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Energy Analysis and the Determination of the Resonant Frequency of a Custom-Designed Scientific Oscillating Body

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ABSTRACT

This paper computes the resonant frequency and energy curves of a harmonically oscillating pendulum. The apparatus was customized in design, and could also be used for demonstrating and validating other oscillatory phenomena with extreme accuracy. The determined resonant frequencies of three pendula of length 21.10 cm, 23.84 cm, and 33 cm, used in this work are 1.07 Hz, 1.0 Hz, and 0.86 Hz respectively, which were found to be close to theoretically calculated values, i.e., the natural frequency of the pendulum. The error analysis of the determined resonant frequencies validated the consistency of the designed laboratory apparatus with the minute errors of 0.009, 0.019, and 0.011 respectively. The potential and kinetic energy calculated for the bob positions ranging from -5.30 cm to +5.30 cm showed an inverse relationship between the two with a total energy of 115.17 J for all the experimental positions of the bob. As an annexure of the study, the paper also determined the g (acceleration due to gravity) value of 9.78 m/s^2 from the experiment, which is very close to its standard value, 9.8 m/s^2.

1. Introduction

Resonance, which occurs when an object experiences a vibratory force close to one of the natural frequencies at which it easily vibrates, has been primarily understood as a property of vibrations of a system. In general the resonant frequency is close to but not necessarily the same as the natural frequency [1]. When an oscillatory force is applied at one resonant frequency of a dynamic system, the system will vibrate with a higher amplitude than if the same force were applied at another non-resonant frequency. The frequency at which the response amplitude is relatively maximum is also called the resonant frequency or system resonant frequency [2]. Forced oscillatory motion has diverse applications in the field of engineering, and its experimental realization of many practical

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applications can be achieved by adding various types of damping forces to the forced vibration [3]. The goal of forced oscillation detection and isolation is to remove the source of the disturbance [4]. The less damping a system has, the higher the amplitude of the forced oscillations near resonance, and a pendulum demonstrates various aspects of forced oscillatory motion and is studied since the time of Galileo [5]. It is used for time keeping and in machines requiring synchronous motion [6-8]. Pendulums are used as a model for demonstrating simple harmonic oscillations, determining acceleration due to gravity and for demonstrating the rotation of earth about its axis [9-11]. Its oscillations can be denoted as simple harmonic when confined to small displacements, and the duration of the oscillations is independent of the mass and angle of displacement [9] [12]. A swing element such as a pendulum used for scientific experiment can as well demonstrate mechanical resonance [13-15]. Mechanical resonance is the tendency of a mechanical system to absorb more energy when the frequency of vibration matches the system's natural vibration frequency than at other frequencies. When designing objects, engineers usually ensure that the mechanical resonant frequencies of the components do not match the driving vibrational frequencies of motors and other vibrating parts. This is nothing but a phenomenon known as resonance catastrophe. Additionally, acoustic resonance, a subset of mechanical resonance, is focused on mechanical vibrations that fall within the range of human hearing, or sound. Hearing is typically restricted to frequencies between 20 Hz and 20,000 Hz for humans (20 kHz) [16].

Due to the relevance of mechanical resonance in research and engineering, the issue of forced oscillations is studied in physics [17]. For instance, resonance must be taken into account when building bridges and other structures. When natural forces like wind and earthquake strike a structure over an extended length of time, it is vulnerable to damages, if its natural frequency is close to that of the natural forces [18]. Most physics textbooks highlight collapsing of Tacoma Narrow Bridge in USA, in the year 1940 as an example of resonance-based disaster [19]. Study on mechanical resonance demand laboratory practical validation. The importance of validating theoretical aspects of physics with practical data is highlighted by Rizal [20]. Similar remarks is being made on the inclusion of technology in teaching and learning of Mathematics by Jafar *et.al.*, [21], showing how the technology enhances learning through direct interaction with the relevant technical equipment like computers and related scientific equipment. In the past, several researchers have made attempts for demonstrating mechanical resonance for physics laboratory using objects such as springs [22] and hack-saw blade [23]. However, there are no reports on the existence of laboratory apparatus to measure resonance in pendulum and for determining energetics of simple harmonic oscillator with a pendulum as an example. The design and development of such apparatus is constrained by inability to manually measure the amplitude of oscillating pendulum [24].

The fundamental procedure of this work might be a good reference for advancing the idea of oscillating bodies into other field of studies like ocean energy analysis. As the problems of the energy crisis and environmental pollution are rapidly getting worse these days [25], the current work dealing with kinetic energy analysis of an oscillating body is capable to emanate explaining many future sources of renewable energy. This work has developed a novel scheme to demonstrate and to measure resonant frequency of a simple pendulum. The apparatus consists of strategic choice of electronic and electrical equipment, placed at appropriate positions for favouring resonance on pendulum and electronically recording data for calculating the resonant frequency. The apparatus is also capable of producing data for calculating energy (potential, kinetic and total energy) of oscillating pendulum and determining acceleration due to gravity (g). Using the data from the apparatus, each of these parameters are calculated and their corresponding energy curves are presented.

2. Methodology

A customized electromagnet and an ultrasound range sensor named HC-SR04 driven with a microcontroller named Atmega-328 develops the setup of the study where the ultrasound range sensor originates. This electromagnet coupled with permanent magnet on the pendulum string have been used to actuate the pendulum in the current work like the way a number of previous studied are done, for example the work of the similar process is being utilized by Adnan *et.al.*, [26]. The software program downloaded on the microcontroller enables the ultrasound range sensor to read nine positions of the oscillating pendulum in one second. The data acquisition continues until the ultrasound system is manually switched off using a switch. The data is emulated from the microcontroller using a freely available *CoolTerm* application [27]. The pendulum for the current study used for determining the resonant frequency is made by attaching a hollow plastic ball (diameter 7.7cm, mass 0.0188 kg) at one end of a light, un-stretchable string. A permanent magnet whose mass is negligibly small compared to the mass of the pendulum bob is attached on the string to actuate the pendulum with magnetic force generated from the electromagnet, placed at the proximity of the permanent magnet. A function generator supplied by PISCO (ISO 9001:2008) is used for driving the electromagnet with a sine signal of desired amplitude and frequency.

2.1. Computation Methods
2.1.1. Resonant frequency

A sine signals of 12 Vpp, ranging in frequency from 0.79 Hz to 1.14 Hz was applied to the electromagnet from the function generator for setting the pendulum on oscillations. At each applied frequency, the time and the corresponding amplitude of the bob were electronically recorded until the pendulum completed 60 oscillations. The data was then visualized by plotting the oscillation curve as a function of time in an Excel spreadsheet and the amplitude of 50th oscillation was selected for plotting the resonance curve. The experiment was repeated for three similar pendula having length 21.10 cm, 23.84 cm and 33.00 cm.

$$\frac{1}{f_0} = 2\pi\sqrt{\frac{l}{g}} \quad (1)$$

The resonant frequency (f) obtained from the resonance curves were compared with the natural frequency (f_0) of all pendula computed from Eq. (1), using standard g value 9.8 m/s^2.

2.1.2. Energy and acceleration due to gravity (g)

The pendulum described earlier was set to oscillations by displacing it manually from its equilibrium position and when the oscillation is stabilized, the bob positions were recorded along with the time of oscillation. The recorded data were plotted to obtain the oscillation curve and the average time period T of five oscillation was obtained. The value of T and the mass m of the pendulum bob were used in Eq. (2) to determine the restoration constant k.

$$\omega = \frac{2\pi}{T} = \sqrt{\frac{k}{m}} \quad (2)$$

The kinetic and the potential energy of the oscillating pendulum were determined using Eq. (3) and Eq. (4) respectively, where x and A respectively defines the pendulum bob position and the maximum amplitude of oscillation [28].

$$U = \left(\frac{1}{2}\right) kx^2 \tag{3}$$

$$K = \left(\frac{1}{2}\right) k(A^2 - x^2) \tag{4}$$

Next, we fitted the curve obtained by plotting T^2 against the length l of the pendulum with the linearized pendulum Eq. (5) to determine the acceleration due to gravity [9].

$$T^2 = \left(\frac{4\pi^2}{g}\right) l \tag{5}$$

$$x = \frac{F_o/m}{\omega_0{}^2 - \omega^2} \cos \omega t \tag{6}$$

The oscillation profile of this curve fits with the curve described by the theoretical Eq. (6) [13], demonstrating the occurrence resonance on the pendulum.

3. Results

The oscillation profile of a pendulum driven with pulsating magnetic force of frequency equal to the resonant frequency of the pendulum is shown in Figure 1.

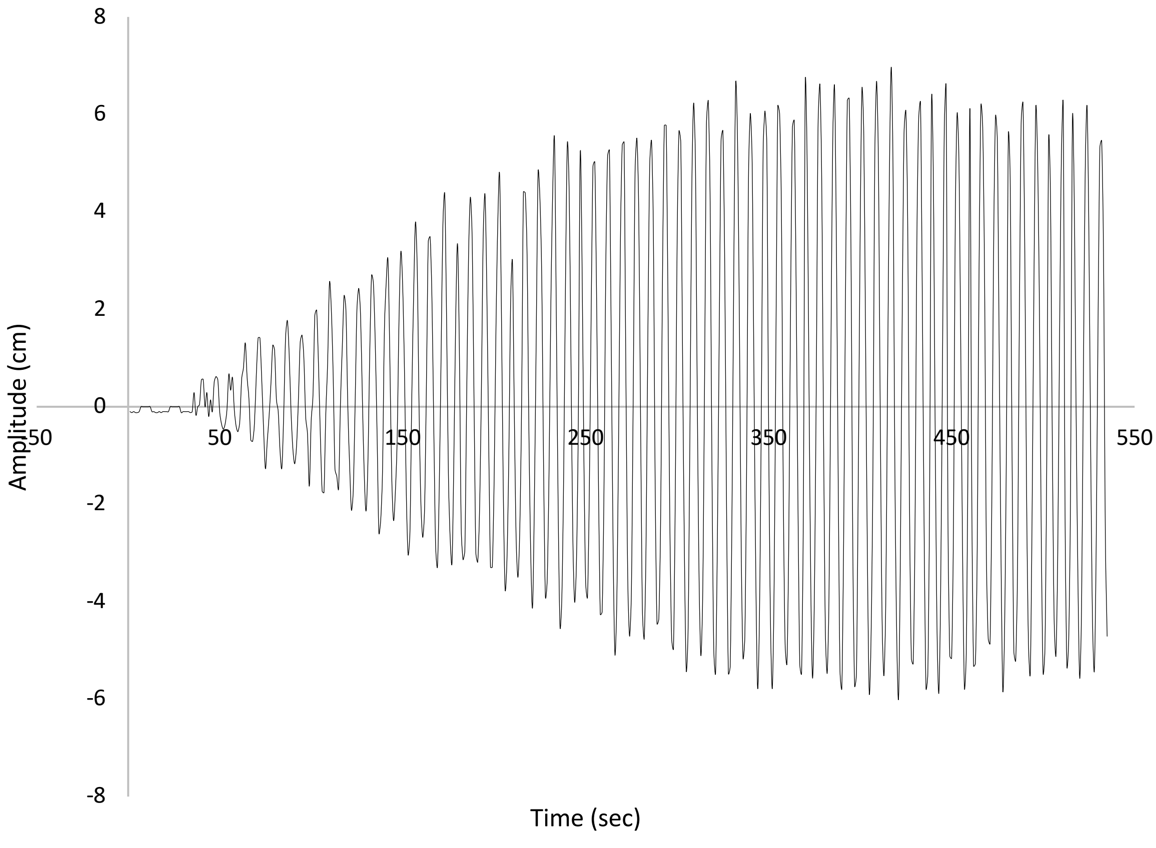

Fig. 1. Time response of the pendulum driven at the resonant frequency

It depicts that the pendulum gradually starts oscillating from its equilibrium position and the amplitude of oscillation increases with time. Figure 2 shows the variation of amplitude of 50th oscillation of three pendula of length 21.10 cm, 23.84 cm and 30.00 cm respectively driven at the frequency range between 0.79 Hz to 1.14 Hz. For each curve there exist the maximum amplitude that correspond to the resonant frequency of the pendulum and value show inverse correlation with the length of the pendulum, i.e., the shorter pendulum resonates with higher frequency. The resonant frequencies from these curves are compared with the calculated natural frequencies of the pendula as shown in Table 1. The measured resonant frequencies are close to the calculated natural frequencies with the maximum percentage error of 1.96 %, confirming the consistency of the experiment with the theoretical prediction.

Table 1
Comparison of resonant frequency with the natural frequency of the pendulum

SN	Length of the pendulum (cm)	Resonant frequency (Hz)	Natural frequency (Hz)	% Error
1	21.10	1.07	1.08	0.92
2	23.84	1	1.02	1.96
3	33.00	0.86	0.87	1.15

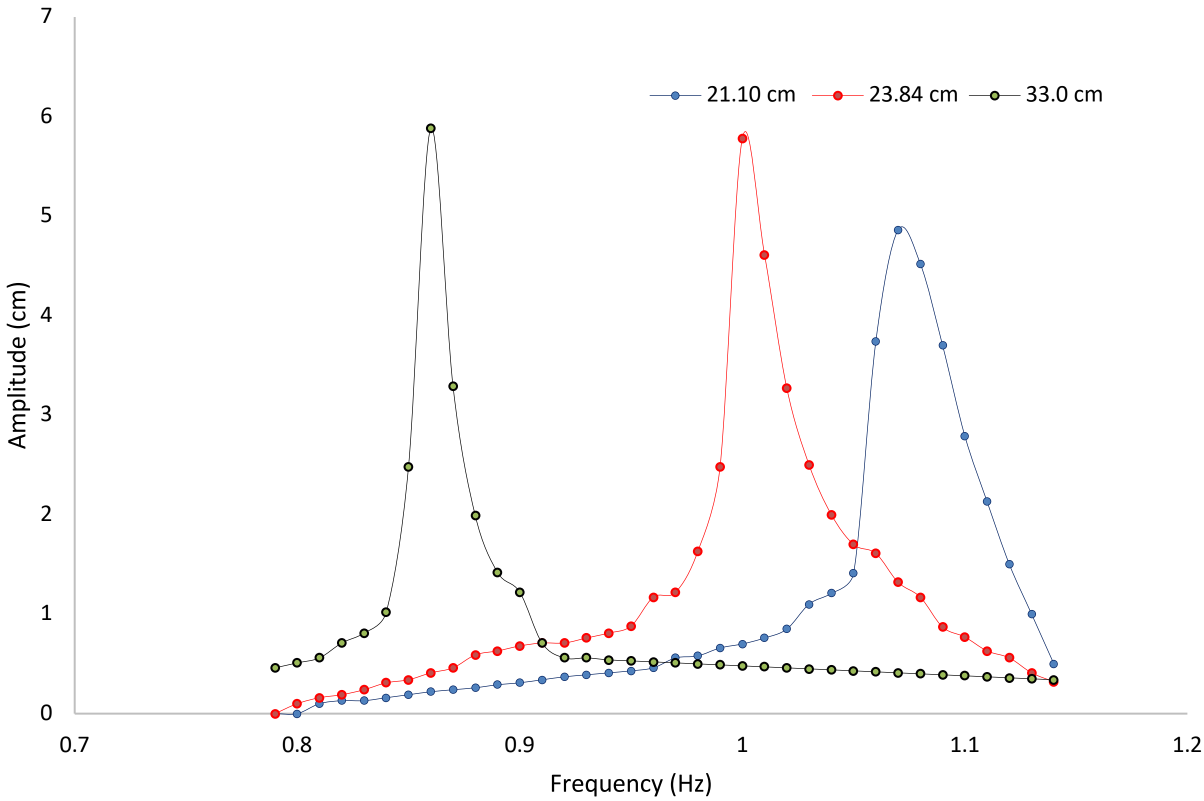

Fig. 2. Resonance curves of three pendulums of length 21.10 cm (blue), 23.84 cm (orange) and 33.00 cm (grey) The frequency corresponding to maximum amplitude represents the resonant frequency of the pendulum

The free oscillation profile of manually displaced simple pendulum (bob mass = 0.27 kg, length = 31.71 cm) is shown in Figure 3. The time period determined from ten oscillations of the pendulum is 1.14 s and the restoration constant k calculated using Eq. (2) is 8.20 kg s-2. The kinetic energy and the potential energy calculated using Eq. (3) and Eq. (4) respectively, for the horizontal bob positions (x) swinging between the amplitudes of -5.3cm and +5.3 cm are shown in Table 2, and the plot for energies as a function of bob position is shown in Figure 4.

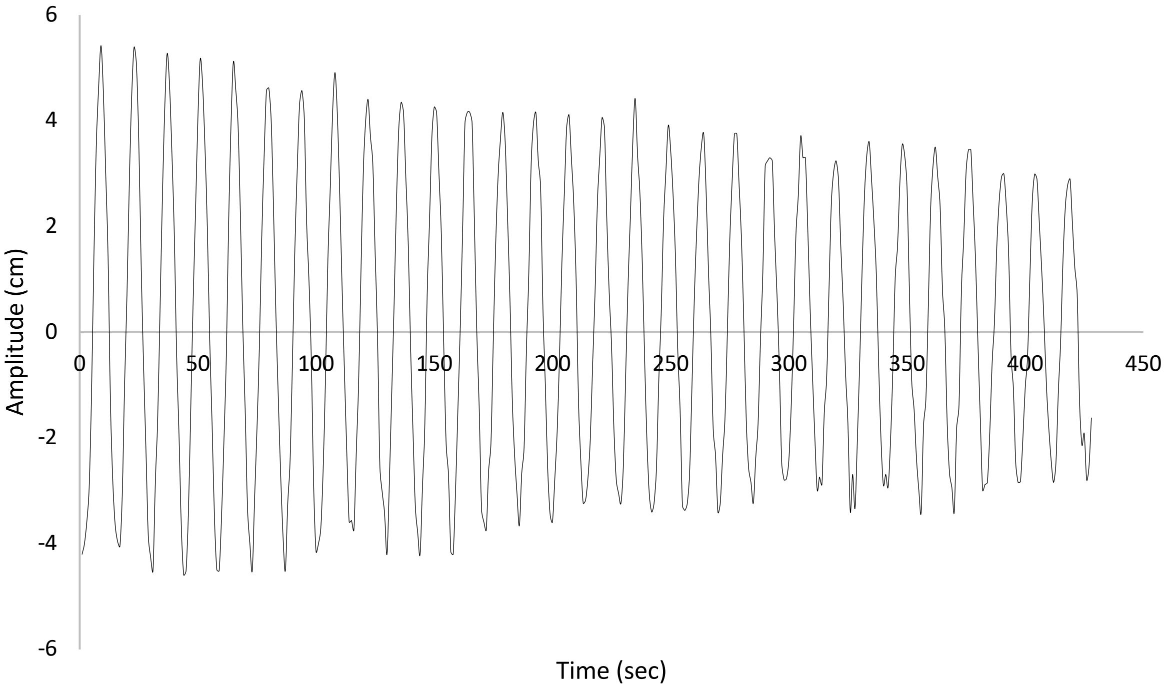

Fig. 3. Time response of the freely oscillating simple pendulum

Table 2
The calculated values of potential energy (U), kinetic energy (K) and total energy (E) at various position of pendulum during the oscillation

SN	Position of bob (cm)	Potential Energy (J)	Kinetic energy (J)	Total Energy (J)
1	-5.30	115.17	0.00	115.17
2	-4.30	75.81	39.36	115.17
3	-3.30	44.65	70.52	115.17
4	-2.30	21.69	93.48	115.17
5	-1.30	6.93	108.24	115.17
6	-0.30	0.37	114.80	115.17
7	0	0	115.7	115.17
8	0.30	0.37	114.80	115.17
9	1.30	6.93	108.24	115.17
10	2.30	21.69	93.48	115.17
11	3.30	44.65	70.52	115.17
12	4.30	75.81	39.36	115.17
13	5.30	115.17	0.00	115.17

The energy curves shown in Figure 4 illustrates that the potential energy of the pendulum gradually increases from zero at the equilibrium position to the maximum value of 115.17 J at the extreme positions where the pendulum is momentarily at rest. On the other hand, kinetic energy is maximum (115.17 J) at the equilibrium position where the velocity of the pendulum is maximum and gradually decreases to zero at the extreme position. In between the two extreme positions, the energy of oscillating pendulum is partly kinetic and partly potential, that sum to total energy of 115.15 J that remain constant throughout, indicating negligible loss of energy during single oscillation. The time period *T* of five simple pendula of different length but same mass and bob size as used earlier, is shown in Table 3.

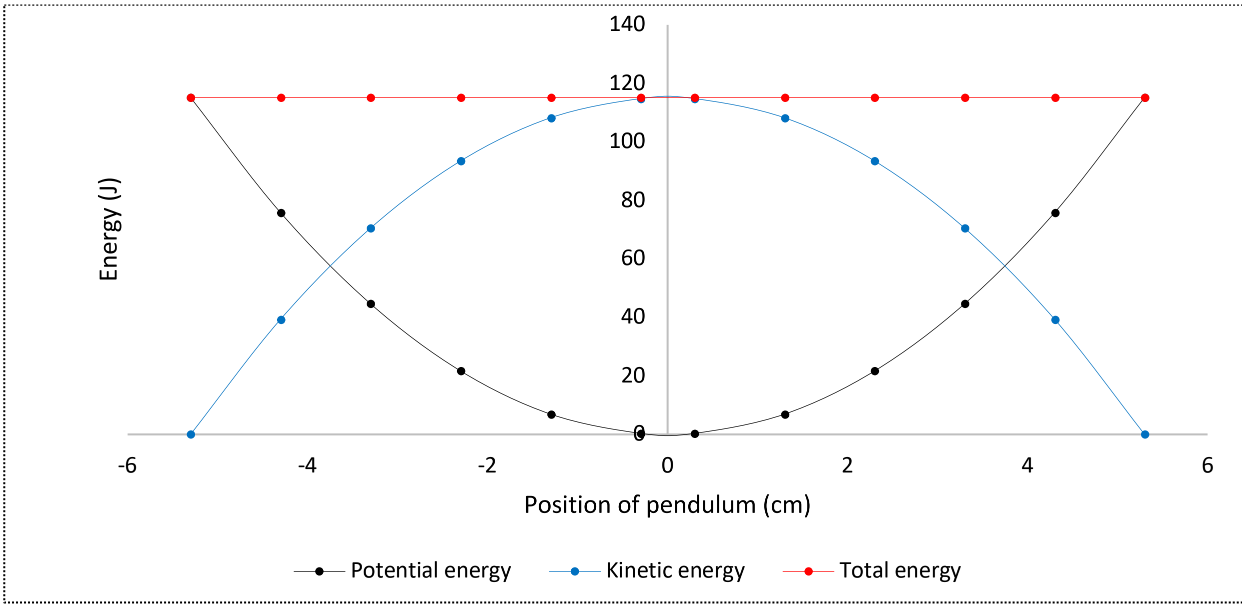

Fig. 4. Experimentally determined energy curves of harmonically oscillating simple pendulum

Table 3
The time period of simple pendulum having different length

SN	Length of the pendulum (m)	Time period (sec.)
1	0.3372	1.1770
2	0.3162	1.1503
3	0.2872	1.0928
4	0.2612	1.0463
5	0.2172	0.9524

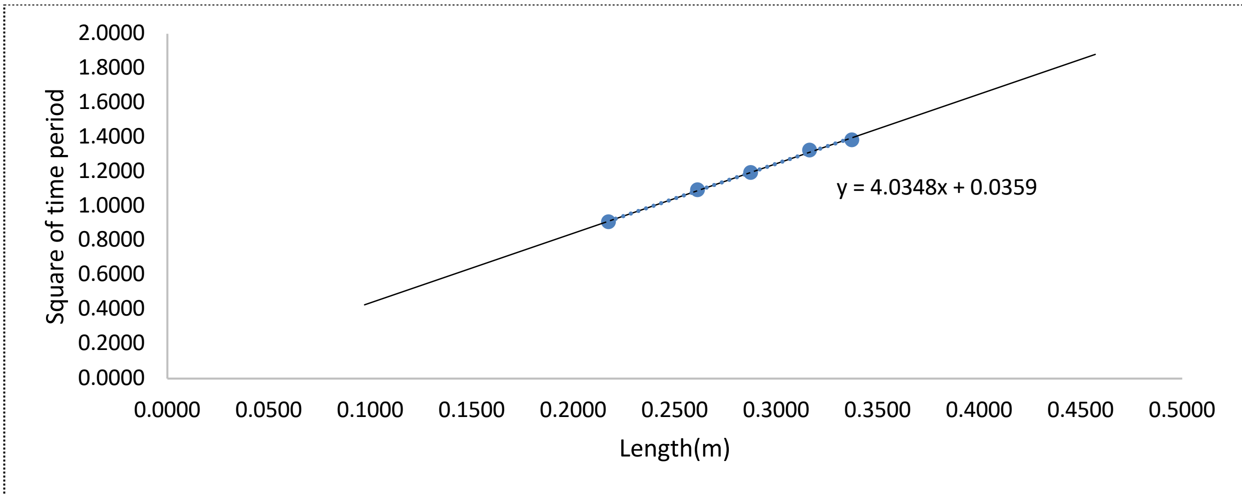

Fig. 5. The linear curve obtained by plotting square of time period as a function of the pendulum length

It shows that the longer pendulum takes longer time to complete an oscillation as compared to the shorter pendulum. Using the data, a linear curve is obtained by plotting T^2 as a function of l. The curve is then fitted with Eq. (5) and from the slope, the acceleration due to gravity g = 9.78 m/s^2 is obtained, which is close to the standard value reported by Tate in 1968 [29].

4. Significance of the Study

The simple experiment presented here serves as a standard model for analysing a broad variety of resonance process in many fields of engineering by providing an indispensable criterion for the design of high buildings, long bridges, ocean liners and rapidly spinning mechanical axles. More importantly, the model may be implemented, without any modification, for determining the physical parameters required for designing prosthetic limbs for humans and animals. The knowledge of resonance from the current work might as well be utilized in stabilizing the floating buoy for ocean wave energy harvesting as in the work of Yusli *et.al.*, [25]. In addition to it, the research may also serve as an easily reproducible experiment for studying oscillatory phenomenon, particularly forced oscillations and related energetics which are essential topics in high school and undergraduate physics curriculum. Further, the experiment is also an extension of the application of commonly used simple pendulum in physics laboratory.

Mechanical resonance being one of the important topics in physics and engineering education, there is still a lack of experimental equipment for providing hands-on experience to the learners. In most physics and engineering text books, experimental verification of mechanical resonance is often substituted with the electrical resonance that occurs on electrical current circulating in a circuit consisting, resistor, capacitor and an inductor. This experiment, do not provide enough information to awake the feel of mechanical resonance as the changes occurring in the magnitude of current can neither be felt nor observed. The current experiment, incorporates the essence of a good experiment that will lead the leaners to appreciate physics of oscillating object.

5. Research Gap

The current work didn't deal with the attenuation in terms of its penetration capability in dissemination process. As a result, the research is not concerned of the adverse effects of the resonant frequency on industrial equipment. It also didn't extend its idea of band gap analysis for different frequency and wavelength. Due to having a simplified model of the structure of the equipment for the research, the analysis of vibration characteristics was also ignored. In addition to it, the gap of the work can be explained relating to time for acquiring the data and ascertaining the length of the pendulum. It takes long time to determine resonant frequency of the oscillating object using the current equipment adjusted for the research. The approximate time required for determining resonant frequency of a single length pendulum is one hour. In order to collect reliable data, the oscillator is required to be confined in a single plane, which can be achieved by placing the equipment in a levelled surface. Misalignment of the oscillating body with the ultrasound range sensor results in skipping of data collection, which in turn may result in miscalculation of the physical parameters.

6. Conclusion

This work was meant to deal with the resonance frequency and energy curve of a harmonically oscillating pendulum given that the design of the instruments were customized. The resonant frequency, energy curves and the acceleration due to gravity determined using the equipment were in close agreement with the theoretical values which were validated with great accuracy. The simplistic design approach of the equipment aided this study in the advancement of the computational skills for data analysis giving the following outcomes:

(i) The experimental equipment designed has proven to be effective in demonstrating physical phenomenon of oscillating pendulum.
(ii) Three different lengths of the pendulums of 21.10 cm, 23.84 cm and 33 cm used for measuring resonant frequencies.
(iii) The frequencies computed were 1.07 Hz, 1.0 Hz and 0.86 Hz, which are close based on theoretically calculated values.
(iv) Estimated errors of 0.009, 0.019 and 0.011 were very small which determined resonance frequencies confirmed the consistency.
(v) The calculated potential and kinetic energies for the positions of the pendulum ranging from − 5.30 cm to +5.30 cm showed two inverse relationships with a total energy of 115.17 J for all experimental positions.

As an annex to the research, this paper also determined a g-value (acceleration of gravity) of 9.78 m/s^2 from experiments. This is very close to the normal value of 9.8 m/s^2. Hence, this equipment may be used as a standard device in the physics laboratory for demonstration and determining various physical parameters related to pendulum oscillations with some limitations though such as, the pendulum used in this study requires longer time to attain oscillations of considerable amplitude due to weak magnetic force produced by the electromagnet driven with low output current function generator. The electromagnet used here retains magnetization when used for longer time resulting jerky oscillations of the pendulum. These problems might be reduced by using a function generator producing higher output current. The rig needs to be placed in a levelled surface, and the electromagnet and ultrasound sensor must align with the pendulum for obtaining quality data.

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