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A SWMR Algorithm Approach for the Vendor Managed Inventory Problem when Static Demand Rates

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ABSTRACT

In a two-phase supply chain with Vendor Managed Inventory (VMI) policies, the supplier takes on the duty of keeping its retailers' inventory up to date and making sure they never run out of stock. This is in addition to his inbound inventory. This study aims to combine the retailers in various set partitions or to maximize their distribution and inventory holding costs within the same set partition. This paper focuses on a two-stage supply chain made of a single warehouse and multiple retailers (SWMR) facing a static demand and implementing VMI. We proposed a model for coordinating the system with a two-phase solution method. The first phase is to minimize the overall inventory costs and, in the second phase, a constructive vehicle routing problem (VRP) heuristic is used to optimize the distribution costs by satisfying some additional restrictions. Results of this implementation of VMI in an illustrative supply chain case are shown and discussed in detail.

Keywords:

Inventory control; Vendor managed inventory; Supply chain optimization; Vehicle routine problem; Static demand

1. Introduction

One of the most important components of business in practically all organizations, and distribution firms in particular, is the efficient monitoring and management of inventory. Since inventory typically accounts for a sizable amount of a company's balance sheet, managing it inside an organization is crucial. Businesses that wish to increase revenue and performance must use a supply chain changing business strategy.

Vendor Managed Inventory (VMI) is a partnering agreement that enables manufacturers and suppliers to keep, track, and refill inventory for their retailers, guaranteeing that they never run out of stock. The supplier decides what the quantities and delivery schedules are for a retailer. Thus, rather than being reactive in reaction to retailers' orders, the replenishment is proactive because it

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is based on the knowledge about inventory that is currently accessible. Understanding the demands of the retailer helps suppliers optimize production cycles, which lowers manufacturing costs and improves planning.

There are numerous benefits to this VMI approach for retailers as well as suppliers. To maximize truck loading and routing costs, the provider can combine numerous deliveries. Furthermore, the quantity of inventory that needs to be kept on hand at the supplier can be significantly decreased as delivery patterns become more consistent. However, retailers are no longer required to commit resources to inventory management.

However, better outcomes are not always the consequence of putting VMI into practice. Failure may occur as a result of the supplier's incapacity to make the best decisions or the lack of the required knowledge. The optimization of this problem is challenging due to the vast volume of data. It entails two extremely difficult issues: supply chain inventory management and distribution optimization.

An optimization model to manage VMI agreements is the inventory routing problem (IRP). We examine in the IRP how deterministic demand affects a two-stage supply system that uses VMI [1]. We concentrate on the single-warehouse and multiple-retailers (SWMR) system coordination problem. It is, thus, assumed that a simple form of supply chain where each retailer $j \in R$ draws required material from a single warehouse *0* to satisfy their given individual demands rate d_j. The warehouse in turn places orders to an outside supplier to fill the orders of the retailers. A fleet of homogenous vehicles $v \in V$ with a limited capacity $k(v)$, is used to deliver a product to each of the retailers which have no storage capacity restrictions. Every time an order is placed by the warehouse, a fixed price is charged. Similar to this, every order has a setup cost that varies depending on the retailer's location. Additionally, each facility in the system has a facility-dependent holding cost for inventory.

In this paper, the objective is to minimize the overall inventory holding and distribution costs of the single-warehouse and multiple-retailers and vendor managed inventory (SWMR-VMI) system, without causing any stockout at none of the retailers. The actions listed below are repeated to solve the problem. In order to reduce the systems' overall inventory holding costs, retailers are clustered in the first step. The direct-shipping tour and the VRP-tour solution are then solved using a vehicle routing problem (VRP) approach (see Figure 1), with the objective of minimizing the distribution costs. Each truck makes a single trip to the warehouse during the direct-shipping tour, which involves leaving the warehouse to serve a retailer and then returning. Then, for the VRP-tour solution, retailers can be clustered within each partition and between different partitions.

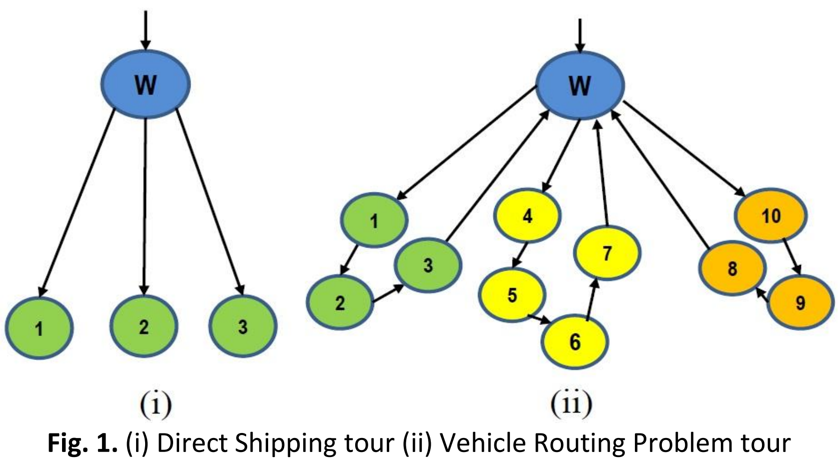


Fig. 1. (i) Direct Shipping tour (ii) Vehicle Routing Problem tour

The remainder of the paper is organized in this manner. Section 2 examines pertinent scholarly works. The integrated inventory-distribution SWMR-VMI system concept is shown in Section 3. The SWMR-VMI system's solution approach is suggested in Section 4. Section 5 provides an example of a thorough investigation of an example supply chain. Conclusions and recommendations for further study are given in Section 6.

2. A Brief Literature

The single warehouse and multiple-retailers inventory models that account for transportation costs are one area of research into this issue. Several authors [2-11] have undertaken examples of these studies.

The expansion of this research line focuses on location-inventory network design models, which combine inventory and location decisions. When a distribution network design is deployed, it could need to make frequent, costly special deliveries to retailers that are constantly running out of stock if it does not incorporate the fluctuation of demand rates and travel times in some way [12]. Barohana *et al.,* [13] investigated a real-world distribution network design issue for computer replacement components. The cost of inventory at the various warehouses is factored into their model. Then, Erlebacher *et al.,* [14] created an analytical model to reduce the overall fixed operation costs, inventory holding costs, and transportation expenses that warehouses incur. It solves the model heuristically.

The costs associated with retailers maintaining inventory are disregarded in all models. The model that we investigated here does not account for the design problem, but it does account for all inventory at the stores and the warehouse. To reduce the multi-echelon ordering and holding costs across the system, the VMI policy deals with the problem of coordinating the warehouse and retailers' inventory replenishment activities.

We examine the model created by Roundy [15] in the first phase, which included the introduction of two new policy types: power-of-two and integer-ratio policies. A subset of integer-ratio policies known as power-of-two policies places orders at multiples of two times the base planning period, or T_B, for each facility [16]. Furthermore, we employ Clarke and Wright's algorithm, which is predicated on a savings concept, as a heuristic in the second phase of our algorithm, where the distribution costs must be minimized. This algorithm's goal is to group the retailers into routes.

3. Modelling Approach

Let R be the set of retailers for the model development, indexed by i. Let $R^+=R\cup\{0\}$, where V is a group of vehicles that are available and *0* is the warehouse. In general, we assumed that vehicle V_1 travels on the shortest route possible, known as the "VRP-tour," which passes through R plus warehouse 0. This route visits the warehouse and all of the retailers in R. This VRP-tour gives a solution for the infinite time horizon. Product deliveries to each of the retailers serviced by vehicle V_1 should be sufficient to cover their consumption over a period of time not shorter than $T^v_{\min}$ in order to prevent stockouts at any of the retailers covered by this vehicle. Therefore, the replenishment cycle of each of the retailers served by vehicle V_1 must be larger or equal to $T^v_{\min}$. The "cycle time," or T, is the interval of time between two successive repeats of the tour. The entire time required to finish a tour provides a lower bound on the cycle time because the tour cannot be resumed before it is finished. Thus, the following formula provides the minimum cycle time for restocking the set of retailers R in a single tour served by vehicle V_1, indicated by $T^v_{\min}$.

$$T^{v}{}_{\min} = \sum_{j\in R^{v}} T_{VRP}(R^{v} \cup \{0\}) \quad (1)$$

Conversely, vehicle V_1's restricted capacity *k(v)* imposes an upper bound on the retailers its serves' replenishment cycle. This upper bound is called maximal cycle time and is denoted by $T^v{}_{max}$. The replenishment cycle must be smaller than or equal to $T^v{}_{max}$ in order to guarantee that the total amount of products given to retailers visited during the single tour does not exceed capacity *k(v)*. The maximum cycle time for vehicle V_1 to restock the set of retailers *R* in a single tour is provided by the following formula.

$$T^{v}{}_{\max} = \min_{j\in R^{v}} \left\{ \frac{k}{\sum_{j\in R^{v}} d_j} \right\} \quad (2)$$

Additionally, we define the following notations:

i. ψ^v: is the fixed operating and maintenance costs of vehicle $v\in V$;
ii. t_{ij}: is the duration of a trip from retailer $i\in R^+= R\cup\{0\}$ to retailer $j\in R^+$;
iii. τ_{ij}: is a per unit transportation cost from the warehouse or retailer i to retailer j;
iv. φ_0: is a fixed ordering cost incurred by the warehouse each time it places an order; the ordering cost is independent of the order quantity;
v. φ_j: is a fixed ordering cost incurred by each retailer $j\in R$ each time it places an order from the warehouse; the fixed ordering cost is independent of the order quantity;
vi. h_0: is the per unit per year inventory holding cost rate in warehouse 0;
vii. h_j: is the per unit per year inventory holding cost rate in retailer j;
viii. T_B: is a base planning period;
ix. T_0: the replenishment interval at warehouse 0;
x. T_j: the replenishment interval at retailer j.

Let R^v be the set of retailers that vehicle *v* serves. Assume that the set of vehicles *v* in V^* serves and clusters the retailers. Vehicle *v* serves consumer *j*; so, $T^v = T_j$. To be optimized is the following objective function:

SWMR-VMI:

$$Z_D= \sum_{v\in V^*} \psi^{v} + \frac{\varphi_0}{T_0} + \sum_{v\in V^*} \left(\frac{1}{2} h_0 \left(\sum_{j\in R^v} d_j \right) \left[\max\left(T_0, T^v\right) - T^v \right] \right) + \sum_{v\in V^*} \left(\frac{1}{T^v} \left(\tau^v + \sum_{j\in R^v} \varphi_j \right) + \frac{1}{2} \left(\sum_{j\in R^v} h_j d_j \right) T^v \right)$$

(3)

The total travel cost for the entire trip made by vehicle v is represented by $\tau^v = \sum_{(i,j)\in \mathrm{Trip}(v)} \tau^{ij}$. This satisfies the requirements that $\sum_{(i,j)\in \mathrm{Trip}(v)} t_{ij} \leq T^v$ and that the total amount delivered to the retailers in each tour the vehicle makes during its trip, Trip(*v*), should not exceed the vehicle's capacity.

For the basic model SWMR-VMI, we can summarize the key findings of Roundy (1985) as is done in [17-19] if we also assume that $(h_j – h_0) > 0$ for every retailer j. By rounding off the result, one can obtain a feasible integer-ratio policy with a cost within 98% of the minimum of Eq. (3). The solution of Eq. (3) represents a lower bound on the average cost of any possible inventory control strategy.

Such a policy can be computed in O($n \times log(n)$) time. Furthermore, in the solution to Eq. (3), the retailers can be divided into three groups: *G*, *L*, and *E*.

The replenishment interval for retailers in *G* is provided by:

$$\hat{T}^v = \sqrt{\frac{2\left[\tau^v + \sum_{j\in R^v} \varphi_j\right]}{\sum_{j\in R^v} h_j d_j}} > \hat{T}_0 \tag{4}$$

The replenishment interval for retailers in *L* is provided by:

$$\hat{T}^v = \sqrt{\frac{2\left[\tau^v + \sum_{j\in R^v} \varphi_j\right]}{\sum_{j\in R^v} (h_j - h_0) d_j}} < \hat{T}_0 \tag{5}$$

Lastly, the replenishment period for retailers in *E* is determined by and is identical to that at the warehouse.:

$$\hat{T}^v = \hat{T}_0 = \sqrt{\frac{2\left(\varphi_0 + \sum_{v\in E}\left[\tau^v + \sum_{j\in R^v} \varphi_j\right]\right)}{\sum_{v\in E}\sum_{j\in R^v} h_j d_j + \sum_{v\in L}\sum_{j\in R^v} h_0 d_j}} \tag{6}$$

If we start from a feasible partition $(R^v)_{v\in V^*}$ of retailers, that satisfies $T^v_{\min} = \sum_{(i,j)\in \mathrm{Trip}(v)} t_{ij} \leq T^v_{\max,}$ defining the smallest cycle time obtained from the total amount delivered to the retailers served during each sub-tour made by the vehicle. Thus, we can determine the optimal values for each of vehicle *v*, as follows:

$$T^v_{opt} = \begin{cases} \hat{T}^v, & \text{if } \; T^v_{\min} \leq \hat{T}^v \leq T^v_{\max} \\ T^v_{\min}, & \text{if } \; T^v_{\min} > \hat{T}^v \\ T^v_{\max}, & \text{if } \; \hat{T}^v > T^v_{\max} \end{cases} \tag{7}$$

To solve the full VRP problem, we first modified Roundy's (1985) algorithm to minimize inventory costs and identify potential retailer set partitions *G*, *E*, and *L*. Next, we used an effective heuristic to solve the constrained VRP problem with the goal of minimizing transportation costs. Until no further retailer clustering is possible, these two phases are repeated with the clustered sets of retailers, at which point the process is terminated. Section 4 provides the detailed steps for the entire algorithm.

4. Solution Approach

The SWMR-VMI system solution approach is covered in this section. The solution's primary goal is to locate new retailer clusters, each of which has a vehicle serving it. There is a determination of the best distribution routes and the best cycle for retailer replenishment. As a result, we can figure out how much the SWMR-VMI system will cost overall for each cluster. Figure 2 depicts the SWMR-VMI system solution framework, which consists of many algorithmic processes and procedures

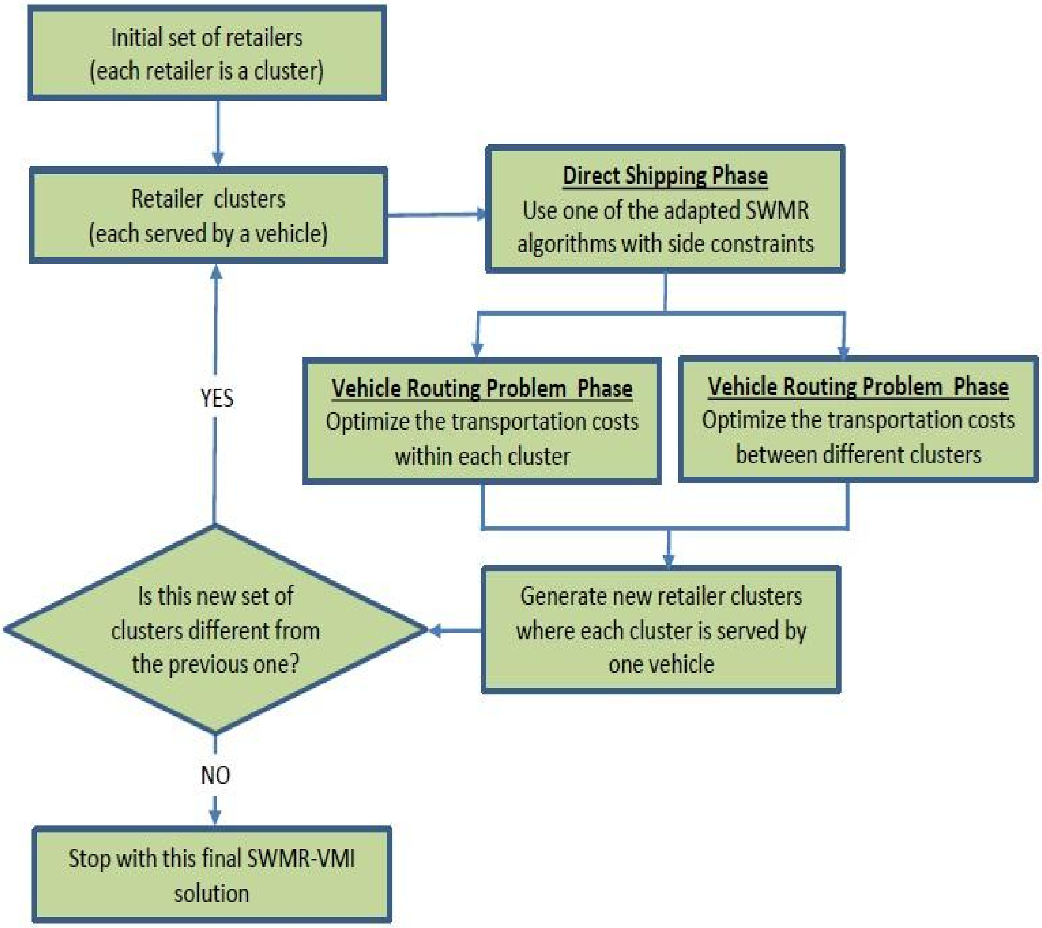


Fig. 2. Solution framework for the SWMR-VMI system

4.1 Roundy's 1985 Algorithm for the Deterministic Situation of G, E and L

We start by initializing a set of retailers into a cluster where the retailers are to be served by a vehicle. In the next step of the process, we have to find the replenishment interval for each retailer as well as the warehouse. In the direct shipping phase, we use the SWMR algorithm developed by Roundy (1985), with the objective of minimizing the inventory costs and clustering all the possible retailers into set partitions *G, E* and *L*. Once the possible retailers' set partitions have been identified, we solve the power-of-two order interval for each retailer as well as the warehouse. Then, we have found the optimal power-of-two policy for the SWMR-VMI system, and all retailers are divided into the set partitions *G, E* and *L*. The detailed steps are as follows:

4.1.1 Calculate and sort the breakpoints

Calculate the breakpoints $\tau_j^{'} = \sqrt{2k_j / h_j . D_j}$ and $\tau_j = \sqrt{2k_j / (h_j - h_0^j) D_j}$

Sort them to form a non-decreasing sequence of *2N* numbers. Label each breakpoint with the value of *n* and with an indicator showing whether it is the left breakpoint τ'_j or the right breakpoint τ_j.

4.1.1 Initialize E, G, L, K and H

Set $E = G = \emptyset$, $L = \{1,\ldots,N\}$, $K = K_0$, and $H = \sum_{j=1}^{n} \frac{1}{2} h_0^j D_j$.

4.1.2 Cross the largest uncrossed breakpoint

Let τ be the largest previously uncrossed breakpoint. If $\tau^2 \geq K/H$ and $\tau = \tau_j$ is a right breakpoint, cross τ and update E, L, K and H by $E \leftarrow E \cup \{n\}$, $L \leftarrow L \backslash \{n\}$, $K \leftarrow K + K_j$ and $H \leftarrow H + h_j$. Then go to step 3. If $\tau^2 > K/H$ and $\tau = \tau'_j$ is a left breakpoint, cross τ and update E, G, K and H by $E \leftarrow E \backslash \{n\}$, $G \leftarrow G \cup \{n\}$, $K \leftarrow K - K_j$ and $H \leftarrow H - h_j - h_0^j$, go to step 3. Otherwise T_j^* is in the current piece. Go to step 4.

*4.1.3 Calculate T**

Set $T^* = \sqrt{K/H}$. Then T_j^* for all retailers $j \in R$.

4.1.4 Rounding the optimal solution T_j^* to the power of two multiple of T_B.

It remains to be shown that step 3 will be executed before the last breakpoint is crossed. Otherwise, we would have $H = 0$ and $E \cup L = \emptyset$. If $\tau = \sqrt{k_j / h_j}$ is the only uncrossed breakpoint, it is a left breakpoint. Then in step 3 $K_0 + K_j$ and $H = h_j$, so $K/H > \tau^2$. Therefore step 4 will be executed and the algorithm will terminate.

4.2 The Savings Algorithm

For the vehicle routing problem phase, we use an adapted version of the method proposed by Clarke and Wright algorithm. Probably this method is the best-known heuristic for the VRP. Clarke and Wright method is a clustering algorithm. In other words, its goal is to choose which retailers will be included in a route by clustering them together and to maximize the transportation costs both within and between different clusters. A different algorithm will then set the order of the route. The Savings Algorithm finds pairs of retailers that are beneficial in a route and links as many of the pairs as possible. The steps for this algorithm are as follows:

i. Start with *n* dedicated routes (round trips that service only one retailer), one for each of the *n* retailers.
ii. Compute the savings in distance, S_{ij}, of combining every possible pair of retailers *i* and *j*, $S_{ij} = C_{i0} + C_{0j} - C_{ij}$.
iii. <u>a:</u> Combine the pairs of retailers in the list which are in the different retailers set partitions. Order the savings in a decreasing fashion. Since negative *S* values are undesirable, omit the negative values from the list.
<u>b:</u> Remove the pairs of retailers in the list which are not in the same retailers set partitions. Start searching at the top of the list and do the following. Order the savings in a decreasing fashion.
iv. Build a route by adding pairs that do not violate any of the set constraints in the order that they appear in the list until the route is full or the list has been exhausted. The resulting retailers form a cluster.

v. Repeat Step 4 until all retailers are routed or the list has been exhausted.
vi. Any retailers that were left unpaired (because of negative savings distances) are left as dedicated routes.

4.3 The Improvement Heuristic

The improving heuristic starts with an arbitrary result and works its way down to a local minimum, beyond which further improvements are not feasible. The 2-opt exchange is one of the improvement heuristics which is very simple, yet very useful. It involves exhaustively considering exchanges of two retailers in different routes; for a given exchange, if the result is lower than before the exchange (and no constraints are violated), the exchange takes place. The process is repeated until no more exchanges are produced a lower cost can be obtained.

5. Computational Example

In this instance, as shown in Figure 3, we take into consideration 15 retailers. With their dispersed locations around the warehouse and their presumed steady demand rates, the combined 15 retailers' hourly output comes to 6.341 tons. Assumed to be ready for product replenishment from the warehouse is a fleet of vehicles. This case's data was attained by Aghezzaf *et al.,* [20].

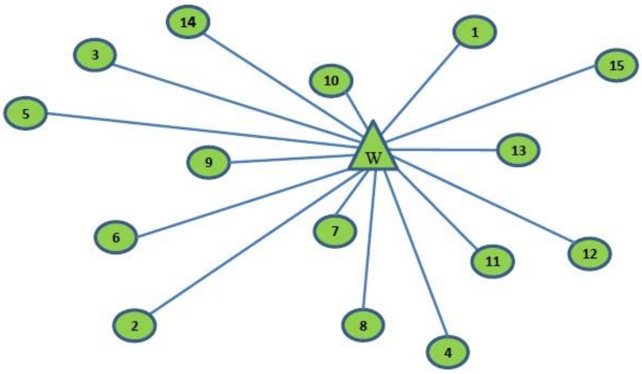


Fig. 3. An Example case with 15 retailers

The distances (in kilometres) between the various retailers are displayed in Table 1. By assuming that each vehicle travels at an average speed of 50 km/h, travel times may be calculated from Table 1. For each vehicle, the following data are provided.

i. A vehicle capacity of 60 tons, 80 tons and 100 tons;
ii. A fixed operating and maintenance cost of \$50 per hour;
iii. A transportation cost of \$0.20 per kilometre;
iv. A fixed ordering cost of the warehouse is \$75;
v. A fixed cost at all retailers per delivery of \$50.

Finally, we assume that there is a difference in inventory holding cost rates at the retailers, with $(h_j - h_0) > 0$ and the retailers have no storage capacity restrictions.

Table 1
Distances in kilometres between the different retailers

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15

0	-	270	480	490	330	550	430	140	260	240	150	240	360	200	430	320
1		-	740	560	580	680	650	420	530	440	210	410	450	260	400	170
2			-	500	490	410	190	350	370	320	540	590	750	660	600	800
3				-	630	160	310	480	630	290	390	720	850	670	180	710
4					-	770	590	290	160	470	490	190	300	350	750	520
5						-	220	540	630	310	490	760	910	740	340	810
6							-	340	430	210	440	610	770	630	430	740
7								-	160	230	250	270	420	310	470	450
8									-	340	400	230	380	350	630	510
9										-	240	450	600	440	320	540
10											-	370	480	280	290	320
11												-	160	170	650	330
12													-	210	770	310
13														-	570	170
14															-	570
15																-

Given the case of the 15 retailers. Rates of demand and distance are provided. There is a 60-ton vehicle accessible to refill products from the warehouse. The vehicle departs the warehouse, serves a retailer, and then returns to the warehouse in a direct shipping tour. The vehicle's maximum cycle duration is 72.9 hours, while its minimum cycle time is its total trip time of 195.6 hours. This solution is infeasible if only a vehicle is used to carry out the tours to all *n* retailers. Therefore, the solution will be feasible, when adding more vehicles or using a vehicle with a larger capacity. Moreover, to optimize the overall inventory costs and distribution costs, some retailers must be clustered and served in one sub-tour of the vehicle.

To get a feasible and better solution, we used the methods by Roundy (1985) to determine the retailers' set partitions *G, E, L* and find the optimal replenishment cycle time for each of the retailers, using the steps of the algorithm presented in section 4.1. We consider the retailer clusters resulting from the set partitions *G, E, L*. From the algorithm in 4.1, we have identified that the retailers set partitions in *G, E, L* are *E* = {1, 3, 5, 6, 7, 8, 9, 10, 13, 14, 15}, *L* = {2, 4, 11, 12}, *G* = {Ø}, and the optimal replenishment cycle time of the tour are given in Table 2.

To minimize transportation costs, we next modified a savings heuristic method created by Clarke and Wright to solve the limited VRP problems using the algorithm in section 4.2. The goal was to cluster retailers as much as feasible. The challenge is deciding which truck will cover which route, how the retailers will be allocated to routes and the order in which they will be visited.

Then, the replenishment cycle time of the tour is multiplied by the retailers' demand rates. Like that, we obtain the quantities that must be delivered from the warehouse to each of the retailers (see Table 2).

Table 2
Quantities delivered to each of the retailers

Retailers	*Demand (ton/hour)*	*Cycle time (hour)*	*Delivery (ton)*

1	0.209	64	13.38
2	0.622	32	19.90
3	0.322	64	20.61
4	0.798	32	25.54
5	0.134	64	8.58
6	0.429	64	27.46
7	0.381	64	24.38
8	0.503	64	32.19
9	0.217	64	13.89
10	0.269	64	17.22
11	0.823	32	26.34
12	0.598	32	19.14
13	0.247	64	15.81
14	0.348	64	22.27
15	0.441	64	28.22

The savings Sij are calculated for every possible pair of retailers i and j, which do not violate the vehicle capacity constraints. The saving heuristic then has constructed two solutions. In the first solution, the pair of retailers is combined within the set partitions G, E, L and in the second one, the pair of retailers is combined between the different set partitions. The distribution results for the VRP-tours of the vehicle are shown in Table 3.

The following solution, which are used to explain the various cost trade-offs and how they are obtained, are summarized in Table 3 based on the distribution results for the vehicle's VRP-tours.

i. Number of vehicles and number of tours;
ii. The optimal cycle time of each tour;
iii. Every tour's truck loading and vehicle usage;
iv. The overall cost of the systems as well as the total cost of each tour;

We have assessed the impacts on the solution's total cost rate in Table 3. For instance, a 60-ton, 80-ton, and 100-ton capacity is used for the vehicle capacity factor. It is estimated that each vehicle will travel at an average speed of 50 km/h. The inclusion of the vehicle capacity factor demonstrates how our method of problem solving can be applied not only to determine the right fleet size but also to choose the right vehicle type for a given set of circumstances.

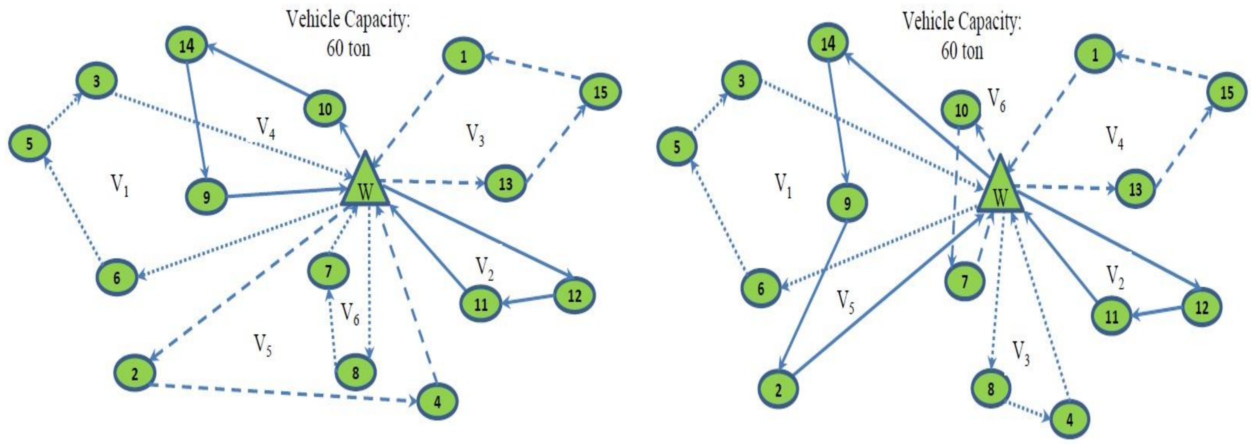
For the retailers' clusters in the same set partitions, the average total cost rate is $364.47 when using the small vehicle of 60 tons, $293.95 when using the vehicle of 80 tons, and $242.45 when using the larger vehicle of 100 tons. For the retailers' clusters in the different set partitions, the average total cost rate is $365.89 when using the small vehicle of 60 tons, $317.30 when using the vehicle of 80 tons, and $262.78 when using the larger vehicle of 100 tons. Therefore, we can summarize that when using the smaller vehicle capacity, the number of deliveries or number of tours increases, and while using the larger vehicle, large quantities can be delivered efficiently to the retailers. Thus, with a smaller vehicle capacity, smaller deliveries are made and less retailers are visited per tour.

Table 3
Distribution results for the VRP-tours of the vehicle

Vehicle	Retailers Clusters in the same set partitions	Retailers Clusters in the different set partitions

Capacity														
	Tour	T^v_{min}	T^v_{max}	T^v	T^v_{opt}	Vehicle Load	Total Cost	Tour	T^v_{min}	T^v_{max}	T^v	T^v_{opt}	Vehicle Load	Total Cost
60 tons	V_1	26.00	67.80	64.00	64.00	56.64	60.06	V_1	26.00	67.80	64.00	64.00	56.64	60.06
	V_2	15.20	42.22	32.00	32.00	57.41	63.26	V_2	15.20	42.22	32.00	32.00	45.47	63.26
	V_3	16.20	66.89	64.00	64.00	53.38	59.14	V_3	15.00	46.12	50.62	46.12	60.00	61.06
	V_4	20.00	71.94	64.00	64.00	56.58	57.48	V_4	16.20	66.89	64.00	64.00	57.41	59.14
	V_5	26.00	42.25	32.00	32.00	45.47	67.46	V_5	31.00	50.55	65.15	50.55	60.00	65.68
	V_6	11.20	67.87	64.00	64.00	45.44	57.07	V_6	10.80	92.31	64.00	64.00	41.60	56.69
						364.47						365.89		
80 tons	V_1	28.40	64.88	64.00	64.00	78.91	58.33	V_1	28.40	64.88	64.00	64.00	78.91	67.84
	V_2	18.40	36.05	32.00	32.00	71.01	63.20	V_2	16.20	47.96	37.22	37.22	62.09	64.89
	V_3	18.00	68.61	64.00	64.00	74.62	58.97	V_3	15.00	61.49	50.62	50.62	65.85	60.85
	V_4	17.60	72.66	64.00	64.00	70.46	57.63	V_4	17.00	87.05	64.00	64.00	58.82	59.45
	V_5	19.20	128.62	32.00	32.00	19.90	55.82	V_5	21.00	65.57	43.63	43.63	53.23	64.27
						293.95						317.30		
100 tons	V_1	28.80	68.97	64.00	64.00	92.80	60.37	V_1	33.20	53.91	44.99	44.99	83.46	70.31
	V_2	33.40	35.20	32.00	33.40	94.89	67.62	V_2	16.20	59.95	37.22	37.22	62.09	64.89
	V_3	18.00	85.76	64.00	64.00	74.62	58.97	V_3	22.40	52.66	37.95	37.95	72.07	68.13
	V_4	11.20	113.12	64.00	64.00	56.58	55.49	V_4	17.00	108.81	64.00	64.00	58.82	59.45
	242.45											262.78		

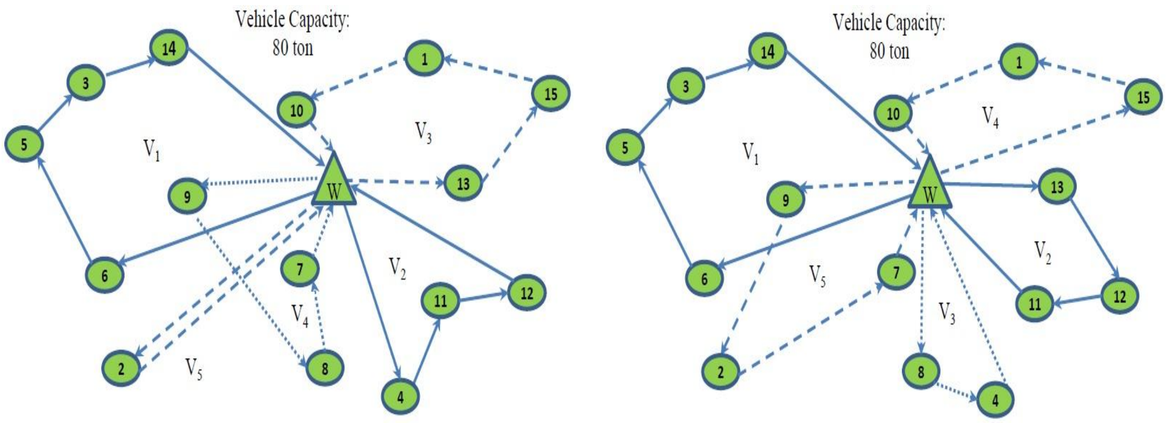
Next, we evaluated the effect between the retailers' cluster in the same set of partitions and the retailers' cluster in a different set of partitions. Figure 4 shows a distribution plan for the 15 retailers with the same vehicle capacity of 60 tons but with different characteristics. The saving heuristic has constructed solutions with six routes for each VRP-tour. When the retailers clustered in the same set of partitions, the total cost rate decreased by $364.47 rather than clustered with the different set partitions with a total cost of $365.89.



(a) (b)

Fig. 4. A distribution plan for the 15 retailers (60 tons)

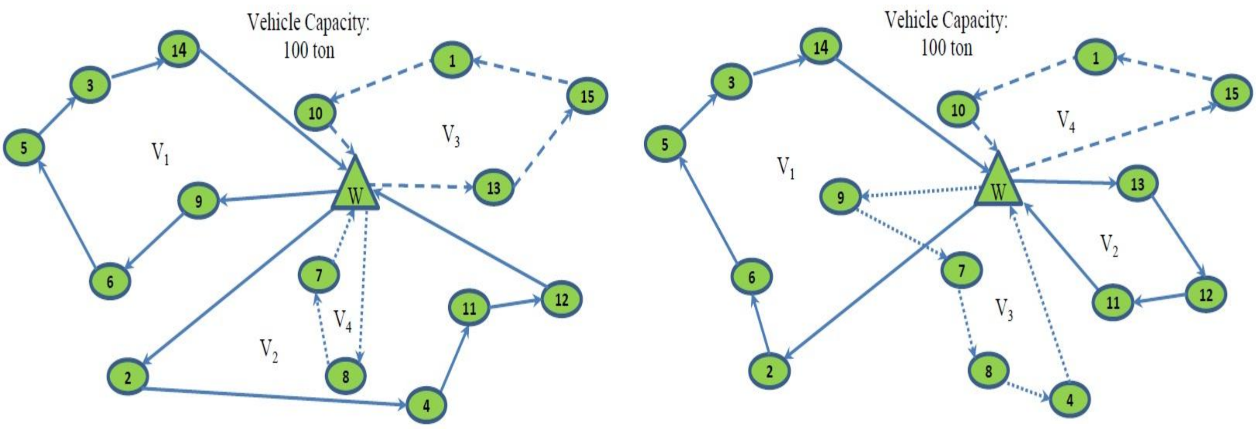
A vehicle with an 80-ton capacity is used to distribute products to the retailer clusters, as shown in Figure 5 above. The saving heuristic has created solutions with five routes for every VRP-tour for this solution strategy. The total cost rate for retailers grouped in different set partitions is $317.30, whereas the total cost rate for retailers clustered with the same set partitions is $293.95. As we can see, we were able to save almost 7.4% because the overall cost rate dropped from $317.30 to $293.95.



(a) (b)

Fig. 5. A distribution plan for the 15 retailers (80 tons)

Figure 6 shows a distribution plan for the 15 retailers with a vehicle capacity of 100 tons. This solution has constructed four routes for each VRP-tour. For the retailers clustered in the different set partitions, the total cost rate is $262.78, while for the retailers clustered with the same set partitions, giving a total cost of $242.45. One can observe that we realized a saving of about 7.6% since the total cost rate decreased from $262.78 for the solution with the different set of partitions to $242.45 for the solution with the same set of partitions. Thus, we also can observe that the average capacity utilization for all tours in the retailers clustered in the same set partitions is utilized efficiently, which is 79.7% compared to the retailers clustered in the different set partitions is 69.1%.



(a) (b)

Fig. 6. A distribution plan for the 15 retailers (100 tons)

6. Conclusions

In a supply network, inventory management and routing present an extremely difficult optimization challenge. In this work, we present a vendor managed inventory (VMI) solution strategy for a two-stage supply chain. In order to maximize inventory holding costs and retailer distribution costs within the same set partition and/or retailer combination amongst the different set partitions, we concentrate on the problem setting with a single warehouse and numerous retailers (SWMR-VMI).

A method to account for predictable demands and reduce the total inventory and distribution expenses of the SWMR-VMI system is presented. The strategy is predicated on a few efficient sub-problems for inventory and routing algorithms. Our solution uses the heuristic algorithms created by Clarke and Wright method to solve the VRP sub-problems and the methods suggested by Roundy (1985) to solve the SWMR problem.

To address more complex problems, such as those involving larger groups of retailers, driving time constraints for both drivers and vehicles, delivery window times at the retailers, multiple tours, heterogeneous fleets of vehicles, multiple warehouses, multiple products, etc., more research will focus on modifying the current solution approach. Lastly, we will look into how the method might be expanded to consider some demand variability and will investigate more on what technology can offer to enhance supply chain performance [21].

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