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Multiple Comparisons of the Dispersion of PM2.5 in Several Locations Utilizing Simultaneous Confidence Intervals for All Pairwise Differences Between the Coefficients of Variation of Inverse Gaussian Distributions

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ABSTRACT

The presence of PM2.5 (particulate matter 2.5) poses a significant threat to human health. Urban activities are where PM2.5 is predominantly produced, and the built environment may have an impact on how it forms and spreads. To analyse PM2.5 data, one can use the coefficient of variation of inverse Gaussian distribution. This study aims to generate simultaneous confidence intervals using the generalized confidence interval, the adjusted generalized confidence interval, the fiducial confidence interval, and the highest posterior density confidence interval methods for all pairs between the coefficients of variation of inverse Gaussian distributions. By using a simulation of Monte Carlo, the effectiveness of the simultaneous confidence interval approaches was evaluated with a focus on two crucial metrics: coverage probabilities and average lengths. The findings demonstrated that the coverage probability conditions were satisfied by the confidence interval obtained by the adjusted generalized confidence interval and the highest posterior density confidence interval methods. The effectiveness of the suggested strategies was shown using PM2.5 data from five Bangkok, Thailand areas.

Keywords:
AGCI; CV; HPD; PM2.5

1. Introduction

Numerous urban and rural locations around the world are experiencing a decline in air quality. According to the World Bank report, the global economy lost nearly $5 trillion in 2013 due to the consequences of air pollution on health [1]. Air pollution being the most significant factor for fatalities worldwide and is currently seen as a serious problem in Thailand [2]. The government places great importance on addressing this issue and has taken proactive measures by implementing several solutions. Dust particles are produced for various reasons, including waste burning, tree burning, and engine exhaust emissions from multiple cars. These particles are responsible for creating hazy and obscured skylines in cities, including Bangkok. It is widely acknowledged that poor air quality and exposure to air pollution have negative impacts on human health, including respiratory and

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cardiovascular disorders [3]. These impacts have garnered significant attention, especially in metropolitan regions with high human activity concentrations and population densities. Accurate forecasting of concentrations of fine PM2.5 with a diameter of less than 2.5 micrometers is crucial for more efficient air pollution control. In order to gather the required information for developing policies aimed at reducing such occurrences, it is imperative to conduct research on the distribution of PM2.5 in each location.

The inverse Gaussian distribution (IG) is a continuous probability distribution that is used in statistics to model data that is continuous and positive-valued. It is particularly useful in modeling waiting times, failure times, and reaction times [4-9]. Furthermore, there have been numerous research studies involving PM2.5 and its association with the IG. For instance, the IG was shown to be appropriate for fitting PM fixed-site data probability density curves by [10]. Gavril *et al.,* [11] discovered that the IG generated better outcomes than the beta, gamma, and Weibull distributions. Authors [12] investigated the PM2.5 concentration data in Chaing Mai at Yupparaj Wittayalai School and city hall, it found that the IG is the most suitable probability density function of the daily average PM2.5 concentration for two stations.

In addition, many scholars are interested in the studies on constructing confidence intervals for a number of inverse Gaussian distribution parameters that are employed in statistical inference. As an illustration, Chhikara and Folks [4] presented a precise method for determining confidence intervals (CIs) for the ratio means when the shape parameters of the two populations are identical. Ye *et al.*, [13] utilized the generalized confidence interval (GCI) approach to calculate intervals and test their predictions concerning the mean of IG populations. Ismail and Auda [14] provided estimates for the scale parameter and posterior density using Gibbs sampling and Jeffreys' prior for the scale parameter when the mean is known. In their effort to conduct a comprehensive survival analysis for IG using the Gibbs sampling methodology, Jayalath and Chhikara [15] employed Bayesian and fiducial methodologies, requiring the utilization of a Monte Carlo Markov Chain (MCMC) method.

The coefficient of variation (CV) is a statistical measure utilized to compare the variability of two or more data sets relative to their mean. The PM2.5 concentration dispersion in different areas can be compared using the CV. Wong and Wu [16] reported that a small sample asymptotic method provided better CIs for the CV. Using the modified method of variance estimates recovery (MOVER), GCI, and large sample approaches, Thangjai and Niwitpong [17] produced CIs for the weighted CVs of two-parameter exponential distributions. To produce a CI for the difference between two CVs for normal distributions, Donner and Zou [18] proposed MOVER. Yosboonruang *et al.,* [19] produced the CIs for the CV of a lognormal distribution using Bayesian and fiducial GCI methods. By generating CIs for the ratio of the CVs of delta-lognormal distributions using both the GCI approach and the MOVER-based Wald interval, Buntao and Niwitpong [20] concluded that the GCI method was the most appropriate. Keawprasert *et al.,* [21] used the scoring technique, the Wald method, and the percentile bootstrap confidence interval to give the CIs for CV in the inverse gamma distribution. Additionally, Chankham *et al.,* [22] presented CIs for the CV of an IG distribution.

Simultaneous estimation of dispersion for numerous locations has been studied using a variety of distributions and parameters because it can either be the same or different for distinct data series for different areas. Donner and Zou [23] constructed simultaneous confidence intervals (SCIs) for multiple contrasts of binomial proportions using a two-step MOVER technique; their proposed method was successful for samples of small to medium size. Zhang [24] offered SCIs for all pairwise IG mean comparisons. According to Li *et al.,* [25], parametric bootstrapping should be used to construct SCIs for all pairwise discrepancies between the means of two-parameter exponential distributions. Using the percentile bootstrap, GCI, and MOVER, Puggard *et al.,* [26] constructed estimation methods for SCIs for all pairwise differences between the CVs and various Birnbaum-

Saunders distributions. These approaches performed well in nearly all of the examples studied. No previous studies developed SCIs for all differences between CVs of the inverse Gaussian distributions. This research tackles a critical deficiency in current literature concerning IG distributions, particularly focusing on CVs. It underscores the lack of SCIs for differences in CVs across IG distributions. To address this gap, the study introduces innovative methods for calculating these SCIs. By advancing statistical techniques in this area, the research aims to enhance the analysis of air pollution data. Ultimately, these developments hold promise for bolstering strategies in air quality management, offering valuable insights for environmental and public health practitioners alike.

The paper is structured as follows. The procedures to estimate SCIs are presented in Section 2. The results are reported in Section 3. Section 4 exhibits the validation of the proposed SCIs. Finally, the conclusion is provided in Section 5.

2. Methodology

Let $W_i = (W_{i1}, W_{i2}, \ldots, W_{in_i})$, $i = 1,2,\ldots,r$ be a random sample from r independent inverse Gaussian distributions, represented by $X_{ij} \sim IG(\mu_i, \lambda_i)$. The distribution function of an inverse Gaussian distribution presented by:

$$f(w_{ij}, \mu_i, \lambda_i) = \left(\frac{\lambda_i}{2\pi w_{ij}^2}\right)^{\frac{1}{2}} exp\left\{-\frac{\lambda_i(w_{ij}-\mu_i)^2}{2{\mu_i}^2 w_{ij}}\right\}, w_{ij} > 0, \mu_i > 0, \lambda_i > 0, \tag{1}$$

where μ_i and λ_i are the mean and scale parameters. Following Ye *et al.,* [13], the corresponding mean and variance of W_i are

$$E(W_i) = \mu_i, \text{ and} \tag{2}$$

$$Var(W_i) = \frac{\mu_i^3}{\lambda_i} \tag{3}$$

In the aftermath, the CV of W_i can be represented as:

$$CV(W_i) = \theta_i = \sqrt{\frac{Var(W_i)}{E(W_i)}} = \sqrt{\frac{\mu_i}{\lambda_i}} \tag{4}$$

Given that we aimed to establish SCIs of the CVs of IGs.

$$\theta_{il} = \theta_i - \theta_l = \sqrt{\frac{\mu_i}{\lambda_i}} - \sqrt{\frac{\mu_l}{\lambda_l}} \tag{5}$$

where$i, l = 1,2,..,r$and$i \neq l$. The maximum likelihood estimators of μ_i and λ_i are shown in Eqs. (6) and (7):

$$\hat{\mu}_i = \bar{W}_i \tag{6}$$

$$\hat{\lambda}_i^{-1} = \frac{1}{n_i}\sum_{j=1}^{n_i}(W_{ij}^{-1} - \bar{W}_i^{-1}) \tag{7}$$

Considering that $\hat{\mu}_i$ and $\hat{\lambda}_i$ are hypothetically independent, the maximum likelihood estimator ofθ_i can be described as:

$$\hat{\theta}_i = \sqrt{\frac{\hat{\mu}_i}{\hat{\lambda}_i}} \tag{8}$$

Similarly,

$$\hat{\theta}_{il} = \hat{\theta}_i - \hat{\theta}_l = \sqrt{\frac{\hat{\mu}_i}{\hat{\lambda}_i}} - \sqrt{\frac{\hat{\mu}_l}{\hat{\lambda}_l}} \tag{9}$$

where $i, l = 1,2,\ldots,r$ and $i \neq l$. The next subsection provides a more thorough explanation of the steps involved in creating the SCIs for θ_{il}.

2.1 GCI

Weerahandi [27] was the first to introduce the GCI technique concept. GCI is a development of the conventional idea of a confidence interval. When a crucial metric has a known distribution and is free of nuisance characteristics, conventional confidence intervals can be created. Suppose that $W_i = (W_{i1}, W_{i2}, \ldots, W_{in}), i = 1,2,\ldots,r$ be a random sample drawn from r independent inverse Gaussian distribution with relevant parameters. $\theta_i = (\mu_i, \lambda_i)$ and nuisance parameterυ_i. Let $w_i = (w_{i1}, w_{i2}, \ldots, w_{in}), i = 1,2,\ldots,r$be an observed value of W_i. The generalized pivotal quantity (GPQ), $R(W_i; w_i, \theta_i, \upsilon_i)$ has the following properties:

i. The distribution of $R(W_i; w_i, \theta_i, \upsilon_i)$ is free of unidentified parameters.
ii. The observed value of $R(W_i; w_i, \theta_i, \upsilon_i)$ is free of nuisance parameter of υ_i.

Subsequently, the $100(1-\alpha)\%$ generalized confidence interval for θ_i is given by$\left[R_{\theta_i}\left(\frac{\alpha}{2}\right), R_{\theta_i}\left(1-\frac{\alpha}{2}\right)\right]$, where $R_{\theta_i}\left(\frac{\alpha}{2}\right)$ is obtained by using $100(\frac{\alpha}{2}) - th$ percentiles of R_{θ_i}. Let $W_{ij} \sim IG(\mu_i, \lambda_i), i = 1,\ldots,r, j = 1,\ldots,n_i$ be an independent random sample from r IG populations. Let $n = \sum_{i=1}^{r} n_i$ be the total sample size,

$$\bar{W}_i = \sum_{j=1}^{n_i} \frac{W_{ij}}{n_i} \text{ and,} \tag{9}$$

$$S_i = \sum_{i=1}^{n_i} \left(\frac{1}{W_{ij}} - \frac{1}{\bar{W}_i}\right) \tag{10}$$

The independent maximum likelihood estimators for $\mu_i$and λ_i are $\hat{\mu}_i = \bar{W}_i$, and $\hat{\lambda}_i = \frac{n_i}{S_i}$. $\bar{W}_i$ and S_i are independent. The GPQs for parameters of an inverse Gaussian distribution are proposed by Ye *et al.,* [13]. The corresponding GPQs for μ_i and λ_i are as follows:

$$R_{\lambda_i} = \frac{n_i \lambda_i V_i}{n_i \upsilon_i} \sim \frac{\chi^2_{n_i-1}}{n_i \upsilon_i}, i = 1,2,\ldots,r \text{ and,} \tag{11}$$

$$R_{\mu_i} = \frac{\bar{w}_i}{\left|1 + \frac{\sqrt{n_i \lambda_i}(\bar{w}_i - \mu_i)}{\mu_i \sqrt{\bar{w}_i}} \sqrt{\frac{\bar{w}_i}{n_i R_{\lambda_i}}}\right|} \underset{\sim}{d} \frac{\bar{w}_i}{\left|1 + Z_i \sqrt{\frac{\bar{w}_i}{n_i R_{\lambda_i}}}\right|} \tag{12}$$

where $\underset{\sim}{d}$ is approximately distributed and $Z_i \sim N(0,1)$. The approximate distribution is generated from the moment matching method of Chhikara and Folk [4]. Note that a limiting distribution of $N(0,1)$ as follow:

$$\frac{\sqrt{n_i \lambda_i}(\bar{w}_{i_i} - \mu_i)}{\mu_i \sqrt{\bar{w}_i}} \tag{13}$$

As a result, μ_i is the observed value of R_{μ_i}. Consequently, the GCI for the CV of an IG distribution is provided by:

$$R_{\theta_i} = \sqrt{\frac{R_{\mu_i}}{R_{\lambda_i}}} \tag{14}$$

In light of this, the GCI for the variations between two independent CVs can be written as:

$$R_{\theta_{il}} = R_{\theta_i} - R_{\theta_l} = \sqrt{\frac{R_{\mu_i}}{R_{\lambda_i}}} - \sqrt{\frac{R_{\mu_l}}{R_{\lambda_l}}} \tag{15}$$

where $i, l = 1,2,\ldots,r$ and $i \neq l$. Therefore, the $100(1-\alpha)\%$ two-sided SCI for θ_{il} based on GCI method can be written as $L_{il} \leq \theta_{il} \leq U_{il}$, where L_{il} and U_{il} are the $\alpha/2$-*th* and $(1-\alpha/2)$-*th* quantiles of $R_{\theta_{il}}$, respectively.

$$SCI_{GCI} = \left[R_{\theta_{il}}\left(\frac{\alpha}{2}\right), R_{\theta_{il}}\left(1 - \left(\frac{\alpha}{2}\right)\right)\right] \tag{16}$$

Algorithm 1 describes the procedures for constructing SCIs using GCI method.

i. Generate datasets W_{ij} based on IG.
ii. Calculate $\hat{\mu}_i$ and $\hat{\lambda}_i$.
iii. Estimate $\chi^2_{n_i-1}$ and Z_i from a Chi-square distribution and a standard normal distribution, respectively.
iv. Compute R_{λ_i}, R_{μ_i}, R_{θ_i}, and $R_{\theta_{il}}$ from Eqs. (11), (12), (14), and (15), respectively.
v. Complete m (m = 2,500) iterations of steps (3) and (4).
vi. Compute $R_{\theta_{il}}\left(\frac{\alpha}{2}\right)$ and $R_{\theta_{il}}\left(1 - \frac{\alpha}{2}\right)$.

2.2 AGCI

Ye *et al.,* [13] claimed that a strategy resembling the GCI method can be applied to the SCIs for all pairwise comparisons of CV from the r IG population. Krishnamoorthy and Tian [28] developed the GPQ as follows:

$$Q_{\mu_i} = \frac{\bar{w}_i}{max\left\{0, 1 + t_{n_i-1}\sqrt{\frac{\bar{w}_i v_i}{n_i - 1}}\right\}} \tag{17}$$

where t_{n_i-1} denotes a t-distribution with $n_i - 1$ degrees of freedom. However, if t_{n_i-1} takes a negative value, the denominator could become zero. Thus Q_{μ_i} can be determined as follows:

$$Q_{\theta_i} = \sqrt{\frac{Q_{\mu_i}}{R_{\lambda_i}}} \quad (18)$$

The GPQs for θ_{il} can then be written as:

$$Q_{\theta_{il}} = Q_{\theta_i} - Q_{\theta_l} = \sqrt{\frac{Q_{\mu_i}}{R_{\lambda_i}}} - \sqrt{\frac{Q_{\mu_l}}{R_{\lambda_l}}} \quad (19)$$

for$i$and $l = 1,2,\dots,r$ $i \neq l$. Hence, SCIs for AGCI is formulated as follows:

$$SCI_{AGCI} = \left[Q_{\theta_{il}}\left(\frac{\alpha}{2}\right), Q_{\theta_{il}}\left(1-\left(\frac{\alpha}{2}\right)\right)\right] \quad (20)$$

where $Q_{\theta_{il}}\left(\frac{\alpha}{2}\right)$ and $Q_{\theta_{il}}\left(1-\left(\frac{\alpha}{2}\right)\right)$ stands for the $100\left(\frac{\alpha}{2}\right) - th$and $100\left(1-\frac{\alpha}{2}\right) - th$ percentiles of the distribution of $Q_{\theta_{il}}$. The notion of Algorithm 1 illustrates the processes to compute SCIs based on the AGCI technique.

2.3 FCI

The fiducial inference was first proposed by Fisher [29]. It serves as an alternative to frequentist and Bayesian inference techniques. Fiducial inference seeks to produce, using observable data, a probability distribution for an unknown parameter. This fiducial distribution depicts the parameter's uncertainty prior to data observation. The observed data are used to build the fiducial distribution, along with a few models or assumptions. Gibbs sampling is a general-purpose MCMC algorithm that can be used to sample from various probability distributions, including those representing fiducial confidence intervals. However, it's important to note that Gibbs sampling itself is not specific to fiducial intervals; rather, it's a method for sampling from complex multivariate distributions [30]. In the case of IG, the sampling distributions of the Maximum Likelihood Estimators (MLEs) for both and are employed. When these MLEs are represented in their respective sample distributions, they can be readily substituted to generate their fiducial distributions.

$\mu_i(f) \sim IG(\hat{\mu}_i, n_i\hat{\lambda}_i)$, and (21)

$$\lambda_i(f) \sim \left(\frac{\hat{\lambda}_i}{n_i}\right)\chi^2_{n_i-1} \quad (22)$$

where $\hat{\mu}_i$and $\hat{\lambda}_i$are MLEs of the $\mu_i$and λ_i. The result is that $\mu_i(f)$ and $\lambda_i(f)$ can be replaced, resulting in:

$$\theta_i(f) = \sqrt{\frac{\mu_i(f)}{\lambda_i(f)}} \quad (22)$$

thus, the $\theta_{il}(f)$ is given:

$$\theta_{il}(f) = \theta_i(f) - \theta_l(f) = \sqrt{\frac{\mu_i(f)}{\lambda_i(f)}} - \sqrt{\frac{\mu_l(f)}{\lambda_l(f)}} \quad (23)$$

Hence, the $100(1-\alpha)\%$ two-sided SCIs for θ_{ij} using FCI method can be defined by:

$$SCI_{FCI} = \left[\theta_{ij}(f)\left(\frac{\alpha}{2}\right), \theta_{ij}(f)\left(1-\left(\frac{\alpha}{2}\right)\right)\right] \quad (24)$$

where $\theta_{ij}(f)\left(\frac{\alpha}{2}\right)$ and $\theta_{ij}(f)\left(1-\left(\frac{\alpha}{2}\right)\right)$ denoted the $100(\frac{\alpha}{2})-th$ and $100(1-\frac{\alpha}{2})-th$ percentiles of $\theta_{ij}(f)$, respectively. The value of $\theta_{ij}(f)$ for two-sided SCI_{FCI} can be estimated using the Gibbs sampling procedure as follows (algorithm 2):

i. Consider the parameter $\left(\mu_i^{(0)}, \lambda_i^{(0)}\right)$ initial values (MLEs).
ii. Generate $\mu_i^{(t)}(f) \sim IG\left(\hat{\mu}_i^{(t-1)}, n_i\hat{\lambda}_i^{(t-1)}\right)$.
iii. Generate $\lambda_i^{(t)}(f) \sim \left(\frac{\hat{\lambda}_i^{(t-1)}}{n_i}\right)\chi^2_{n_i-1}$.
iv. Repeat Steps 2-3, T(T=20,000) Times, where T is the quantity of MCMC replications.
v. Calculate the desired parameter after burning in 1,000 samples.
vi. Using the fiducial inference Eq. (24), calculate the 95% SCIs.

2.4 HPD

The HPD method can be used to estimate credible intervals after an MCMC algorithm has generated a batch of samples from the posterior distribution [31]. The narrowest interval that encompasses a specific proportion of the posterior distribution is known as the HPD interval. It offers a method to encapsulate the parameter estimate uncertainty, indicating the range of tenable values [32]. Therefore, $100(1-\alpha)\%$ the two-sided SCIs for all pairwise differences between CVs of an IG distribution based on the HPD can be obtained using the R statistical program with the HDInterval package as follows:

$$SCI_{HPD} = \left[\theta_{il}(h)\left(\frac{\alpha}{2}\right), \theta_{il}(h)\left(1-\frac{\alpha}{2}\right)\right] \quad (25)$$

where $\theta_{il}(h)\left(\frac{\alpha}{2}\right)$ and $\theta_{il}(h)\left(1-\frac{\alpha}{2}\right)$ are the lower and upper bound of HPD interval for θ_{il}. The HPD technique employs the following algorithm to build a confidence interval for the SCIs of an IG distribution (algorithm 3):

i. Consider using the initial values, also known as maximum likelihood estimates (MLEs), for the parameters $\left(\mu_i^{(0)}, \lambda_i^{(0)}\right)$.
ii. Calculate the parameter of interest using algorithm 2.
iii. Compute the 95% SCIs based on HPD according to Eq. (25).

3. Results

To assess the finite sample qualities of the proposed GCI, AGCI, FCI, and HPD methods, it is acceptable to use the coverage probability (CP) values that are greater than or comparable to the nominal confidence level at 0.95. Additionally, the SCI should have a small average length (AL), as demonstrated below.

$$CP = \frac{\sum_{M=1}^{5,000} c^M(L_{il}^M \leq \theta_{il} \leq U_{il}^M)}{5,000}, \text{ and} \quad (26)$$

$$AL = \frac{\sum_{M=1}^{5,000}(U_{il}^M - L_{il}^M)}{5,000} \quad (27)$$

where $c^M(L_{il}^M \leq \theta_{il} \leq U_{il}^M)$ is the number of θ_{il} contained in the interval, the lower and upper boundaries of the interval are L_{il}^M and U_{il}^M, respectively, and M is the number of simulations. The R statistical software package was used to conduct the Monte Carlo simulation investigation. The factors used in the simulation study were as follows: (1) cases: r = 3 and 5; (2) sample sizes are 10,50, and 100; (3) population means: $\mu_i = 0.5,1$; and (4) population scales: $\lambda_i = 1,5,10.$ The simulations were conducted 5,000 times for each set of parameters. Additionally, 2,500 replications of the GCI, AGCI, and FCI techniques were carried out for each parameter combination.

Tables 1 and 2, correspondingly, show the outcomes for $r = 3$ and 5. The ALs of the approaches with different sample sizes and the CP are summarized in Figures 1 and 2. In every situation, the ALs of all the techniques tended to get shorter as sample numbers were raised. In all cases, the GCI, AGCI, FCI, and HPD CPs were above or very close to 0.95. The shortest length is therefore thought to be the optimal CI based on the ALs of these CIs. We found that the ALs of FCI-HPD were the shortest lengths CP > 0.95 in practically all cases.

Table 1
CPs and ALs for the 95% SCIs for θ_{il} of inverse Gaussian distributions (r = 3)

n	μ	λ	CPs (ALs)			
			GCI	AGCI	FCI	HPD
10^3	0.5^3	1^3	0.9770	**0.9540**	0.9570	0.9750
			(1.7627)	**(1.1817)**	(1.2660)	(1.2580)
		5^3	0.9530	0.9430	0.9410	**0.9613**
			(0.5712)	(0.5299)	(0.5376)	**(0.5332)**
		10^3	0.9600	**0.9550**	0.9533	0.9690
			(0.3894)	**(0.3752)**	(0.3760)	(0.3740)
		1:5:10	0.9670	0.9477	0.9563	**0.9620**
			(0.9534)	(0.7228)	(0.7618)	**(0.7386)**
	1^3	1^3	0.9890	0.9480	0.9593	**0.9817**
			(3.6509)	(1.6757)	(1.8984)	**(1.8816)**
		5^3	0.9553	0.9430	0.9447	**0.9627**
			(0.8663)	(0.7449)	(0.7699)	**(0.7625)**
		10^3	0.9533	0.9463	0.9457	**0.9663**
			(0.5632)	(0.5234)	(0.5325)	**(0.5263)**
		1:5:10	0.9753	0.9490	0.9517	**0.9523**
			(1.8087)	(1.0296)	(1.1338)	**(1.0963)**
50^3	0.5^3	1^3	0.9857	**0.9523**	0.9540	0.9610
			(0.5182)	**(0.4157)**	(0.4587)	(0.4574)
		5^3	0.9630	**0.9540**	0.9543	0.9573
			(0.1950)	**(0.1852)**	(0.1894)	(0.1889)
		10^3	0.9473	0.9417	0.9450	**0.9503**
			(0.1345)	(0.1310)	(0.1325)	**(0.1321)**
		1:5:10	0.9723	0.9487	0.9490	**0.9500**
			(0.3082)	(0.2607)	(0.2819)	**(0.2794)**
	1^3	1^3	0.9923	0.9487	0.9500	**0.9560**
			(0.8707)	(0.5872)	(0.7111)	**(0.7087)**
		5^3	0.9630	0.9470	0.9450	**0.9513**
			(0.2884)	(0.2615)	(0.2731)	**(0.2723)**
		10^3	0.9593	0.9473	0.9443	**0.9510**

Table 1. Continued
CPs and ALs for the 95% SCIs for $\theta_{il}$of inverse Gaussian distributions (r = 3)

n	μ	λ	CPs (ALs)			
			GCI	AGCI	FCI	HPD
			(0.1956)	(0.1858)	(0.1897)	**(0.1895)**
		1:5:10	0.9800	**0.9533**	0.9550	0.9507
			(0.4996)	**(0.3698)**	(0.4277)	(0.4237)
100^3	0.5^3	1^3	0.9853	**0.9520**	0.9497	0.9523
			(0.3522)	**(0.2850)**	(0.3170)	(0.3163)
		5^3	**0.9567**	0.9437	0.9460	0.9480
			(0.1341)	(0.1276)	(0.1303)	(0.1301)
		10^3	0.9527	0.9493	0.9507	**0.9503**
			(0.0924)	(0.0901)	(0.0912)	**(0.0909)**
		1:5:10	0.9750	**0.9540**	0.9460	0.9473
			(0.2113)	**(0.1798)**	(0.1942)	(0.1932)
	1^3	1^3	0.9913	**0.9510**	0.9510	0.9540
			(0.5830)	**(0.4038)**	(0.4913)	(0.4901)
		5^3	0.9650	0.9457	0.9487	**0.9513**
			(0.1982)	(0.1802)	(0.1888)	**(0.1884)**
		10^3	0.9627	0.9497	0.9507	**0.9537**
			(0.1342)	(0.1277)	(0.1306)	**(0.1303)**
		1:5:10	0.9820	**0.9523**	0.9497	0.9500
			(0.3396)	**(0.2550)**	(0.2964)	(0.2947)
10:50:100	0.5^3	1^3	0.9807	**0.9500**	0.9580	0.9577
			(0.9359)	**(0.6744)**	(0.7299)	(0.7008)
		5^3	0.9620	**0.9537**	0.9563	0.9567
			(0.3234)	**(0.3032)**	(0.3074)	(0.2943)
		10^3	0.9550	0.9467	0.9483	**0.9547**
			(0.2201)	(0.2129)	(0.2146)	**(0.2055)**
		1:5:10	0.9567	**0.9667**	0.9733	0.9600
			(0.8075)	**(0.5890)**	(0.5989)	(0.5620)
	1^3	1^3	0.9897	0.9373	0.9523	**0.9533**
			(1.8282)	(0.9590)	(1.1117)	**(1.0682)**
		5^3	0.9697	**0.9510**	0.9620	0.9610
			(0.4910)	**(0.4301)**	(0.4451)	(0.4264)
		10^3	0.9597	0.9497	0.9487	**0.9517**
			(0.3216)	(0.3015)	(0.3065)	**(0.2935)**
		1:5:10	0.9867	**0.9507**	0.9580	0.9440
			(1.5830)	**(0.8163)**	(0.9196)	(0.8617)

Note: Bold denotes the best performing method

Table 2
CPs and ALs for the 95% SCIs for $\theta_{il}$of inverse Gaussian distributions (r = 5)

n	μ	λ	CPs (ALs)			
			GCI	AGCI	FCI	HPD
10^5	0.5^5	1^5	0.9786	**0.9561**	0.9556	0.9754
			(1.7438)	**(1.1727)**	(1.2605)	(1.2493)
		5^5	0.9643	**0.9552**	0.9577	0.9719
			(0.5658)	**(0.5251)**	(0.5323)	(0.5284)
		10^5	0.9503	0.9458	0.9478	**0.9659**
			(0.3863)	(0.3721)	(0.3747)	**(0.3719)**
		1^2:5:10^2	0.9729	**0.9578**	0.9604	0.9650
			(1.0112)	**(0.7529)**	(0.7958)	(0.7731)
	1^5	1^5	0.9876	0.9491	0.9547	**0.9758**
			(3.6160)	(1.6643)	(1.9180)	**(1.8968)**
		5^5	0.9668	**0.9518**	0.9560	0.9753

Table 2. Continued
CPs and ALs for the 95% SCIs for θ_{il} of inverse Gaussian distributions (r = 5)

n	μ	λ	CPs (ALs)			
			GCI	AGCI	FCI	HPD
			(0.8626)	**(0.7433)**	(0.7659)	(0.7600)
		10^5	0.9601	**0.9542**	0.9534	0.9705
			(0.5688)	**(0.5278)**	(0.5348)	(0.5307)
		1^2:5:10^2	0.9751	0.9498	0.9496	**0.9528**
			(1.9607)	(1.0761)	(1.1837)	**(1.1479)**
50^5	0.5^5	1^5	0.9793	0.9479	0.9519	**0.9581**
			(0.5171)	(0.4140)	(0.4584)	**(0.4572)**
		5^5	0.9609	**0.9505**	0.9533	0.9570
			(0.1947)	**(0.1851)**	(0.1891)	(0.1886)
		10^5	0.9562	**0.9520**	0.9508	0.9550
			(0.1345)	**(0.1310)**	(0.1322)	(0.1318)
		1^2:5:10^2	0.9709	0.9455	0.9481	**0.9509**
			(0.3258)	(0.2725)	(0.2961)	**(0.2937)**
	1^5	1^5	0.9925	0.9493	0.9540	**0.9605**
			(0.8673)	(0.5863)	(0.7077)	**(0.7056)**
		5^5	0.9680	0.9499	0.9523	**0.9569**
			(0.2893)	(0.2621)	(0.2743)	**(0.2736)**
		10^5	0.9594	0.9491	0.9497	**0.9535**
			(0.1947)	(0.1851)	(0.1888)	**(0.1883)**
		1^2:5:10^2	0.9831	**0.9520**	0.9555	0.9550
			(0.5300)	**(0.3851)**	(0.4479)	(0.4441)
100^5	0.5^5	1^5	0.9818	0.9470	0.9473	**0.9502**
			(0.3518)	(0.2847)	(0.3170)	**(0.3162)**
		5^5	0.9612	0.9533	0.9531	**0.9558**
			(0.1340)	(0.1276)	(0.1305)	**(0.1302)**
		10^5	0.9461	0.9400	0.9431	0.9453
			(0.0924)	(0.0902)	(0.0912)	(0.0910)
		1^2:5:10^2	**0.9741**	0.9484	0.9479	0.9476
			(0.2231)	(0.1882)	(0.2048)	(0.2037)
	1^5	1^5	0.9946	**0.9515**	0.9523	0.9551
			(0.5817)	**(0.4029)**	(0.4898)	(0.4886)
		5^5	**0.9645**	0.9480	0.9468	0.9490
			(0.1983)	(0.1805)	(0.1888)	(0.1883)
		10^5	0.9580	0.9494	0.9499	**0.9521**
			(0.1338)	(0.1274)	(0.1303)	**(0.1300)**
		1^2:5:10^2	**0.9825**	0.9470	0.9474	0.9480
			(0.3588)	(0.2656)	(0.3110)	(0.3093)
10^2:50:100^2	0.5^5	1^5	0.9787	0.9472	0.9553	**0.9565**
			(1.0118)	(0.7154)	(0.7730)	**(0.7446)**
		5^5	0.9546	0.9456	0.9490	**0.9531**
			(0.3435)	(0.3218)	(0.3272)	**(0.3146)**
		10^5	0.9505	0.9450	0.9502	**0.9533**
			(0.2331)	(0.2256)	(0.2275)	**(0.2187)**
		1^2:5:10^2	0.9718	**0.9501**	0.9582	0.9507
			(0.8873)	**(0.6290)**	(0.6723)	(0.6363)
	1^5	1^5	0.9914	**0.9538**	0.9601	0.9556
			(1.9580)	**(1.0092)**	(1.1631)	(1.1212)
		5^5	0.9681	**0.9505**	0.9559	0.9579
			(0.5155)	**(0.4515)**	(0.4661)	(0.4486)
		10^5	0.9574	0.9481	0.9509	**0.9548**
			(0.3407)	(0.3187)	(0.3235)	**(0.3111)**
		1^2:5:10^2	0.9810	**0.9511**	0.9538	0.9436
			(1.7812)	**(0.8976)**	(1.0129)	(0.9583)

Note: Bold denotes the best performing method

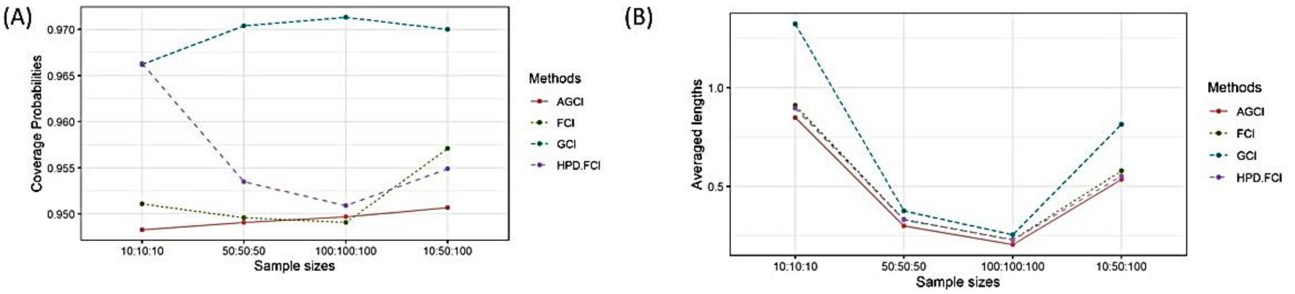

Fig. 1. Graphs of (A) coverage probability and (B) averaged length of SCIs for the difference of the coefficients of variation of inverse Gaussian distributions (*k* = 3)

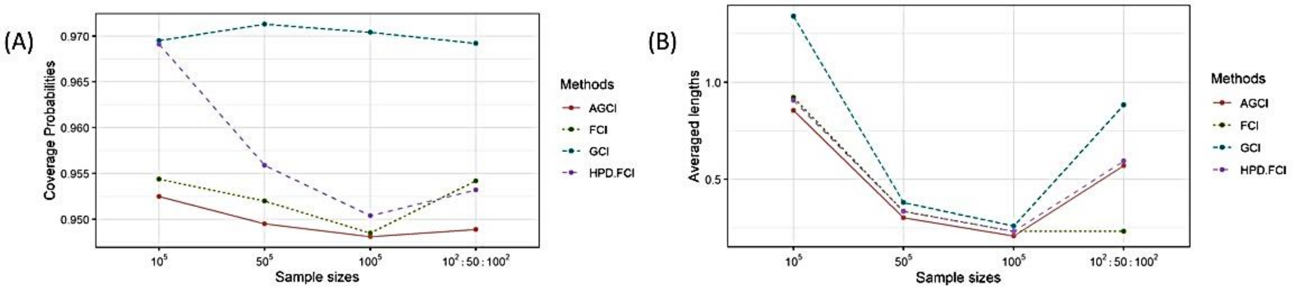

Fig. 2. Graphs of (A) coverage probability and (B) averaged length of SCIs for the difference of the coefficients of variation of inverse Gaussian distributions (*k* = 5)

4. Empirical study

In this section, we consider empirical examples PM2.5 dataset from five districts in Bangkok Thailand, (http://air4thai.pcd.go.th/webV3/#/History) i.n., Bangkapi district (D1), Wang Thong lang district (D2), Thon Buri district (D3), Bang Na district (D4), and Pathum wan district (D5). PM2.5 datasets from these regions in January and February 2022 were used to validate the effectiveness of suggested approaches. The summary statistics of PM 2.5 datasets from five districts are present in Table 3. As a result, the least Akaike information criterion (AIC) and the lowest Bayesian information criterion (BIC) assessed how well the distributions fit with the data. The IG was appropriate for all of the PM2.5 datasets from the five districts, according to the AIC and BIC results as shown in Table 4. Additionally, Figure 3 shows Q-Q plots for the positive PM2.5 data based on IG.

The 95% SCIs for of PM2.5 datasets from five districts in Bangkok, Thailand, are shown in Table 5. The findings support the model findings in that the average Table 5 displays the 95% SCIs for PM2.5 datasets from five Bangkok, Thailand, districts. The AL of the AGCI was the shortest, which is consistent with the model results. It is a good alternative for creating the SCIs for all of the dispersion of PM2.5 datasets from the five districts of Bangkok, Thailand.

Table 3
The summary statistics of PM 2.5 datasets from five districts in Bangkok Thailand

Districts	η	$\hat{\mu}$	$\hat{\lambda}$	$\hat{\theta}$
Bangkapi	50	24.98	198.2181	0.355
Wang Thong Lang	50	25.96	293.2322	0.298
Thon Buri	50	28.94	174.9499	0.407
Bang Na	50	30.62	228.3665	0.366
Pathum Wan	50	30.74	227.9669	0.367

Table 4
The results of AIC and BIC

Districts	AIC				BIC			
	Normal	Cauchy	Weibull	Inverse Gaussian	Normal	Cauchy	Weibull	Inverse Gaussian
Bangkapi	360.4302	376.4821	359.6558	**355.3920**	364.2543	380.3062	363.4798	**359.2160**
Wang Thong Lang	352.9858	364.6568	354.0612	**343.8110**	356.8098	368.4808	357.8853	**347.6351**
Thon Buri	386.7206	405.1236	384.0753	**381.1828**	390.5447	408.9476	387.8993	**385.0069**
Bang Na	385.0997	401.9569	383.4442	**378.1737**	388.9237	405.7810	387.2682	**381.9977**
Pathum wan	375.9883	393.5688	375.6609	**370.5355**	379.8123	397.3928	379.4850	**374.3596**

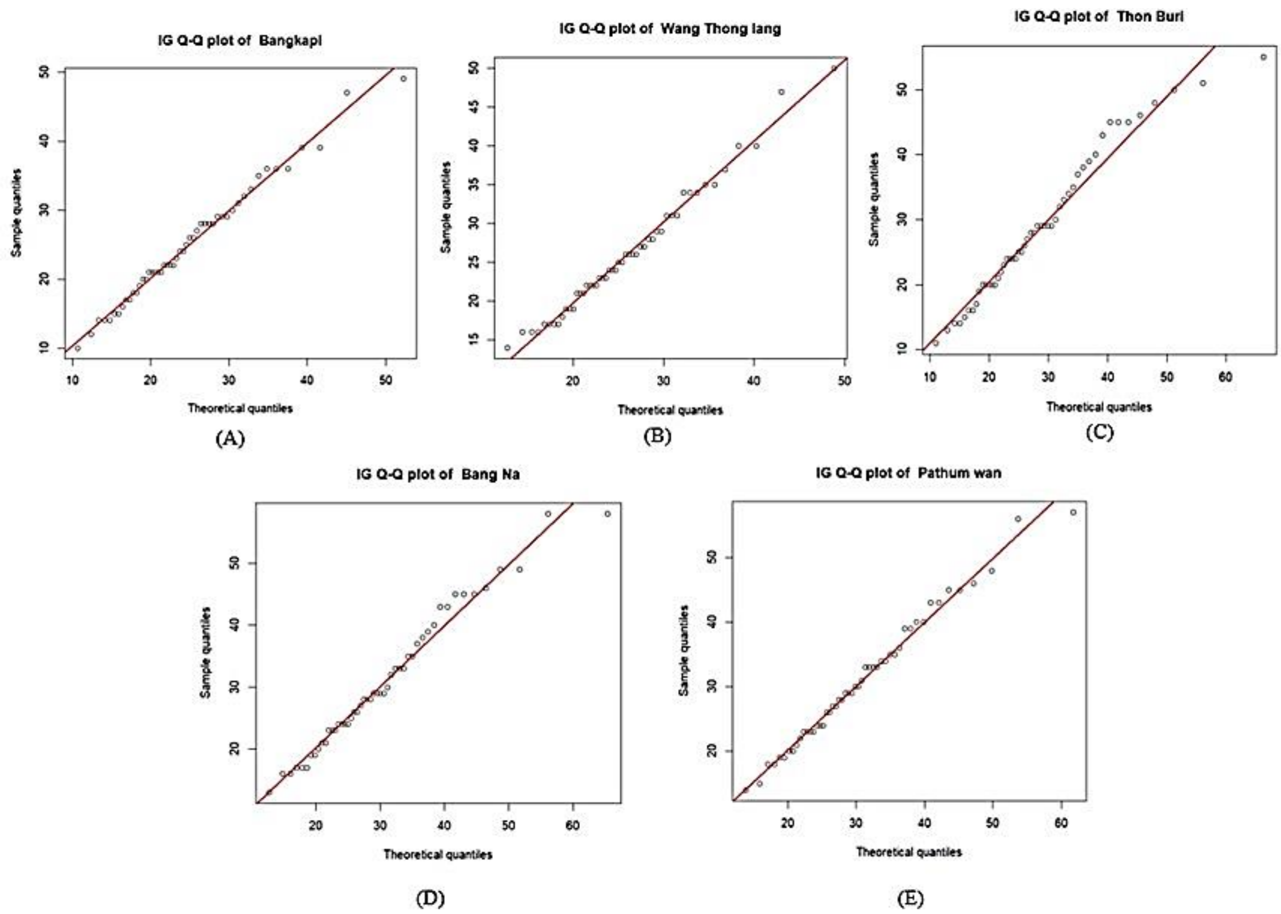

Fig. 3. Q-Q plots of the daily PM 2.5 data from five districts in Bangkok Thailand

Table 5
The 95% SCIs for θ_{il} of PM 2.5

Comparisons	GCI			AGCI			FCI			HPD		
	L	U	Le	L	U	Le	L	U	Le	L	U	Le
D1-D2	-0.0504	0.1555	0.2059	-0.0406	0.1593	0.1999	-0.0388	0.1620	0.2008	-0.0410	0.1595	0.2005
D1-D3	-0.1795	0.0598	0.2393	-0.1650	0.0560	0.2210	-0.1718	0.0626	0.2344	-0.1706	0.0634	0.2340
D1-D4	-0.1295	0.0950	0.2245	-0.1177	0.0940	0.2117	-0.1204	0.0994	0.2198	-0.1232	0.0957	0.2189
D1-D5	-0.0834	0.1259	0.2093	-0.0779	0.1251	0.2030	-0.0831	0.1282	0.2113	-0.0865	0.1236	0.2101
D2-D3	-0.2404	-0.0054	0.2350	-0.2247	-0.0075	0.2172	-0.2255	-0.0070	0.2185	-0.2208	-0.0042	0.2166
D2-D4	-0.1769	0.0332	0.2101	-0.1671	0.0238	0.1909	-0.1746	0.0307	0.2053	-0.1706	0.0331	0.2037
D2-D5	-0.1386	0.0611	0.1997	-0.1345	0.0551	0.1896	-0.1354	0.0605	0.1959	-0.1395	0.0561	0.1956
D3-D4	-0.0810	0.1767	0.2577	-0.0688	0.1640	0.2328	-0.0770	0.1630	0.2400	-0.0815	0.1579	0.2394
D3-D5	-0.0422	0.2026	0.2448	-0.0284	0.1891	0.2175	-0.0367	0.1924	0.2291	-0.0416	0.1867	0.2283
D4-D5	-0.0782	0.1464	0.2246	-0.0704	0.1387	0.2091	-0.0716	0.1419	0.2135	-0.0732	0.1402	0.2134

Notes: L=Lower; U=Upper; Le=Length

5. Conclusions

We constructed SCIs for of the IG obtained from GCI, AGCI, FCI, and HPD. The CP of the best performing CI was found to be near to or greater than the nominal confidence level of 0.95 and to have the lowest average length. The outcomes show that AGCI and HPD are the best techniques in every situation. Furthermore, these techniques may be used to develop SCIs for all pairs differences between the CVs of PM2.5 concentration datasets from five different Bangkok, Thailand, districts. The obtained results and the simulation results are identical.

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