Journal of Advanced Research in Applied Sciences and Engineering Technology

Journal homepage:
https://semarakilmu.com.my/journals/index.php/applied_sciences_eng_tech/index
ISSN: 2462-1943

Hungarian Method and Branch and Bound Method for Solving Travelling Salesman Problem in Interval Number in Rice Distribution

Ihda Hasbiyati [1,*], Esra Fridani Syalom Siregar[1], Ahriyati[2], Moch Panji Agung Saputra[3], Sukono[4], Yasir Salih[5]

[1] Department of Mathematics, Faculty Mathematics and Sciences, University of Riau, Pekanbaru , Provinsi of Riau, 28293, Indonesia
[2] Department of Mathematics, Faculty Mathematics and Sciences, University of Palangkaraya, Palangkaraya , Provinsi Kalimantan Tengah, 27111, Indonesia
[3] Doctoral Program of Mathematics, Faculty of Mathematics and Natural Sciences, Universitas Padjadjaran, Sumedang, Provinsi Jawa Barat, 45363, Indonesia
[4] Department of Mathematics, Faculty Mathematics and Sciences, Universitas Padjadjaran, Sumedang, Provinsi Jawa Barat, 45363, Indonesia
[5] Department of Mathematics, Faculty of Education, Red Sea University, Port Sudan, Sudan

ARTICLE INFO

Article history:
Received 22 November 2023
Received in revised form 2 February 2024
Accepted 15 June 2024
Available online 15 July 2024

Keywords:
Travelling salesman problem; Hungarian method; Branch and Bound Method; fuzzification

ABSTRACT

The aim of this research is to minimize the travel time costs for transport cars starting from the warehouse in the sub-district, and delivering rice to 6 sub-districts in Pekanbaru only once and the transport car returning to the warehouse location. Minimizing travel time costs is carried out using the Hungarian method and the Branch and Bound method to solve the traveling salesman problem in interval numbers. The traveling salesman problem has interval costs because it depends on several obstacles experienced by the salesman, for example travel traffic constraints, transportation conditions, weather, and other costs. Then convert the interval numbers into trapezoidal fuzzy numbers using the fuzzification method. The results of the analysis show that using the Hungarian method and the Branch and Bound method to solve the traveling salesman problem at interval numbers can provide an optimal travel route for visiting 6 sub-districts in Pekanbaru using rice transport cars. This optimal route is expected to minimize costs and also save transportation time in rice distribution activities to the 6 sub-districts.

1. Introduction

Travelling salesman problem are widely applied in everyday life, for example, the efficiency of goods delivery, food delivery service, transportation problem, etc. The travelling salesman aims to find the shortest route or minimum distance carried out by a salesman in going around all cities exactly once returning to the city of origin [12,22]. In the travelling salesman problem, given a route n cities, the salesman wants to visit all citiesexactly once and return to the initial city at the cost of the distance or travel time between each city, which is assumed to be a number of intervals because it depends on traffic conditions, weather, cargo, etc [17]. In this study the type of interval used a

[*] *Corresponding author.*
E-mail address: ihdahasbiyati@lecturer.unri.ac.id

https://doi.org/10.37934/araset.48.2.7891

closed interval. To place a number in a closed interval is the same as limiting it to a lower bound and an upper bound [13,19].

Kumar and Gupta [10], conducted a study of several fuzzy assignment problems and fuzzy traveling salesman problems that could not be solved using the fuzzy ranking method. In the study, two new methods are proposed to solve the fuzzy assignment problem and the fuzzy traveling salesman problem, which can be used to solve problems that cannot be solved using the fuzzy ranking method. The two proposed methods show that they are easy to understand and apply to find optimal solutions to fuzzy assignment problems and fuzzy traveling salesman problems that occur in everyday life [4]. Dhanasekar *et al.,* [5] has improved the Hungarian method for solving fuzzy assignment problems. This improved Hungarian method can be used to solve fuzzy assignment problems, when the minimum number of lines crossing the fuzzy zero is not the same as the order of the fuzzy cost matrix. They say that improved methods can be used to obtain optimal solutions with less computational work. Apart from that, this method can also be used to search for Hamiltonian series with minimum fuzzy costs in the fuzzy traveling salesman problem. [18] Conducted a study on the full interval assignment problem solving procedure (FIAP), the weighted Hungarian method. In the study, the weighted method, FIAP is broken down into an Interval Assignment Problem, which is solved using existing methods and using the optimal solution, so that the optimal interval assignment solution for FIAP is obtained. In fact, the assignment problem can be solved as an ordinary transportation problem or as a linear programming problem, but the uniqueness of its structure can be exploited, resulting in a special-purpose algorithm, the so-called Hungarian method.

Based on several previous research studies, although it can be used to solve the traveling salesman problem, there are still problems that need to be developed in this research. In this research, the Hungarian method and branch and bound method for solving traveling salesman problems in interval numbers are used to solve rice distribution. Based on the study, economic activities in Pekanbaru City need to be optimized because they play a large role in the Riau Province region. For this reason, it needs to be optimized because by optimizing this part of the economy, other sectors will also have a good impact. One economic component that can be optimized is rice distribution activities. Distribution must be carried out optimally, namely when all regions receive a supply of rice according to demand with minimum operating costs. To minimize operating costs, you can optimize the transportation section because most distribution activities are related to transportation. Finding a route that has the optimal (shortest) distance in visiting each sub-district exactly once and returning to the sub-district that was the start of the journey is analogous to a problem known as the Traveling Salesman Problem (TSP). Among the approaches that can be used is the Hungarian Branch and Bound method. Using an approach with the Hungarian method and branch and bound method for solving traveling salesman problems in interval numbers, it is hoped that it can provide an optimal travel route for visiting 6 sub-districts in Pekanbaru using a carrier car. This optimal route can certainly reduce costs and also save transportation time in optimized distribution activities. Rice distribution activities are also optimized, the economic sector also gets a positive impact.

2. Fuzzification Process

Assuming two intervals $X = [\underline{X}, \overline{X}]$ and $Y = [\underline{Y}, \overline{Y}]$, the arithmetic operations on the intervals are as follow [20,24]:

i. Addition : $X + Y = [\underline{X} + \underline{Y}, \overline{X} + \overline{Y}]$,
ii. Subtraction : $X - Y = [\underline{X} - \underline{Y}, \overline{X} - \overline{Y}]$,

iii. Multiplication : $X.Y = [\min \underline{X}\underline{Y}, \underline{X}\overline{Y}, \overline{X}\underline{Y}, \overline{X}\overline{Y},\ \max \underline{X}\underline{Y}, \underline{X}\overline{Y}, \overline{X}\underline{Y}, \overline{X}\overline{Y}]$.

Then fuzzification is the process of changing a crisp set into a fuzzy set [16,27]. To fuzzify a given interval cost into a trapezoidal fuzzy number, the trisectional approach fuzzification method can be used. Assume interval cost $[L, H]$. The trisectional approach fuzzification method of interval cost is a follow [2,6,30]:

$$d = \frac{(H-L)}{3} \quad (1)$$

Assume $\tilde{A} = (a, b, c, d)$ is a trapezoidal fuzzy number. Given the following α-cut: $(\alpha_\alpha^L, \alpha_\alpha^U) = [(b-a)\alpha + a\ ,\ d-(d-c)\alpha]$, which $\alpha \in [0,1]$. In the decision making process, the ordering of fuzzy number is very important. Robus't ranking method is given to determine the rank of trapezoidal fuzzy number. Robus't ranking method is transforming trapezoidal fuzzy number into crisp number. If $\tilde{A} = (a, b, c, d)$ is a trapezoidal fuzzy number, then the Robus't ranking is as follow [1,23,28]:

$$R(\tilde{A}) = \int_0^1 (0.5)(\alpha_\alpha^L, \alpha_\alpha^U) d\alpha \quad (2)$$

where $(\alpha_\alpha^L, \alpha_\alpha^U)$ is the α-cut of the trapezoidal fuzzy number. In the case of trapezoidal fuzzy number, the Robus't ranking becomes as follow [1,26,31]:

$$R(\tilde{A}) = \frac{(a+b+c+d)}{4}. \quad (3)$$

After becoming a crisp number through the ranking method. The travelling salesman problem can be solved using the Hungarian method and the branch and bound method.

3. Research Methods

In the following subchapter discuss about Hungarian Method, Branch and bound Method and travelling salesman problem.

3.1. Hungarian Method

The Hungarian method can solve the travelling salesman problem. The Hungarian method is a method modifies the rows and columns of the cost matrix so that a single zero component appears in each row or column to be selected as an allocation in the travelling salesman problem. Assume that Z is $n \times n$ matrix. The Hungarian method algorithm for the travelling salesman problem is as following steps [3,9]:

1) Fore each row of the matrix Z, find the smallest element and substract it from every element in its row.
2) Do the same (step 1) for all columns.
3) Cover all zeros in the matrix using minimum number of horizontal and vertical lines.
4) If the minimum number of covering lines is n, an optimal assignment is possible and finished. Else if line are lesser than n, haven't found the optimal assignment, and must proceed to step 5

5) Determine the smallest element not covered by any line. Substract this element from each uncovered row, and then add it to each covered column. Return to step 3.

3.2 Branch and Bound Method

Skeleton branch and bound work is a fundamental and widely used methodology for production solutions of optimization problems [15]. The branch and bound method used a search strategy tree to enumerate all possible solutions to a given problem salesman. The branch and bound method is an algorithm that divide the problem into two smaller problem subsets by doing branch and bound to produce the optimal solution. The branching process is carried out to build a tree branch towards a solution, the limiting process is carried out by calculating the cost of the nodes taking into account the bound [12]. Branch and bound method algorithm for travelling salesman problem are as follows [21,25]:

1) Identify the problem in the form of a travelling salesman matrix.
2) Reducing row. Determine the smallest cost of each row, then measure the smallest cost sequences in each row.
3) Reducing the column. Determine the smallest cost of each column, then subtract the smallest cost in each column.
4) Add up all smallest cost of each row and column so that resulting in a root node bound cost $\tilde{C}(\gamma)$.
5) Determine root node and branch node to perform branching only. Change the root, branch, and $\tilde{C}(1,j)$ node to infinity (∞).
6) Re-do row reduction and column reduction up to each rows and columns have 0 element.
7) To calculate the limit cost, you can use the formula on the ratio of the (4)
8) Repeat step 5 for aech expanded node have a limit cost.
9) Draw a branch tree and its limit costs. Then charge the minimum found on the branch is used as the rood node $\tilde{C}(\gamma)$ to do the next branching.
10) Do it by repeating step 5 until the toot node is only has one branch.

The formula for calculating limit cost is as follow:

$$\tilde{C}(S) = \tilde{C}(\gamma) + \tilde{C}(i,j) + \sum r \tag{4}$$

$\tilde{C}(S)$ = minimum travel cost for node S in the tree.
$\tilde{C}(\gamma)$ = minimum travel cost through node γ. γ is a node root of S.
$\tilde{C}(i,j)$= side cost (i,j).
$\sum r$ = sum of all reduction in the row and column reductiom process for node S.

3.3 Travelling Salesman Problem

The travelling salesman problem was first introduced by Irish Mathematician W.R. Hamilton [7,21]. The Traveling Salesman Problem has long history since 18th century [23]. The travelling salesman is a classic optimization problem combinatorial [4,29]. The travelling salesman problem is the problem of optimizing and finding trips with the shortest route, from a number of cities that must be passes by the salesman. Each city may only be passed once in its journey, and the journey must end at the city of departure, with the distance between each city being known. To solve problem of travelling salesman must pay attention to several basic aspects, namely the number of cities to be passed, travel routes, and must pay attention to the distance between cities to be visited [8,11].

Next is the mathematical formulation for the travelling salesman problem can be written in the form sigma as follows [14,16]:

$$\min z = \sum_{i=1}^{n}\sum_{j=1}^{n} c_{ij}x_{ij}$$

subject to,

$$\sum_{j=1}^{n} x_{ij} = 1, \qquad i = 1,2,\dots,n, \sum_{i=1}^{n} x_{ij} = 1, \qquad j = 1,2,\dots,n, x_{ij} + x_{ji} \le 1,\ \ 1 \le i \ne j \le n, x_{ij} + x_{jk} + x_{ki} \le 1,\ \ 1 \le i \ne j \ne k \le n, \vdots\ x_{ip1} + x_{p1p2} + x_{p2p3} + \cdots + x_{p_{n-2}i} \le (n-2),\ \ 1 \le i \ne p1 \dots \ne p_{n-2} \ne k \le n, \boldsymbol{x_{ij}} = \mathbf{1}\text{atau}\boldsymbol{x_{ij}} \tag{5}$$

Research method contains explanations in the form of paragraphs about the research design or descriptions of the experimental settings, data sources, data collection techniques, and data analysis conducted by the researcher. This guide will explain writing headings. If your headers exceed one, use the second level of headings as below.

4. Results

In this section, we look for the optimal solution to the traveling salesman problem in the PT problem. Koto Independent. PT. Koto Mandiri is a rice warehouse located in Tampan District. The rice will be sent (distributed) to 6 sub-districts in Pekanbaru using transport vehicles. This distribution has distribution cost constraints in sending rice to 6 sub-districts in Pekanbaru. Decisions regarding rice delivery routes to 6 sub-districts in Pekanbaru have not yet optimized distribution costs. These distribution costs can be minimized by planning distribution appropriately so that the distribution costs incurred are optimal. This obstacle can be seen as a traveling salesman problem, so the Hungarian method and Branch and Bound Method can be used to find the best solution for distributing rice from warehouses to the 6 sub-districts in Pekanbaru at the lowest cost.

Therefore, the aim of the traveling salesman problem in this case is to minimize the travel time costs for the transport car starting from the warehouse in the sub-district and delivering rice to 6 sub-districts in Pekanbaru only once and the transport car returns to the location (warehouse). The estimated cost of leaving the transport vehicle is not the same as the cost of returning to the warehouse because it depends on road conditions, cargo, weather, etc. Therefore, travel time is a number that lies in an interval, so it is necessary to convert the interval number into a trapezoidal fuzzy number using the fuzzification method. There is a method for solving the traveling salesman problem based on interval data, where the traveling salesman problem is converted into a fuzzy trapezoidal transportation problem with trisectional fuzzification then by using the proposed ranking technique based on the in-center concept of the triangle, crisp values are obtained so that initial and optimal solutions can be determined. traveling salesman problem. The following is an estimate of the transport car's journey time interval in minutes.

A : Sub-district Tampan
B : Sub-district Rumbai
C : Sub-district Kulim
D : Sub-district Sukajadi
E : Sub-district Tenayan Raya
F: Sub-district Marpoyan Damai
G : Sub-district Senapelan

Table 1 shows the estimation of the travel time interval for every sub-district from A to G.

Table 1
Estimate of the Travel Time Interval

	A	B	C	D	E	F	G
A	∞	[40,43]	[48,54]	[23,26]	[36,42]	[20,23]	[27,30]
B	[42,45]	∞	[35,41]	[17,23]	[35,38]	[24,30]	[19,22]
C	[46,52]	[26,32]	∞	[35,38]	[27,33]	[28,31]	[34,37]
D	[25,31]	[25,28]	[29,32]	∞	[19,22]	[17,20]	[6,9]
E	[35,41]	[40,46]	[15,18]	[20,23]	∞	[16,19]	[21,24]
F	[26,32]	[24,27]	[30,33]	[15,21]	[15,18]	∞	[17,20]
G	[21,24]	[15,21]	[35,38]	[9,12]	[22,25]	[13,19]	∞

Then use the fuzzification method in Eq (1). The results of the fuzzification method are shown in Table 2 as follows.

Table 2
Fuzzification Method

	A	B	C	D	E	F	G
A	∞	(40,41,42,43)	(48,50,52,54)	(23,24,25,26)	(36,38,40,42)	(20,21,22,23)	(27,28,29,30)
B	(42,43,44,45)	∞	(35,37,39,41)	(17,19,21,23)	(35,36,37,38)	(24,26,28,30)	(19,20,21,22)
C	(46,48,50,52)	(26,28,30,32)	∞	(35,36,37,38)	(27,29,31,33)	(28,29,30,31)	(34,35,36,37)
D	(25,27,29,31)	(25,26,27,28)	(29,30,31,32)	∞	(19,20,21,22)	(17,18,19,20)	(6,7,8,9)
E	(35,37,39,41)	(40,42,44,46)	(15,16,17,18)	(20,21,22,23)	∞	(16,17,18,19)	(21,22,23,24)
F	(26,28,30,32)	(24,25,26,27)	(30,31,32,33)	(15,17,19,21)	(15,16,17,18)	∞	(17,18,19,20)
G	(21,22,23,24)	(15,17,19,21)	(35,36,37,38)	(9,10, 11,12)	(22,23,24,25)	(13,15,17,19)	∞

Then use Robus’t ranking to convert fuzzy number to crisp number as in Eq. (3). The cost estimation results using Robus't ranking are shown in Table 3 as follows.

Table 3
Estimate Cost

	A	B	C	D	E	F	G
A	∞	41.2	51	24.5	39	21.5	28.5
B	43.5	∞	38	20	36.5	27	20.5
C	49	29	∞	36.5	30	29.5	35.5
D	28	26.5	30.5	∞	20.5	18.5	7.5
E	38	43	16.5	21.5	∞	17.5	22.5
F	29	25.5	31.5	18	16.5	∞	18.5
G	22.5	18	36.5	10.5	23.5	16	∞

Use Table 3 to solve the travelling salesman problem using the Hungarian method. The result of the Hungarian method are obtained in Table 4 as follows.

Table 4
The Optimal Solution for the Hungarian Method

	A	B	C	D	E	F	G
A	∞	20	29.5	3	17.5	0*	7
B	11.5	∞	18	0*	16.5	7	0.5
C	8	0*	∞	7.5	1	0.5	6.5
D	8.5	19	23	∞	13	11	0*
E	9.5	26.5	0*	5	∞	1	6
F	0.5	9	15	1.5	0*	∞	2
G	0*	7.5	26	0	13	5.5	∞

The optimal route solution use the Hungarian method for carrier cars :

$$\begin{aligned} &= [27,30] + [15,18] + [15,18] + [26,32] + [26,32] + [17,23] + [6,9] + [21,24] \\ &= [27+15+15+26+26+17+6+21\ ,30+18+18+32+32+23+9+24] \\ &= [153,186]\ . \end{aligned} \tag{6}$$

Based on the optimal solution for determining the route using the Hungarian method, the minimum total travel time taken by the transport car in the case of minimizing the problem of travelling salesman delivering rice in Table 4 is [153,186] minute.

Use Table 3 to solve the travelling salesman problem using the branch and bound method. The first is to calculate the original root node, namely by reducing the rows and columns of the cost matrix so that each row or column has a zero component. Obtained for the root node :

$$\tilde{C}(A) = \sum r = 21.5 + 20 + 29 + 7.5 + 16.5 + 16.5 + 10.5 + 12 = 133.5 \tag{7}$$

i. Branch out from Sub-district A to B in Table 4. Change (A, B) and B_{11} as ∞, obtained:

$$\tilde{C}(B) = \tilde{C}(A) + \tilde{C}(A,B) + \sum r = 133.5 + 20 + 0.5 = 154\ . \tag{8}$$

ii. Branch out from Sub-district A to C in Table 4. Change (A, C) and C_{11} as ∞, obtained:

$$\tilde{C}(C) = \tilde{C}(A) + \tilde{C}(A,C) + \sum r = 133.5 + 29.5 + 1 = 164\ . \tag{9}$$

iii. Branch out from Sub-district A to D in Table 4. Change (A, D) and D_{11} as ∞, obtained:

$$\tilde{C}(D) = \tilde{C}(A) + \tilde{C}(A,D) + \sum r = 133.5 + 3 + (0.5 + 0.5) = 137.5\ . \tag{10}$$

iv. Branch out from Sub-district A to E in Table 4. Change (A, E) and E_{11} as ∞, obtained:

$$\tilde{C}(E) = \tilde{C}(A) + \tilde{C}(A,E) + \sum r = 133.5 + 17.5 + (0.5 + 0.5) = 152\ . \tag{11}$$

v. Branch out from Sub-district A to F in Table 4. Change (A, F) and F_{11} as ∞, obtained:

$$\tilde{C}(F) = \tilde{C}(A) + \tilde{C}(A,F) + \sum r = 133.5 + 0 + 0 = 133.5\ . \tag{12}$$

vi. Branch out from Sub-district A to G in Table 4. Change (A, G) and G_{11} as ∞, obtained:

$$\tilde{C}(G) = \tilde{C}(A) + \tilde{C}(A, G) + \sum r = 133.5 + 7 + (8.5 + 0.5) = 149.5 . \quad (13)$$

Can be seen that the total cost is the limit the minimum problem for travelling salesman is from Sub-district A to F, obtained Table 5.

Table 5
Branch out from Sub-district A to F

	A	B	C	D	E	F	G
A	∞	∞	∞	∞	∞	∞	∞
B	11.5	∞	18	0	16.5	∞	0.5
C	8	0	∞	7.5	1	∞	6.5
D	8.5	19	23	∞	13	∞	0
E	9.5	26.5	0	5	∞	∞	6
F	∞	9	15	1.5	0	∞	2
G	0	7.5	26	0	13	∞	∞

Sub-district F is set as the root node. Do it by repeat branch and bound.

i. Branch out from Sub-district F to B in Table 5. Change (F, B) and B_{11} as ∞, obtained:

$$\tilde{C}(B) = \tilde{C}(F) + \tilde{C}(F, B) + \sum r = 133.5 + 9 + 1 = 143,5 . \quad (14)$$

ii. Branch out from Sub-district F to C in Table 5. Change (F, C) and C_{11} as ∞, obtained:

$$\tilde{C}(C) = \tilde{C}(F) + \tilde{C}(F, C) + \sum r = 133.5 + 15 + (5 + 1) = 154.5 . \quad (15)$$

iii. Branch out from Sub-district F to D in Table 5. Change (F, D) and D_{11} as ∞, obtained:

$$\tilde{C}(D) = \tilde{C}(F) + \tilde{C}(F, D) + \sum r = 133.5 + 1.5 + (0.5 + 1) = 136.5 . \quad (16)$$

iv. Branch out from Sub-district F to E in Table 5. Change (F, E) and E_{11} as ∞, obtained:

$$\tilde{C}(E) = \tilde{C}(F) + \tilde{C}(F, E) + \sum r = 133.5 + 0 + 0 = 133.5 . \quad (17)$$

v. Branch out from Sub-district F to G in Table 5. Change (F, G) and G_{11} as ∞, obtained:

$$\tilde{C}(G) = \tilde{C}(F) + \tilde{C}(F, G) + \sum r = 133.5 + 2 + (8.5 + 1) = 145 . \quad (18)$$

Can be seen that the total cost is the limit the minimum problem for travelling salesman is from Sub-district F to E, obtained Table 6.

Table 6
Branch out from Sub-district F to E

	A	B	C	D	E	F	G
A	∞	∞	∞	∞	∞	∞	∞
B	11.5	∞	18	0	∞	∞	0.5
C	8	0	∞	7.5	∞	∞	6.5
D	8.5	19	23	∞	∞	∞	0
E	∞	26.5	0	5	∞	∞	6
F	∞	∞	∞	∞	∞	∞	∞
G	0	7.5	26	0	∞	∞	∞

Sub-district E is set as the root node. Do it by repeat branch and bound.

i. Branch out from Sub-district E to B in Table 6. Change (E,B) and B_{11} as ∞, obtained:

$$\tilde{C}(B) = \tilde{C}(E) + \tilde{C}(E,B) + \sum r = 133.5 + 26.5 + (6.5 + 18) = 184{,}5\ . \tag{19}$$

ii. Branch out from Sub-district E to C in Table 6. Change (E,C) and C_{11} as ∞, obtained:

$$\tilde{C}(C) = \tilde{C}(E) + \tilde{C}(E,C) + \sum r = 133.5 + 0 + 0 = 133.5\ . \tag{20}$$

iii. Branch out from Sub-district E to D in Table 6. Change (E,D) and D_{11} as ∞, obtained:

$$\tilde{C}(D) = \tilde{C}(E) + \tilde{C}(E,D) + \sum r = 133.5 + 5 + (0.5 + 18) = 157\ . \tag{21}$$

iv. Branch out from Sub-district E to G in Table 6. Change (E,G) and G_{11} as ∞, obtained:

$$\tilde{C}(G) = \tilde{C}(E) + \tilde{C}(E,G) + \sum r = 133.5 + 6 + (8.5 + 18) = 166\ . \tag{22}$$

Can be seen that the total cost is the limit the minimum problem for travelling salesman is from Sub-district E to C, obtained Table 7.

Table 7
Branch out from Sub-district E to C

	A	B	C	D	E	F	G
A	∞	∞	∞	∞	∞	∞	∞
B	11.5	∞	∞	0	∞	∞	0.5
C	∞	0	∞	7.5	∞	∞	6.5
D	8.5	19	∞	∞	∞	∞	0
E	∞	∞	∞	∞	∞	∞	∞
F	∞	∞	∞	∞	∞	∞	∞
G	0	7.5	∞	0	∞	∞	∞

Sub-district C is set as the root node. Do it by repeat branch and bound.

i. Branch out from Sub-district C to B in Table 7. Change (C,B) and B_{11} as ∞, obtained:

$$\tilde{C}(B) = \tilde{C}(C) + \tilde{C}(C,B) + \sum r = 133.5 + 0 + 0 = 133{,}5\ . \tag{23}$$

ii. Branch out from Sub-district C to D in Table 7. Change (C,D) and D_{11} as ∞, obtained:

$$\tilde{C}(D) = \tilde{C}(C) + \tilde{C}(C, D) + \sum r = 133.5 + 7.5 + (0.5 + 7.5) = 149 \,. \quad (24)$$

iii. Branch out from Sub-district C to G in Table 7. Change (C, G) and G_{11} as ∞, obtained:

$$\tilde{C}(G) = \tilde{C}(C) + \tilde{C}(C, G) + \sum r = 133.5 + 6.5 + (8.5 + 7.5) = 156 \,. \quad (25)$$

Can be seen that the total cost is the limit the minimum problem for travelling salesman is from Sub-district C to B, obtained Table 8.

Table 8
Branch out from Sub-district C to B

	A	B	C	D	E	F	G
A	∞	∞	∞	∞	∞	∞	∞
B	∞	∞	∞	0	∞	∞	0.5
C	∞	∞	∞	∞	∞	∞	∞
D	8.5	∞	∞	∞	∞	∞	0
E	∞	∞	∞	∞	∞	∞	∞
F	∞	∞	∞	∞	∞	∞	∞
G	0	∞	∞	0	∞	∞	∞

Sub-district B is set as the root node. Do it by repeat branch and bound.

iv. Branch out from Sub-district B to D in Table 8. Change (B, D) and D_{11} as ∞, obtained:

$$\tilde{C}(D) = \tilde{C}(B) + \tilde{C}(B, D) + \sum r = 133.5 + 0 + 0 = 133{,}5 \,. \quad (26)$$

v. Branch out from Sub-district B to G in Table 8. Change (B, G) and G_{11} as ∞, obtained:

$$\tilde{C}(G) = \tilde{C}(B) + \tilde{C}(B, G) + \sum r = 133.5 + 0.5 + 8.5 = 142.5 \,. \quad (27)$$

Can be seen that the total cost is the limit the minimum problem for travelling salesman is from Sub-district B to D, obtained Table 9.

Table 9
Branch out from Sub-district B to D

	A	B	C	D	E	F	G
A	∞	∞	∞	∞	∞	∞	∞
B	∞	∞	∞	∞	∞	∞	∞
C	∞	∞	∞	∞	∞	∞	∞
D	∞	∞	∞	∞	∞	∞	0
E	∞	∞	∞	∞	∞	∞	∞
F	∞	∞	∞	∞	∞	∞	∞
G	0	∞	∞	∞	∞	∞	∞

Sub-district D is set as the root node. Do it by repeat branch and bound.

vi. Branch out from Sub-district D to G in Table 9. Change (D, G) and G_{11} as ∞, obtained:

$$\tilde{C}(G) = \tilde{C}(D) + \tilde{C}(D, G) + \sum r = 133.5 + 0 + 0 = 133{,}5 \,. \quad (28)$$

Can be seen that the total cost is the limit the minimum problem for travelling salesman is from Sub-district D to G, obtained Table 10.

Table 10
Branch out from Sub-district D to G

	A	B	C	D	E	F	G
A	∞	∞	∞	∞	∞	∞	∞
B	∞	∞	∞	∞	∞	∞	∞
C	∞	∞	∞	∞	∞	∞	∞
D	∞	∞	∞	∞	∞	∞	∞
E	∞	∞	∞	∞	∞	∞	∞
F	∞	∞	∞	∞	∞	∞	∞
G	∞	∞	∞	∞	∞	∞	∞

Since it is no longer possible to form new nodes, then the optimal solution is obtained for the travelling salesman problem using branch and bound method is:

$$\begin{aligned} &= [27,30] + [15,18] + [15,18] + [26,32] + [26,32] + [17,23] + [6,9] + [21,24] \\ &= [27+15+15+26+26+17+6+21, 30+18+18+32+32+23+9+24] \\ &= [153,186]\,. \end{aligned} \quad (29)$$

Hungarian method and branch and bound method both produce optimal solutions. The optimal solution based on both methods is shown in Table 11 as follows.

Table 11
Optimal Solution of Method

Hungarian Method	Branch and Bound Method
$A \to F \to E \to C \to B \to D \to G \to A$	$A \to F \to E \to C \to B \to D \to G \to A$
[153,186]	[153,186]

5. Discussion

In this section, we discuss the comparison between the Hungarian method and the Branch and Bound method, especially with regard to solving the Traveling Salesman Problem in Rice Distribution in Riau Province. The Hungarian method can be used to solve the Asymmetric Traveling Salesman Problems (ATSP) problem. It is still possible to complete ATSP using the Hungarian method, if done manually with a number of cities less than 10. However, the Hungarian method has advantages and disadvantages. The Hungarian algorithm shows simpler calculations, because it uses a cost matrix, and calculation errors can be minimized.

The Branch and Bound method is quite effective in solving the Traveling Salesman Problem (TSP). Where one of the steps will not expand and will kill the node which is unlikely to lead to a solution. This method is a fairly efficient algorithm for solving TSP. However, in solving this problem, the Branch and Bound method has a weakness, namely: this method still calculates possible solutions with real number variable types even though in the end this possible solution will not be considered. But this causes the computing time to increase.

If you look at the performance comparison against TSPLIB instances, with a maximum data request of 10, the Branch and Bound method has the largest calculation time, and the Hungarian method has the smallest simulation time. However, as the data increases, the calculation time of the

Hungarian method remains stable with a small increase, and the calculation time of the Branch and Bound method experiences a considerable increase.

The traveling salesman problem in this research is minimizing the travel time costs of the rice transporting car starting from the warehouse in the sub-district, and delivering rice to 6 sub-districts in Pekanbaru only once and the car transporting it back to the location. So with only less than 10 data requests, the traveling salesman problem is more suitable to be solved using the Hungarian method.

6. Conclusions

The travelling salesman problem relates to the journey of a salesman in finding the shortest route from the city of origin around the entire city exactly once and back to the city of origin. The travelling salesman problem has an interval cost because it depends on several constraints experienced by the salesman, for example constraint on traffic travel, transportation conditions, weather, and other costs. Then convert the interval numbers into trapezium fuzzy number with the fuzzification method. Futhermore, the travelling salesman problem can be solved by the Hungarian method and branch and bound method in finding the optimal solution. The travelling salesman problem with interval numbers can be solved using the Hungarian method and branch and bound method. Both methods give the same optimal route and the optimal solution which is also the same value. The optimal route is obtained $A \to F \to E \to C \to B \to D \to G \to A$ and the optimal solution is [153,186] in the form of interval numbers.

References

[1] Ahmed, Nizam Uddin, Aminur Rahman Khan, and Md Sharif Uddin. "Solution of mixed type transportation problem: a fuzzy approach." *Buletinul Institutului Politehnic Din Iaşi* 61, no. 2 (2015): 19-31. https://doi.org/10.1016/j.jksus.2018.04.013

[2] Bisht, Dinesh CS, and Pankaj Kumar Srivastava. "Trisectional fuzzy trapezoidal approach to optimize interval data based transportation problem." *Journal of King Saud University-Science* 32, no. 1 (2020): 195-199. https://doi.org/10.4236/ajcm.2019.92005

[3] Czopik, Janusz, and Sugar Land. "An application of the Hungarian algorithm to solve traveling salesman problem." *American Journal of Computational Mathematics* 9, no. 02 (2019): 61. https://doi.org/10.5120/12942-9921

[4] Dhanasekar, S., S. Hariharan, and P. Sekar. "Classical travelling salesman problem (TSP) based approach to solve fuzzy TSP using Yager's ranking." *International journal of computer applications* 74, no. 13 (2013): 1-4. https://doi.org/10.37418/amsj.9.11.46

[5] Dhanasekar, S., V. Parthiban, and A. David Maxim Gururaj. "Improved hungarian method to solve fuzzy assignment problem and fuzzy traveling salesman problem." *Advances in Mathematics: Scientific Journal* 9, no. 11 (2020): 9417-9427. https://doi.org/10.26637/MJM0S20/0060

[6] Ganesh, A. Hari, and A. Helen Shobana. "On optimizing interval data based critical path problem for the analysis of various fuzzy quantities using fuzzification and centroid based defuzzification techniques." *Mat. S (1)* (2020): 319-330. https://doi.org/10.37418/amsj.9.8.29

[7] Kaur J., and Pal A. "An analysis of different metaheuristic approaches for Solving travelling salesman problems". *Advances in Mathematics: Scientific Journal* 9, No. 8 (2020): 5601-5610. https://doi.org/10.37418/amsj.9.8.29

[8] Kulenović, Fatka, and Azra Hošić. "Application of genetic algorithm in solving the travelling Salesman problem." In *IOP Conference Series: Materials Science and Engineering*, vol. 1208, no. 1, p. 012032. IOP Publishing, 2021. https://doi.org/10.1142/S0218488519500314

[9] Khan, Indadul, Sova Pal, and Manas Kumar Maiti. "A hybrid PSO-GA algorithm for traveling salesman problems in different environments." *International Journal of Uncertainty, Fuzziness and Knowledge-Based Systems* 27, no. 05 (2019): 693-717. https://doi.org/10.3390/app10186180

[10] Kumar, Amit, and Anila Gupta. "Assignment and travelling salesman problems with coefficients as LR fuzzy parameters." *International journal of Applied Science and engineering* 10, no. 3 (2012): 155-170.

[11] Liu, Meijiao, Yanhui Li, Qi Huo, Ang Li, Mingchao Zhu, Nan Qu, Liheng Chen, and Mingyi Xia. "A two-way parallel slime mold algorithm by flow and distance for the travelling salesman problem." *Applied Sciences* 10, no. 18 (2020): 6180. https://doi.org/10.3390/app10186180

[12] Mataija, Mirta, Mirjana Rakamarić Šegić, and Franciska Jozić. "Solving the travelling salesman problem using the Branch and bound method." *Zbornik Veleučilišta u Rijeci* 4, no. 1 (2016): 259-270.

[13] Patterson, Mike, and Daniel Friesen. "Variants of the traveling salesman problem." *Studies in Business and Economics* 14, no. 1 (2019): 208-220. https://doi.org/10.2478/sbe-2019-0016

[14] Mohammed, S. Yahya, and M. Divya. "Solving fuzzy travelling salesman problem using octagon fuzzy numbers with α-cut and ranking technique." *IOSR Journal of Mathematics (IOSR-JM)* 12, no. 6 (2016): 52-56.

[15] Morrison, David R., Sheldon H. Jacobson, Jason J. Sauppe, and Edward C. Sewell. "Branch-and-bound algorithms: A survey of recent advances in searching, branching, and pruning." *Discrete Optimization* 19 (2016): 79-102. https://doi.org/10.1016/j.disopt.2016.01.005

[16] Azlin Mohd Nordin, Noraimi, S. Sarifah Radiah Shariff, Mohd Omar, and Siti Suzlin Supadi. "Optimizing order picker problem using dynamic programming method." *International Journal of Nonlinear Analysis and Applications* 13, no. 1 (2022): 1985-1998.

[17] Hernández-Pérez, Hipólito, Juan José Salazar-González, and Beatriz Santos-Hernández. "Heuristic algorithm for the split-demand one-commodity pickup-and-delivery travelling salesman problem." *Computers & Operations Research* 97 (2018): 1-17. https://doi.org/10.1016/j.cor.2018.04.016

[18] Priya, D., and G. Ramesh. "The Hungarian Method for the Assignment Problem, With Generalized Interval Arithmetic and Its Applications." In *Journal of Physics: Conference Series*, vol. 1377, no. 1, p. 012046. IOP Publishing, 2019. https://doi.org/10.1088/1742-6596/1377/1/012046

[19] Rajarajeswari, P., and D. Maheswari. "Solving Travelling Salesman Problem with Interval Cost Constraints." *Journal of Information and Computational Science* 10, no. 4-2020 (2020).

[20] Rajarajeswari, P., and D. Maheswari. "A Comparative Study for Solving Interval Linear Assignment Problem." *IJMTT* 66, no. 4 (2020). https://doi.org/10.14445/22315373/IJMTT-V66I4P518

[21] Rastogi, Archit, Ankur Kumar Shrivastava, Nitisha Payal, Ramander Singh, and Aditi Sharma. "A proposed solution to Travelling Salesman Problem using Branch and Bound." *International Journal of Computer Applications* 65, no. 5 (2013).

[22] Selvamalar, N., and V. Vinoba. "Fuzzy Branch and Bound Solution for a Flow Shop Scheduling Problem." *Journal of Computer and Mathematical Sciences* 8, no. 3 (2017): 85-95.

[23] Stoilova, K., and T. Stoilov. "Transportation modelling and solving Travelling Salesman problem." In *IOP Conference Series: Materials Science and Engineering*, vol. 878, no. 1, p. 012026. IOP Publishing, 2020. https://doi.org/10.1088/1757-899X/878/1/012026

[24] Suneela, S., and S. Chakraverty. "New ranking function for fuzzy linear programming problem and system of linear equations." *Journal of information and optimization sciences* 40, no. 1 (2019): 141-156. https://doi.org/10.1080/02522667.2018.1450196

[25] Subasri, S., and K. Selvakumari. "Neutrosophic travelling salesman problem in trapezoidal fuzzy number using branch and bound technique." In *Journal of Physics: Conference Series*, vol. 1362, no. 1, p. 012098. IOP Publishing, 2019. https://doi.org/10.1088/1742-6596/1362/1/012098

[26] Sudha, A., G. Angel, M. Priyanka, and S. Jennifer. "An Intuitionistic Fuzzy Approach for Solving Generalized Trapezoidal Travelling Salesman Problem." *International Journal of Mathematics Trends and Technology* 29, no. 1 (2016): 9-12. https://doi.org/10.14445/22315373/IJMTT-V29P502

[27] Mariappan, Punithavili, Mohd Zahuri Khairani, Muhammad Nidzam Yaakob, Maran Chanthiran, and Andy Noces Cubalit. "Technology Applications through Cooperative Learning in Visual Arts Education among Students in Inclusive Education Programs: A Fuzzy Delphi Approach." *Journal of Advanced Research in Applied Sciences and Engineering Technology* 33, no. 3 (2023): 407-419.

[28] Xiao, Guixia, Normalia Samian, Muhammad Faiz Mohd Faizal, Muhammad Alif Zakwan Mohd As'ad, Muhammad Firdaus Mohamad Fadzil, Azizol Abdullah, Winston Khoon Guah Seah, Mastura Ishak, and Irman Hermadi. "A Framework for Blockchain and Internet of Things Integration in Improving Food Security in the Food Supply Chain." *Journal of Advanced Research in Applied Sciences and Engineering Technology* 34, no. 1 (2024): 24-37.

[29] Noor, Norlenda Mohd, Zuraida Alwadood, Nuramelissa Ahmad Murad, N. Maisarah, and M. Termizi. "Optimization of Private Bus Scheduling in UiTM Shah Alam Using Integer Linear Programming." *Journal of Advanced Research in Computing and Applications* 15 (2019): 26-34.

[30] Gundurouthula, Srikanth, M. Gopal Naik, and Togiti Dhilip. "Construction Delay Assessment using Compound Index and Fuzzy Optimization Technique: A Case Study of Hyderabad Construction Industry." *Journal of Advanced Research in Applied Sciences and Engineering Technology* 34, no. 2 (2023): 30-49.

[31] Ramli, Ahmad Kamal. "An Active Cyber Insurance Policy Against Cybersecurity Risks Using Fuzzy Q-Learning." *Journal of Advanced Research in Applied Sciences and Engineering Technology* 30, no. 3 (2023): 212-221.