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Heat Transfer Study in Cylindrical Cavity with Heat Absorption or Generation

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ABSTRACT

Lattice Boltzmann mesoscopic (LBM) is applied to solve energy equation of a transient conduction radiation heat transfer problem in a two-dimensional cylindrical participating (absorbing, emitting and scattering) medium in the presence of heat generation/absorption coefficient. Control volume finite element method (CVFEM) formulation is used to obtain the radiative information. To study the effectiveness of the LBM-CVFEM combination on unsteady conduction-radiation problems in cylindrical media, the energy equation of the problem is also solved using the finite difference method (FDM) in which the CVFEM is used to compute radiative information. The effects of heat generation/absorption coefficient on temperature distributions in the medium are studied. Results of the present work are benchmarked against those available in the literature. The hybrid numerical model's results are also compared with those obtained by the FDM-CVFEM combination. All the results presented in this work show that the present method is accurate and valuable for the analysis of cylindrically axisymmetric radiative heat transfer problems.

Keywords:
LBM, lattice Boltzmann Method, conduction, radiation, cylindrical media, heat absorption, heat Generation

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1. Introduction

During the past five decades, much effort has been expended to solve coupled unsteady coupled conduction and radiation heat transfer problems in semitransparent cylindrical media. The motivation for this effort arises from extensive engineering applications, such as industry combustion chambers, the design of reactors, heat pipes, rocket propulsion systems, etc. Therefore, research on coupled conduction and radiation heat transfer problems in cylindrical media appears to be of practical significance.

For many engineering applications such as boilers, combustors, and rocket propulsion systems, axisymmetric assumption is usually made due to its geometric and theoretical simplicity

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and, thereby, economic benefits because it physically describes three dimensional phenomena with two-dimensional procedure. Therefore, substantial efforts are exerted to analyze the axisymmetric problems in the field of radiation as well as flow and heat transfer including combustion. During the past few decades, numerous methods have been proposed to solve the RTE in the axisymmetric and cylindrical geometry. Among others, the Monte Carlo method (MCM) [1], the discrete transfer method (DTM) [2,3], the discrete ordinates method (DOM) [4], the finite-volume method (FVM) [5-10], the collapsed dimension method (CDM) [3], and the control volume finite element method (CVFEM) [11-23]. Analyses of coupled conduction-radiation heat transfer in cylindrical media have also been reported by many researchers [24-26].

Over the past few years, lattice Boltzmann method (LBM) has found wide-ranging applications in science and engineering [27-29]. This surge in interest is mainly attributed to its ability, direct discretization, computational simplicity, ability and efficiency. Unlike other conventional computational fluid dynamics (CFD) solvers, such as finite difference method (FDM), Finite Element Method (FEM) and Finite Volume Method (FVM), which are based on macroscopic models, the LBM is a mesoscopic approach and it describes and captures physics better. This method includes simple calculations procedure, efficient implementation for a parallel architecture, and simplicity of boundary condition’s implementation, easy and robust handling of complex geometry, and others.

More recently, the application of LBM has gained momentum in the solution of transient conduction-radiation problems [20-23, 27-29]. In the present work, we extend the application of LBM in solving both energy equation of a 2-D transient conduction-radiation heat transfer problem in a cylindrical enclosure and radiative transfer equation (RTE) in the presence of uniform heat absorption or generation effect.

2. Physical Model and Formulation

We consider combined transient conduction radiation heat transfer in a 2D system with a homogeneous absorbing, emitting, and scattering participating medium. Figure 1 and 2 depicts the system and coordinates.

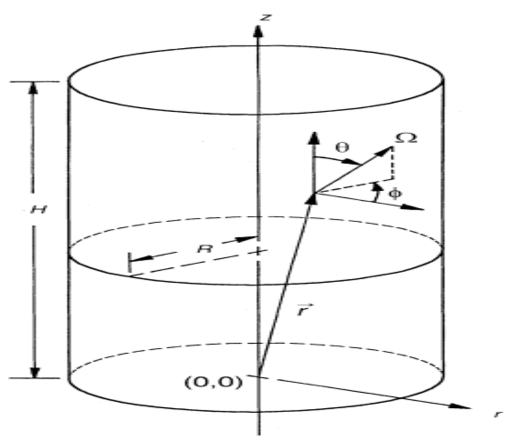


Fig. 1. Geometry and coordinates

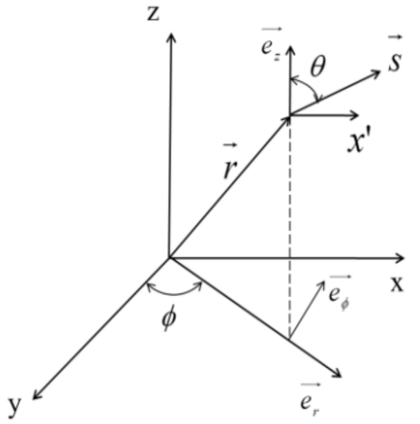


Fig. 2. Schematic of the Cartesian and cylindrical coordinates for the equation of radiative transfer

Equation governing unsteady heat transfer in a finite axisymmetric cylindrical medium, is as follow

$$\rho c_p \frac{\partial T}{\partial t} = \frac{k}{r}\frac{\partial}{\partial r}(r\frac{\partial T}{\partial r}) + k\frac{\partial^2 T}{\partial z^2} + Q \quad (1)$$

$$Q = Q_R + q_a \quad (2)$$

where ρ is the density, c_p is the specific heat and k is the thermal conductivity.

Q_R represents the radiative heat flux which given by

$$Q_R + \nabla\overrightarrow{q_R} = 0 \quad (3)$$

where

$$\vec{q}_R = \int_{4\pi} I\vec{\Omega}d\Omega \quad (4)$$

and I is the radiative intensity which can be obtained by solving the Radiative Transfer Equation (RTE).

For an absorbing, emitting and scattering grey medium the RTE can be written as

$$\vec{\nabla}.(I(s,\vec{\Omega}).\vec{\Omega}) = -(k_a + k_d)I(s,\vec{\Omega}) + k_a I_b(s) + \frac{k_d}{4\pi}\int_{\Omega'=4\pi} I(s,\overrightarrow{\Omega'})P(\overrightarrow{\Omega'} \to \vec{\Omega})d\Omega' \quad (5)$$

where $I(s,\vec{\Omega})$ is the radiative intensity, which is a function of position s and direction $\vec{\Omega}$; k_a and k_d are absorption and scattering coefficients, respectively; $I_b(s)$ is the blackbody radiative intensity at the temperature of the medium; and $P(\overrightarrow{\Omega'} - \vec{\Omega})$ is the scattering phase function from the incoming $\overrightarrow{\Omega'}$ direction to the outgoing direction $\vec{\Omega}$. The term on the left-hand side represents the gradient of the intensity in the direction $\vec{\Omega}$. The three terms on the right-hand side represent the changes in intensity due to absorption and out-scattering, emission, and in-scattering, respectively.

The radiative boundary condition for Eq. (5), when the wall bounding the physical domain is assumed grey and emits and reflects diffusely, can be expressed as

$$I_w(\vec{\Omega}) = \frac{\varepsilon_w \sigma T_w^4}{\pi} + \frac{1-\varepsilon_w}{\pi} \int_{\vec{\Omega}'.\vec{n}_w<0} I_w(\vec{\Omega}') \left| \vec{\Omega}'.\vec{n}_w \right| d\Omega' \quad \text{if } \vec{\Omega}'.\vec{n}_w > 0 \tag{6}$$

$\vec{n}_w$ represent the unit normal vector on the wall and ε_w represents the wall emissivity.

3. Numerical methods

The CVFEM is used to discretize the RTE. In the CVFEM, the spatial and angular domains are divided into a finite number of control volumes and control solid angles. The direction of propagation $\vec{\Omega}$ is defined in Cartesian spatial coordinates ($\vec{e_x}, \vec{e_y}, \vec{e_z}$) as shown in figure 1.

$$\vec{\Omega} = \sin\theta\cos\phi\vec{e_x} + \sin\theta\sin\phi\vec{e_y} + \cos\theta\vec{e_z} \tag{7}$$

In cylindrical spatial coordinates ($\vec{e_x}, \vec{e_y}, \vec{e_z}$), $\vec{\Omega}$ is expressed as

$$\vec{\Omega} = \sin\theta\cos\psi\vec{e_r} + \sin\theta\sin\psi\vec{e_\varphi} + \cos\theta\vec{e_z} = \Omega_r\vec{e_r} + \Omega_\varphi\vec{e_\varphi} + \Omega_z\vec{e_z} \tag{8}$$

where

$$\psi = \phi - \varphi \tag{9}$$

The total solid angle is subdivided into $N_\theta \times N_\psi$ control solid angles as depicted in figure 2, where

$$\Delta\psi = (\psi^+ - \psi^-) = 2\pi / N_\psi \tag{10}$$

$$\Delta\theta = (\theta^+ - \theta^-) = \pi / N_\theta \tag{11}$$

The control solid angle $\Delta\Omega^{mn}$ is given by (Fig. 2):

$$\Delta\Omega^{mn} = \int_{m\Delta\theta}^{(m+1)\Delta\theta} \int_{n\Delta\Psi}^{(n+1)\Delta\Psi} \sin\theta \, d\theta \, d\psi \tag{12}$$

R is the radius of the cylinder and H is its height.

For the energy equation, the detail may be referred in References [20,23,27,29]. The kinetic equation in the LBM for a two dimensional enclosure is given

$$\frac{\partial f_i(\vec{r},t)}{\partial t} + \vec{c_i}.\nabla f_i(\vec{r},t) = \Omega_i, \qquad i = 0,..,8 \tag{13}$$

For a 2-D cylindrical geometry and taking into account the single time relaxation model of the Bhatanagar–Gross–Krook (BGK) approximation, the discrete Boltzmann equation is given by

$$\frac{\partial f_i(\vec{r},t)}{\partial t} + \vec{c_i}.\nabla f_i(\vec{r},t) = -\frac{1}{\tau}[f_i(\vec{r},t) - f_i^{eq}(\vec{r},t)], \qquad i = 0,..,8 \tag{14}$$

where τ is the relaxation time and f^{eq} is the equilibrium distribution function.

In heat transfer problems, the relaxation time τ for the D2Q9 lattice (Fig. 3-4) is computed from

$$\tau = \frac{3k}{(\rho c_p)c^2} + \frac{\Delta t}{2} \tag{15}$$

The nine velocities and their corresponding weights in the D2Q9 lattice are the following:

$$c_0 = (0,0) \tag{16}$$

$$\vec{c}_{i=1,4} = (\cos(\frac{i-1}{2}\pi), \sin(\frac{i-1}{2}\pi)).c \quad ; \vec{c}_{i=5,8} = \sqrt{2}(\cos(\frac{2i-1}{4}\pi), \sin(\frac{2i-1}{4}\pi)).c \tag{17}$$

$$\vec{c}_{i=5,8} = \sqrt{2}(\cos(\frac{2i-1}{4}\pi), \sin(\frac{2i-1}{4}\pi)).c \tag{18}$$

$$\omega_0 = \frac{4}{9} \quad \omega_{1,2,3,4} = \frac{1}{9} \quad \omega_{5,6,7,8} = \frac{1}{36} \tag{19}$$

It is to be noted that in the above equations, $c = \Delta r / \Delta t = \Delta z / \Delta t$ and the weights satisfy the relation $\sum_{i=0}^{8} \omega_i = 1$.

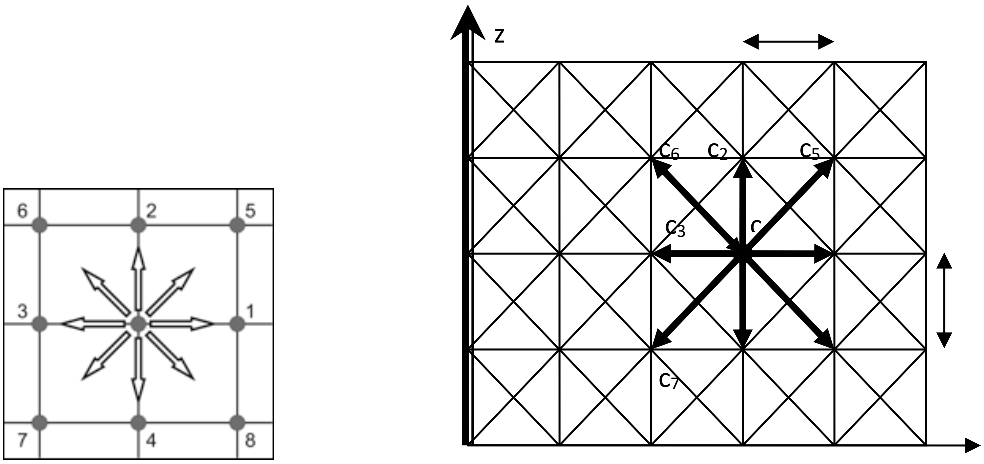


Fig. 3. Discretisation in axisymetric configuration

After discretization, Eq. (14) can be written as

$$f_i(\vec{r}+\vec{c}_i\Delta t, t+\Delta t) = f_i(\vec{r},t) - \frac{\Delta t}{\tau}[f_i(\vec{r},t) - f_i^{eq}(\vec{r},t)] \quad (20)$$

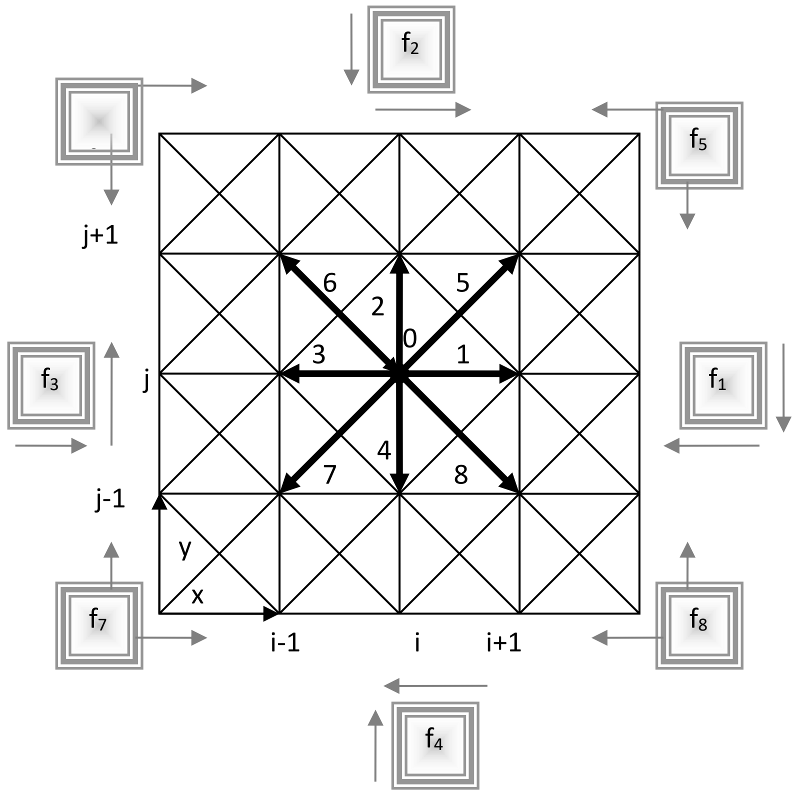


Fig. 4. Propagation in the LBM

In case of heat transfer problems, the temperature is obtained after summing f_i over all direction, i.e,

$$T(\vec{r},t) = \sum_{i=0}^{8} f_i(\vec{r},t) \quad (21)$$

To process Eq. (20), an equilibrium distribution function is required. For heat conduction problems, this is given by

$$f_i^{eq}(\vec{r},t) = \omega_i T(\vec{r},t) \quad (22)$$

$$f_i(\vec{r}+\vec{c}_i\Delta t, t+\Delta t) = f_i(\vec{r},t) - \frac{\Delta t}{\tau}[f_i(\vec{r},t) - f_i^{(0)}(\vec{r},t)] + \omega_i(\frac{\Delta t}{\rho c_p})(\frac{\lambda}{r}\frac{\partial T}{\partial r} - div(\vec{q}_R)) \quad (23)$$

where the divergence of radiative heat flux is given by:

$$\beta(1-\omega)(4\pi\frac{\sigma T^4}{\pi} - G) \quad (24)$$

G is the incident radiation.

Equation (23) is the equivalent form of the energy equation Eq. (1) in the LBM formulation, taking into account the presence of the volumetric radiation and the axisymmetric configuration. The boundary conditions are based on the properties of the known and unknown populations on each side as shown on figure 5. To express these conditions the bounce-back concept in the LBM in which particle fluxes are balanced at any point on the boundary was used.

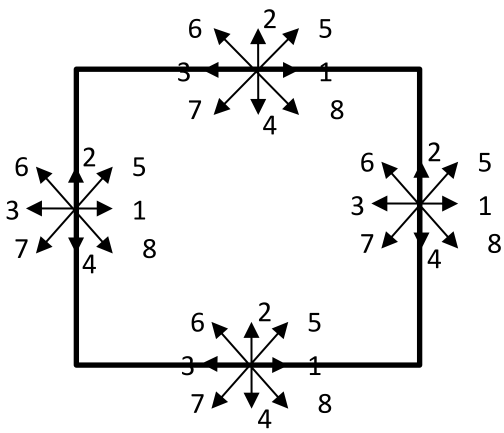


Fig. 5. Boundary conditions with known and unknown populations

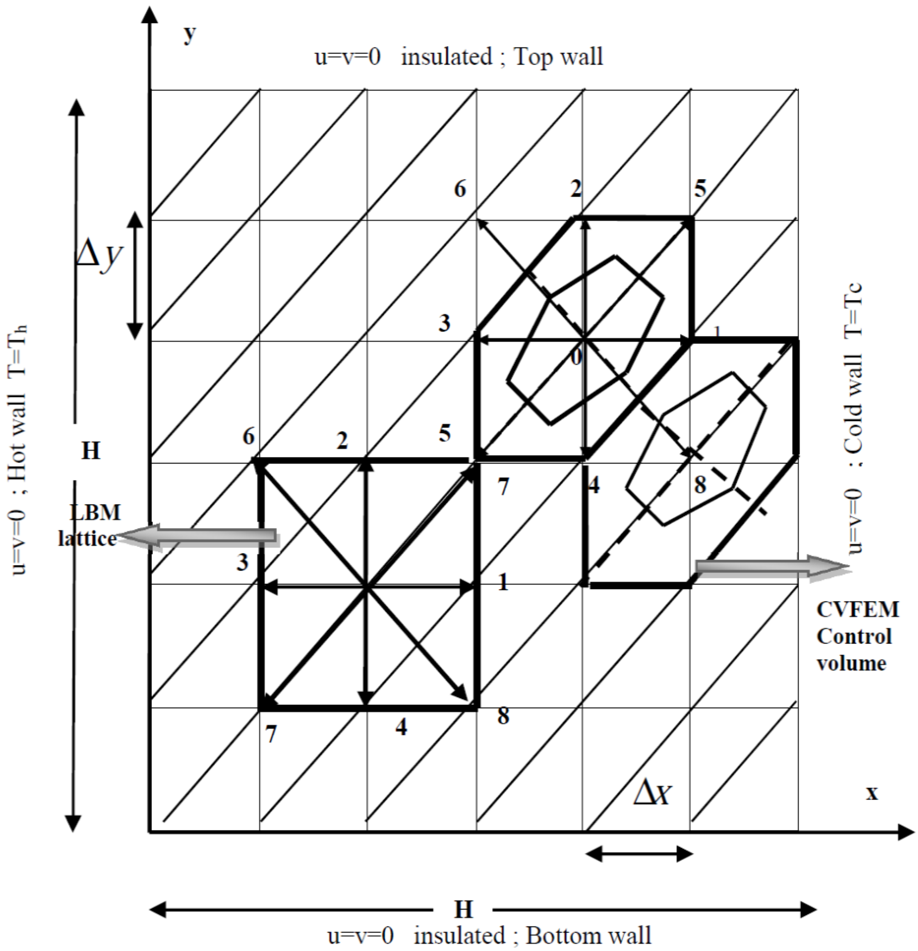


Fig. 6. Arrangement of lattices and control volumes in the domain

4. Results and discussion

4.1 Case 1: Transient heat conduction in an infinite cylinder

The problem of determining the distribution of temperature in a infinite cylinder in the presence of heat generation is considered [30-34]. The temperature must satisfy the partial differential equation

$$\frac{\partial T}{\partial t} = \frac{\alpha}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) + \frac{\alpha Q}{k} \tag{25}$$

The cylinder is of radius r_0, the faces are kept at T_s temperature and the flow of heat results from the initial distribution of temperature T_0 inside the cylinder.

The analytic solution of the problem is given by:

$$\theta = 1 - (r / r_0)^2 \tag{26}$$

where

$$\theta = \frac{T}{(4k(T - T_s) / Qr_0^2)} \tag{27}$$

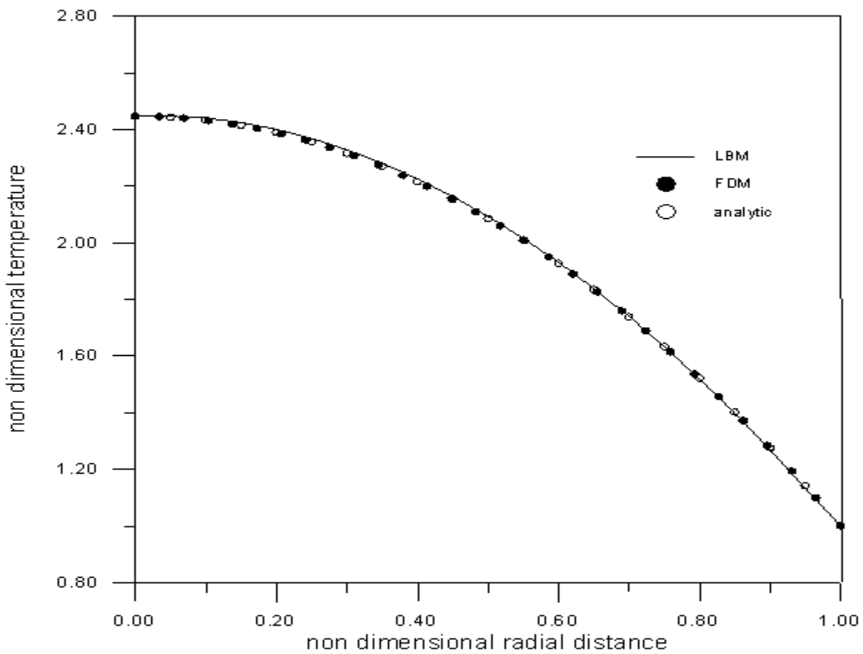
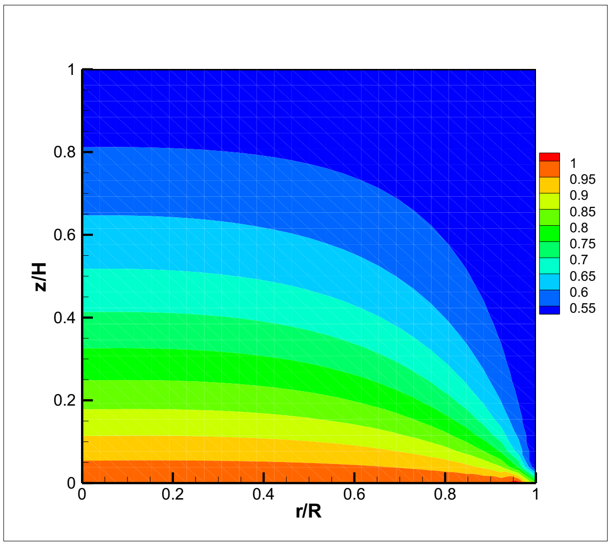


Fig.7. Non-dimensional temperature (case1)

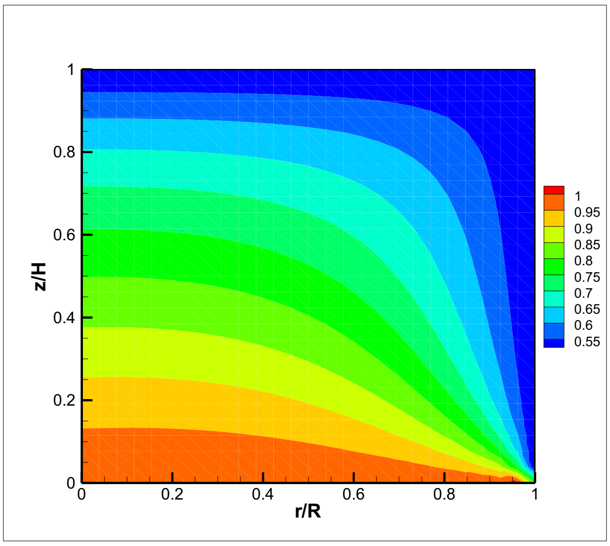
Case 2: *Transient heat conduction radiation in axisymmetric cylinder with heat generation/absorption*

After a grid and direction independency tests, in figure 100, we present steady state isotherms for a conduction-radiation parameter $N = 0.1$, For an extinction coefficient $\beta = 1$, a scattering albedo $\omega = 0.5$. First, the effect of an uniform heat generation is depicted in figure 8(a) and this, for g*=0,3,5. In figure 8(b), we highlight a heat absorption source on a transient conduction radiation axisymetric cylindrical participating medium.

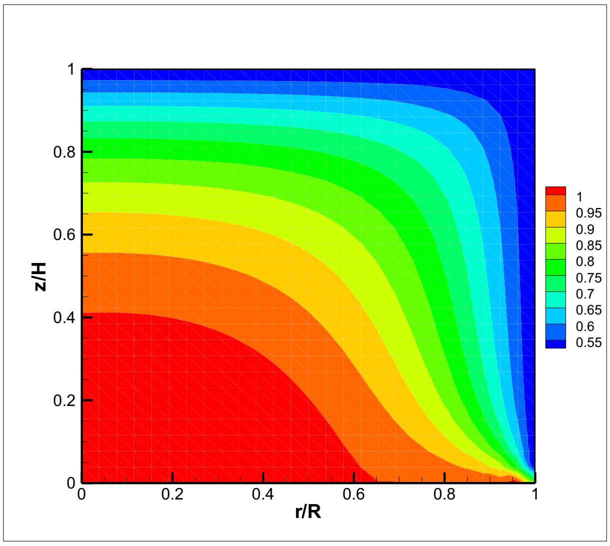
The bottom surface $(z/H = 0)$ at a high temperature $(T_b = T_{ref})$ and the remaining surfaces ($r/R = 1$ and $z/H = 0$) at a lower temperature $T = T_b/2$. The study reveals that (27×27) control volumes/lattices and $(N_\theta, N_\varphi) = (6,8)$ directions are sufficient to achieve the grid and ray independency.



N=0.1, g*=0, ω=0.5, β=1

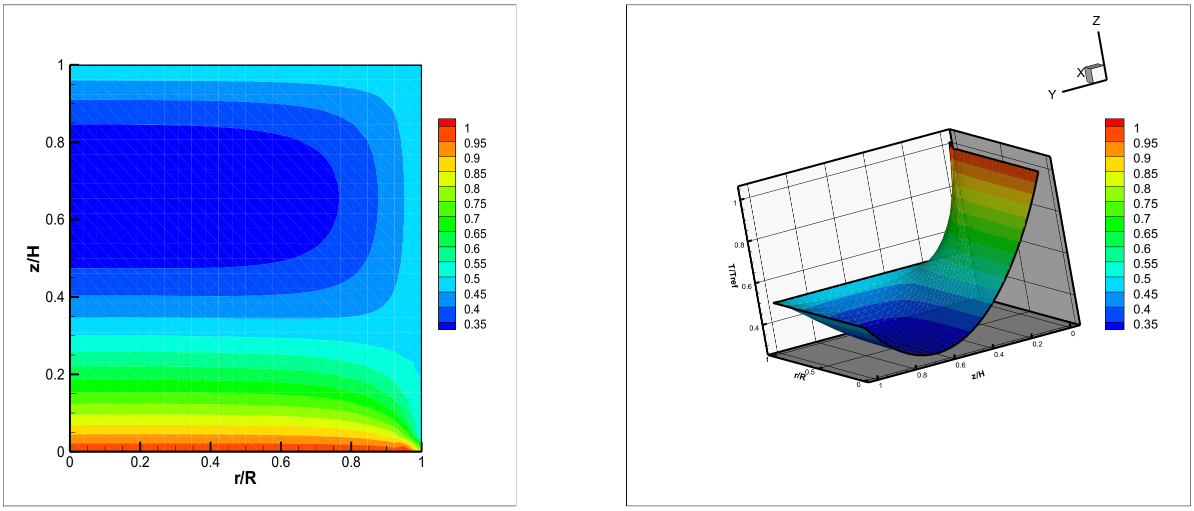


N=0.1, g*=2, ω=0.5, β=1

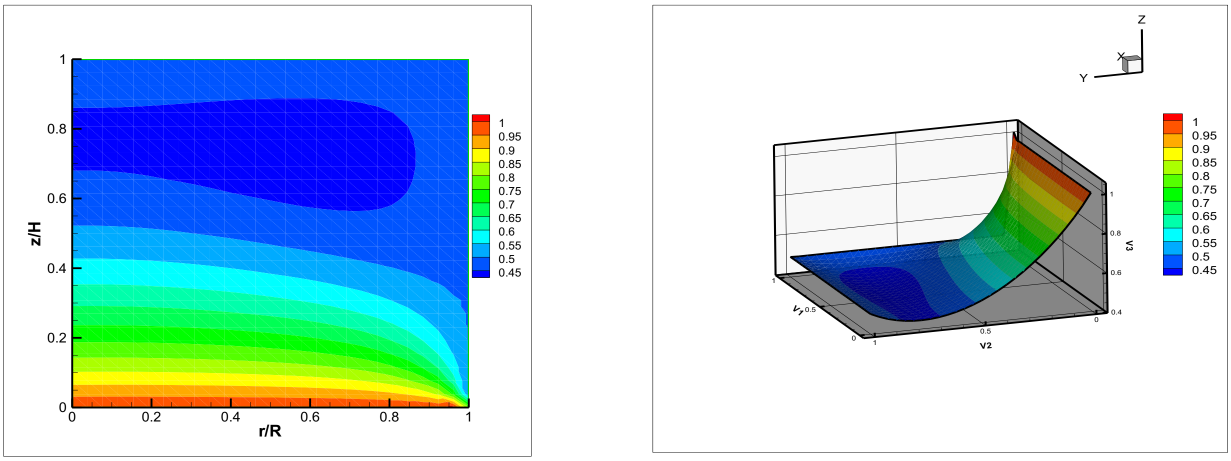


N=0.1, g*=5, ω=0.5, β=1

Fig. 8(a). Dimensionless steady state isotherms (case1)



N=0.1, g*=-5, ω=0.5, β=1



N=0.1, g*=-3, ω=0.5, β=1

Fig. 8(b). Dimensionless steady state isotherms

5. Conclusions

In this work, the usage of the LBM to solve transient conduction-radiation problem was extended to a 2-D cylindrical enclosure. The CVFEM was used to compute the radiative information required for the energy equation. To compare the performance of the LBM-CVFEM hybrid method, the problems were also solved using the FDM-CVFEM combinations. The study shows that internal heat generation modifies significantly temperature fields. The increase in the value of the heat generation parameter leads to increase in the temperature inside cavity also increases and hence that negates the heat transfer from the heated surface.

On the other hand, the presence of heat source within the enclosure causes an increase in the fluid temperature, leading to a reduction radiative heat transfer on the hot wall. while heat absorption produces lower temperature distributions inside the cylinder.

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