflow stations represent the strength between them. By taking into account a cross-correlation values (between two stations) and by comparing with the selected threshold value between two stations, it is possible to determine whether or not there is a link between any two stations [19]. The adjacency matrix will then be formed based on how each node in the network is connected to each other node. The hallmark stages of the EB method will continue to be used in the MDEB method. To find the optimal split for the network, the highest value of modularity density function, or D value, is calculated instead of the modularity function (Q value). Consequently, the MDEB method [12] applied the modularity density function (D value) as shown in Eq. (1).

$$D = \sum_i \left(\frac{2l_i}{n_i} - \frac{l_i^{ext}}{n_i} \right) \tag{1}$$

where n_i is the count of nodes in subgraph *i*, l_i^{ext} is the count of external links in subgraph *I,* and l_i is the count of internal links in subgraph *i*.

3. Study Area

The application of the MDEB method for catchment classification will be carried out on monthly streamflow data from 30 stations across Sabah as shown in Figure 1. Different colors are used to represent districts across Sabah and the black circles represent the location of streamflow monitoring stations that are considered. Sabah is divided with 5 divisions which are Kudat division, west coast division, Sandakan division, interior division and Tawau division. These 5 divisions then are divided into 24 districts in Sabah; West coast division covers Tuaran, Kota Belud, Ranau, Kota Kinabalu, Penampang, Putatan, and Papar; Interior division covers Beaufort, Kuala Penyu, Tambunan, Keningau, Sipitang, Nabawan and Tenom; Kudat division covers Kota Marudu, Kudat and Pitas ; Sandakan division covers Sandakan, Beluran, Tongod and Kinabatangan; and Tawau division covers Lahad Datu, Tawau, Semporna and Kunak.

The monthly streamflow data of 30 streamflow stations in Sabah are obtained and maintained by the Department of Irrigation and Drainage (DID). Some stations obtained from the DID database are excluded in this study, due to missing data and hence, 8 districts are also not considered in this study which are Kudat, Kota Kinabalu, Putatan, Kuala Penyu, Semporna, Kunak, Lahad Datu and Sandakan.

The average monthly streamflow values considered in this study consists of that are observed over a period of 20 years, which is from January 2001 to December 2020. The 30 observed streamflow stations are varied in their characteristics as shown in Table 1. For example: (1) the basin drainage area is in the range of 0 to 12300 km^2; (2) the elevation for all stations ranges from 0 to 1600 m; and (3) the flow mean ranges from 0.709875 m^3/s to 687.395 m^3/s.

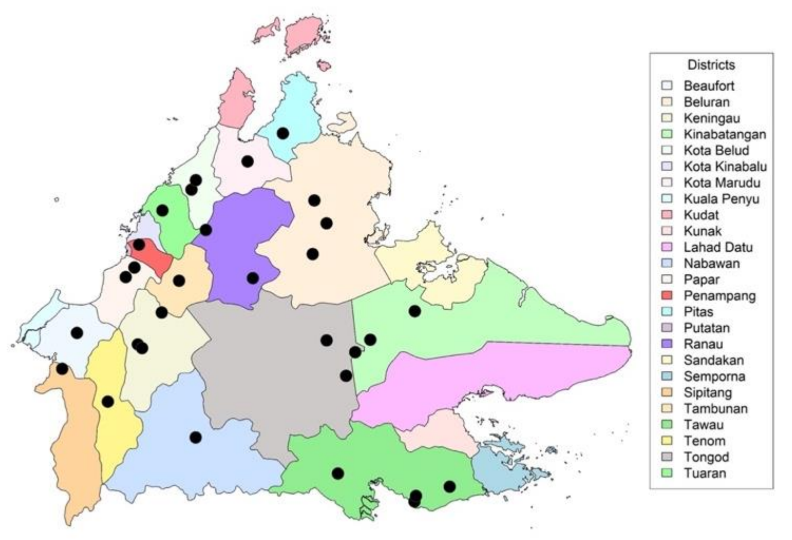


Fig. 1. Location of the 30 hydrologic monitoring stations across Sabah

Table 1
Variations of Sabah streamflow data

	Minimum	Maximum	Station (Districts)
Latitude	4.2823°	6.620778°	Minimum: 4278402 (TWU)[a] Maximum: 6670401 (PTS)
Longitude	115.6325°	118.1064°	Minimum: 5156403 (SPTG) Maximum: 4381401 (TWU)
Drainage area (km^2)	0	12300	Minimum: 4378401 (TWU), 5663402 (TBN) Maximum: 5478403 (KNBTGN)
Elevation (m)	0	1600	Minimum: 4378401 (TWU), 5275401 (TGD), 5478403 (KNBTGN), 5663402 (TBN), 6073402 (BLRN), 6162404 (TRN) Maximum: 6065401 (RNU)
Flow mean (m^3/s)	0.709875	687.395	Minimum: 6065401 (RNU) Maximum: 5478403 (KNBTGN)
Flow standard deviation (m^3/s)	0.544047	515.729	Minimum: 6065401 (RNU) Maximum: 5478403 (KNBTGN)
Flow CV	0.520896	2.344676	Minimum: 5668401 (RNU) Maximum: 4474401 (TWU)

[a] BLRN Beluran; KNBTGN Kinabatangan; PTS Pitas; RNU Ranau; TBN Tambunan; TGD Tongod; TRN Tuaran; TWU Tawau.

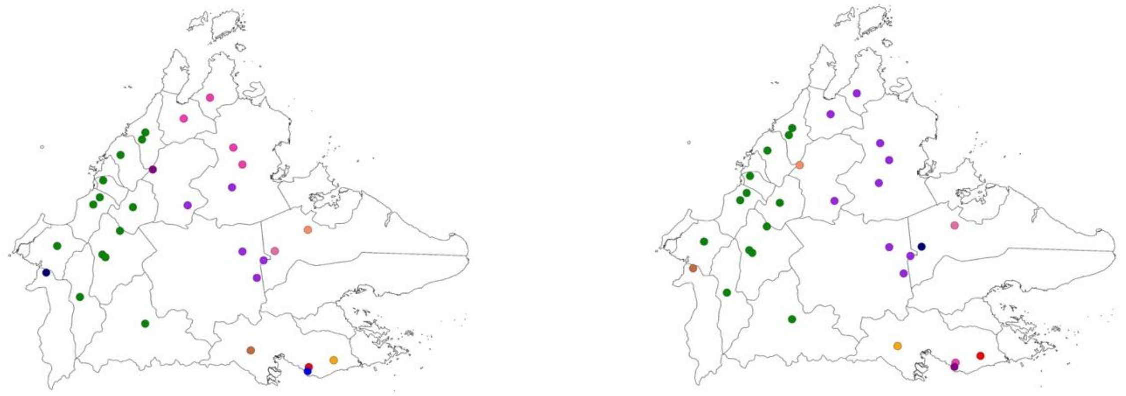
4. Results

The Pearson correlation coefficient between the streamflow stations is used to assign linkages between node pairs (i.e., stations) in the implementation of the MDEB technique to classify monthly streamflow data of 30 catchments in Sabah. The correlation threshold's selection range is based on network-based methodologies for network-based analysis of streamflow (and other hydrologic) data in order to more accurately reflect the effect of the threshold. T = 0.50, 0.55, 0.6, and 0.65 are the threshold values that are taken into account [19,20]. Any node pair with a correlation coefficient greater than a specified threshold is granted a connection.

The communities identified for the four selected threshold values: T = 0.5, 0.55, 0.6, and 0.65 are shown in Figure 2 where all communities identified are represented with different colors to visualize different communities and have no significance when comparing thresholds. A high number of linkages are detected when the threshold is too low leads to form big communities and span high percentage of the research region; however, this is not beneficial for examining the various features of the catchments. An excessively high threshold, on the other hand, will result in fewer linked linkages, eventually breaking the network into closer and smaller neighbors which then forming small communities also is meaningless to study catchment behaviors. Therefore, an ideal threshold value selection is critical in order to assess each community established in the network by investigate the catchment characteristics/properties and to cover big proportion of study area as feasible.

In order to comprehend the hydrologic similarities and to provide convincing explanations of the classification of catchments, Table 2 presents a precise calculation of amount of communities with the count of stations is obtained. The greatest community's number of stations declines as the threshold value rises while the count of communities with just at most two catchments rises. Nonetheless, the overall number of detected communities differs when the threshold values are increased.In light of the aforementioned findings, the communities designated for the threshold value T = 0.6 (Figure 2(c)) are picked based on their geographic limits and areas to allow for a more accurate evaluation of the catchment behaviours and trends across the catchments. Based on Figure 2(c), there are three communities have at least four stations (blue, orange and red colors) are studied. Figure 3 shows the largest three communities identified plotted with the colored districts for better visualization of the communities' locations by referring regions and boundaries across Sabah. Table 2 shows that, for T = 0.6, the largest community, which has 10 stations, accounts for almost 33% of the overall number of stations (10 out of 30), and that the three largest communities, which have a combined total of 14 stations, account for nearly 63% of the total count of stations (19 out of total 30). This would appear to indicate that, despite of distance or position in different basins/regions, each catchment within a major community has important connections with the other catchments within that community. Communities consist only one catchment (11 communities) have account for over 80% of the total count of identified communities (14) but account nearly 36% of total station count (11 out of 30). This appears to imply that, regardless of their being inside the same basin/region, each streamflow station in these tiny communities interacted loosely with the other catchments. These findings show that catchment and flow characteristics (which are impacted by physical and demographic closeness) have a key role in community development, i.e., catchment classification. This may be further investigated by connecting the selected communities to catchment/flow characteristics. This is carried out in the next sections with regard to station elevation and station drainage area (as catchment characteristics) as well as station flow coefficient of variation (CV) and station flow mean (as flow characteristics).

(a) T = 0.50 (b) T = 0.5



(c) T = 0.60 (d) T = 0.65

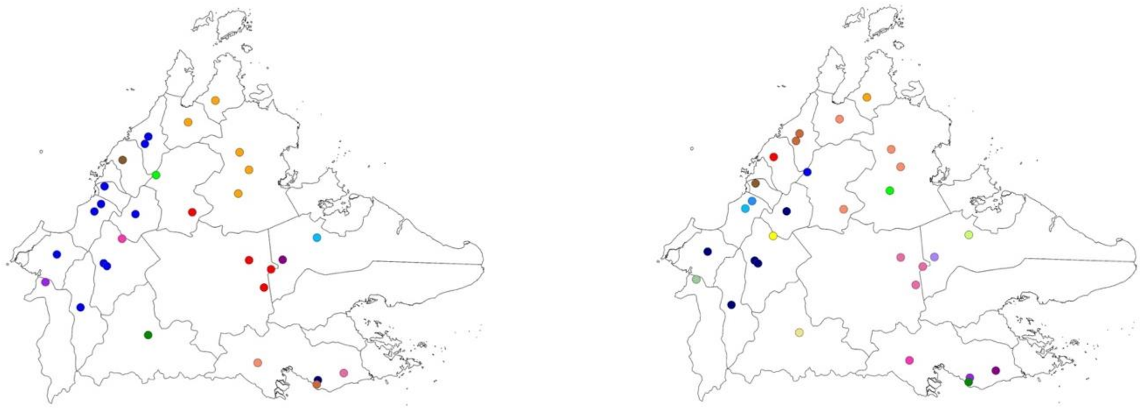


Fig. 2 (a-d). Classification using the MDEB method with four thresholds values: (a) T = 0.50; (b) T = 0.55; (c) T = 0.60; and (d) T = 0.65

Table 2

Count of identified communities across Sabah using the MDEB method at four correlation thresholds (T=0.50,0.55,0.60 and 0.65). (NS is the count of stations, NSC is the count of stations in communities and NC is the count of communities)

$\mathbf{T = 0.50}$			$\mathbf{T = 0.55}$			$\mathbf{T = 0.60}$			$\mathbf{T = 0.65}$		
NSC	NC	NS	NSC	NC	NS	NSC	NC	NS	NSC	NC	NS
1	8	8	1	8	8	1	11	11	1	16	16
4	1	4	9	1	9	4	1	4	2	1	2
5	1	5	13	1	13	5	1	5	3	1	3
13	1	13	Total	10	30	10	1	10	4	1	4
Total	11	30				Total	14	30	5	1	5
									Total	20	30

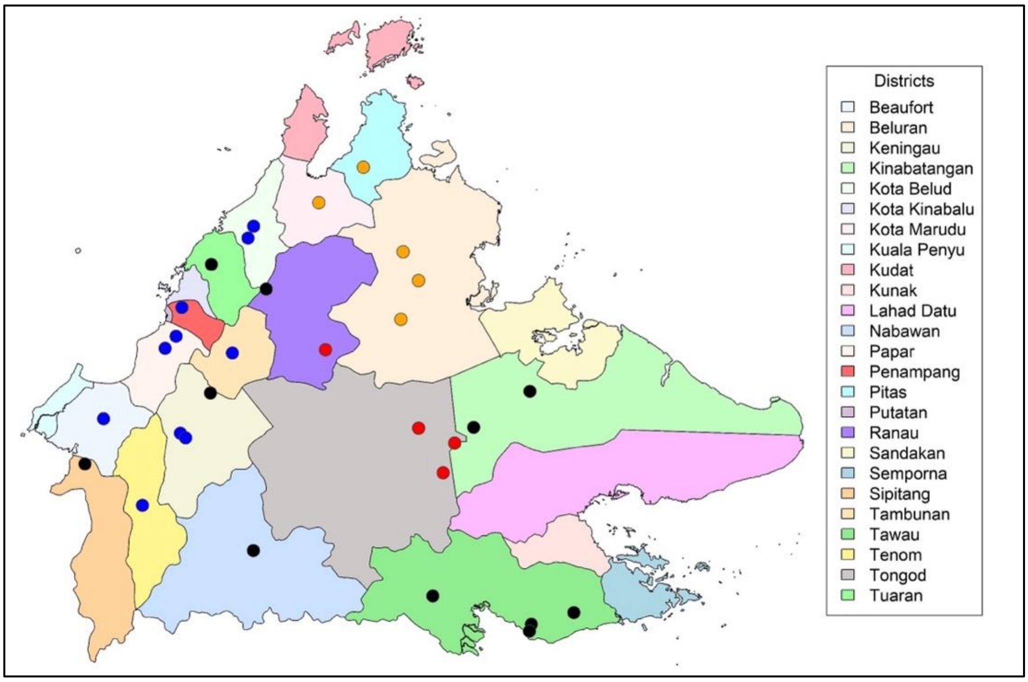


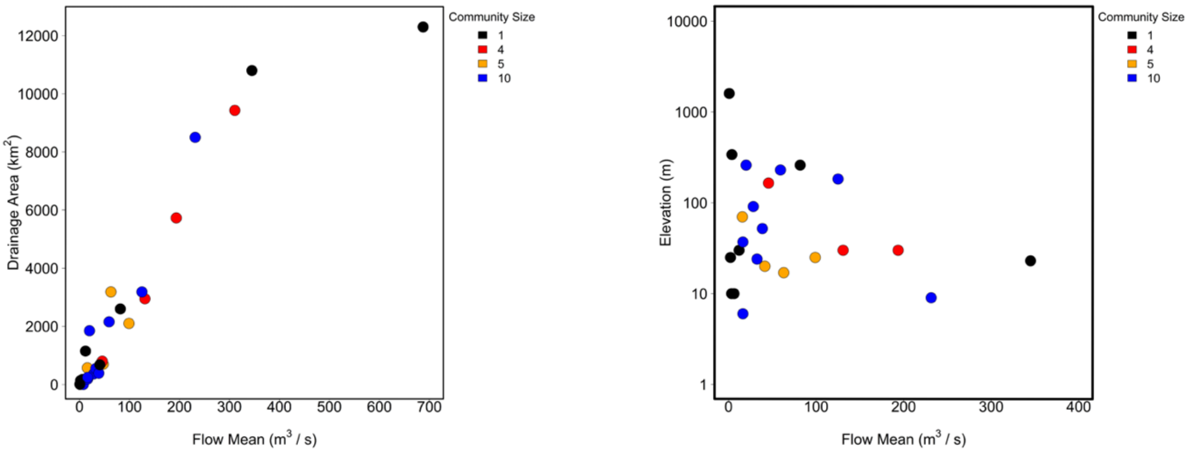
Fig. 3. Locations of the identified communities using the MDEB method at correlation threshold, T = 0.60. The largest communities are plotted in different colors (red, blue, and orange), while the individual communities (consist only one station) is colored in black. All districts are colored to distinguish regions and boundaries

Only the three biggest communities (19 stations) are colored in Figure 4 as depiction of the relationship between flow mean and the catchment parameters (i.e., elevation and drainage area). Figure 5 depicts their association with the flow CV. In order to better visualize the locations of the selected communities, the colors used in these figures for those communities match those in Figure 3. As shown in Figure 3, the communities that are located in some parts of West Coast and Interior Divisions (blue-community 10), Kudat and part of Sandakan Divisions (orange-community 5), and also parts of West Coast and Sandakan Divisions (red-community 4) are analyzed. The relationship (drainage area vs. flow mean) are directly proportional as seen in figure 4(a), indicates that when stations' drainage area increases, the flow mean also increases. On the other hand, the relationship in Figure 4(b) (elevation vs. flow mean) shows that the stations clustered at flow mean ranged mostly within 0 to 100 m^3/s when the elevation is ranging between 10 m to 1000 m.

Figure 4(a) and 5(a) describe the relationships between drainage area against flow mean and drainage area against flow CV are worth mentioning here, as there are significantly different for most communities, especially when the drainage area are compared with the flow mean as in Figure 4(a), the pattern is linearly strong proportional. While, the distribution of drainage area when compared with the flow CV (Figure 5(a)) are more clustered within lower range in both drainage area and flow CV, albeit the community in blue (community 10) is more clustered than the communities colored in red (community 4) and orange (community 5). It is also important to note in Figure 5(a) that the community colored in red (community 4) is hugely varied in terms of drainage area but almost similar values in flow CV (i.e. around 0.5), while community colored in orange (community 5) are almost similar in size of drainage area but varied in terms of flow CV which is range within 1.0 to 1.5. This seems to suggest that the directly proportional relationship in Figure 4(a) is most likely due to the river morphology (river network) between the catchments in the communities in Sabah catchments, as a whole. The water discharges from upstream to downstream through the river channel and contribute to increased size of drainage areas at downstream and hence the increasing flow mean as well. It also can be seen from the relationships (elevation vs. flow mean and elevation vs. flow CV) in Figures 4(b) and 5(b) are almost similar in pattern for most communities, where most stations formed almost 'C' shape distribution. This seems to prove the river network factor in the network where the tributaries from upstream started from higher elevations and flow downstream assisted with gravity hence formed higher flow rate (either in flow mean or flow CV).

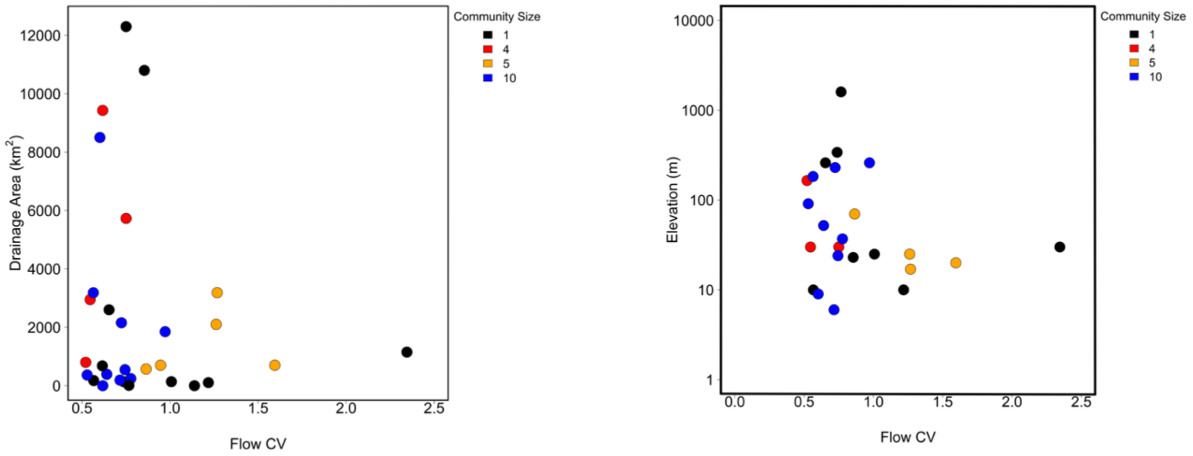
Overall, The MDEB method and its capacity to identify catchments on the basis of connectedness, without prior knowledge of catchment physics and purely depending on linkages in streamflow, has shown to be beneficial. Furthermore, most communities vary in elevation implying that the catchments have significant links among themselves, despite of elevation variations.

By analysing the distance and correlation between the stations, the efficacy of the MDEB approach for classification is also investigated. Figure 6 compares the distance-correlation data for the three communities mentioned previously. From the relationship, communities 10 (blue), 5 (orange) and 4 (red) (Figure 6(a), (b), (c)) decrease in correlations as the distance increases. In this present study, as the number of stations considered is relatively small (30 stations) hence it is not surprising for relatively smaller communities have almost similar behaviors in terms of distance and correlation relationship between the stations in a community. Apart from that, all communities with marginally strong correlations are determined to be connected, maybe because of the close proximity as depicted by community colored in orange and red (Figure 6 (a-b)). Therefore, geographical closeness and the river system may also be crucial considerations for classifying catchments.



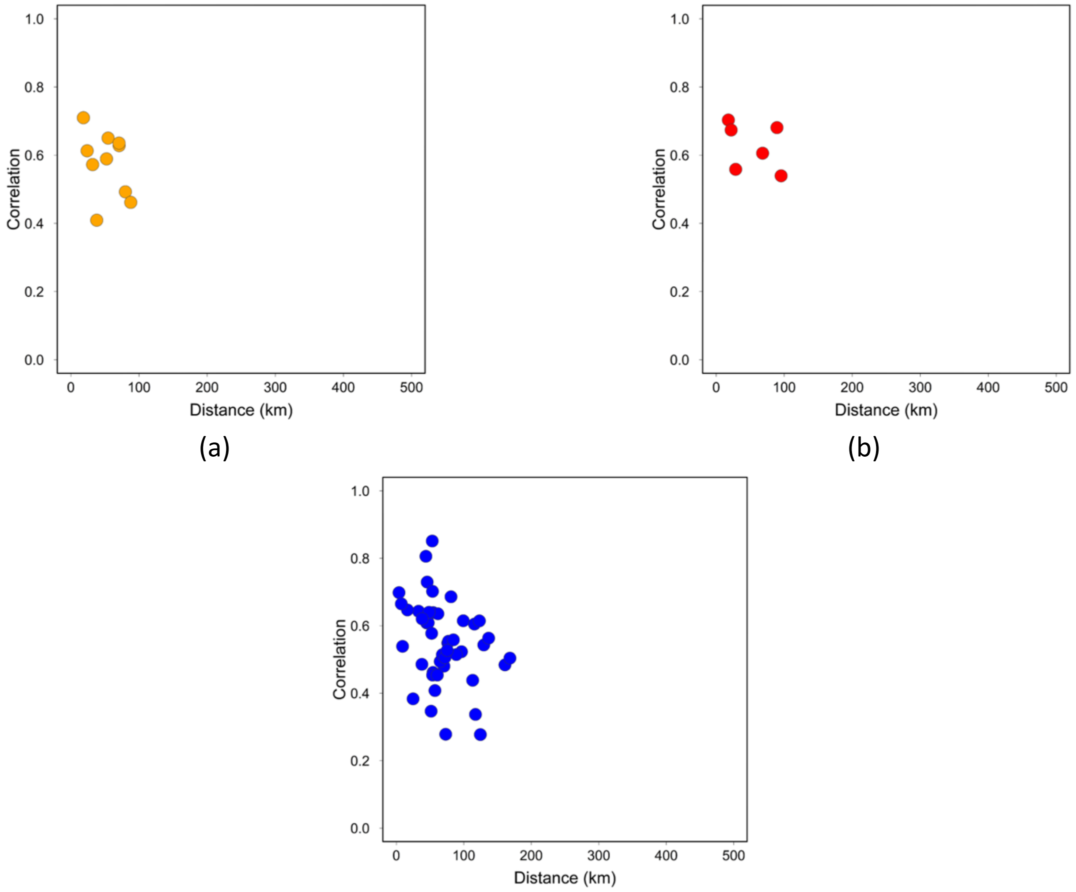
(a) (b)

Fig. 4 (a-b). Scatterplot between station drainage; (a) area and (b) elevation with flow mean for all the communities (30 stations) across Sabah. Different colors to represent communities based on the color scheme in Figure 2(c)



(a) (b)

Fig. 5 (a-b). Scatterplot between station drainage; (a) area and (b) elevation against flow CV for all the communities (30 stations) across Sabah. Different colors to represent communities based on the color scheme in Figure 2(c)



(a) (b) (c)

Fig. 6 (a-c). Scatterplot of distance-correlation for three selected communities across Sabah, following the coloring scheme in Figure 2(c)

5. Conclusions

The Sabah catchments were classified using the Modularity Density based-Edge Betweenness (MDEB) technique in the current study. The method's application to monthly streamflow data from a network of 30 catchments in Sabah produced encouraging catchment classification findings. The research typically turned out the following results: A significant minority of communities that contain at least four stations; for instance, only 20% out of total communities, but they account major portion for nearly 63% of all catchments. A significant count of communities contains only one catchment. For instance, 11 communities (totaling 11 catchments) combine to represent as much as 80% of all communities. Some noteworthy findings came from comparing the identified settlements to a number of important catchment/flow parameters (elevation, drainage area, flow CV and flow mean), as well as from distance and correlation relationship. The findings also showed that it is still important to analyze monthly streamflow data for small-area classification when the correlation threshold value is more than 0.5.

The current study is the first such attempt to use the MDEB approach to classify a relatively limited number of catchments in Sabah while accounting for variations in meteorological and demographic parameters. The evaluation of the MDEB approach has provided some insight into the method's broad applicability for catchment categorization. The results are very positive since they show that the MDEB approach (and community structure methods more generally) are appropriate and effective for catchment classification.

It is also important to note that the community structure concept was used to analyze streamflow data for the classification of catchments in the current study. Such a variable for watershed classification is indeed helpful, and this is particularly true when the variable is streamflow because streamflow at a catchment's outflow is a good indicator of how whatever occurs inside the catchment will turn out. However, it is vital to take into account other variables that, in one way or another, affect the catchment dynamics (such as rainfall), in order to assess the applicability and utility of the MDEB approach for classification towards precision and reliability. As a result, rainfall analysis employing the MDEB approach, particularly when combined with the community structure idea for the general framework of hydrology, may provide more demanding requirements and more trustworthy results.

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