Journal of Advanced Research in Applied Sciences and Engineering Technology

Journal homepage:
https://semarakilmu.com.my/journals/index.php/applied_sciences_eng_tech/index
ISSN: 2462-1943

Investigation of Inner Elliptical Pin-Fins Configuration on Magnetoconvective Heat Transfer

Raoudha Chaabane[1,*], Nor Azwadi Che Sidik[2], Hong Wei Xian[2]

[1] Laboratory of Thermal and Energetic Systems Studies (LESTE) at the National School of Engineering of Monastir, University of Monastir, Tunisia
[2] Malaysia–Japan International Institute of Technology (MJIIT), Universiti Teknologi Malaysia, Jalan Sultan Yahya Petra, 54100 Kuala Lumpur, Malaysia

ABSTRACT

This paper deals with Lattice Boltzmann computation of two-dimensional incompressible flow past two heat elliptical sinks vertically attached to the horizontal walls of a cavity in the presence of magnetic field. A two dimensional nine-velocity square lattice Boltzmann (D2Q9) model is used. The obtained results show that the applied Lattice Boltzmann method (LBM) can effectively capture vortex shedding structures and other features inside such complex flow. The present study has been carried out for Hartmann numbers with panoply of heat sinks location. Results show that the heat sinks positions greatly influence the heat transfer rate depending on the Hartmann number.

Keywords:
Lattice Boltzmann method; D2Q9 model; convection; Heat transfer; MHD; curved boundary; elliptical sinks

Received: 26 June 2022	Revised: 11 August 2022	Accepted: 13 August 2022	Published: 21 August 2022

1. Introduction

Natural convection in MHD (magneto hydrodynamic) cavities is an important phenomenon in engineering systems and in various industries of high-performance insulation for packed sphere beds, grain storage, float glass production, buildings, injection molding chemical catalytic reactors, air-conditioning in rooms, cooling of electronic devices and geophysical problems. Extensive research studies using various numerical simulations were conducted into natural convection [1-24]. Some of the latter works have acquired a basic understanding of free convection flows and heat transfer characteristics MHD free convection flows because those configurations can be encountered in numerous problems with industrial and technological interest, covering a wide range of basic sciences such as nuclear engineering, fire research, crystal growth, astrophysics and metallurgy. In MHD free convection flows, the balance is achieved by inertial, viscous, electromagnetic and buoyancy forces, rendering the solution more complicated.

---

[*] *Corresponding author*
E-mail address: raoudhach@gmail.com

https://doi.org/10.37934/araset.27.2.2838

In order to simulate such engineering complex configuration with mixed boundary conditions and in the presence of internal elliptical heat sinks placed at different positions on the south and the north walls of the MHD cavity, the Lattice Boltzmann Method (LBM) is used. It is to be known that the (LBM) is a new method for simulating fluid flow and modeling physics in fluids. It has been applied extensively within the last decade. Based on kinetic theory for simulating fluid flows and modeling the physics in fluids [25-32], it becomes a powerful, effective and easy numerical method, for simulation of complex flow problems with different boundary conditions [33, 34].

In comparison with the conventional CFD methods, the LBM isbased on the microscopic models, mesoscopic kinetic equations. The macroscopic dynamics of a fluid is the result of collective behavior of many microscopic particles in the system. The LBM has been proved to recover the Navier-Stokes equation by using the Chapman-Enskog expansion. The major advantage of this method is its explicit feature of the governing equation, easy for parallel computation and easy for implementation of irregular boundary conditions. To our best knowledge, no previous study on effects of elliptical heat sinks on natural convection in an MHD cavity with the LBM had already been studied so far. The main aim of this paper is to study this complex configuration in cavity filled with liquid gallium (Pr=0.025) and a pure fluid (Pr=0.7) and also to highlight the ability of the LBM for solving problems with various complex boundary conditions. The qualitative effects of Hartmann number on horizontal velocity, vertical velocity streamlines, isotherms, mean square velocity and pressure are investigated.

2. Mathematical formulation

2.1 Study Cases

The considered model is shown in Figs.1-3. It displays a two-dimensional cavity with side length of H. the semi elliptical obstacles are with a radius (r) equal to the semi minor axis (a) and (b) the semi major axis equal to 0.25H. The left vertical is maintained at high temperature (T_H) while the three other walls are insulated and impermeable to mass transfer. The cavity can be filled with a liquid gallium with Pr number of 0.025 or a non MHD fluid (Pr=07). The gravitational acceleration acts downward. The uniform external magnetic field with a constant magnitude B_0 is applied in the transverse x-direction. It is assumed that the induced magnetic field produced by the motion of an electrically conducting fluid is negligible compared to the applied magnetic field. Thermo-physical properties of the fluid are assumed to be constant, and the density variation in the buoyancy force term is handled by the Boussinesq approximation. The flow is two-dimensional, laminar and incompressible; in addition, it is assumed that the viscous dissipation and Joule heating are neglected.

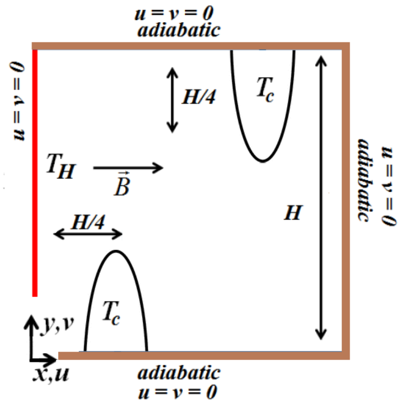


Fig. 1. Configuration 1 and mixed boundary conditions

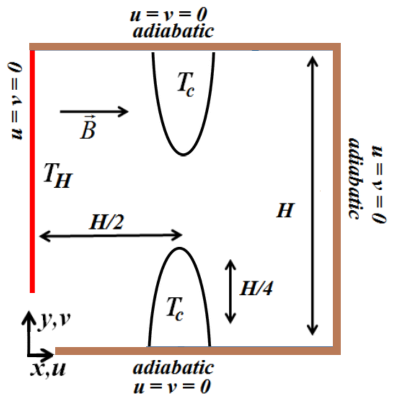


Fig. 2. Configuration 2 and mixed boundary conditions

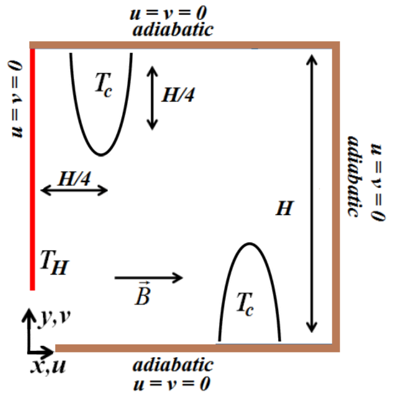


Fig. 3. Configuration 3 and mixed boundary conditions

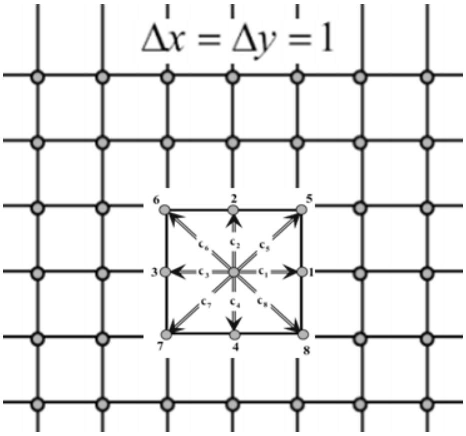


Fig. 4. The standard D_2Q_9 lattice for velocity and temperature mode

2.2 Numerical Method: Lattice Boltzmann Method (LBM)

A scheme of the standard D2Q9 (Figure 4) for flow and for temperature LBM method are used in this work [33-34] hence only brief discussion will be given in the following paragraphs, for completeness. The governing equations for MHD free convection are written in terms of the macroscopic variable depending on horizontal and vertical positions respectively as:
Continuity equation

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

Momentum equations

$$\rho(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y})=-\frac{\partial p}{\partial x}+\mu(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2})+F_x \tag{2}$$

$$\rho(u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y})=-\frac{\partial p}{\partial y}+\mu(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2})+F_y \tag{3}$$

Energy equation

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}) \tag{4}$$

where $v=\mu/\rho$ is the kinematic viscosity, F_x and F_y are the body forces at horizontal and vertical directions respectively and they are defined as follows [25]:

$$F_x = R(v\sin\gamma\cos\gamma - u\sin^2\gamma) \tag{5}$$

$$F_y = R(u\sin\gamma\cos\gamma - v\cos^2\gamma)+\rho g\beta(T-T_m) \tag{6}$$

where the Ha number is defined as

$$Ha = L\,B_x\sqrt{\sigma / \mu} \tag{7}$$

In the LBM the total force is:

$$F = F_x + F_y \tag{8}$$

$$F_x = 3w_k\,\rho\left[R(v\sin\gamma\cos\gamma) - u\sin^2\gamma)\right] \tag{9}$$

$$F_y = 3w_k\rho\begin{bmatrix} g_y\beta(T - T_m) \\ +R(u\sin\gamma\cos\gamma) - v\cos^2\gamma \end{bmatrix} \tag{10}$$

where $R = \mu Ha^2$ and γ is the direction of the magnetic field.

The functional density, velocity and temperature in the mesoscopic approach are:

$$\rho(\mathbf{r},t) = \sum_k f_k(\mathbf{r},t) \tag{11}$$

$$\mathbf{u}(\mathbf{r},t) = \sum_k \mathbf{c}_k f_k(\mathbf{r},t) / \rho(\mathbf{r},t) \tag{12}$$

$$T = \sum_k g_k(\mathbf{r},t) \tag{13}$$

3.3 Solution Method and Algorithm

To ensure that the code simulates an approximated incompressible regime, the characteristic velocity of the flow ($V = \sqrt{g\beta_T H(T_w - T_\infty)}$) at the free convection regime must be compared with the fluid speed of sound c_s. In the present study, the characteristic velocity is selected as 0.1 of sound speed. By fixing Ra number $Ra = \beta g_y H^3 \Pr(T_H - T_C)/\nu^2$, Pr number $\Pr = \alpha\nu$ and Mach number ($Ma = u/c$), the kinematic viscosity $\nu = Ma\,Ha\,c_S\sqrt{\Pr/Ra}$ and thermal diffusivity α are deduced. Nusselt number *Nu* is one of the most important dimensionless parameters in describing the convective and thermic heat transport. For the example, the local Nusselt number and the average value at the left side wall are calculated as;

$$Nu_y = -\frac{H}{T_H - T_C}\frac{\partial T}{\partial x}\bigg|_{x=0} \tag{14}$$

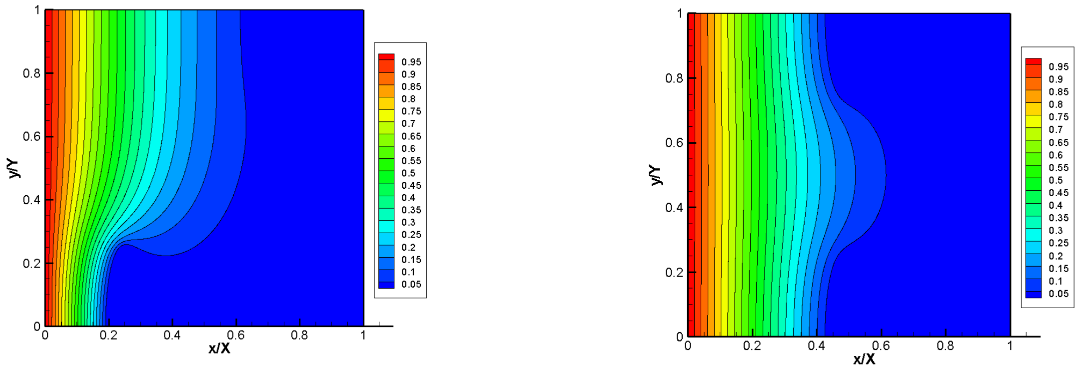
$$Nu_{av} = \frac{1}{H}\int_0^H Nu_y\,dy \tag{15}$$

In the following section, we present the steps that must be executed for the numerical implementation:

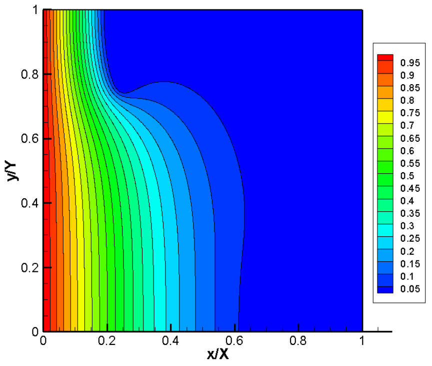
- Input constant parameters.
-Calculate the relaxation times.
- Initialize the temperature, and velocity fields for the entire computational domain and the equilibrium distribution functions.
- Calculate the current distribution function for propagation of temperature and velocity.
- Propagate all current distribution functions to the neighbor lattices in all directions.
- Impose the boundary conditions.
- Calculate the current temperature and velocity field for the entire computational domain and the current equilibrium distribution functions using the current temperature and velocity fields
- The error convergence is checked, if the error doesn't satisfies the convergence criterion, go to step

4. Results and Discussion

Figures 5 and 6 highlight the great influence of the positions of the heat sinks, with three proposed different configurations, on the flow intensity by the modification of the temperature distribution and the streamlines inside the cavity (Pr=0.7 and Ra = 10^5). For the first configuration, the cold zone is located at the bottom of the cavity next to cold wall, for the third configuration I the cold zone is located at the top of the cavity next to cold wall and for the second one, the cold zone is located symmetrically in the medium of the cavity next to the right wall (Fig. 5). The position and the size of these cold zones influence the flow intensity in the cavity (Fig. 6)

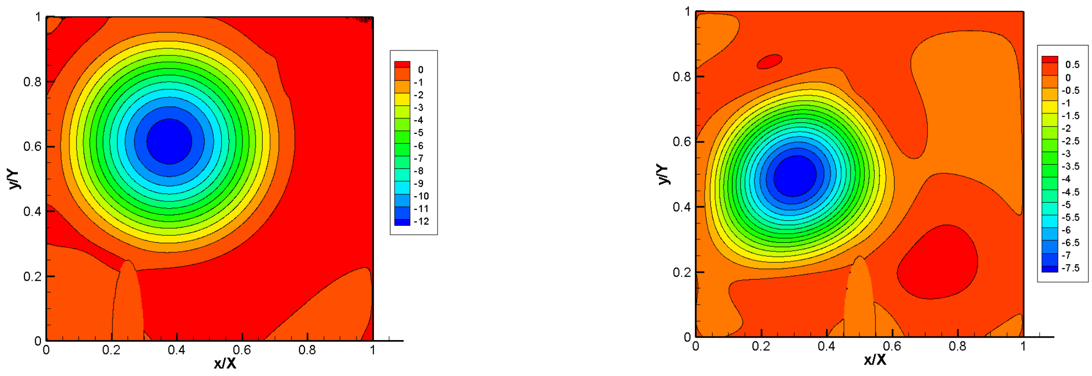


Configuration 1 Configuration 2



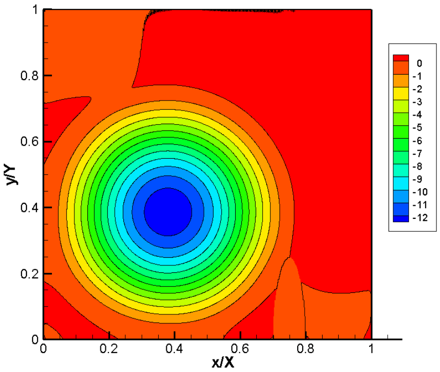
Configuration 3

Fig. 5. isotherms for aspect ratio AR=10 (a=r and b=0.25H) when Ra = 10^5 and Pr=0.7 for different configurations of heat sink's positions



Configuration 1

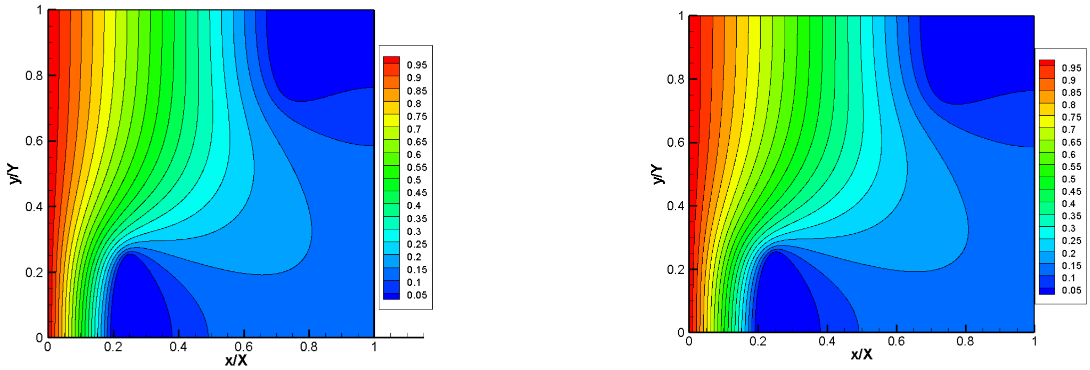
Configuration 2



Configuration 3

Fig. 6. isotherms for aspect ratio AR=10 (a=r and b=0.25H) when Ra = 10^5 and Pr=0.7 for different configurations of heat sink's positions

Figures (7-12) shows the qualitative effect of the Hartmann (Ha=0 , absence of MHD) and (Ha=150) number with a high Rayleigh number (Ra=10^5) in the case of liquid gallium, on the horizontal velocity (fig. 7), the vertical velocity (Fig. 8), the streamlines (Fig. 9), the isotherms (Fig. 10), the mean square velocity (Fig. 10) and the pressure (Fig. 11) of the liquid gallium with Pr=0.025. This simulation is for a specific placement configuration of the heat sinks (configuration 1).

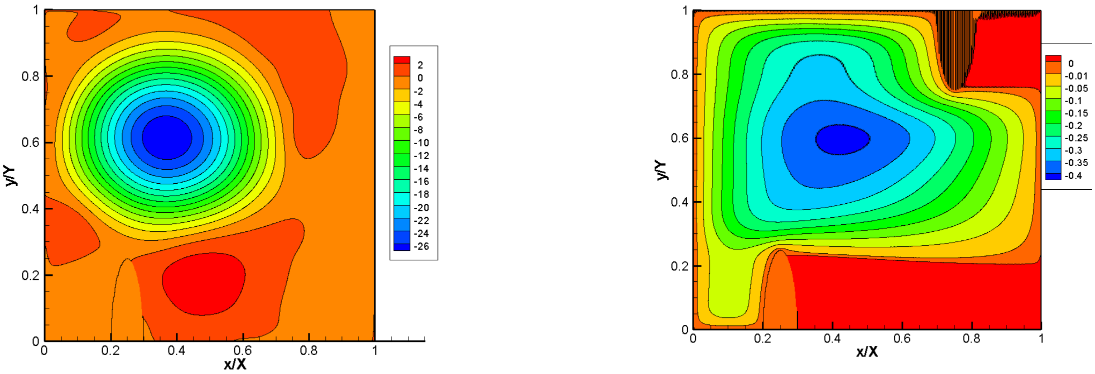


Ha=0

Ha=150

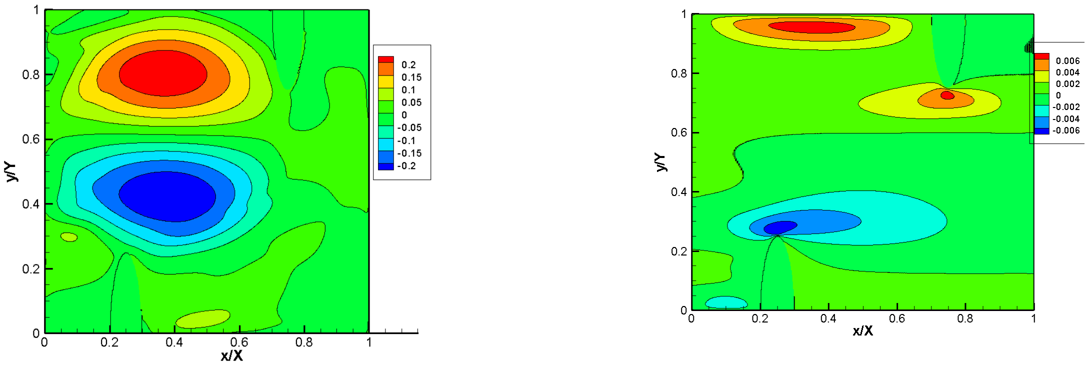
Fig. 7. Isotherms for aspect ratio AR=10 (a=r and b=0.25H) when Ra = 10^5 and Pr=0.025. (configuration 1)

The isotherms show, for all Hartmann number, the same patterns while for the streamlines a great variation is highlighted. In fact, a circular big cell in clockwise rotation is formed inside the cavity for Ha=0 that increase and extend differently when Hartmann increase (Figs. 8-11). We notice also that, the presence of heat sinks produces inactive areas next to the right vertical wall (Fig. 7). We highlight an intense temperature gradient between the hot wall and the cold semi elliptical sinks, indicating strong heat transfer rate in this region. The patterns of pressure are highly influenced by increasing Hartmann number (Fig. 12).



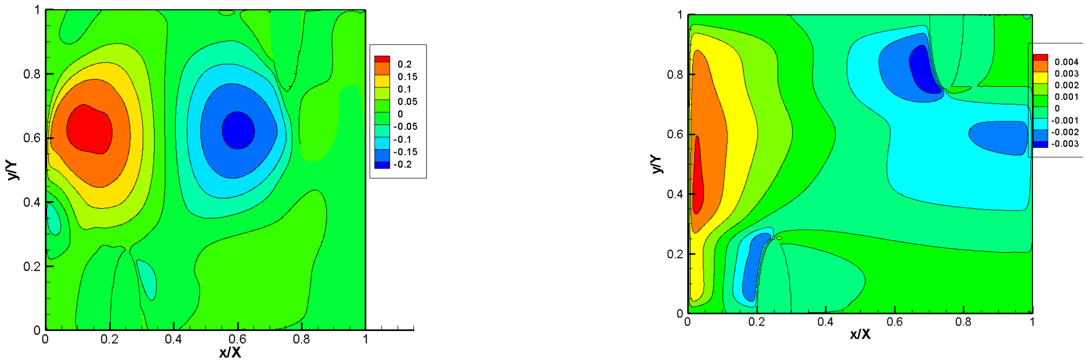
Ha=0 Ha=150

Fig. 8. Streamlines for aspect ratio AR=10 (a=r and b=0.25H) when Ra = 10^5 and Pr=0.025. (Configuration 1)



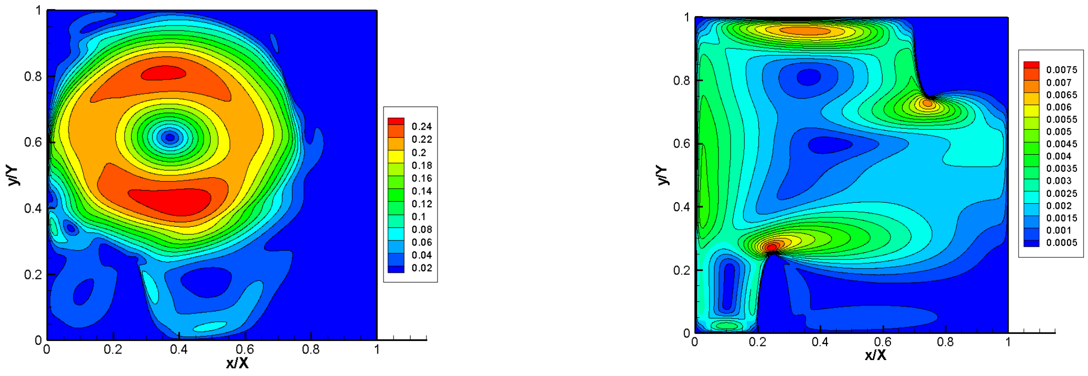
Ha=0 Ha=150

Fig. 9. Horizontal velocity for aspect ratio AR=10 (a=r and b=0.25H) when Ra = 10^5 and Pr=0.025. (Configuration 1)



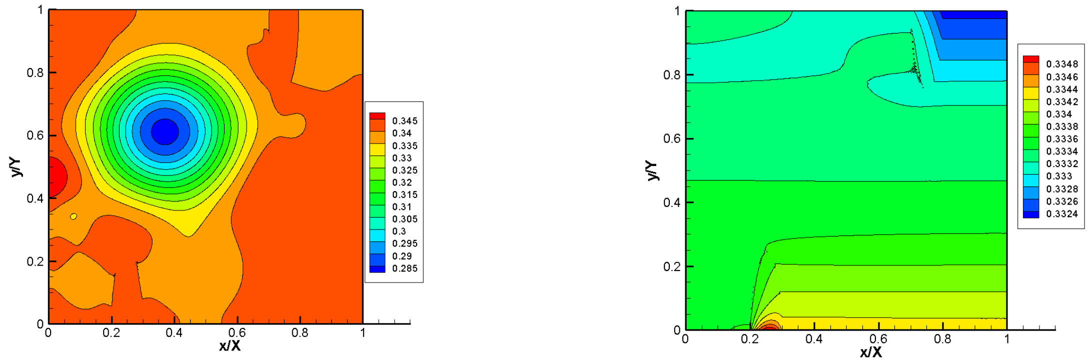
Ha=0 Ha=150

Fig. 10. Vertical velocity for aspect ratio AR=10 (a=r and b=0.25) when Ra = 10^5 and Pr=0.025. (Configuration 1)



Ha=0 Ha=150

Fig. 11. Mean square velocity for aspect ratio AR=10 (a=r and b=0.25H5) when Ra = 10^5 and Pr=0.025. (Configuration 1)



Ha=0 Ha=150

Fig. 12. Pressure for aspect ratio AR=10 (a=r and b=0.25H) when Ra = 10^5 and Pr=0.025. (Configuration 1)

Figure 13 show the local Nusselt number along the hot wall for Ra=10^5 , Ha= 0 Pr=0.7and Pr=0.025 respectively. The results show that for a high Rayleigh number, the local Nusselt number increase with the decrease of Prandtl number in the absence of magnetic effect.

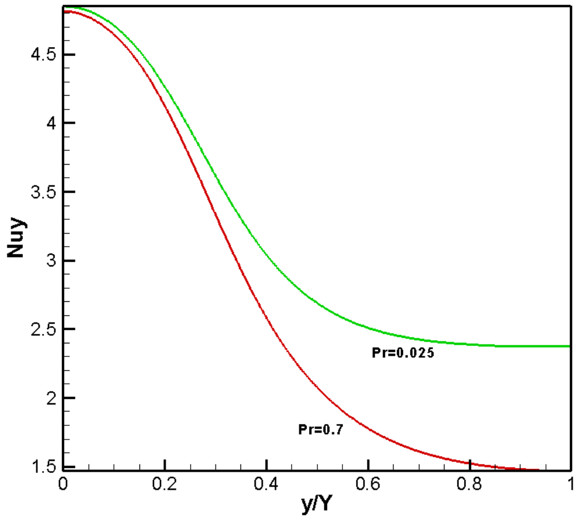


Fig. 13. Variation of the local Nusselt number for e for aspect ratio AR=10 (a=r and b=0.25H) when Ra = 10^5 and Ha=0. (Configuration 1)

5. Conclusions

The present study considered steady state, laminar, two-dimensional MHD free convection within a liquid gallium filled square enclosure in the presence of in-plane magnetic field for mixed boundary conditions in the presence of semi elliptical heat sinks.

In the present paper the effects of the positions of semi elliptical heat sinks and Hartmann number has been analyzed with LBM. This investigation demonstrated ability of LBM for simulation such complex engineering configuration. In addition, this method helps to obtain stream function and isotherm counters smoothly. This investigation was performed for various positions of semi elliptical heat sinks and Hartmann number and we deduce that The heat sinks positions greatly influence the heat transfer rate depending on the Hartmann number for a high Rayleigh number of $Ra=10^5$. Besides in the absence of MHD effect the local Nusselt number increase with the decrease of the Prandtl number.

References

[1] Tey, Wah Yen, Yutaka Asako, Nor Azwadi Che Sidik, and Rui Zher Goh. 2017. “Governing Equations in Computational Fluid Dynamics: Derivations and A Recent Review”. *Progress in Energy and Environment* 1 (June):1-19.

[2] Polat, O. and Bilgen, E., 2002. Laminar natural convection in inclined open shallow cavities. *International Journal of Thermal Sciences*, *41*(4), pp.360-368. https://doi.org/10.1016/S1290-0729(02)01326-1

[3] Ayegbusi, Olutobi Gbenga, Abdullah Sani Ahmad, and Yaik Wah Lim. 2018. “Overall Thermal Transfer Value of Naturally Ventilated Double Skin Façade in Malaysia”. *Progress in Energy and Environment* 5 (May):16-26.

[4] Mohamad, A. A., R. Bennacer, and Mohammed El-Ganaoui. "Double dispersion, natural convection in an open end cavity simulation via Lattice Boltzmann Method." *International Journal of Thermal Sciences* 49, no. 10 (2010): 1944-1953. https://doi.org/10.1016/j.ijthermalsci.2010.05.022

[5] Rahimah Mahat, Muhammad Saqib, Imran Ulah, Sharidan Shafie, and Sharena Mohamad Isa. 2022. “MHD Mixed Convection of Viscoelastic Nanofluid Flow Due to Constant Heat Flux”. *Journal of Advanced Research in Numerical Heat Transfer* 9 (1):19-25.

[6] Fidaros, D., A. Grecos, and N. Vlachos. "Development of numerical tool for 3D MHD natural convection." *Annex xx*(2017): 73-74.

[7] Abdulhafid M. A. Elfaghi, Alhadi A. Abosbaia, Munir F. A. Alkbir, and Abdoulhdi A. B. Omran. 2022. “CFD Simulation of Forced Convection Heat Transfer Enhancement in Pipe Using Al2O3/Water Nanofluid”. *Journal of Advanced Research in Numerical Heat Transfer* 8 (1):44-49.

[8] Alchaar, S., P. Vasseur, and E. Bilgen. "Natural convection heat transfer in a rectangular enclosure with a transverse magnetic field." (1995): 668-673. https://doi.org/10.1115/1.2822628

[9] Garandet, J. P., T. Alboussiere, and R. Moreau. "Buoyancy driven convection in a rectangular enclosure with a transverse magnetic field." *International journal of heat and mass transfer*35, no. 4 (1992): 741-748. https://doi.org/10.1016/0017-9310(92)90242-K

[10] Mohamad WafirulHadi, Titin Trisnadewi, and Nandy Putra. 2021. “Thermal Management System Based on Phase Change Material (PCM) and Heat Pipe in Lithium-Ion Electric Vehicle Batteries”. *Journal of Advanced Research in Experimental Fluid Mechanics and Heat Transfer* 3 (1):26-35.

[11] Cowley, M. D. "Natural convection in rectangular enclosures of arbitrary orientation with vertical magnetic field." *Magnetohydrodynamics* 32, no. 4 (1996): 390-424.

[12] Akbar, Ronald, A. S. Pamitran, and J. T. Oh. 2021. “Two-Phase Flow Boiling Heat Transfer Coefficient With R290 in Horizontal 3 Mm Diameter Mini Channel”. *Journal of Advanced Research in Experimental Fluid Mechanics and Heat Transfer* 3 (1):1-8.

[13] Ozoe, Hiroyuki, and Kazuto Okada. "The effect of the direction of the external magnetic field on the three-dimensional natural convection in a cubical enclosure." *International Journal of Heat and Mass Transfer* 32, no. 10 (1989): 1939-1954. https://doi.org/10.1016/0017-9310(89)90163-4

[14] Ece, Mehmet Cem, and Elif Büyük. "Natural-convection flow under a magnetic field in an inclined rectangular enclosure heated and cooled on adjacent walls." *Fluid Dynamics Research* 38, no. 8 (2006): 564. https://doi.org/10.1016/j.fluiddyn.2006.04.002

[15] Abdulhafid M A Elfaghi, Alhadi A Abosbaia, Munir F A Alkbir, and Abdoulhdi A B Omran. 2022. “CFD Simulation of Forced Convection Heat Transfer Enhancement in Pipe Using Al2O3/Water Nanofluid”. *Journal of Advanced Research in Micro and Nano Engineering* 7 (1):8-13.

[16] Jalil, J. M., and K. A. Al-Tae’y. "MHD turbulent natural convection in a liquid metal filed square enclosure." *Emirates Journal for Engineering Research* 12, no. 2 (2007): 31-40.

[17] Gelfgat, A. Yu, and P. Z. Bar-Yoseph. "The effect of an external magnetic field on oscillatory instability of convective flows in a rectangular cavity." *Physics of Fluids* 13, no. 8 (2001): 2269-2278. https://doi.org/10.1063/1.1383789

[18] Aleksandrova, Svetlana, and Sergei Molokov. "Three-dimensional buoyant convection in a rectangular cavity with differentially heated walls in a strong magnetic field." *Fluid Dynamics Research* 35, no. 1 (2004): 37. https://doi.org/10.1016/j.fluiddyn.2004.04.002

[19] Mohd Zokri, Syazwani, Nur Syamilah Arifin, Abdul Rahman Mohd Kasim, and Mohd Zuki Salleh. 2020. “Free Convection Boundary Layer Flow of Jeffrey Nanofluid on a Horizontal Circular Cylinder With Viscous Dissipation Effect”. *Journal of Advanced Research in Micro and Nano Engineering* 1 (1):1-14.

[20] Kefayati, GholamReza, Mofid Gorji, Hasan Sajjadi, and Davood Domiri Ganji. "Investigation of Prandtl number effect on natural convection MHD in an open cavity by Lattice Boltzmann Method." *Engineering Computations* (2013). https://doi.org/10.1108/02644401311286035

[21] Bilgen, E., and R. Ben Yedder. "Natural convection in enclosure with heating and cooling by sinusoidal temperature profiles on one side." *International Journal of Heat and Mass Transfer* 50, no. 1-2 (2007): 139-150. https://doi.org/10.1016/j.ijheatmasstransfer.2006.06.027

[22] Waqar Ahmed, Nor Azwadi Che Sidik, and Yutaka Asako. 2022. “Metal Oxide and Ethylene Glycol Based Well Stable Nanofluids for Mass Flow in Closed Conduit”. *Journal of Advanced Research in Micro and Nano Engineering* 6 (1):1-15.

[23] Varol, Yasin, Hakan F. Oztop, and Ioan Pop. "Numerical analysis of natural convection for a porous rectangular enclosure with sinusoidally varying temperature profile on the bottom wall." *International Communications in Heat and Mass Transfer* 35, no. 1 (2008): 56-64. https://doi.org/10.1016/j.icheatmasstransfer.2007.05.015

[24] Saeid, Nawaf H., and Yusli Yaacob. "Natural convection in a square cavity with spatial side-wall temperature variation." *Numerical Heat Transfer, Part A: Applications* 49, no. 7 (2006): 683-697. https://doi.org/10.1080/10407780500359943

[25] Chaabane, Raoudha, Nor Azwadi Che Sidik, and Abdelmajid Jemni. 2021. “Convective Boundary Conditions Effect on Cylindrical Media With Transient Heat Transfer”. *Journal of Advanced Research in Fluid Mechanics and Thermal Sciences* 82 (2):146-56. https://doi.org/10.37934/arfmts.82.2.146156

[26] Chaabane, Raoudha, Faouzi Askri, and Sassi Ben Nasrallah. "Parametric study of simultaneous transient conduction and radiation in a two-dimensional participating medium." *Communications in Nonlinear Science and Numerical Simulation* 16, no. 10 (2011): 4006-4020. https://doi.org/10.1016/j.cnsns.2011.02.027

[27] Ismail, A. ., Jahanshaloo, L. ., & Fazeli, A. . (2020). Lagrangian Grid LBM to Predict Solid Particles’ Dynamics immersed in Fluid in a Cavity. *Journal of Advanced Research in Fluid Mechanics and Thermal Sciences*, *3*(1), 17–26.

[28] Chaabane, Raoudha, Faouzi Askri, and Sassi Ben Nasrallah. "Analysis of two-dimensional transient conduction–radiation problems in an anisotropically scattering participating enclosure using the lattice Boltzmann method and the control volume finite element method." *Computer Physics Communications* 182, no. 7 (2011): 1402-1413. https://doi.org/10.1016/j.cpc.2011.03.006

[29] Perumal, D. Arumuga, Gundavarapu V.S. Kumar, and Anoop K. Dass. 2021. “Lattice Boltzmann Simulation of Viscous Flow past Elliptical Cylinder”. *CFD Letters* 4 (3):127-39.

[30] Chaabane, Raoudha, Faouzi Askri, and Sassi Ben Nasrallah. "Application of the lattice Boltzmann method to transient conduction and radiation heat transfer in cylindrical media." *Journal of Quantitative Spectroscopy and Radiative Transfer*112, no. 12 (2011): 2013-2027. https://doi.org/10.1016/j.jqsrt.2011.04.002

[31] Chaabane, Raoudha, Faouzi Askri, Abdelmajid Jemni, and Sassi Ben Nasrallah. "Numerical study of transient convection with volumetric radiation using an hybrid lattice Boltzmann bhatnagar–gross–krook–control volume finite element method." *Journal of Heat Transfer* 139, no. 9 (2017). https://doi.org/10.1115/1.4036154

[32] Chaabane, Raoudha, Faouzi Askri, Abdelmajid Jemni, and Sassi Ben Nasrallah. "Analysis of Rayleigh-Bénard convection with thermal volumetric radiation using Lattice Boltzmann Formulation." *Journal of Thermal Science and Technology* 12, no. 2 (2017): JTST0020-JTST0020. https://doi.org/10.1299/jtst.2017jtst0020

[33] Perumal, D. Arumuga, Gundaravarapu V.S. Kumar, and Anoop K. Dass. 2021. “Application of Lattice Boltzmann Method to Fluid Flows in Microgeometries”. *CFD Letters* 2 (2):75-84.

[34] Succi, Sauro. *The lattice Boltzmann equation: for fluid dynamics and beyond*. Oxford university press, 2001.

[35] Series, R. W., and D. T. J. Hurle. "The use of magnetic fields in semiconductor crystal growth." *Journal of crystal growth* 113, no. 1-2 (1991): 305-328. https://doi.org/10.1016/0022-0248(91)90036-5