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Hanning's Performance in 4253HT Smoother on Four Signals of Different Noise Levels

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ABSTRACT

Smoothing is a method of data analysis that aims to provide well-defined patterns or signals by removing noise or unstructured patterns from data sets. To obtain clean and smooth data, Hanning is applied as one of the important components in smoothing. However, Hanning is not resistant to outliers. Therefore, this study aims to determine the best type of Hanning that is able to obtain the greatest performance of 4253HT smoother in signal recovery. Functions of Linear, Complex Sinusoidal, Custom Pulse Train, and Sawtooth signals corrupted with five levels of contaminated normal noise were used as signals in the smoothing process. All signals were applied to assess three different Hanning types, which were Tukey, Husain and Shitan. Besides, a Root Mean Square Error (RMSE) was used as an evaluator to determine and assess the performance of 4253HT smoother when utilizing an alternative Hanning. Based on the overall performance of 4253HT smoother, Husain Hanning presented the best outcome and worked most efficiently at all levels of noise except at the lowest noise (10%), which Tukey Hanning executed better. The findings of this study could benefit other researchers to decide the best Hanning to be used before performing forecasting and further analysis to improve the accuracy of predicting.

1. Introduction

Smoothing is a method of obtaining a pattern from a large amount of noise, which is heavy noise. Heavy noise in a time series causes blurry patterns and reduces the accuracy and ease of predicting. According to Velleman [1], it is crucial to look for data smoothers that are unaffected by noise with irregular "spikes" or a long-tailed distribution. Many previous studies have proven that compound smoothers exhibit excellent performance and effectively eliminate heavy noise from a data set (see [2-4]). The applications of 4253HT smoother, as well as the methods of enhancing their performances, were reported in numerous previous studies. Alam and Alam [5,6] had proven this in their studies by applying a non-linear smoother in forecasting of Malaysian crude palm oil prices.

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Meanwhile, Azmi [7] described precisely the performance of 4253HT smoother and its application in forecasting that provides outstanding outcomes.

A compound smoother, which is a non-linear method of smoothing, was first introduced by Tukey [8]. A compound smoother is a group of numerous smoothing algorithms, including weighted moving averages, median smoothers of various span sizes, rough splitting, and re-smoothing. Moreover, a compound smoother is a useful tool for smoothing a data series without causing huge damage to its details. Following that, a compound smoother that consists of running the median of even and odd span sizes, Hanning, and ‘twice’ was proposed by Vellemen [1], which is 4253HT smoother. The study reported that long-tail noise disturbs the 4253HT just minimally, while Gaussian white noise does not affect it.

An attempt to enhance a compound smoother was made by Sargent [4] to fit the performance of Australian football players, where they combined the smoothing algorithms of running median of varied span sizes, Hanning, and ’twice’. Using the smoother output, forecasting was performed utilizing the exponential smoothing method. The findings showed that smoothed data is more suitable to be used in forecasting compared to actual data. In order to reconstruct Normalized Difference Vegetation Index (NDVI) time series data, Jin [9] developed RMMEH, a compound smoother that includes maximum smoother, median smoother, moving average, and Hanning. Besides, Jin [9] agreed that 4253HT is a good smoother amongst others despite the fact that RMMEH has been recognized to be superior at smoothing NDVI data based on specified conditions.

Another study was done by Azmi [10] on the modification of 4253HT smoother which focuses on the estimation of middle point of running median for even spans by applying geometric, harmonic, quadratic, and contra harmonic mean in terms of running smoothing. Nevertheless, investigations on other modifications of 4253HT smoother’s component are highly encouraged.

The determination of 4253HT smoother’s performance is crucial in ensuring the preservation of edges from various noises and in developing a new smoother which has higher resistance towards outliers. This study empirically demonstrated the performance of 4253HT smoother by applying noise to certain signals. Besides, the efficacy of the 4253HT smoother in eliminating noise while preserving the accurate pattern signal was assessed. At the same time, this study also evaluated its ability to perform as a robust smoother while not affecting the outliers and edges. Moreover, this study aims to determine the best type of Hanning that is able to obtain the greatest performance of 4253HT smoother in signal recovery. A 4253HT smoother algorithm was performed using the Hanning coefficient by Tukey [8], Shitan [11] and Husain [12], where all the related procedures are further elaborated in the next section.

2. Methodology

This section describes the procedures for conducting 4253HT smoother, the types of Hanning applied in this study, and four signals used to assess each type. Throughout this study, there were four Hanning types used, including Hanning Tukey, Shitan, and Husain. In addition, four different signals, Linear Sinusoidal, Complex Sinusoidal, Custom Pulse Train, and Sawtooth, were applied to conduct this study. Following that, the performance of all smoothers was assessed through data simulation, where a further description of the simulation procedure is included in this section. Throughout this study, the results were obtained using R software.

2.1 4253HT

Over the succeeding years, Tukey [8] proposed various types of smoothers, each of which possesses robust characteristics. Afterwards, Velleman [1] extensively modified these smoothers into diverse versions. One of the modified smoothers was 4253HT smoother, a compound smoother that combines running median, weighted moving average, and re-smoothing of the rough. Let infinite real data be denoted as Y= …,Y_{t-1}, Y_t, Y_{t+1},…, and a smoother, $S_a(Y_t)$ is defined as an algorithm that works on Y with a is the steps to construct a new series known as smoothed values. The algorithms consisted in 4253HT smoother are as follows:

i. Algorithm $1(a = 1)$: Perform running median of span size four

$$S_1(Y_t) = median[Y_{t-2}, Y_{t-1}, Y_t, Y_{t+1}]$$

ii. Algorithm 2 $(a = 2)$: Perform running median of span size two for re-centre

$$S_2(Y_t) = median[S_1(Y_t),\ S_1(Y_{t+1})]$$

iii. Algorithm 3 $(\mathrm{a} = 3)$:Re-smooth $\mathrm{S_2(Y_t)}$ by applying median smoother with span size five

$$S_3(Y_t) = median[S_2(Y_{t-2}),\ S_2(Y_{t-1}),\ S_2(Y_t), S_2(Y_{t+1}),\ S_2(Y_{t+2})]$$

iv. Algorithm 4 $(\mathrm{a} = 4)$:Perform running median of span size three

$$S_4(Y_t) = median[S_3(Y_{t-1}),\ S_3(Y_t), S_3(Y_{t+1})]$$

v. Algorithm 5 $(a = 5)$: Apply Hanning with coefficients from Tukey for illustration

$$S_5(Y_t) = \tfrac{1}{4}S_4(Y_{t-1}) + \tfrac{1}{2}S_4(Y_t) + \tfrac{1}{4}S_4(Y_{t+1})$$

vi. Algorithm 6 $(a = 6)$: Re-smooth the rough and add the rough into the smoothed values in Algorithm 5.

$$S_6(Y_t) = S_5(Y_t) + S_5\big(Y_t - S_5(Y_t)\big)$$

2.2 Hanning

Hanning is a weighted moving average named after an Austrian meteorologist, Julius von Hann. Generally, Hanning is essential in smoothing to form clean data. Nevertheless, Hanning is easily affected by outliers. Due to this reason, outliers are eliminated by running the median before applying Hanning. As can be seen in a study by Tukey [8], the outliers were initially stabilized by running median smoother before a symmetric coefficient of form 1/4, 1/2, 1/4 of the running weighted average was applied as a gentle smoother [13].

The list of Hanning coefficient, h with its algorithm, H_t used in 4253HT smoother operation is as follows:

i. Tukey [8]:

$$h = \left\{\frac{1}{4}, \frac{1}{2}, \frac{1}{4}\right\}$$
$$H_t = \frac{1}{4}\, Y_{t-1} + \frac{2}{4}\, Y_t + \frac{1}{4}\, Y_{t+1}$$

ii. Shitan [11]:

$$h = \left\{\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right\}$$
$$H_t = \frac{1}{3}\, Y_{t-1} + \frac{1}{3}\, Y_t + \frac{1}{3}\, Y_{t+1}$$

iii. Husain [12]:

$$h = \left\{\frac{3}{8}, \frac{2}{8}, \frac{3}{8}\right\}$$
$$H_t = \frac{3}{8}\, Y_{t-1} + \frac{2}{8}\, Y_t + \frac{3}{8}\, Y_{t+1}$$

This study assessed three types of Hanning coefficients with four types of signals: Linear Sinusoidal, Complex Sinusoidal, Custom Pulse Train, and Sawtooth. Instead of using only high frequencies for Linear Sinusoidal signals, diverse frequencies ranging from 10 - 100 were applied to the signals.

2.3 Simulation Procedure

A simulation process was performed according to Conradie [14], where $N = 200$ times was used in this study. Data is generally expressed as:

$$Y_t = G_t + W_t$$

where Y is data or input, G is signal and W is noise at t^{th} time. Moreover, the signals used in this simulation process were Linear Sinusoidal, Complex Sinusoidal, and Custom Pulse Train and Sawtooth signals, which are given as follows:

i. <u>Linear Sinusoidal Function:</u> A Linear Sinusoidal function $\mathrm{G_t}$ consists of a wave and trend pattern, which is expressed as follows:

$$G_t = Acos\left(\frac{2\pi t}{f} + B\pi\right) + C_t.$$

The parameters $A = 20$, $B = 0.6$, and $C = 0.7$ were selected to attain a preferred curve. Moreover, the time points, t, were selected between 0 and 200, while the frequencies were $10, 20, \dots, 100$. Basically, the first signal used is a simple signal.

ii. Complex Sinusoidal Function: A complex signal is the summation of two sinusoids and a squared trend, which is written as:

$$G_t = 0.01t^2 + \cos(0.5t) + \cos(2t)$$

The second signal is more complex, with quadratic terms considered.

iii. Custom Pulse Train: The signal custom was retrieved from package gsignal, Boxtel (2021), in which R code pulstran was used for the custom signal.

iv. Sawtooth Signal: Signal Sawtooth used function from sawtooth() in R code.

Aside from that, noise is created by combining two normal distributions, which results in a contaminated normal noise. Hence, the noise distributions are expressed as follows:

$$W_{1t} \sim N(0,1^2)$$
$$W_{2t} \sim N(0, 5.06^2)$$

where the first and second noise is represented as W_{1t} and W_{2t}, respectively. Besides, the parameters chosen were suitable for forming noise with a heavy spike and high kurtosis [14]. The noise obtained was introduced into a signal at five distinct levels, which are $10\%, 25\%, 50\%, 75\%$, and 90%. Furthermore, the simulation of 90% of contaminated normal distribution refers to 90% total noise from $W_{1t} \sim N(0,1^2)$, whereas the remaining 10% is from $W_{2t} \sim N(0, 5.06^2)$. A similar procedure was applied for the other four noise levels.

The performance of the 4253HT smoother using an alternate Hanning was evaluated by applying an evaluator called Root Mean Square Error (RMSE). Lower RMSE values suggest that the smoother is effective in eliminating unwanted noise. RMSE formula is written as:

$$RMSE = \left[\frac{1}{N}\sum_{i=1}^{N}\frac{1}{n}\sum_{t=1}^{n}(G_{ti} - S_t)^2\right]^{\frac{1}{2}}$$

where G_{ti} denotes the original noise-free signal, S_t is 4253HT smoother, constant n represents data length, whereas constant N is the number of simulations.

3. Results and Discussion

This section presents and discusses the results obtained from the study performed on four different signals, including Linear Sinusoidal, Complex Sinusoidal, Custom Pulse Train, and Sawtooth signals. Each signal was contaminated by normal noise levels of 10%, 25%, 50%, 75%, and 90%. Furthermore, the results were presented using plots, where the red line represents the signal, the green line is 4253HT smoother, and the black line is signal plus noise or signal which is corrupted by noise. Since all Hannings generated similar performances based on naked-eye observation, the plots for each signal are only provided for Hanning Tukey's performance, which only illustrated the signals' pattern. Hence, the RMSE values for each signal used in all types of Hanning are tabulated in Tables 1, 3, and 5 to determine the difference precisely.

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3.1 Linear Sinusoidal Signal

Figure 1 illustrates the efficacy of smoother in eliminating the noise of different frequencies used. As a comparison, only frequencies of 10 (left) and 100 (right) are used to indicate the differences. With only 10% of contaminated normal noise, 4253HT smoother is capable of reaching the original trail efficiently. This is proven when the smoother line (green) and signal line (red) overlap in the plot.

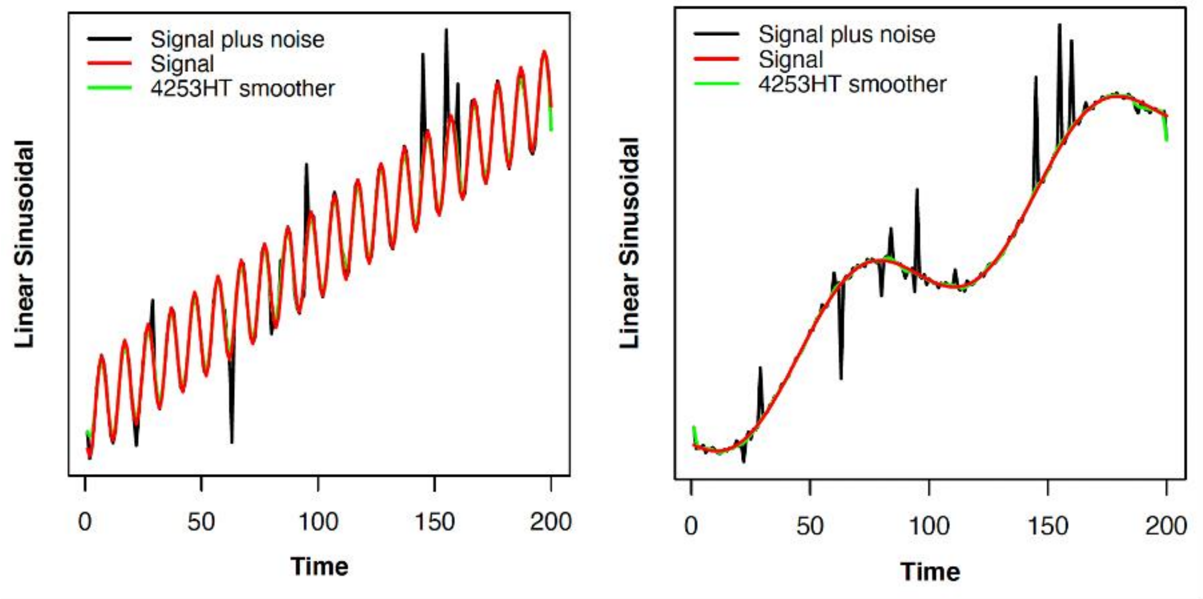

Fig. 1. Performance of 4253HT smoother using Tukey's Hanning in extracting Linear Sinusoidal signal with 10% of contaminated noise at the frequency of 10 (left) and 100 (right)

Moreover, this smoother is excellent in eliminating noise and outliers (heavy spike). Although the signal is interrupted by heavy noise that is 90% of contaminated normal in Figure 2, the original trail is still detectable by smoother but with limited capability. This can be seen obviously when frequency 100 is applied to the signal. The graph depicts the red line as closely traveling with a green line across time. Besides, the smoother is robust towards outliers (heavy spike) and remains the original signal edge simultaneously.

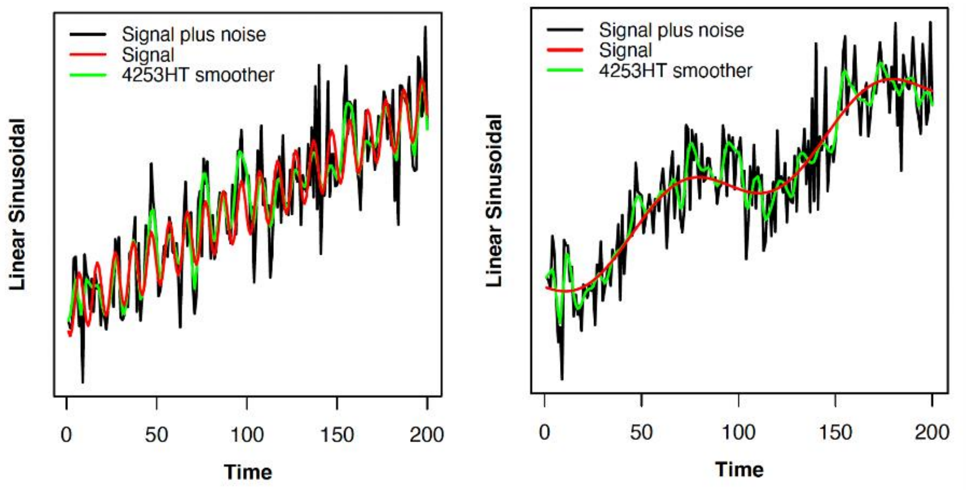

Fig. 2. Performance of 4253HT smoother using Tukey's Hanning in extracting Linear Sinusoidal signal with 90% of contaminated noise at the frequency of 10 (left) and 100 (right)

Table 1 presents an RMSE value of 4253HT smoother when a Linear Sinusoidal signal is inserted with 10%, 25%, 50%, 75%, and 90% of contaminated normal noise. The range of RMSE values obtained is between 1.1382 (min) and 12.3542 (max). These RMSE values are influenced by two factors, which are frequency and percentage of noise. At all frequencies, the RMSE value increases as the percentage of noise increases. For instance, smoother that uses Hanning Tukey at frequency 10 recorded an increasing RMSE value from 2.3537 for 10% of noise to 12.3542 for 90% of noise. The same trend is observed for all frequency levels. Nothing much can be discussed on RMSE value according to frequency value due to its inconsistency.

Table 1
RMSE values of each combination of smoother and Hanning in Linear Sinusoidal signal

Hanning	Frequency, f	Contaminated Normal Noise				
		10	25	50	75	90
Tukey	10	2.3537	4.5125	7.3390	10.4559	12.3542
	20	1.4241	2.6863	6.2145	9.5917	11.6965
	30	1.3196	2.8150	6.3420	9.5541	11.8859
	40	1.2217	2.4514	6.1882	9.3302	11.5708
	50	1.1798	2.2989	6.2211	9.5020	11.6693
	60	1.2368	2.4374	6.2702	9.4519	11.6904
	70	1.1544	2.3084	6.1339	9.3466	11.6030
	80	1.1860	2.3915	6.2130	9.3434	11.5964
	90	1.1382	2.3840	6.1480	9.3332	11.5884
	100	1.1395	2.3122	6.1614	9.4058	11.6039
Shitan	10	2.3612	4.5617	7.3632	10.4023	12.2403
	20	1.3922	2.4319	6.0764	9.3762	11.5021
	30	1.3011	2.7563	6.2485	9.3996	11.7087
	40	1.2117	2.4412	6.1655	9.1801	11.4041
	50	1.1781	1.1781	6.1131	9.3296	11.5133
	60	1.2321	2.3947	6.1270	9.3095	11.5312
	70	1.1535	2.2631	6.0063	9.2154	11.4633
	80	1.1871	2.3593	6.0882	9.1701	11.4223
	90	1.1463	2.3422	6.0181	9.1658	11.4346
	100	1.1452	2.2756	6.0332	9.2446	11.4531
Husain	10	2.4157	4.6308	7.4008	10.4052	12.2069
	20	1.3853	2.5640	6.0127	9.2852	11.4190
	30	1.2951	2.7315	6.2113	9.3307	11.6263
	40	1.2108	2.3801	5.9858	9.1127	11.3342
	50	1.1816	2.2390	6.0655	9.2504	11.4490
	60	1.2319	2.3752	6.0664	9.2436	11.4614
	70	1.1593	2.2485	5.9454	9.1542	11.4066
	80	1.1894	2.3490	6.0323	9.0918	11.3515
	90	1.1526	2.3249	5.9581	9.0903	11.3724
	100	1.1512	2.2618	5.9750	9.1703	11.3911

Table 2 summarizes different Hannings' performance in 4253HT smoother when the Linear Sinusoidal signal is inserted with 10%, 25%, 50%, 75%, and 90% of contaminated normal noise. According to the table, Tukey can only work mostly at lower frequencies and noises, while Shitan shows an inconsistent result. However, Husain performs excellent overall levels of contaminated normal noise at frequencies $20, 30$ and 40. It also performs well at frequencies 50 to 100 over above 10% of contaminated normal noise. Thus, these prove that Husain is the most efficient Hanning coefficient among them.

Table 2
Summary of Hannings' performances in 4253HT smoother when Linear Sinusoidal signal is inserted with 10%, 25%, 50%, 75%, and 90% of contaminated normal noise

Frequency	Contaminated Normal Noise				
	10	25	50	75	90
10	Tukey	Tukey	Tukey	Shitan	Husain
20	Husain	Shitan	Husain	Husain	Husain
30	Husain	Husain	Husain	Husain	Husain
40	Husain	Husain	Husain	Husain	Husain
50	Husain	Shitan	Husain	Husain	Husain
60	Husain	Husain	Husain	Husain	Husain
70	Shitan	Husain	Husain	Husain	Husain
80	Tukey	Husain	Husain	Husain	Husain
90	Tukey	Husain	Husain	Husain	Husain
100	Tukey	Husain	Husain	Husain	Husain

3.2 Complex Sinusoidal Signal

Figure 3 depicts the performance of 4253HT smoother using Tukey's Hanning in extracting Complex Sinusoidal signal with 10% (left) and 90% (right) of contaminated normal noise. At noise 10% and 90%, it can be seen that the performance of 4253HT smoother still follows the trend of both signals.

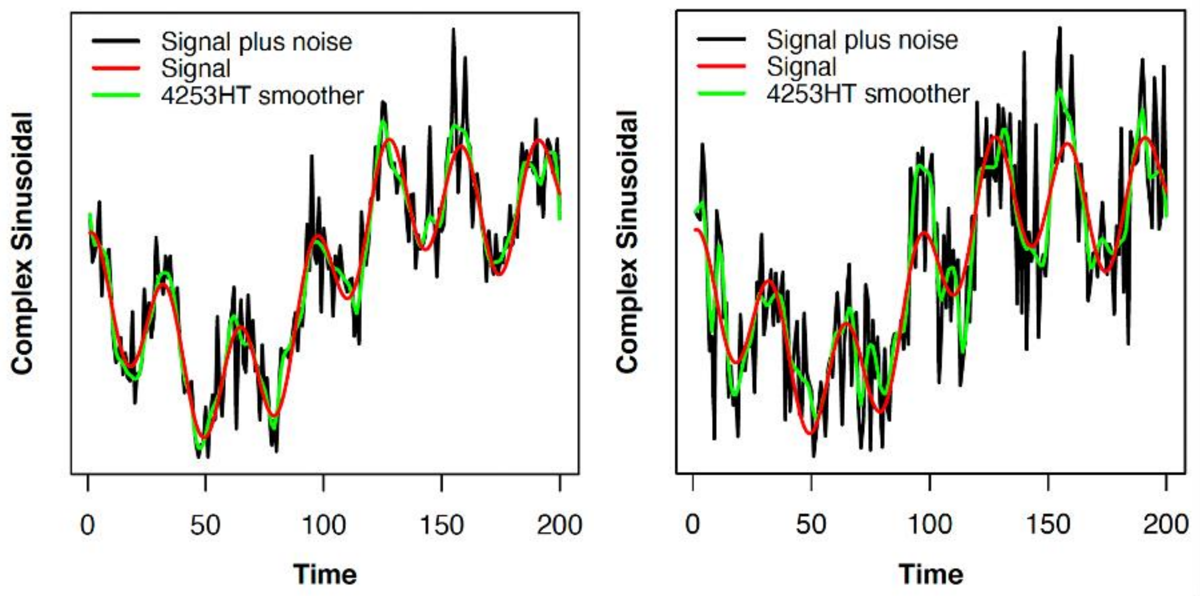

Fig. 3. Performance of 4253HT smoother using Tukey's Hanning in extracting Complex Sinusoidal signal with 10%(left) and 90% (right) of contaminated normal noise

Table 3 presents the RMSE value of 4253HT smoother when a Complex Sinusoidal signal is inserted with 10%, 25%, 50%, 75%, and 90% of contaminated normal noise. It provides that RMSE values for all Hanning types increase as the noise level increases. Moreover, the RMSE values obtained are between 1.1042 (min) and 11.6010 (max).

Table 3
SE value of 4253HT smoother when Complex Sinusoidal signal is inserted with 10%, 25%, 50%, 75%, and 90% of contaminated normal noise

Signal	Hanning	Contaminated Normal Noise (%)				
		10	25	50	75	90
Complex	Tukey	1.1042	2.2543	6.1013	9.3805	11.6010
	Shitan	1.1176	2.2211	5.9749	9.2144	11.4399
	Husain	1.1265	2.2122	5.9172	9.1400	11.3718

Table 4 presents the summary of different Hannings' performance in 4253HT smoother when a Complex Sinusoidal signal is inserted with 10%, 25%, 50%, 75%, and 90% of contaminated normal noise. Based on the results, Husain performs better (or more efficiently) at all levels of noise except at the lowest noise percentage (10%), where Tukey executes better.

Table 4
Summary of Hannings' performances in 4253HT smoother when Complex Sinusoidal signal is inserted with 10%, 25%, 50%, 75%, and 90% of contaminated normal noise

Signal	Contaminated Normal Noise (%)				
	10	25	50	75	90
Complex	Tukey	Husain	Husain	Husain	Husain

3.3 Custom Pulse Train and Sawtooth Signals

The performances of 4253HT smoother using Tukey's Hanning in extracting Custom Pulse Train signal with contaminated normal noise of 10% (left) and 90% (right) are plotted in Figure 4. The plots obtained for this signal indicate a trend like heartbeats or a periodic input. Hence, the 4253HT smoother was tested to extract the periodic inputs while maintaining the original signal. Figure 4 proves that 4253HT smoother can eliminate noise excellently at 10% of noise level, as the signal and 4253HT smoother lines were discovered to travel closely together over time. However, at 90% of noise level, the smoother is less sensitive to spike or impulse yet still able to trace the original trail. In addition, Table 5 clearly depicts that the RMSE value decreases as the noise level increases.

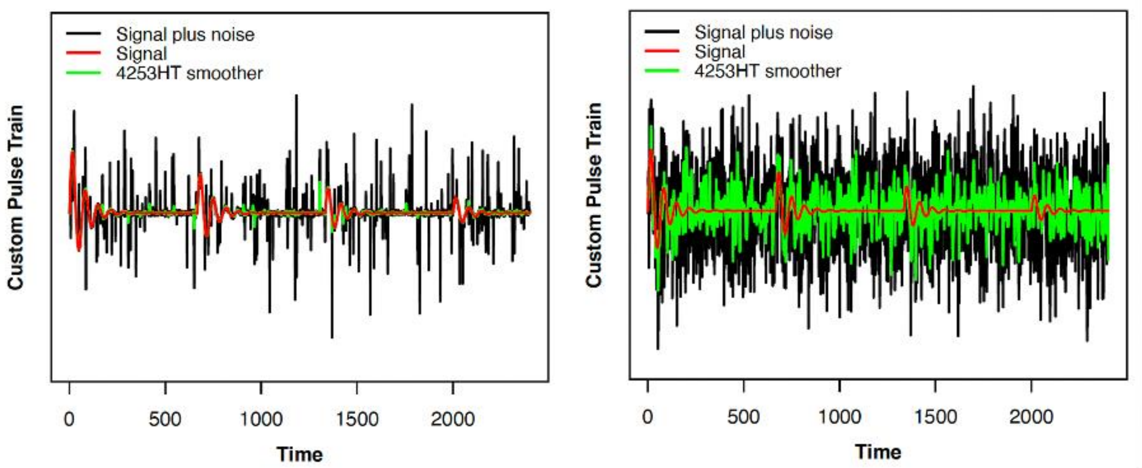

Fig. 4. Performance of 4253HT smoother using Tukey's Hanning in extracting Custom Pulse Train signal with 10% (left)and 90% (right) contaminated normal noise

Figure 5 displays the plots of performance of 4253HT smoother using Tukey's Hanning in extracting Sawtooth signal with 10% (left) and 90% (right) contaminated noise. It is observed that the smoother performs almost perfectly at low noise (10%), as the plot shows the green line that represents 4253HT smoother is overlapping with a red line which represents signal except at several parts with lower density or smaller sparks. Meanwhile, at high volatility (90%), smoother performance is not consistent as the disperse from the signal (red line) is large. Nevertheless, the 4253HT smoother can still detect the signal's original pattern.

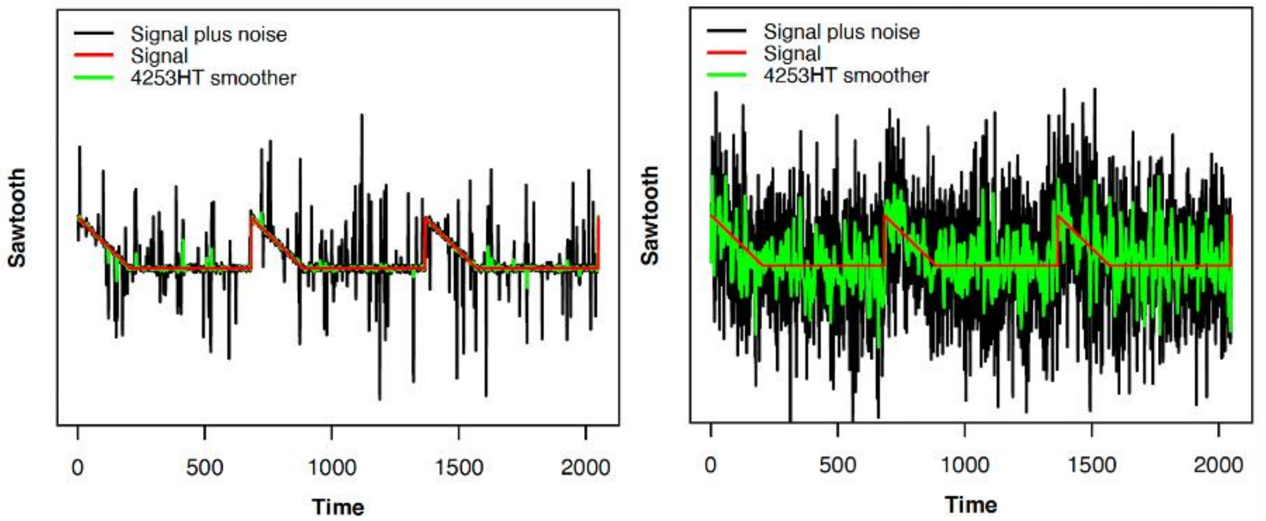

Fig. 5. Performance of 4253HT smoother using Tukey's Hanning in extracting Sawtooth signal with 10%(left) and 90% (right) contaminated normal noise

Table 5 presents the RMSE values of 4253HT smoother's performance on Custom Pulse Train and Sawtooth signals with 10%, 25%, 50%, 75%, and 90% of contaminated normal noise. Overall, both signals and all Hanning methods obtain increasing RMSE values as the noise level increases. As for the Custom Pulse Train signal, Tukey Hanning performs slightly better at the lowest noise level, while Husain Hanning is more efficient at noise levels 25%, 50%, 75%, and 90% Moreover, as for the Sawtooth signal, Husain Hanning exhibits the most excellent performance as it obtains the lowest RMSE values at all levels of noise.

Table 5
RMSE values of 4253HT smoother's performance on Custom Pulse Train and Sawtooth signals with 10%, 25%, 50%, 75%, and 90% of contaminated normal noise

Signal	Hanning	Contaminated Normal Noise				
		10	25	50	75	90
Custom Pulse Train	Tukey	1.23472	2.94793	6.3471	9.94453	12.0838
	Shitan	1.32044	2.87272	6.24355	9.82440	11.9496
	Husain	1.30298	2.84294	6.20161	9.77278	11.8921
Sawtooth	Tukey	1.32878	3.03846	6.69228	9.49997	11.3006
	Shitan	1.29733	2.97687	6.59580	9.38476	11.1595
	Husain	1.28592	2.95238	6.55125	9.33352	11.0974

Table 6 summarizes Hannings' performances in 4253HT smoother on Custom Pulse Train and Sawtooth signals with 10%, 25%, 50%, 75%, and 90% of contaminated normal noise. As for both signals, it is indicated that smoother with Husain Hanning performs efficiently at all levels of noise, except for Custom Pulse Train at 10% contaminated normal noise, as the smoother with Tukey Hanning works better.

Table 6
Summary of Hannings' performances in 4253HT smoother on Custom Pulse Train and Sawtooth signals with 10%,25%, 50%, 75%, and 90% of contaminated normal noise

Signal	Contaminated Normal Noise				
	10	25	50	75	90
Custom Pulse Train	Tukey	Husain	Husain	Husain	Husain
Sawtooth	Husain	Husain	Husain	Husain	Husain

4. Conclusions

The smoothing method has been necessary for statistical studies to provide reliable patterns when a data series contains a large number of noises. According to Gabbouj [15], the linear smoothing method has been used broadly in studies since the procedure is not complicated to be applied in data analysis. Many scholars used 4253HT in preliminary analysis to study trend trajectory in various fields, such as image signal processing [16], medical [17,18], agriculture [19], microbiology [20], climatology [21] and finance [22]. The main purpose of this study is to investigate the performance of 4253HT smoother with different types of Hanning (Tukey, Shitan, Husain) in capturing Linear, Complex Sinusoidal, Custom Pulse Train, and Sawtooth trend signals while inserting various levels of noise volatility $(10\%, 25\%, 50\%, 75\%\ and\ 90\%)$. Hence, the performance of 4253HT smoother was evaluated based on the RMSE values obtained. According to the results, 4253HT smoother with Husain Hanning showed the best performance at all levels of noise on Linear Sinusoidal signal. As for the Complex Sinusoidal signal, Tukey Hanning worked better at the lowest noise (10%), while Husain Hanning is more efficient at the higher levels of noise $(25\%, 50\%, 75\%\ and\ 90\%)$. Moreover, for the Custom Pulse Train signal, Tukey Hanning is excellent at 10%,, while Husain Hanning dominated the higher noise levels. Next, for the Sawtooth signal, Husain Hanning undoubtedly is assumed as the best due to the lowest RMSE values obtained at all noise levels. Thus, the study concludes that Husain Hanning is the most efficient Hanning in extracting all four signals at almost all levels of noise. It is hoped that the findings of this study could benefit other researchers in deciding the best Hanning to be used before performing forecasting and further analysis to improve the accuracy of predicting. The 4253HT smoother's performance is crucial to be determined to ensure the edges are preserved from various noises and to develop a new smoother which has higher resistance towards outliers.

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