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AdaBoost Algorithm-Based Channel Estimation: Enhanced Performance

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ABSTRACT

A combination of a group of rules characterized by weakness and imprecision, to reach a prediction rule known for its high precision, forms the concept of boosting, a machine learning approach. From this concept, the Adaptive Boosting (AdaBoost) algorithm spun. It is the first of its kind, and remains in use, under study, and involved in practical applications in various fields to this day. This algorithm is involved in modulation techniques, as it works on improving the bit error rate (BER). Inputting a noisy signal received from a sender into AdaBoost yields the original signal after removing noise. This is done through the reconstruction of the constellation diagram of the modulation technique, by eliminating the noise filling the data's signal space. The result is an enhancement of the BER of 8 quadrature amplitude modulation (8QAM) and 16 quadrature amplitude modulation (16QAM), through AdaBoost. The AdaBoost algorithm is then added to the channel estimation techniques like the Least Squares (LS), Least Mean Squares (LMS) and Recursive Least Squares (RLS) and is utilized to enhance the BER performance of different estimation techniques in a Rayleigh fading environment. The AdaBoost technique allows benefiting from its learning capabilities as a machine learning algorithm in overcoming the effect of noise and fading channel on the received signals.

1. Introduction

Wireless communication is a field with constant developments related to technology. It has experienced much growth in later years and represents just a single part of a more sophisticated field, namely communication systems. It is referred to by the term wireless as this is its nature, as opposed to wireline communication. Advances in this field work in tandem with other technological developments of this era. In this century, telecommunications are increasingly dependent on wireless channels. The reason for this being that these types of channels have allowed for various services, including voice, data and more recently, multimedia.

The physical properties of a wireless channel have undesired effects on the signal running through this channel. The relation between the transmitted signals and the environment surrounding them is

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marked by complexity. As the signal runs from transmitter to receiver, it is subjected to different actions. This includes reflections, which take place as large objects are present in the way. There is also signal scattering and diffraction of electromagnetic waves, which results from the presence of objects that hinder these waves. For all of the above, the receiver might receive an attenuated, distorted, or delayed signal or one with phase-shifting. Signals subjected to these types of interruptions are called multipath signals or signal copies. Multipath signals can construct or destruct one another during interaction. In the latter case, a significant reduction can affect the power of the signal. This is known as fading. Deep fading occurs when the interference between signals is strongly destructive and can cause a failure in communication. This is usually the effect of a major reduction in the channel's SNR.

Wireless communication channels are subjected to two types of fading effects, i.e., large and small-scale fading. Movement over a large area causes fading in the average signal power or path loss. This is known as large-scale fading. As for small-scale fading, it is attributed to small changes in the space between sender and receiver and is caused by major variations in both amplitude and phase of the signal under-transmission. Small-scale fading is also known by a different name, Rayleigh fading. This name stands as long as there is a large number of multiple reflective paths. Another condition for the name to apply is the absence of a line-of-sight signal component. The Rayleigh probability density function (pdf) is a statistical method used to define the envelope of the received signal. However, in case of the presence of a main non-fading signal component, the Rician pdf is applied to define the envelope of this small-scale fading. An example of the non-fading signal component includes the line-of-sight propagation path. [1].

Wireless communication channels possess different properties, among which there is the Doppler shift. The relative motion between sender and receiver results in a Doppler shift. However, it can also take place through the motion of any type of object in the wireless communication channel. This type of channel possesses a time-varying nature resulting from the Doppler shift. In Figure 1, there is a photo showing the multipath effect present in a regular wireless communication channel [2].

Due to the different paths within the environment that each signal has to pass through, multipath signals, communicating between remote dominant reflectors, mobile senders, and local scatterers and the base station, vary in Doppler shift, carrier phase shift, amplitude, and time delay. Should the sender be on the move, then multipath signals will acquire the property of time-variance.

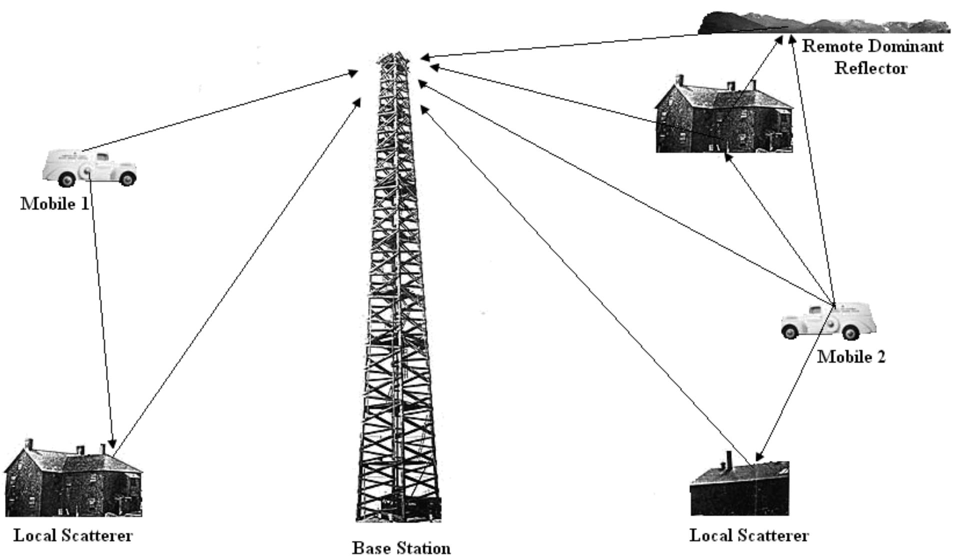

Fig. 1. Multipath propagation in a wireless communication channel [2]

For a wireless communication system to have improved performance, it must be able to overcome the challenge of furnishing a specific CSI at the receiver's end, to be able to coherently detect the transmitted signal [3]. In the absence of this CSI, differential demodulation, a non-coherent method, is used to demodulate and detect the signal. The use of such a method results in a loss of approximately 3 to 4 dB in signal-to-noise ratio (SNR), as opposed to the application of a coherent method. This represents such an immense loss that researchers have been working hard to eliminate by developing coherent methods for channel estimation [4].

This research uses the AdaBoost algorithm to enhance the BER of various channel estimation techniques, such as the Least Squares (LS), Least Mean Squares (LMS) and Recursive Least Squares (RLS). The AdaBoost algorithm is added to enhance the BER performance in a Rayleigh fading environment. The technique allows benefiting from its learning capabilities as a machine learning algorithm in overcoming the effect of noise and fading channel on the received signals. The application of AdaBoost helps detect the various characteristics of a signal, as it recovers the originally sent data from the received signal, containing noise. The result is an improvement in BER. This algorithm is well known in the field of image processing and feature recognition and the novelty of this paper is the application of this algorithm in the field of wireless communication and transmitted signals. This application has many benefits, seen in its boosting capabilities, leading to an improvement in the results reached.

This paper is divided into five sections, with the previous part making up Section 1. Section 2 involves an explanation of the pilot-assisted channel estimation technique. Section 3 contains an explanation of channel estimation and equalization. As for Sections 4 and 5, they consist of simulation results and conclusions, respectively

2. Pilot-assisted Channel Estimation Technique

One of the challenges faced by those working in the field of telecommunication is the ability to estimate communication channels to a certain degree of optimization and update. This is necessary, so as to be able to decode, equalize, and demodulate the signal, as well as perform a series of applications related to baseband processing. Different modulation and transmission methods have been recently devised to provide efficient methods of channel estimation [5]. These methods, which can be applied in wireless communication systems, including WiMAX, LTE, and WiFi, can be classified into three categories: decision-directed channel estimation (DDCE), pilot-assisted (training-based) channel estimation, and blind and semi-blind channel estimation [6].

Among the three types specified above, pilot-assisted or training-based channel estimation has garnered a lot of attention. In this technique, multiplexing takes place between training sequences recognized by the receiver with symbols of the sent information. This happens at a pre-determined position prior to transmission. The receiver uses this training data to estimate the CSI related to its position. The CSI equivalent to the information data positions is then obtained by means of interpolating different channel estimates earlier received from the training data sequence [7].

3. Channel Estimation and Equalization

Channel estimation techniques give the receiver an indication about the channel status and hence will give the decision in detecting the sent information. Any medium through which a signal is sent can cause corruption in said signal. This can take place through various means, such as intersymbol interference, frequency distortion, thermal noise, phase distortion, and others. As such, the result can be a corrupted signal at the end of the receiver. Channel estimation is the act of predicting, detecting or approximately calculating, and describing the characteristics of the effects of the physical aspects of a communication channel on the data sequence inputted. Once the conditions of error minimization are fulfilled, a channel is said to be well-estimated.

Among the benefits of techniques related to channel estimation is their ability to elaborate the behavior of a channel and to allow receivers to approximate its impulse response. Error minimization takes place through the equalization method. It enables channels to reach an ideal setting for voice, data, and video transmission [8].

Channel equalization is a technique that achieves amplitude reduction and frequency reduction, as well as phase distortion reduction, in a communication channel with the intent of enhancing transmission performance. The main operation of channel equalization is to reverse the effect of the channel. Channel equalization is always carried out after channel estimation.

Adaptive equalization is a technique that adapts automatically to the time varying properties of the channel. The following adaptive algorithms are considered among the most famous prediction algorithms used in the equalizer to estimate the channel responses [9].

3.1 Least Square (LS)

A regular method, usually employed in regression analysis, is the LS technique. This method is used in over-determined systems, for the estimation of their solution. In these systems, the set of equations has more equations than it has unknowns. Least mean squares in this instance, has to do with the minimization, by the total solution of the sum of the squares, of the remainder resulting from every equation [10].

To estimate the relation between different variables, regression analysis is used in statistical modeling as a group of processes. These processes focus on the relationship between a certain dependent variable and a single or multiple independent variables, by employing various types of modeling and analysis techniques. This type of analysis investigates the changes that take place in the dependent variable, upon fixing all independent variables, except one [11].

Data fitting is the field where this model has the most application. The minimization of the sum of squared residuals is considered the best fit within the LS approach. In defining a residual, it is said that it is the variation between an observed value and the value most fitted delivered by the model. Simple regression and LS techniques are not the best approach, when there are uncertainties in the independent variable. As such, for these cases, models used for fitting errors-in-variables are most appropriate [10].

There are two main divisions of LS problems: linear LS and nonlinear LS. This division is based on whether residuals in all unknowns are linear or not. Statistical regression analysis holds a linear LS problem within its components and provides a solution in closed form. In case of a nonlinear problem, iterative refinement is employed to reach a solution. During each iteration, a linear problem is used for the approximation of the system. As such, both cases are alike in their core calculation [12].

From a statistical point of view, the mean square error (MSE) of an estimator determines the average of the squares of errors. This estimator is related to a procedure dealing with the estimation of an unseen quantity. This implies that the MSE measures the average squared difference between the estimated value and the actual value. The value to be expected from a squared error loss is the MSE, which is a risk function. In most cases, MSE has a positive value. This is due to randomness. It could also be due to the estimator not bearing into consideration any information that might yield an increasingly accurate estimate [13].

The quality of the estimator can be determined through the calculation of the MSE. When its values are closer to zero, it is said that the estimator is of better quality than when its values are further away from zero. The value of the MSE is always non-negative. MSE, as the second moment related to the origin of the error, shows an estimator's variance and its bias. The former deals with the broadness of the spread of estimates among various data samples. As for the latter, it marks the difference between the real and the estimated average values. MSE represents the variance of the estimator, in case of an unbiased estimator. MSE is similar to variance in the fact that they both have the same unit of measurement as the square of the estimated quantity. Same as standard deviation, calculating the square root of the MSE yields the root-means-square error (RMSE) or the root-mean-square deviation (RMSD). Both have the same unit of measurement as the estimated quantity. The standard error in the case of an unbiased estimator is the RMSE, which is the square root of the variance [14].

3.2 Minimum Mean Square Error (MMSE)

In statistics and signal processing, there is a prediction method that works on minimizing the MSE, it is entitled the minimum mean squared error (MMSE) estimator. It is a common method that measures the quality of an estimator, as well as the values of the fitted data for a dependent variable. MMSE is mainly a reference to estimation with the help of a quadratic loss function. This takes place in the Bayesian setting. Should this happen, then the posterior mean of the estimated variable yields the MMSE estimator. This drives the MMSE estimator to take a form within a certain type of functions, as it is difficult to calculate the posterior mean. A preferable type of MMSE estimators is linear MMSE. This is because it is simple in its application, in its calculation, and it is quite flexible. It has also been the precursor of many commonly used estimators, including the Wiener–Kolmogorov filter and Kalman filter [15].

This leads to the minimization of the squared error. The keyword in this process is the "mean", since a mean squared error is calculated and used to minimize the deterministic squared error. This mean of the squared error is also employed as a random variable. In statistics, this is the Bayesian approach and it is a very important issue. Thus, Bayesian is the resultant of a MMSE. This approach considers the sent symbol vector ($\bar{x}$) to be a random quantity. As such, another random quantity would be $\bar{y}$ and thus, the minimization of the error in the mean takes place [16].

3.3 Least Mean Square (LMS)

One class of adaptive filters is the LMS technique. It is employed so as to have a similar effect as the desired filter. This takes place by reaching the coefficient of the desired filter. This is the coefficient related to the production of the LMS of the error signal. This LMS is the variation between two signals: the desired and the actual ones. The LMS algorithm is one of the popular techniques that can be utilized for adaptive channel equalization. The criterion used in this technique is to minimize the MSE between the desired equalizer output and the actual equalizer output [17].

The LMS filter depends on the concept of approaching the optimum filter weights. This takes place through the update of the filter weights in a manner so as they could all move towards the optimum filter weight. The basis of this method is the gradient descent technique. In the beginning of this process, the algorithm takes up small weights (mostly zero). The weights are then updated at each stage, upon the determination of the gradient of the MSE. In case of a positive MSE-gradient, the error would continue its increase in the positive direction. If the same weight is used for more iterations, this means that weights need to be reduced. In the same way, if the gradient is negative, the weights need to be increased. The LMS adaptive filter is described by the equations [9]:

$$W(n+1) = W(n) - \mu(n)e(n)X(n) \tag{1}$$

$$e(n) = d(n) - W^T(n)X(n) \quad (2)$$

where $W(n) = [w_0(n)\ w_1(n) \ldots\ldots w_{L-1}(n)]^T$ is the coefficient vector, $X(n) = [x(n)\ x(n-1) \ldots\ldots x(n-L+1)]^T$ is the input signal vector, d(n) is the desired signal, e(n) is the error signal, μ(n) is the step size (convergence coefficient), L is the filter length (number of filter taps), and n is the actual input sample's number [9].

The popularity of the LMS adaptive filter stems from the fact that its algorithm employs the existing data to get estimates for the gradient vector. The same algorithm also includes a process that applies iteration to sequentially correct the weight vector in the negative direction of the gradient vector, so as to minimize MSE. The LMS algorithm is known for its simplicity, as opposed to other algorithms [8].

3.4 Recursive Least Square (RLS)

Another type of adaptive filter algorithm is the RLS. It is an algorithm that seeks to determine the coefficients needed for a minimum weighted linear least squares cost function, on a recursive basis. This function is related to input signals. RLS works in a manner that is the complete opposite to LMS and other algorithms. These algorithms seek the reduction of the MSE. To reach the RLS, deterministic input signals are used. On the other hand, inputs for LMS and similar algorithms are stochastic. Among the advantages of RLS is its extremely fast convergence, as opposed to most other algorithms used for the same purpose. However, a major disadvantage arising from this benefit is increased computational complexity [18].

A brief explanation of the RLS algorithm for a p-th order RLS filter can be summarized as

Parameters:

p = Filter order

λ = Forgetting factor

δ = Value of initialize P(0)

Initialization: $w_n = 0$

$P(0) = \delta^{-1}I$, where I is the (p+1)-by-(p+1) identity matrix

Computation: For n =0,1,2…

Then, the weight update can be given by the following equation:

$$w(n) = w(n-1) + \alpha(n)\, g(n) \quad (3)$$

where,

$$\alpha(n) = d(n) - w(n-1)^T x(n) \quad (4)$$

$$g(n) = p(n-1)z(n)\{\lambda + x^T(n)P(n-1)x(n)\}^{-1} \quad (5)$$

4. Simulation Results

In this section, the simulation results are presented after adding the AdaBoost to the different estimation techniques. The results are presented in the form of a comparison between the BER curve of the system with and without the AdaBoost algorithm. The system is simulated for different modulation techniques such as 8QAM and 16QAM in a Rayleigh fading environment. After applying the above explained channel estimation techniques to the simulated system, the following results of enhancement are obtained. The values calculated in dB are obtained from each figure. This is done by choosing a value of SNR and then obtaining the BER corresponding values to it from both curves (with and without

AdaBoost). After that, the dB=10 log(x) rule is used to change the value of BER to dB; therefore, it becomes dB=10 log (BER). Then the value of enhancement is calculated by subtracting the BER values in dB for both curves (with and without AdaBoost).

4.1 Least Square (LS)

Figure 2, shows the simulation results for the 8QAM system of 105 bits. As for SNR, it falls between 1 and 30 dB using LS channel estimation in a Rayleigh fading environment. The number of channel taps=4 and number of AdaBoost classifiers =20. Starting from SNR value equal 1 to 5 dB, the AdaBoost simulated system appears to go above the LS system. After that, both systems curves match with each other from SNR value of 6 dB to SNR value of 9 dB. At SNR of 11 dB, it can be seen that the system with AdaBoost shows some enhancement of 2.4 dB. After that, at SNR of 15 dB, the enhancement is 2.7 dB. Starting from SNR of 19 dB up to SNR of 25 dB, the AdaBoost simulated system shows an enhancement ranging from 5.8 dB to 15.7 dB. The enhancement in the results comes from adding the AdaBoost to the simulated system which helps in enhancing its BER in comparison to the BER of the system without AdaBoost. The overall BER performance of the system gets better, since the AdaBoost works on recovering the originally emitted signal from the noise accompanying it.

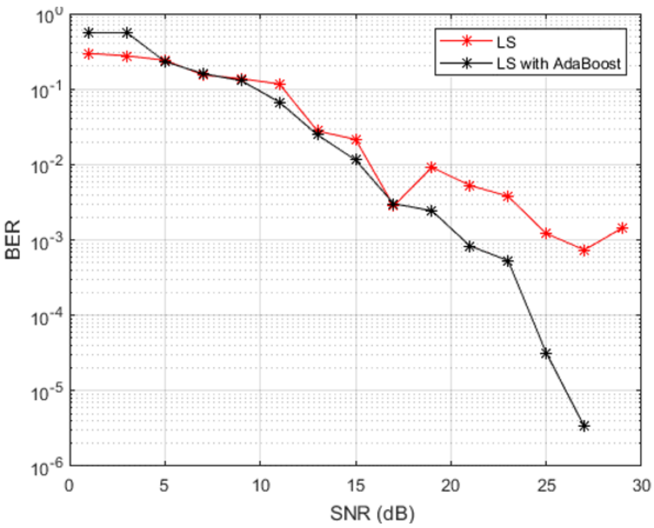

Fig. 2. BER vs. SNR in the case of an 8QAM system using LS channel estimation

The simulation result for the same system but using 16QAM modulation can be seen in Figure 3. Starting from SNR of 1 to 11 dB, the system with AdaBoost appears to be worse than the conventional LS system. After that, from SNR value of 13 dB to SNR value of 23 dB, it can be seen that the system with AdaBoost shows some enhancement in the overall BER performance that ranges from 1.2 dB to 5.2 dB than the main system. At SNR value of 27 dB, the system recorded the highest enhancement, which is 17dB. Similarly, the enhancement in the results comes from adding the AdaBoost algorithm to the simulated system.

The LS iterative channel estimation method with soft decision feedback shows an improvement of 0.15 dB for the least square estimation. So when compared with the proposed model, least square with AdaBoost, it showed better results.

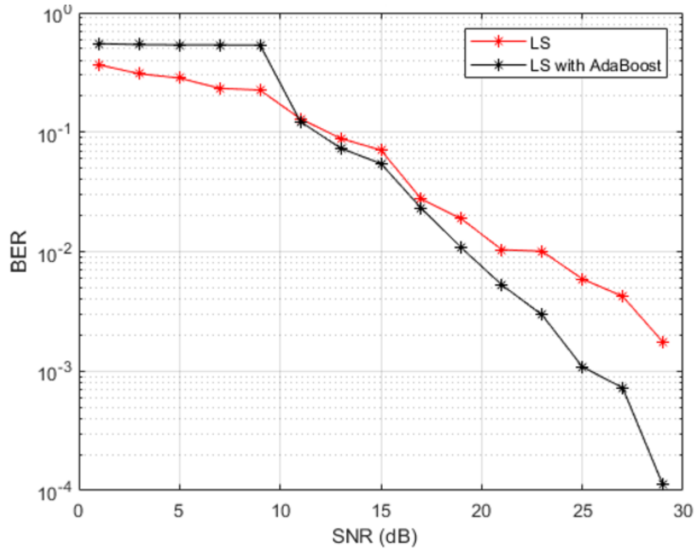

Fig. 3. BER vs. SNR in the case of a 16QAM system using LS channel estimation

4.2 Least Mean Square (LMS)

Now, we move on to the next channel estimation technique used which is the LMS. As shown in Fig. 4, the simulation result is illustrated for the 8QAM system with LMS estimation in a Rayleigh fading environment. The number of channel taps=4 and number of AdaBoost classifiers =20 classifiers. For SNR equal 1dB to 3dB, the system with AdaBoost appears to be worse than the conventional LMS system. Then, starting from SNR of 5 dB to 30 dB, the AdaBoost curve shows an enhancement of around 0.64 dB up to 7.62 dB. As previously stated, the AdaBoost algorithm works by recovering the original signal sent by the sender from the noisy signal received by the receiver. This effect leads to the enhancement of the simulated BER. Thus, the BER curve simulated by the AdaBoost yields better results than those of the same curve without the application of said algorithm.

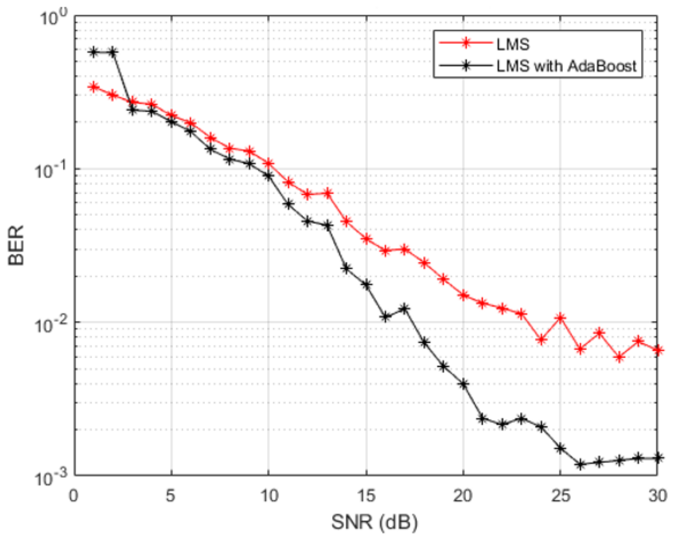

Fig. 4. BER vs. SNR in case of an 8QAM system using LMS channel estimation

Similarly, Fig. 5 shows the simulation results for the same 16QAM system using LMS estimation. At the beginning, the system with AdaBoost appears to be worse than the conventional LMS system from SNR value of 1 dB to almost the value of 3 dB. Then, starting from SNR value of 11 dB to 25 dB, the AdaBoost shows an enhancement in results of around 0.51 dB to 4.74 dB. At SNR of 30 dB, an enhancement of 5.65 dB is noticed. Again the AdaBoost technique is able to achieve an enhancement of different values throughout the entire BER curve.

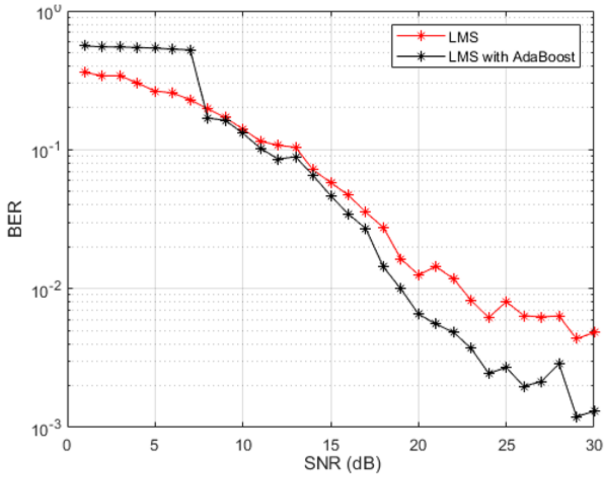

Fig. 5. BER vs. SNR in the case of a 16QAM system using LMS channel estimation

4.3 Recursive Least Square (RLS)

Figure 6 shows the performance of 8QAM system using RLS estimation at the same previously specified parameters. The Adaboost algorithm causes the simulated system to show enhancements in BER, when compared with the system without the AdaBoost. As seen, the AdaBoost curve appears to be worse than the conventional RLS system at value of SNR ranging from 1 dB to 4 dB. Then, from SNR of 7 dB to 11 dB, the enhancement is equal to 1.2 dB. After that, starting from SNR of 13dB to 30 dB, the performance of AdaBoost appears to be better than the RLS system by almost 3.7 dB to 9.3 dB.

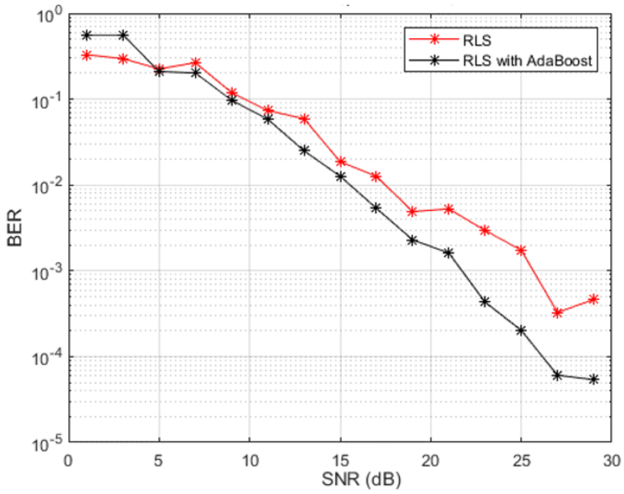

Fig. 6. BER vs. SNR in the case of an 8QAM system using RLS channel estimation

Simulating the same system but for 16QAM modulation, Fig. 7 shows the AdaBoost simulated system versus the RLS system. From SNR of 1 dB to 9 dB, the AdaBoost appears to be worse than the conventional the RLS. Starting from SNR of 13 dB to 25 dB, the enhancement is ranging from 1.2 dB to 4.9 dB.

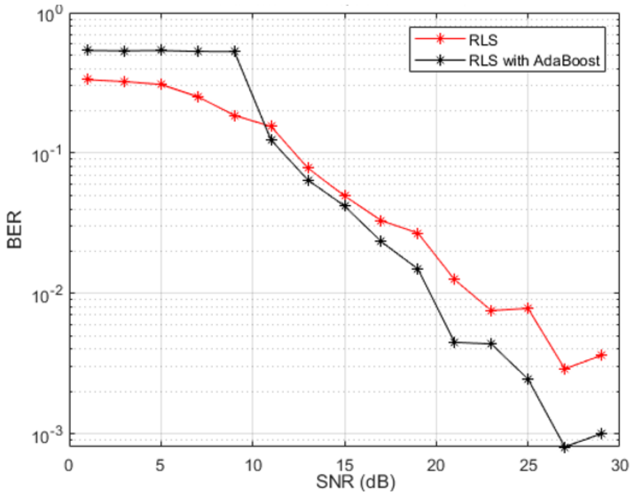

Fig. 7. BER vs. SNR in the case of a 16QAM system using RLS channel estimation

5. Conclusion

It can be concluded from all the previously shown simulation results that using the AdaBoost algorithm is able to achieve an enhancement with the different types of channel estimation techniques. The highest value of enhancement comes from using the LS technique with both the 8QAM and the 16QAM modulation. The next best results came from using the RLS technique with 8QAM, followed by the case of using LMS technique also with 8QAM modulation. The least improved results came from using LMS with 16QAM and using also RLS with the 16QAM. A comparison of all the results in this chapter is summarized in Table 1.

Table 1
Comparison between the results of the channel estimation techniques with AdaBoost

	LS with AdaBoost					LMS with AdaBoost		
	8QAM			16QAM		8QAM	16QAM	
SNR(dB)	11	15	19 to25	13 to 23	27	5 to 30	11 to 25	30
Enhancement in BER(dB)	2.4	2.7	5.8 up to 15.7	1.2 up to 5.2	17	0.64 up to 7.62	0.5 up to 4.7	5.6

	RLS with AdaBoost		
	8QAM		16QAM
SNR(dB)	7 to 11	13 to 30	13 to 25
Enhancement in BER(dB)	1.2	3.7 up to 9.3	1.2 up to 4.9

As seen from Li *et al.*, [4], the channel estimator using decoded/undecoded dual mode reference shows an enhancement of 3 dB. The system with channel estimation has about 1.5 dB improvement to the one without channel estimation which still is worse than the proposed model. Computer simulation demonstrates that channel estimation gives about 2.5 dB improvement when the Doppler frequency is

40 Hz and about 1.5 dB improvement when Doppler frequency is as large as 200 Hz. When comparing the boosted MMSE (B-MMSE) receiver in [15], it showed an improvement of about 2 dB over the MMSE detector. The turbo coded performance of B-MMSE showed a gain of 5 dB in comparison with MMSE receiver.

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