Journal of Advanced Research in Applied Sciences and Engineering Technology 56, Issue 1 (2026) 300-307

SEMARAK ILMU PUBLISHING

Journal of Advanced Research in Applied Sciences and Engineering Technology

Journal homepage: https://semarakilmu.com.my/journals/index.php/applied_sciences_eng_tech/index
ISSN: 2462-1943

Theoretical Comparison of Wavelet Transform and Fourier Transform in SARIMA-GANN Forecasting Model

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ABSTRACT

Seasonal Autoregressive Integrated Moving Average (SARIMA) model which combine seasonal differencing with an ARIMA model are used when the time series data exhibits periodic characteristics. It is popular in modelling and forecasting demand as its ability in identifying the patterns and seasonality of the series and grasping the linear trend of the series effectively. However, the assumption of linearity in many time series events may not be satisfied in practice and thus the accuracy needs to be improved. Besides, it is unable to extract the non-linearity of the series. Genetic Algorithm based Neural Network (GANN) is proposed to combine with SARIMA to overcome its shortcomings but it might occur the overfitting event. Therefore, it is important to reduce the complexity of the input data of the neural network overhead to overcome this problem. This paper contains the section of explanation of both Wavelet Transform (WT) and Fourier Transform (FT) and the discussion of the comparative pros and cons among each other in analysing signals especially in handling the overfitting problem by neural network.

Keywords:
Autoregressive integrated moving average; fourier transform; genetic algorithm; neural network; seasonal autoregressive integrated moving average; wavelet transform

1. Introduction

The previous study by the authors concluded the possibility of constructing a vegetable price forecasting model in helping the national issues of Malaysia in agricultural sector [1] because it is important for an effective decision making in marketing and businesses. To the consideration of all possible aspects, SARIMA is preferable to be used in doing the price forecasting as it composed of trend and seasonal elements of the series although it is appropriate only for a stationary time series and only able to extract the linearity of the series. In the proposed model of the previous research by the authors, the non-linear components (or residuals) of the SARIMA model will be handled by the proposed GANN model but before that the residuals need to be processed and separate into sub-frequencies to easier it to be the input data of neural network.

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https://doi.org/10.37934/araset.56.2.300307

The process of breaking the residuals of SARIMA, which is also the non-linear components of the series into its sub-frequencies is called decomposition in signal processing. The signal decomposition is effective to identify the model information in time-domain signals. Basically, the step of decomposing or breaking a given signal into a sum of simpler signals is so called signal decomposition. The goal of signal decomposition is extraction and separation of signal components from composite signals, which should preferably be related to semantic units. However, it is challenging to separate single components from mixed signals, where the composite signals consist of a sample-wise superposition from multiple components. Signal decomposition methods are more about the classification of underlying features, which characterize the component to be separated, for example, the details and approximate components would be modeled by GANN after the decomposition technique.

Most of the previous studies focus more on the comparing the performance between WT and FT. FT is best known to break a waveform whether it is a function or signal into an alternate representation, characterized by sine and cosines whereas WT decomposes a function into a set of wavelets. Both of them have their pros and cons. Therefore, this paper aims to compare their performance of decomposing and denoising a signal based on the related previous studies by theoretically before starting to construct the price forecasting model as targeted.

This paper is organized as follows: Section 2 is literature review which describes the previous research studies related to the WT and FT that are referred. The summary of comparison between WT and FT are explained in Section 3. Section 4 is the conclusion.

2. Literature Review

Signal processing is useful in data science and machine learning because it helps to collect the data and then analyze and transmit it to destination effectively where the received signal at the destination is then reconstructed or transformed and being stored. This process utilizes the use of efficient resources to transmit and store the needed information [2]. The Fourier and wavelet analysis is usually used to simplify the functions into a simpler function which it means the transformation of equations into a coordinate system where its expressions simplify, decouple, and are amendable to computation and analysis [3].

2.1 Fourier Transform (FT)

In the early 1800s which a French mathematician named Joseph Fourier have introduced the most foundational and ubiquitous coordinate transformation according to the frequency content or spectrum in terms of Fourier series [4]. He introduced the concept that sine and cosine functions of increasing frequency provide an orthogonal basic for the space of solution functions [3]. He showed that any periodic function can be represented as an infinite sum of periodic complex exponential functions and named as Fourier transform (FT) [5]. Basically, FT is a continuous form of Fourier series. It decomposes a signal defined on infinite time interval into a λ-frequency component where λ can be either real or complex number [6].

Theoretically, the Fourier series allows the function f to be decomposed as function below:

$$f(t) = \sum_{j=-\infty}^{+\infty} a_j e^{i2\pi\omega_j t}, a_j \in \mathbb{R} \tag{1}$$

where ω j = j/T is a constant. The equation above is an orthogonal basis representation of the function f [7]. It states that any periodic function can be decomposed as an infinite sum of periodic functions no matter if it is a sine or cosine function and it ease the analysis of the frequencies present on the function *f* which the frequencies included the fundamental frequency, and all of the other frequencies which are the integer multiplies ω_j, $j \in \mathbb{Z}$, of this fundamental frequency [7].

FT which decomposes a signal into orthogonal trigonometric basis functions of different frequencies and phases is often called the Fourier spectrum. The FT of a continuous function *f(t)* can be expressed as the following equation [4]:

$$\hat{f}(\omega) = \langle f, e^{i\omega t} \rangle = \int_{-\infty}^{\infty} e^{-i\omega t} f(t) dt \tag{2}$$

However, this transform is not suitable if the signal is localized and nonstationary because of the utilized time axis to have the global spectral analysis and thus it is unable to provide a local picture of signal variations. Therefore, the short-time FT (STFT) was introduced by Dennis Gabor which there will be a window function to extract data over an interval and then the FT will be computed [2]. This scheme is so called time-dependent spectral analysis, because the window slides along the time axis to compute FT of different segments of the signal. Hence, it is suitable for functions which are locally stationary but globally un-stationary.

If the window function be *g(t-τ)*, and *||g||2* is the *L2(ℝ)* norm of the window function *g(t)*, then the STFT is given by F (ω , τ) [1].

$$F(\omega, \tau) = \int_{-\infty}^{\infty} f(t) g(t-\tau) e^{-i\omega t} dt \tag{3}$$

$$f(t) = \frac{1}{2\pi \|g\|_2^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} F(\omega, \tau) \overline{g(t-\tau)}\, e^{i\omega t} d\omega d\tau \tag{4}$$

The short-time FT is also called as continuous Gabor transform (CGT) and it occurred the shortcomings which is caused by Heisenberg's uncertainty principle that a window of fixed size provides an upper bound on the frequency resolution. Ultimately, it is said to have the constant width of the window which is used and hence it provides only the fixed resolution [5, 8].

2.1.1 Discrete Fourier Transform (DFT)

In order to compute and work with real data, the FT is compulsory to be approximated on discrete vectors of data and thus it caused the resulted Discrete Fourier Transform (DFT). It is a discretized version of Fourier series for vectors of data obtained by discretizing the function f(x) at a regular spacing, Δx and it is useful for numerical approximation and computation [3]. The DFT is given by:

$$\widehat{f_k} = \sum_{j=0}^{n-1} f_j \, e^{-i2\pi jk/n} \tag{5}$$

and the inverse DFT is given by:

$$f_k = \frac{1}{n} \sum_{j=0}^{n-1} \hat{f}_j e^{i2\pi jk/n} \tag{6}$$

2.1.2 Fast Fourier Transform (FFT)

While computing a DFT of *n* points by using only its definition, takes $O(n^2)$ time, a Fast Fourier Transform (FFT) can do this in only $O(n \log n)$ steps. The revolutionary FFT was developed by James W. Cooley and John W. Tukey in 1965 which this algorithm based on a fractal symmetry in the FT that allows an *n* dimensional DFT to be solved with a number of smaller dimensional DFT computations [3]. More specifically, DFT is the discrete version of the FT that transforms a signal from the time domain representation to its representation in the frequency domain, whereas FFT is any efficient algorithm for the evaluation of DFT.

FFT decomposition has to be done in order to compute the DFT efficiently to reduce the arithmetic to operations $O(n \log n)$. There are two methods for FFT decomposition which are the decimation-in-time method and the decimation-in-frequency method. The former method decomposes the n-point data into the even-numbered $n/2$-point data and the odd-numbered $n/2$-point data while the latter divides the n-point data into the first half of the $n/2$-point data and the second half of the $n/2$-point data [9].

2.2 Wavelet Transform (WT)

According to the problem faced by STFT which it provides only fixed resolution, the wavelet transform was developed to provide a solution for multiresolution [4, 5]. In WT, a new window or basis function is introduced which can be enlarged or compressed to capture both low frequency and high frequency component of the signal (which relates to scale) [5]. There are two key concepts in WT which are scaling and shifting. By scaling, the wavelet will be stretched or sink in the time domain while the process of shifting performed the delaying or advancing of wavelet along the length of the signal. Its key strength is that there are different sizes and shapes of wavelets, for example, the Daubechies wavelet that can be differentiated into few types of wavelets according to the number of vanishing moments such as *db2*.

The basic idea in wavelet analysis is to start with a function $\psi(t)$, known as the mother wavelet, and generate a family of scaled and translated versions of the function [3]:

$$\psi_{a,b}(t) = \frac{1}{\sqrt{a}}\psi(\frac{t-b}{a}) \tag{7}$$

The parameters a and b are the scaling and translation factor for scaling and translating the function ψ, respectively.

2.2.1 Continuous Wavelet Transform (CWT)

Both the CWT and STFT (also called as CGT) targeted to extract the time-frequency characteristics of the one-dimensional signal. Therefore, they may be described in terms of representation (or an inner product) of a signal with respect to a specific family of (atomic) functions that are generated by a single analysing function [10].

CWT decompose the signals in terms of families of atoms consisting of all the translated and dilated versions of the analysing function while STFT decompose function in terms of families of atoms consisting of all the translated and modulated versions of the analysing function. The main difference among these two transforms is that the Gabor atoms have a fixed bandwidth and the wavelet atoms have bandwidths that range continuously from arbitrarily small to arbitrarily large.

The equation of CWT is expressed as follow:

$$W(a,b) = \frac{1}{\sqrt{|a|}} \int_{-\infty}^{\infty} x(t)\psi\left(\frac{t-b}{a}\right) dt \tag{8}$$

The parameter W(a,b) is called the wavelet coefficient and parameter a and b are scaling parameter and shifting or translational parameter respectively [5]. ψ(t) is the mother wavelet and different dilations and translations may lead to different daughter wavelets.

2.2.2 Discrete Wavelet Transform (DWT)

The DWT is implemented to handle the problem of CWT which it is hard to be implemented as a redundant transform because the translation parameter *b* and scaling parameter *a* found to be infinite [5]. In order to reconstruct the *f* completely from the transform values on specific subsets only by selecting proper discrete subset of the t-a-plane for the CWT and of the t-ω -plane for the CGT respectively, it may lead to the use of the DWT [11]. The key difference between CWT and DWT is that CWT uses every possible wavelet over a range of scales and locations which means an infinite number of scales and locations while DWT uses a finite set of wavelets, for example, those that defined at a particular set of scales and locations.

The representation of DWT is expressed as below:

$$W_{m,n} = \int_{-\infty}^{\infty} x(t)\psi_{m,n}(t)\, d(t) \tag{9}$$

where the parameters *m* and *n* control the wavelet dilation and wavelet translation respectively and these two control parameters are contained in the set of all integers in both positive and negative. Hence, the $W_{m,n}$ are the DWT values given on a scale-location grid of index *m, n* and also known as the wavelet coefficients or detail coefficients [12].

2.3 Related Works of Fourier Transform (FT) and Wavelet Transform (WT)

By using and comparing the use of Fast Fourier Transform and DWT for signal compression [6], the Haar wavelets performed the best for signals with step or block functions, Daubechies is the best for “Mishmash” type of signals while FFT gives a good result for sine or cosine-based signals. After few observations of the discrete wavelet function done by Rahman and his teams [13] graphically, the authors concluded that the discrete functions which in terms of Haar wavelet is the best in detecting those non-stationary or transitory characteristics and hence the user can scale and translate the function and approximate the function by only a few coefficients.

The authors from previous study [14] compared the performance of WT, FT and STFT on analysis of non-stationary processes and then they preferred WT the most as it allows them to claim the hypothesis about existence of quasi-harmonic components in non-stationary signals with some frequencies and also it provides a full and precise image of the quasi-harmonic components’ dynamics in signal. In a research study [15], the author evaluates the similarities and differences between STFT and WT by not only their theoretical comparison but also the experiments done by using both transforms on de-noising application and also on de-noising a recorded white-noise original signal. She concluded that STFT has advantage for those programs with huge data length and strong external interferences while WT is the best choice for the program which do not have strong interferences and data length such as voice recording.

In the process of using FT and WT on an ECG signal from a previous study [16], it can be concluded that the adaptive, multiresolution capability of the wavelet transform makes it well suited for decomposing and de-noising signals for dynamical structure. In the case study of ECG signals also, the author from a research study [17] has carried out the comparative analysis where the Cubic Spline Wavelet shown up the most in the best method among few research papers on analysing and comparing the performance of different wavelet transform. On the other hand, there was research [18] compared the performance of both FFT and WT on separating the information of EEG beta band from the real-time health assessment of 3DTV. They concluded that both FFT and WT are consistent in feature extraction of EEG.

The authors of another research [19] have done the comparative study between FT and WT on detecting the periodicity of both summer and winter and the result turned out well where both transforms are collaborated between each other and hence the seasonal behaviour of wind speed data of summer and winter is completely out of phase. Besides, the comparative analysis done by Kaur [20] summarised the fact that any application using by FT can also be formulated by WT to provide more accurately localized time and frequency information and hence they concluded that wavelet analysis is more suitable and reliable than Fourier analysis in term of signal analysis.

In a research study done in 2020 [21], the authors aimed to compare the WT and FT regarding fault detection in wind turbine drivetrain bearings and they found out that the Wavelet Packet Transform (WPT) which is an extension of DWT has higher performance as a field vibration measurement analysis tool than FFT. Last but not least, Yavuzdeğer and his team [22] analysed and determined the power quality disturbances through different signal processing techniques included the FT, STFT and WT. WT is superior to other used transform techniques for signal processing and it enable us to obtain the low frequency information in long time intervals and high-frequency information in a short time intervals.

3. Summary of Comparison between Fourier Transform (FT) and Wavelet Transform (WT)

In general, FT convert signal from time domain to frequency domain and it provide two-dimensional information about any signal that the frequency component presented in a signal and their respective amplitudes while WT gives a complete three-dimension information about any signal i.e. the frequency components that are presented in any signal, their respective amplitude, and the time axis of the frequency components located at.

In the fact that FT is suitable only for stationary signal, the WT is suitable for both stationary and non-stationary signal. The FT has zero-time resolution and very high frequency resolution but WT has the composite side compared to FT which it has high time resolution, high frequency resolution as well as stationary time and frequency resolutions. In Fourier analysis, the signal is converted into sine and cosine waves of different amplitudes and frequencies whereas the signal is converted into scaled and translated version of mother wavelets in wavelet analysis.

WT is very suitable for studying the local behaviour of the signal such as irregularities and spikes compared to FT which is not suitable for that because the wavelet analysis is very irregular and cannot be predicted while the shape of sine and cosine wave in Fourier analysis are well defined, regular, smooth and predictable. The input can be a real or complex function for both FT and WT, however, the output of FT is always complex while the output of WT may be real or complex.

Table 1 [23, 24] tabulates the comparison between FT and WT. There are various qualities being considered for this comparison.

Table 1
Summary of comparison between Fourier transform and wavelet transform

Method	Fourier transform	Wavelet transform
Basis	Non-adaptive	Non-adaptive
Frequency	Convolution global uncertainty	Convolution regional uncertainty
Non-linearity	No	No
Non-stationary	No	Yes
Feature extraction	No	Discrete: No; Continuous: Yes
Linearity	Yes	Yes
Stationary	Yes	Yes
Presentation	Energy-frequency	Energy-time-frequency
Theoretical base	Theory complete	Theory complete

4. Conclusions

In this paper, the comparison of two different signal analysis tools which are the Fourier Transform (FT) and Wavelet Transform (WT) has been discussed. It can be said that the existence and development of WT is to resolve the few difficulties inherent in FT. There is one important thing about FT and WT is that WT can do most of the operations of FT and it can even perform better than FT that it provides more accurately localized time and frequency information. This is the reason of how the WT became popular in many fields such as the electrical engineering field to do signal analysis and data compression and so on. Due to the key advantages of WT which it able to extract local spectral and temporal information simultaneously and provide variety of wavelets in different sizes and shapes, the WT is then more suitable to be used in the signal decomposition of the proposed SARIMA-GANN price forecasting model.

Acknowledgement
This research was funded by a grant from Ministry of Higher Education of Malaysia (FRGS Grant FRGS/1/2020/ICT06/UNIMAP/02/1).

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