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Comparative Analysis of Inventory Management System with Two-Phase Time-dependent Demand and Shortages under Tolerable Slack of Payment for Deteriorating Goods

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ABSTRACT

In commercial transactions, the supplier frequently gives the merchant a maximum permitted settlement delay to boost sales. In order to account for these variations and assume time-varying holding costs and shortages, this research analyses a time-dependent two-phase ramp curve demand under the allowable slack of payment model for goods with a limited life span. The demand rate for the nonlinear ramp curve is deterministic, changes over time until a pivotal moment, and remains constant. It is presumed that the breakthrough point happened within the cycle duration. In this research, two scenarios are considered: One is that the slack of the payment period is shorter than the demand turning point, and on the other hand, the payment defer period is higher than the demand turning point. The outcomes of numerical results are used to compare the best optimality. The sensitivity analysis is then employed to exhibit the extent of the applicability of the inventory parameters.

1. Introduction

Demand is a significant constraint in determining the appropriate inventory strategy, an essential component of our manufacturing, distribution, and retail. To construct the inventory models, researchers made the following assertions about the demand for the items: constant, linearly increasing or declining, exponentially expanding or decreasing with time, stock-dependent, etc. Following that, it was found that the preceding demand patterns did not accurately reflect demand for some goods, including recently introduced fashion items, the latest technologies, clothing, cosmetics, and automobiles, for which the demand increases gradually as they are introduced to the market and subsequently stays at a fixed value. Ramp-type demand is a notion that is presented to consider the demand for these types. Ramp-type demand describes an increase in demand that lasts for a set number of times before stabilising and becoming steady.

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In commercial settings, deterioration is inevitable. The importance of the analysis of production inventories of deteriorating goods with and without shortages has currently increased. Deterioration's constraint has significance in many inventory management systems. Deterioration is characterised as modifications, harm, decay, spoiling, obsolescence, theft, loss of use, or loss of marginal value of items that reduce the original's usefulness. The holding cost has been considered in the majority of inventory models as a constant function. However, several businesses observe fluctuations in supply, demand, and price breaks. Because of this, the provider has a stock and holding cost that changes with the passing of time. Therefore, our current work examines how holding costs vary with time. Trade credit is a strategy used in administration by businesses or wholesalers to encourage retailers to increase sales and make an adequate profit. Contrarily, trade credit lowers the cost of purchases and is a significant source of short-term external financing for businesses. Beyond this time, the corporation or company begins to charge interest in accordance with the terms and conditions agreed upon, but no interest is charged during this time.

1.1. Aim of this study

The development of the inventory model discussed in this article seeks to further explore an inventory model that includes a two-phase demand pattern with a demand rate function, holding costs that rise with time, a constant rate of deterioration, a waiting period for payments that is acceptable, and shortages that are authorised. Numerous academics study demand rate functions that ramp up linearly over time before becoming constant. This study contributes to the nonlinear ramp-type demand rate function, which increases over time until it becomes constant in the presence of shortages.

2. Literature Survey

Aggarwal and Jaggi [1] created the optimal ordering policy under permissible delay in payments. Jamal *et al*., [2] developed an ordering policy for goods that decay with shortages and trade credit. Chu *et al*., [3] derived the economic order quantity for the deteriorating goods under acceptable delay in payments. Sarker *et al*., [4] examined the optimal payment time under permissible delay in payment for deteriorated goods. Abad and Jaggi [5] presented an integrated model for determining unit price and credit period length for price-dependent demand. Chung [6] developed an inventory system with partial backlogs and allowable payment delays. Chung and Huang [7] constructed the EPQ model under permissible delay. Tripathy *et al*., [8] explored an EOQ model for linearly degrading rates with shortages and acceptable payment delays. Tripathy *et al*., [9] examined a model of the optimal price and ordering approach for three-parameter Weibull degradation under trade credit. Skouri *et al*., [10] created an inventory model with parameters for tolerable payment delays, a general ramp-type demand rate, a partial backlog of unfulfilled demand, and more. A model for perishable products with time-dependent quadratic demand and variable deterioration under allowable payment delay was investigated by Singh *et al*., [11]. Uthayakumar *et al*., [12] created a pharmaceutical inventory model with a quadratic demand rate and variable holding cost with tolerable late payment. Yadav *et al*., [13] examined the inventory model with variable holding costs for non-instantaneous deteriorating items.

The ideal ordering rules with ramp-type demand rates and the permitted payment delay with many potential scenarios were created by Y. Shi *et al*., [14]. Singh *et al*., [15] investigated the inventory model while accounting for the varying rate of deterioration and the quadratic function as the demand rate. Shah *et al*., [16] derived the replenishment policy for quadratic demand under the trade credit policy. Tripathy and Sukla [17] developed the inventory mode under imprecision and trade credit. A generalized order level with time-varying demand and a Weibull distributed rate of deterioration with an allowable stay in payment across numerous trade-credit periods was presented by Mondal B. *et al.,* [18]. The inventory model was developed by Sharma *et al.*, [19] for the trade credit policy for degrading items. Khan *et al.*, [20] investigated the optimal lot-size model with a linear time-varying holding cost. The deterministic model for Giffen goods, in which deteriorated with linearly growing demand, was developed by Palanivelu *et al.*, [21]. Kaushik [22] examined the inventory models under permissible delay with different interest rates for the due amount. Saranya, P and Chandrsekaran [23] derived the inventory system with time-dependent demand and advertisement cost. The heart disease prediction system is developed and solved using convolution neural network algorithm by Gopalakrishnan *et al.,* [24]. Mansor *et al.*, [25] created the IoT based Inventory management system using load cell and Node MCU.

The article is structured as follows: Section 3 explains the assumptions to be made, and symbols are used for developing the mathematical inventory model. The mathematical model and its components are described in Section 4. The methodology for solving the inventory model is detailed in Section 5. The numerical and sensitivity results of the proposed model are illustrated in section 6. Section 7 concludes with the implications and extension ideas of inventory models.

3 Symbols and Assumptions

3.1 Assumptions

- The inventory system takes only one item into consideration.
- Demand rate is represented by a time-dependent non-linear ramp curve function.
- Time horizon is unbounded.
- Replacement rate is not constrained.
- Zero lead time.
- The pace of deterioration is fixed and there is no substitute through the cycle time.
- Shortages are acceptable and completely backlogged.
- The suppliers provide the retailers a legally permitted payment delay.
- Model I, Allowable delay period occurs after the demand breakthrough point.
- Model II Allowable slack period occurs before the demand breakthrough point.

3.2 Symbols

- A – Purchasing cost/order.
- $RC(t)$- Deterministic Demand and which is assumed as $RC(t) = \begin{cases} at - bt^2 & , 0 < t \le \tau \\ DC(\tau) & , \tau \le t < T \end{cases}$

Where $DC(\tau) = a\tau - b\tau^2$ *and* $\tau \le \frac{a}{2b}$.

- η - Rate of deterioration.
- $h_1 t + h_2$ - Inventory Holding cost per unit per unit time.
- $In(t), In_1(t), In_2(t)$- the level of inventory at any time t.
- ICC- Inventory holding cost per cycle.
- IDC – Inventory Deterioration Cost per cycle.

- IE_{11}, IE_{12} - Interest earned during the time of allowable slack in payments.
- IC_{11}, IC_{12} - Interest charged after the time of allowable slack in payments.
- TIC_{11}, TIC_{12} - Total cost per unit of time.
- T_{11}, T_{12} - Replenishment period of cycle time.

4. Inventory Modelling and Methodology

4.1 Mathematical Model

The governing differential equation of inventory model is given by:

$$\frac{dIn(t)}{dt} = -\eta I(t) - RC(t) \tag{1}$$

And by the assumption of the two-phase demand, Equation (1) can be written as three different equations:

$$\frac{dIn(t)}{dt} = -\eta I(t) - (at - bt^2), \qquad 0 < t \le \tau \tag{2}$$

$$\frac{dIn(t)}{dt} = -\eta I(t) - (a\tau - b\tau^2), \qquad \tau < t \le t_1 \tag{3}$$

$$\frac{dIn(t)}{dt} = -(a\tau - b\tau^2), \qquad t_1 < t \le T \tag{4}$$

With initial and boundary conditions,

$$In(0) = Q \quad \text{And} \quad In(t_1) = 0 \tag{5}$$

On solving Equation (2), (3) and (4), we get,

$$In(t) = \frac{1}{\eta}\left(e^{-\eta T}\left(-b\tau^2 + a\tau + \eta q\right) - a\tau + b\tau^2\right), \qquad 0 < t \le \tau \tag{6}$$

$$In(t) = e^{-\eta T} q - \frac{2b + a\eta}{\eta^3}\left(1 + e^{-\eta T}\right) + \frac{T^2 b}{\eta} - \frac{T\left(2b + a\eta\right)}{\eta^2}, \qquad \tau \le t < T \tag{7}$$

$$In(t) = -\left(a\tau - b\tau^2\right)\left(T - t_1\right), \qquad t_1 \le t \le T \tag{8}$$

By applying Equation 5 in Equation (6) and Equation (7), the initial order quantity is obtained as follows:

$$Q=\frac{\left(-b\tau^2\left(1-e^{\eta T}\right)-a\tau\left(1+e^{\eta T}\right)\right)}{\eta} \tag{9}$$

Then Equation (7) and Equation (8) will become,

$$In(t)=\frac{2b+a\eta}{\eta^3}\left(1-e^{-\eta t}\right)+\frac{e^{-\eta t}}{\eta}\left(\left(b\tau^2-a\tau\right)\left(1-e^{\eta T}\right)\right)+\frac{bt^2}{\eta}-\frac{t\left(2b+a\eta\right)}{\eta^2}, \qquad 0\le t\le\tau \tag{10}$$

$$In(t)=\frac{1}{\eta}\left(b\tau^2-a\tau-e^{-\eta(t+T)}\left(b\tau^2-a\tau\right)\right), \qquad \tau\le t\le t_1 \tag{11}$$

The inventory carrying cost over the cycle is calculated with the linear holding cost rate and the inventory level at any time t,

$$\begin{aligned}IHC&=\int_0^T\left(h_1t+h_2\right)In(t)\,dt\\&=\int_0^\tau\left(h_1t+h_2\right)In(t)\,dt+\int_\tau^T\left(h_1t+h_2\right)In(t)\,dt\\&=\int_0^\tau\left(h_1t+h_2\right)\left(\frac{1}{\eta}\left(e^{-\eta T}\left(-b\tau^2+a\tau+\eta q\right)-a\tau+b\tau^2\right)\right)dt\\&\quad+\int_\tau^T\left(h_1t+h_2\right)\left(e^{-\eta T}Q-\frac{2b+a\eta}{\eta^3}\left(1+e^{-\eta T}\right)+\frac{T^2b}{\eta}-\frac{T\left(2b+a\eta\right)}{\eta^2}\right)dt\end{aligned} \tag{12}$$

Inventory deterioration cost over the inventory cycle is obtained by the remaining quantities available after met the demand in cycle time and deterioration cost per unit.

$$\begin{aligned}IDC&=d\times\left(Q-\int_0^{t_1}RC(t)\,dt\right)\\&=d\times\left(Q-\int_0^\tau RC(t)\,dt-\int_\tau^{t_1}RC(t)\,dt\right)\\&=d\times\left(Q-\int_0^\tau\left(at-bt^2\right)dt-\int_\tau^{t_1}\left(a\tau-b\tau^2\right)dt\right)\end{aligned} \tag{13}$$

Since shortages are allowed and fully backlogged, the shortage cost per cycle is found by using the shortage of demand over the cycle and shortage cost per unit,

$$ISC=s\int_{t_1}^T\left[\left(a\tau-b\tau^2\right)\left((t_1-T\right)\right]dt \tag{14}$$

4.1.1 MODEL -I $A_d \le T$

The allowable slack of payment A_d occurs after the turning point (τ). The allowable slack of duration in model I is presented in Figure 1 in connection to the point at which demand breakthroughs and the cycle length of time.

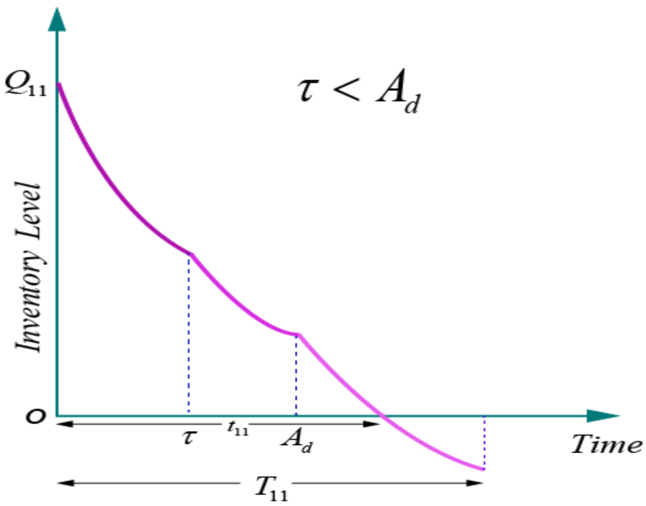

Fig. 1. Inventory Model I

If the allowable slack of period occurs after the demand turning point,

Interest which can be earned during $[0, A_d]$,

$$
\begin{aligned}
EI_1 &= PI_e \int_0^{A_d} RC(T)\,dt \\
&= PI_e \int_0^{\tau} RC(t)\,dt + \int_{\tau}^{A_d} RC(t)\,dt \\
&= PI_e \int_0^{\tau} \left(at - bt^2\right) dt + \int_{\tau}^{A_d} \left(a\tau - b\tau^2\right) dt
\end{aligned}
\tag{15}
$$

Interest charges that payable by the retailer during the period $[A_d, T]$

$$
\begin{aligned}
CI_{11} &= cI_c \int_{A_d}^{T} In(t)\,dt \\
&= cI_c \left\{ \int_{A_d}^{\tau} \frac{1}{\eta}\left(e^{-\eta T}\left(-b\tau^2 + a\tau + \eta Q\right) - a\tau + b\tau^2\right) dt \right. \\
&\qquad \left. + \int_{\tau}^{T} \left(e^{-\eta T} Q - \frac{2b + a\eta}{\eta^3}\left(1 + e^{-\eta T}\right) + \frac{T^2 b}{\eta} - \frac{T\left(2b + a\eta\right)}{\eta^2} \right) dt \right\}
\end{aligned}
\tag{16}
$$

4.1.2 Model II $\tau \geq A_d$

In model II, the inventory level due to the demand varies by time to time and the allowable slack of time appears before the breakthrough point of the demand, which is cleared by Figure 2.

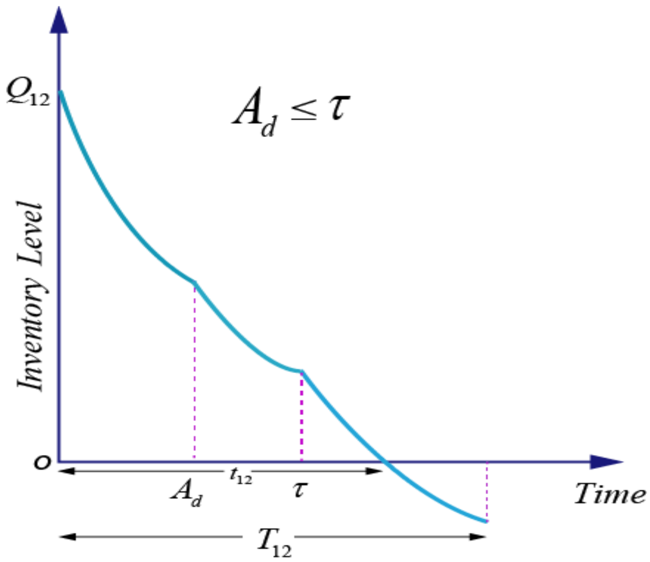

Fig. 2. Inventory Model II

Interest earned during $\left[0, A_D\right]$,

$$EI_{12} = PI_e \int_0^{A_D} RC(t)\,dt$$
$$= PI_e \int_0^{P_D} \left(at - bt^2\right)dt \tag{17}$$

Total interest which will be charged to retailer during $\left[A_d, T\right]$

$$CI_{12} = cI_c \int_{A_d}^{T} In(t)\,dt$$
$$= cI_c \left\{ \int_{A_d}^{\tau} In(t)\,dt + \int_{\tau}^{T} In(t)\,dt \right\}$$
$$= cI_c \left\{ \int_{A_d}^{\tau} \frac{1}{\eta}\left(e^{-\eta T}\left(-b\tau^2 + a\tau + \eta Q\right) - a\tau + b\tau^2\right)dt \right.$$
$$\left. + \int_{\tau}^{T}\left(e^{-\eta T}Q - \frac{2b + a\eta}{\eta^3}\left(1 + e^{-\eta T}\right) + \frac{T^2 b}{\eta} - \frac{T\left(2b + a\eta\right)}{\eta^2}\right)dt \right\} \tag{18}$$

Inventory total cost per of unit time is obtained with the inventory ordering cost, carrying cost, deterioration cost, shortage cost, interest charged and interest gained through the allowable slack of payment.

Total Inventory Cost for model I is obtained by:

$$TIC_{11} = \frac{1}{T_{11}}\left(OC + ICC + IDC + ISC - EI_{11} + CI_{11}\right)$$

$$= \frac{1}{T_{11}}\left\{k + \int_0^{\tau}\left(h_1 t + h_2\right)\left(\frac{1}{\eta}\left(e^{-\eta T}\left(-b\tau^2 + a\tau + \eta Q\right) - a\tau + b\tau^2\right)\right)dt\right.$$

$$+ \int_{\tau}^{T}\left(h_1 t + h_2\right)\left(e^{-\eta T}Q - \frac{2b + a\eta}{\eta^3}\left(1 + e^{-\eta T}\right) + \frac{T^2 b}{\eta} - \frac{T\left(2b + a\eta\right)}{\eta^2}\right)dt$$

$$+ d\left(Q - \int_0^{\tau}\left(at - bt^2\right)dt - \int_{\tau}^{T}\left(a\tau - b\tau^2\right)dt\right)$$

$$+ s\int_{t_1}^{T}\left[-\left(a\tau - b\tau^2\right)\left(T - t_1\right)\right]dt$$

$$+ cI_c\left\{\int_{A_d}^{\tau}\frac{1}{\eta}\left(e^{-\eta T}\left(-b\tau^2 + a\tau + \eta Q\right) - a\tau + b\tau^2\right)dt\right.$$

$$\left. - PI_e\int_0^{\tau}\left(at - bt^2\right)dt + \int_{\tau}^{A_d}\left(a\tau - b\tau^2\right)dt\right\}$$

$$\left. + \int_{\tau}^{T}\left(e^{-\eta T}Q - \frac{2b + a\eta}{\eta^3}\left(1 + e^{-\eta T}\right) + \frac{T^2 b}{\eta} - \frac{T\left(2b + a\eta\right)}{\eta^2}\right)dt\right\} \tag{19}$$

Total inventory cost for model II is found by:

$$TIC_{12} = \frac{1}{T_{12}}\left(OC + ICC + IDC + ISC - EI_{12} + CI_{12}\right)$$

$$
\begin{aligned}
&= \frac{1}{T_{12}}\Bigg\{ k + \int_0^{\tau} (h_1 t + h_2)\left(\frac{1}{\eta}\left(e^{-\eta T}\left(-b\tau^2 + a\tau + \eta Q\right) - a\tau + b\tau^2\right) \right) dt \\
&\quad + \int_{\tau}^{T} (h_1 t + h_2)\left(e^{-\eta T} Q - \frac{2b + a\eta}{\eta^3}\left(1 + e^{-\eta T}\right) + \frac{T^2 b}{\eta} - \frac{T\left(a2b + a\eta\right)}{\eta^2} \right) dt \\
&\quad + d\left(Q - \int_0^{\tau} \left(at - bt^2\right) dt - \int_{\tau}^{T} \left(a\tau - b\tau^2\right) dt \right) \\
&\quad + s\int_{t_1}^{T} \left[-\left(a\tau - b\tau^2\right)\left(T - t_1\right) \right] dt - PI_e \int_0^{A_d} \left(at - bt^2\right) dt \\
&\quad + cI_c \Bigg\{ \int_{A_d}^{\tau} \frac{1}{\eta}\left(e^{-\eta T}\left(-b\tau^2 + a\tau + \eta Q\right) - a\tau + b\tau^2\right) dt \\
&\quad + \int_{\tau}^{T} \left(e^{-\eta T} Q - \frac{2b + a\eta}{\eta^3}\left(1 + e^{-\eta T}\right) + \frac{T^2 b}{\eta} - \frac{T\left(2b + a\eta\right)}{\eta^2} \right) dt \Bigg\}\Bigg\}
\end{aligned}
\tag{20}
$$

5. Methodology

In this study, the goal to achieve the lowest total inventory cost possibly. The necessary condition to be minimizing is,

$$
\begin{aligned}
&\frac{\partial TIC_{11}}{\partial t_{11}} = 0 \;\; and \;\; \frac{\partial TIC_{11}}{\partial T_{11}} = 0, \\
&\frac{\partial TIC_{12}}{\partial t_{12}} = 0 \;\; and \;\; \frac{\partial TIC_{12}}{\partial T_{12}} = 0, \;\; .
\end{aligned}
\tag{21}
$$

The sufficient condition to be minimize is Hessian matrix of TIC_{11} *and* TIC_{12} with respect to t_1 *and* T should be positive definite or positive semi definite. Analytical solutions are challenging because the equations in Equation (21) are highly non-linear in t_1 *and* T. The mathematical model's various variables can be assigned numerical values in order to compute the solutions to the equations Equation (19) and Equation (20). To numerically solve the problem MATLAB software is used.

6. Numerical Results and Sensitivity Investigation

Numerical examples are afforded by assigning various values to the model parameters.

6.1 Numerical Results

6.1.1 Example: 1 Model – I: ($\tau \le A_d \le T_{11}$)

By assuming the values for the model parameters, $a = 200,\; b = 100,\; \eta = 0.05,\; h_1 = 0.5,\; h_2 = 3,$ $\tau = 0.25,\; A_D = 0.35,\; k = 500,\; d = 20, p = 50, i_e = 0.08, i_c = 0.25$, cycle lengths are obtained as

$t_{11}^{*}=0.7124$ *and* $T_{11}^{*}=4.5605$. With this optimum cycle time value, optimal total inventory cost $TIC_{11}^{*}=252.53$ and economic order quantity $q_{11}^{*}=71.32$ are obtained.

For the proposed inventory model, where the permitted delay duration is greater than the demand breakthrough point, the total cost curve is represented in Figure 3, which is obtained by the sensitivity numerical results in MATLAB (R2020) software and clarifies the convexity of the total inventory cost function.

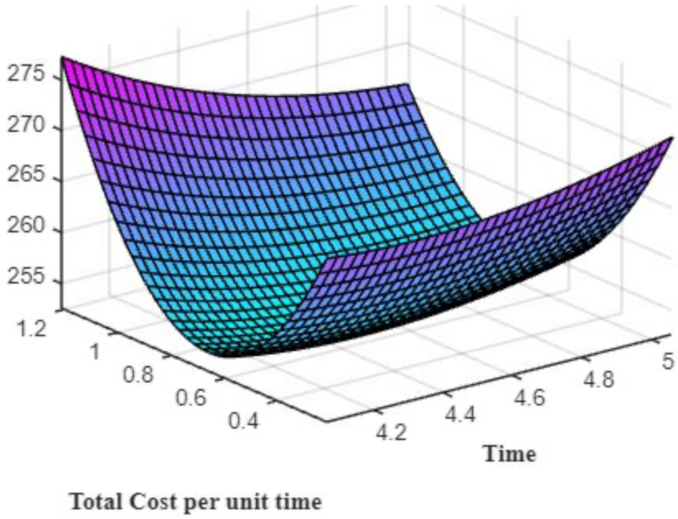

Fig. 3. Model-I Total Cost Curve

6.1.2 Example: 2 Model – II ($A_d \leq \tau \leq T_{12}$*)*

Consider the values in which are assumed in example 1 except the values of τ *and* A_D. Assume $\tau = 0.35$ *and* $A_D = 0.25$. The cycle lengths are obtained as $t_{12}^{*}=0.6250$ *and* $T_{12}^{*}=4.2764$. Optimal total inventory cost $TIC_{12}^{*}=316.31$ and economic order quantity $q_{12}^{*}=68.22$ are attained.

The convexity of the total inventory cost of model II, in which the permitted delay duration is less than the demand breakthrough point, is described in Figure 4.

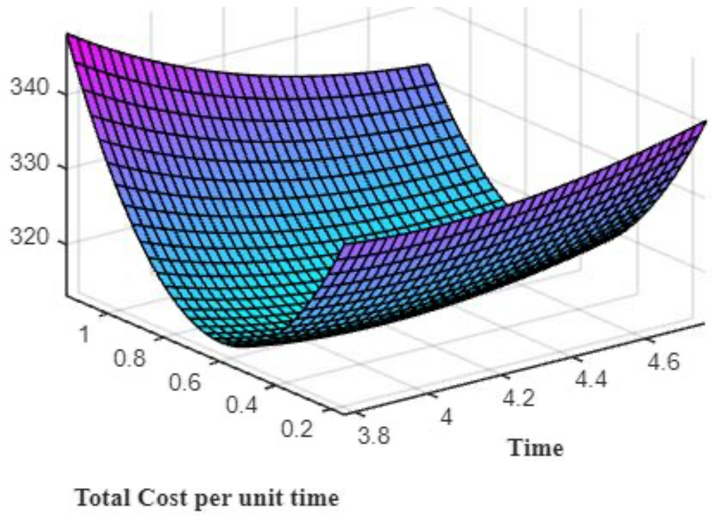

Fig. 4. Model-II Total Cost Curve

6.2 Sensitivity Investigation

The numerical sensitivity results are listed in Table 1 and Table 2 for the inventory models I and II respectively and it explains the stability of both models.

Table 1
Model I Sensitivity Results

Parameter	Value	t_{11}^{*}	TIC_{11}^{*}	Q_{11}^{*}
a	160	0.7695	218.26	59.44
	180	0.7380	235.75	65.60
	200	0.7124	252.53	71.32
	220	0.6910	268.73	76.73
	240	0.6728	284.45	81.92
b	80	0.7071	256.87	73.91
	90	0.7097	254.70	72.63
	100	0.7124	252.53	71.32
	110	0.7151	250.35	70.00
	120	0.7208	245.96	67.28
η	0.03	0.7377	251.45	74.26
	0.04	0.7249	252.00	72.77
	0.05	0.7124	252.53	71.32

	0.06	0.7003	253.04	69.92
	0.07	0.6887	253.54	68.56
h_1	0.3	0.7164	252.41	71.89
	0.4	0.7144	252.47	71.61
	0.5	0.7124	252.53	71.32
	0.6	0.7104	252.59	71.05
	0.7	0.7084	252.65	70.77
h_2	1	0.8415	246.34	89.94
	2	0.7716	249.67	79.75
	3	0.7124	252.53	71.32
	4	0.6616	255.01	64.28
	5	0.6175	257.19	58.33

Table 2
Model II Sensitivity Results

Parameter	Value	t_{11}^*	TIC_{11}^*	Q_{11}^*
a	160	0.6742	265.73	56.72
	180	0.6469	291.39	62.60
	200	0.6250	316.31	68.22
	220	0.6071	340.65	73.65
	240	0.5921	364.53	78.96
b	80	0.6194	325.62	70.55
	90	0.6221	320.97	69.39
	100	0.6250	316.31	68.22
	110	0.6280	311.63	67.03
	120	0.6311	3.6.94	65.83
η	0.03	0.6474	315.15	71.04
	0.04	0.6360	315.74	69.60

	0.05	0.6250	316.31	68.22
	0.06	0.6144	316.86	66.88
	0.07	0.6041	317.39	65.58
h_1	0.4	0.6265	316.24	68.45
	0.5	0.6250	316.31	68.22
	0.6	0.6235	316.37	67.99
	0.7	0.6220	316.44	67.77
h_2	2	0.6770	312.91	76.03
	3	0.6250	316.31	68.22
	4	0.5805	319.30	61.70
	5	0.5419	321.98	56.21

6.3 Sensitivity Diagrams

The sensitivity diagrams Figure 5 to Figure 16 explain the influence of the parameters of inventory model on cycle length, total inventory cost and economic order quantity.

6.3.1 Model-I sensitivity diagrams

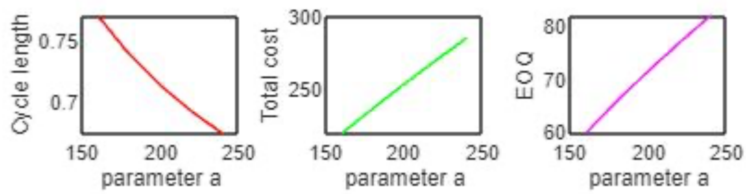

Fig. 5. Constraint of parameter a on $t_{11}^*, T_{11}^*\ and\ EOQ$

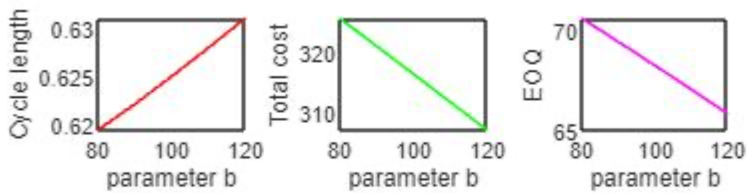

Fig. 6. Constraint of parameter b on $t_{11}^*, T_{11}^*\ and\ EOQ$

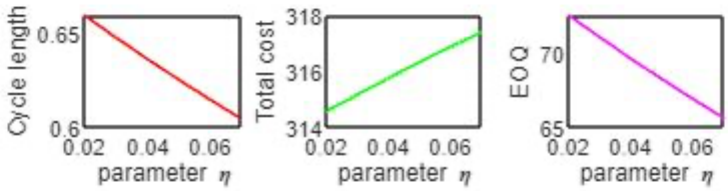

Fig. 7. Constraint of parameter η on t_{11}^*, T_{11}^* and EOQ

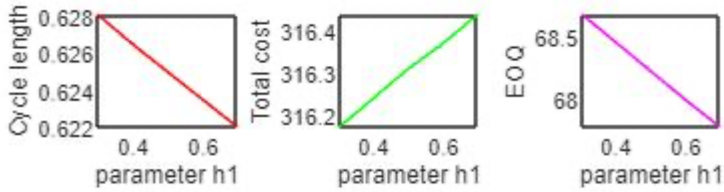

Fig. 8. Constraint of parameter h_1 on t_{11}^*, T_{11}^* and EOQ

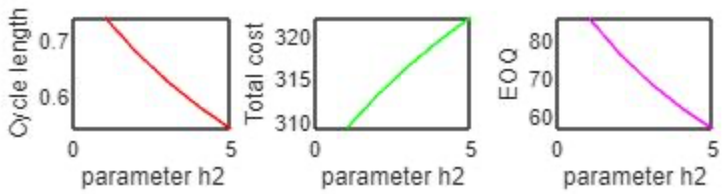

Fig. 9. Constraint of parameter h_2 on t_{11}^*, T_{11}^* and EOQ

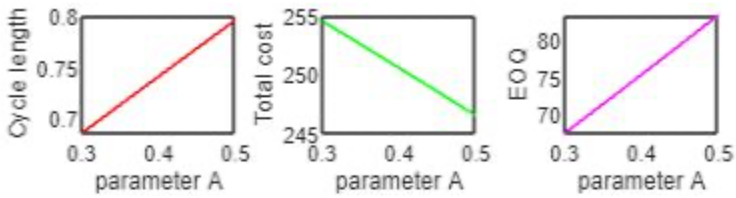

Fig. 10. Constraint of parameter A_d on t_{11}^*, T_{11}^* and EOQ

6.3.2 Case (ii) Sensitivity Diagrams

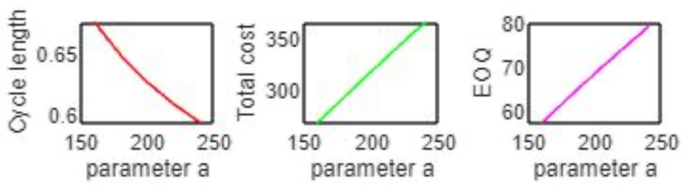

Fig. 11. Constraint of parameter a on t_{12}^*, T_{12}^* and EOQ

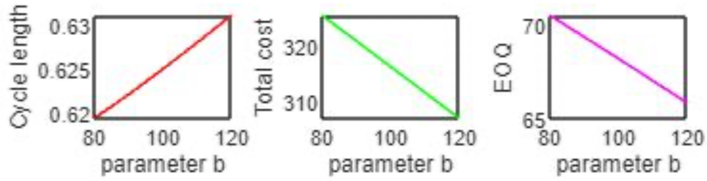

Fig. 12. Constraint of parameter b on t_{12}^{*}, T_{12}^{*} *and* EOQ

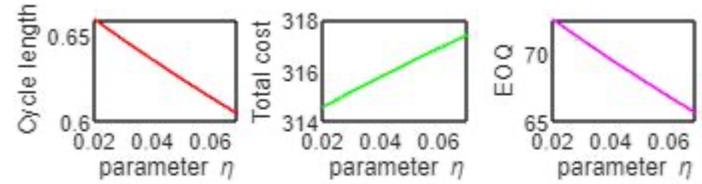

Fig. 13. Constraint of parameter η on t_{12}^{*}, T_{12}^{*} *and* EOQ

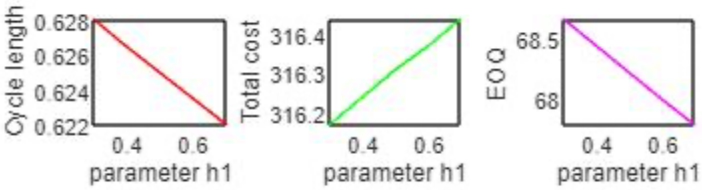

Fig. 14. Constraint of parameter h_1 on t_{12}^{*}, T_{12}^{*} *and* EOQ

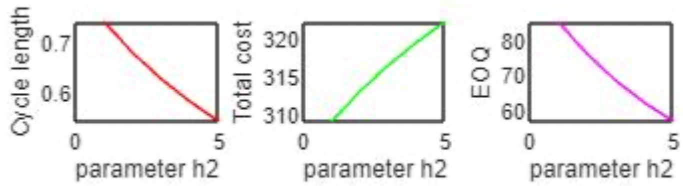

Fig. 15. Constraint of parameter h_2 on t_{12}^{*}, T_{12}^{*} *and* EOQ

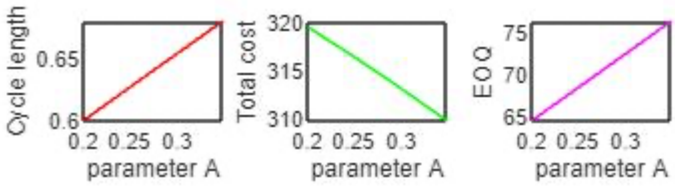

Fig. 16. Constraint of parameter A_d on t_{12}^{*}, T_{12}^{*} *and* EOQ

6.4 Discussion of Results

- The convexity of inventory total cost for the models I and II is revealed with the figures 3 and 4.
- When the demand parameter ‘a’ rises, inventory cost and order quantity tend to increase but cycle length is inversely changed.
- Another demand parameter ‘b’ controls on total cost and order quantity nominally.

- If the deterioration rate increases further, the total inventory cost increases but EOQ and cycle length inversely proportional to it.
- Linearly increasing holding cost parameter h_1 *and* h_2 influence on the cost as well as in the cycle length.
- It is clear that from the outcomes of the numerical examples and the values in table 1 and table 2 model I is best optimal comparatively with the model II. The minimum overall inventory cost is obtained in the model I. The permissible delay period occurs after the demand breakthrough point in the model I. The demand pivotal moment τ and the allowable delay in payment play the vital role on maintaining the inventory cost.

7. Conclusion

The benefits of a reasonable payment delay are examined in this study in order to assist businesses and customers in making payment decisions. The aforementioned tactic works for products with a stable rate of deterioration and variable demand. This methodology can be used to study a variety of topics, including the most recent scientific advancements, drugs, food products, and pharmaceuticals. Calculus methods can be used to determine the overall cost of the inventory using the acquired optimal cycle length. The numerous numerical results show the suggested model's practicality. The suggested course of action can be extended to encompass the products with linearly rising demand, demand that influenced by supply and by price, and demand that is influenced by all three. The investigation of two-phase demand (first phase no-linear time-dependent and second phase constant) with varying holding cost and shortages under a tolerable payment delay will begin with this study.

References

[1] Aggarwal, S. P., and C. K. Jaggi. "Ordering policies of deteriorating items under permissible delay in payments." *Journal of the operational Research Society* 46, no. 5 (1995): 658-662. https://doi.org/10.1057/jors.1995.90

[2] Jamal, A. M. M., B. R. Sarker, and S. Wang. "An ordering policy for deteriorating items with allowable shortage and permissible delay in payment." Journal of the operational research society 48 (1997): 826-833. https://doi.org/10.1057/palgrave.jors.2600428

[3] Chu, Peter, Kun-Jen Chung, and Shaw-Ping Lan. "Economic order quantity of deteriorating items under permissible delay in payments." Computers & Operations Research 25, no. 10 (1998): 817-824. https://doi.org/10.1016/S0305-0548(98)00006-9

[4] Sarker, Bhaba R., A. M. M. Jamal, and Shaojun Wang. "Optimal payment time under permissible delay in payment for products with deterioration." Production Planning & Control 11, no. 4 (2000): 380-390. https://doi.org/10.1080/095372800232117

[5] Abad, Prakash L., and C. K. Jaggi. "A joint approach for setting unit price and the length of the credit Period for a seller when end demand is price sensitive." International Journal of Production Economics 83, no. 2 (2003): 115-122. https://doi.org/10.1016/S0925-5273(02)00142-1

[6] Chung, K. J. "A complete proof on the solution procedure for non-instantaneous deteriorating items with permissible delay in payment," Computers & Industrial Engineering, 56(1) (2009): 267-273. https://doi.org/10.1016/j.cie.2008.05.015

[7] Chung, K. J., & Huang, Y. F. "The optimal cycle time for EPQ inventory model under permissible delay in payments," International Journal of Production Economics, 84(3) (2003): 307-318. https://doi.org/10.1016/S0925-5273(02)00465-6

[8] Tripathy, P. K., & Pradhan, S. "An integrated partial backlogging inventory model having Weibull demand and variable deterioration rate with the effect of trade credit," International Journal of Scientific and Engineering Research, 2(4) (2011): 01-05.

[9] Tripathi, R. P., S. S. Misra, and H. S. Shukla. "A cash flow oriented EOQ model under permissible delay in payments." International Journal of Engineering, Science and Technology 2, no.11 (2010).

https://doi.org/10.4314/ijest.v2i11.64560
[10] Skouri, K., Konstantaras, I., Papachristos, S., & Teng, J. T. "Supply chain models for deteriorating products with ramp type demand rate under permissible delay in payments," Expert Systems with Applications, 38(12) (2011): 14861-14869. https://doi.org/10.1016/j.eswa.2011.05.061
[11] Singh, T., & Pattnayak, H. "An EOQ model for a deteriorating item with time dependent quadratic demand and variable deterioration under permissible delay in payment," Applied Mathematical Science, 7(59) (2013): 2939-2951. https://doi.org/10.12988/ams.2013.13261
[12] Uthayakumar, R., & Karuppasamy, S. K. "A pharmaceutical inventory model for variable demand and variable holding cost with partially backlogged under permissible delay in payments in health care industries," International Journal of Applied and Computational Mathematics, 3 (2017): 327-341. https://doi.org/10.1007/s40819-017-0358-9
[13] Yadav, A. S., & Swami, A. "An inventory model for non-instantaneous deteriorating items with variable holding cost under two-storage," International journal of procurement management, 12(6) (2019): 690-710. https://doi.org/10.1504/IJPM.2019.102928
[14] Shi, Y., Zhang, Z., Zhou, F., & Shi, Y., "Optimal ordering policies for a single deteriorating item with ramp-type demand rate under permissible delay in payments," Journal of the Operational Research Society, 70(10) (2019): 1848-1868. https://doi.org/10.1080/01605682.2018.1468865
[15] Singh, T., Muduly, M. M., Asmita, N., Mallick, C., & Pattanayak, H. "A note on an economic order quantity model with time-dependent demand, three-parameter Weibull distribution deterioration and permissible delay in payment," Journal of Statistics and Management Systems, 23(3) (2020):643-662. https://doi.org/10.1080/09720510.2019.1681676
[16] Shah, N. H., Chaudhari, U., & Cárdenas-Barrón, L. E. "Integrating credit and replenishment policies for deteriorating items under quadratic demand in a three-echelon supply chain," International Journal of Systems Science: Operations & Logistics, 7(1) (2020): 34-45. https://doi.org/10.1080/23302674.2018.1487606
[17] Tripathy, P. K., & Sukla, S. "Interactive inventory model under permissible delay and imprecision," International Journal of Operation Research, 17(2) (2020): 9-23.
[18] Mondal, B., Garai, A., & Roy, T. K. "Optimization of generalized order-level inventory system under fully permissible delay in payment.," RAIRO-Operations Research, 55 (2021): S195-S224. https://doi.org/10.1051/ro/2019079
[19] Sharma, A., & Kaushik, J. "Inventory model for deteriorating items with ramp type demand under permissible delay in payment," International Journal of Procurement Management, 14(5) (2021): 578-595. https://doi.org/10.1504/IJPM.2021.117292
[20] Khan, M. A. A., Ahmed, S., Babu, M. S., & Sultana, N. "Optimal lot-size decision for deteriorating items with price-sensitive demand, linearly time-dependent holding cost under all-units discount environment," International Journal of Systems Science: Operations & Logistics, 9(1) (2022): 61-74. https://doi.org/10.1080/23302674.2020.1815892
[21] Palanivelu, Saranya, and E. Chandrasekaran. "A Deterministic Replenishment Policy for Constant m Deteriorating Goods with Time-Dependent Demand." In Proceedings of 2nd International Conference On Mathematical Modeling and Computational Science: ICMMCS 2021, Singapore: Springer Nature Singapore, (2022): pp. 271-280. https://doi.org/10.1007/978-981-19-0182-9_28
[22] Kaushik, J. "An inventory model with permissible delay in payment and different interest rate charges. Decision Analytics Journal, 6, (2023): 100180. https://doi.org/10.1016/j.dajour.2023.100180
[23] Saranya, Palanivelu, and Ekambaram Chandrasekaran. "Optimal inventory system for Deteriorated Goods with time-varying demand rate function and advertisement cost." Array 19 (2023): 100307. https://doi.org/10.1016/j.array.2023.100307
[24] Gopalakrishnan, S., M. Sahaya Sheela, K. Saranya, and J. Jasmine Hephzipah. "A Novel Deep Learning-Based Heart Disease Prediction System Using Convolutional Neural Networks (CNN) Algorithm." *International Journal of Intelligent Systems and Applications in Engineering* 11, no. 10s (2023): 516-522. https://ijisae.org/index.php/IJISAE/article/view/3306
[25] Mansor, Muhammad Naufal, Noor Anida Abu Talib, Syahrul Affandi Saidi, Wan Azani Mustafa, and Nurul Fakhirah Zamri. "Arduino IOT Based Inventory Management System Using Load Cell and Node MCU." *Journal of Advanced Research in Applied Sciences and Engineering Technology* 32, no. 3 (2023): 12-25. https://doi.org/10.37934/araset.32.3.1225