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Comparison of Modified and Classical Hamilton-Crosser Models for MWCNT-Water Nanofluid Heat Transfer Rate

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ABSTRACT

The classical Hamilton-Crosser model is used to calculate the thermal conductivity of nanofluids. This thermal conductivity is controlled by the volume fraction of nanoparticles. However, the classical model encounters limitations when dealing with situations that include high concentrations of particles and a wide range of nanoparticle shapes. Researchers have acknowledged these limitations and have made modifications to the classical model to improve its accuracy and applicability. This research aims to compare the modified model with the classical Hamilton-Crosser model, focusing on the heat transfer rate of multi-walled carbon nanotube (MWCNT) water-based nanofluid. The governing equations were converted into ordinary differential equations using similarity variables and solved using the bvp4c function in MATLAB. The numerical solutions generated using bvp4c investigate the impact of a magnetic field, viscous dissipation, nanoparticle volume fraction, surface transpiration rate, length of MWCNT, and diameter of MWCNT. The findings suggest that the modified model reliably forecasts elevated heat transfer rates in comparison to the classical model. In addition, increased lengths of MWCNTs result in elevated rates of heat transfer. In contrast, as the diameter of MWCNTs increases, there is a progressive reduction in heat transmission rates. Therefore, the research suggests that the revised model is very well suited for identifying the ideal diameters of nanotubes to improve heat transfer efficiency. The results enhance the accuracy of thermal conductivity models and further the comprehension of nanofluid heat transfer properties.

1. Introduction

The concept of nanotechnology was introduced in the late 1950s. Nanotechnology refers to the scientific, technical, and technological activities that take place at the nanoscale, which is a range of 1 to 100 nanometers. The development of nanofluids was initiated in the 1990s as a direct response to the emergence of nanotechnology. In 1995, Choi and Eastman [1] initially introduced the concept of nanofluids at the Argonne Laboratory in the United States. A nanofluid is a type of liquid that is produced by combining materials with a size range of 1–100 nanometers with base fluids in such a way that improves the thermophysical characteristics of the base fluids, such as density, thermal

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conductivity, heat capacity, and so on. Nanofluids have attracted considerable interest in recent years because of their distinctive thermal and fluidic characteristics, making them very promising for various applications, especially in enhancing heat transfer. Nanofluids are widely used as thermal transportation media in various important areas of engineering and research. These include radiator coolant, engine lubricant, brake fluid, spacecraft, electronic microchannels, nuclear energy, and solar energy [2–4].

Increasing thermal conductivity in thermal and industrial engineering processes depends on the shape of the surfaces and the addition of liquid additives, because cooling needs to happen quickly. This is because thermal conductivity may be enhanced by controlling external forces using electromagnetic fields. Carbon nanotubes (CNTs) are cylindrical structures made up of carbon atoms. Iijima Sumio first introduced them to the scientific community in 1991, identifying both multi-walled carbon nanotubes (MWCNTs) and single-walled carbon nanotubes (SWCNTs) [5]. These structures provide exceptional thermal, electrical, and mechanical characteristics, resulting in extensive use across several fields. Researchers are interested in CNTs because they have many useful properties, such as storing energy, absorbing energy, dampening vibrations, and improving electrical and thermal conductivities [6].

CNT has wide-ranging uses in several fields, such as electronics, materials science, and medicine. In nanofluids, CNTs have demonstrated exceptional promise in improving both thermal conductivity and fluidic characteristics. Recent research, shown by articles [7] and [8], demonstrates the versatility of CNT in improving the efficiency of heat transfer and the behaviour of fluids in many applications. CNTs can be used as nanoparticles in a working fluid and possess enhanced thermophysical, mechanical, and chemical properties. Their notable characteristics include their expansive surface area, arrangement, rigidity, chemical durability, and relative superiority compared to other nanoparticles. Furthermore, carbon successions in CNTs have not posed any environmental hazards [9]. CNTs possess unique attributes that render them the most cutting-edge material presently accessible. They find utility throughout a diverse range of disciplines within material sciences and engineering, including nanotechnology, energy storage, hardware, biomedicine, ceramics, optics, and thermal defence. Scientists have extensively studied the dispersion of CNTs in nanofluids because of their significant consequences [10–12].

Thermal conductivity refers to the capacity of a substance to conduct or transfer heat [13]. Nanofluids possess high thermal conductivity, leading to improved heat transfer efficiency [14]. The Maxwell model provided the theoretical model that served to assess the synthesized nanofluid's thermal conductivity [15]. In 1962, Hamilton and Crosser [16] modified the Maxwell model to incorporate the influence of nanoparticle shape. The Hamilton-Crosser model provides a fundamental framework for evaluating the effective thermal conductivity of nanofluids. This classical model offers a straightforward and comprehensive approach to evaluating the increase in thermal conductivity in nanofluids. The classical Hamilton-Crosser model is advantageous due to its simplicity and ease of implementation, making it an ideal starting point for researchers venturing into the realm of nanofluid modelling. Joshi *et al.,* [17] used the classical Hamilton-Crosser model to investigate the impact of MWCNTs on the thermophysical characteristics of polyester and mineral oil. The study found that the dispersion of MWCNT nanoparticles improves the thermophysical properties of polyester and mineral oil.

Nevertheless, the classical Hamilton-Crosser model exhibits shortcomings when presented with circumstances involving substantial particle concentrations and distinct nanoparticle shapes. These restrictions may cause calculations about the thermal conductivity of nanofluids to be inaccurate under certain circumstances. Researchers have acknowledged these constraints and, as a result, have made modifications to the classical Hamilton-Crosser model to improve its accuracy and applicability.

The recent adjustments to the thermal conductivity model are designed to tackle the issues presented by nanofluid systems specifically. Among the things that are thought about in these changes are the shape factor of CNTs, how particles group together, Brownian motion, Kapitza resistance, and the particle nanolayer. Researchers have examined several modifications to the classical model in order to enhance its accuracy and applicability in predicting the thermal conductivity of nanofluids [18–22]. The main purpose of these modifications is to provide a more precise depiction of the thermal conductivity of nanofluids, considering the various difficulties posed by nanoscale particles and their interaction with the base fluid. This enables more accurate predictions and a more profound comprehension of the mechanisms behind heat conductivity in nanofluid systems.

Recent scientific studies, such as those by Yang and Xu [18] and Yang *et al.,* [19], have sought to address these constraints by suggesting modifications to the classical Hamilton-Crosser model. Drawing inspiration from the classical Hamilton-Crosser model, scholars have made diverse adjustments in an effort to improve its precision and practicality. In these changes, specific problems caused by nanofluid systems are fixed, leading to better predictions and knowledge of thermal conductivity. Nayak *et al.,* [20] did important research that looked at how to use a modified Hamilton-Crosser model to study how shape and the interfacial layer affect water nanofluid and carbon nanotubes. Wang *et al.,* [21] rebuilt the classical Hamilton-Crosser model to create a better model for the thermal conductivity of nanofluids. They did this by taking into account the effects of particle aggregation, Brownian motion, Kapitza resistance, and the nanolayer of particles. Additionally, Farbod and Ahangarpour [22] proposed a newly modified Hamilton-Crosser model. Based on their research, it was determined that the classical Hamilton-Crosser model was inadequate for predicting the experimental thermal conductivities. As a result, they suggested that the shape factor in the classical Hamilton-Crosser model be modified from $n = 6$ to $n = 6 + \gamma L/D$, where *L* and *D* are the length and diameter of the carbon nanotubes, giving a prediction that is very close to the experimental data. This indicates that the shape of nanoadditives significantly influences the properties of nanofluids.

According to the literature review, no previous study has theoretically compared the modified Hamilton-Crosser model by Farbod and Ahangarpour [22] and the classical Hamilton-Crosser model in terms of MWCNT water-based nanofluid heat transfer rate. Therefore, the main objective of the current study is to analysis the comparison of modified and classical Hamilton-Crosser models with regard to the rate of heat transfer of MWCNT water-based nanofluid. This leads to a theoretical study that uses governing equations from Idris *et al.,* [23] to look into the rate of heat transfer of an MWCNT water-based nanofluid along with the effects of a magnetic field, viscous dissipation, and suction/injection across a moving interface that is permeable. The equations were solved numerically with the help of bvp4c. This study is anticipated to offer a fresh viewpoint on the use of modified thermal conductivity models for MWCNT nanofluid, which is considered beneficial.

2. Mathematical Formulation

This study examined the flow and heat transfer of a nanofluid produced by dispersing MWCNTs of different lengths and diameters into water. The nanofluid flow is characterised as incompressible, laminar, steady, and two-dimensional, and it is also influenced by magnetic field, viscous dissipation, suction, and injection effects. Additionally, we assume a constant velocity U_w moving in the same direction to the free stream U_∞ of the two-dimensional surface. The governing equations as given by Idris *et al.,* [23], expressed as partial differential equations (PDEs), comprise the continuity equation, the momentum equation, and the energy equation, are given by

$$u_x + v_y = 0, \tag{1}$$

$$uu_x + vu_y = \frac{\mu_{nf}}{\rho_{nf}} u_{yy} - \frac{\sigma_{nf}}{\rho_{nf}} \big(B(x)\big)^2 u, \tag{2}$$

$$uT_x + vT_y = \alpha_{nf} T_{yy} + \frac{\mu_{nf}}{(\rho C_p)_{nf}} \big(u_y\big)^2. \tag{3}$$

The boundary conditions are (see Idris *et al.,* [23])

$$\begin{aligned} &u = U_w, \quad v = V_w(x), \quad T = T_w, \quad \text{at } y = 0, \\ &u \to U_\infty, \quad T \to T_\infty, \quad \text{as } y \to \infty. \end{aligned} \tag{4}$$

Here $B(x)$ indicates the uniform magnetic field's strength and $V_w(x)$ refers to mass flux velocity, which are given in the following form:

$$B(x) = B_0 \sqrt{\frac{1}{2x}}, \qquad V_w(x) = -\sqrt{\frac{U\nu_f}{2x}} S, \tag{5}$$

where B_0 is a constant, U is a composite velocity where U is introduced as $U = U_w + U_\infty$, and the dimensionless constant determines the surface transpiration rate, denoted by S. A positive or negative S represents suction and injection, respectively, while $S = 0$ indicates an impermeable surface.

The thermophysical properties of the nanofluid, such as viscosity μ_{nf}, density ρ_{nf}, electrical conductivity σ_{nf}, thermal diffusivity α_{nf} , and heat capacity $\big(\rho C_p\big)_{nf}$ are given as (refer to Tiwari and Das [24])

$$\mu_{nf} = \frac{\mu_f}{(1-\phi)^{2.5}}, \tag{6.1}$$

$$\rho_{nf} = (1-\phi)\rho_f + \phi\rho_s, \tag{6.2}$$

$$\frac{\sigma_{nf}}{\sigma_f} = 1 + \frac{3(\sigma_s/\sigma_f - 1)\phi}{\sigma_s/\sigma_f + 2 - (\sigma_s/\sigma_f - 1)\phi}, \tag{6.3}$$

$$(\rho C_p)_{nf} = (1-\phi)(\rho C_p)_f + \phi(\rho C_p)_s. \tag{6.4}$$

Note that ϕ refers to the nanoparticles volume fraction for MWCNT. Meanwhile the subscript nf represents the nanofluid (MWCNT-water), f represents the base fluid (water), and s reflects the solid nanoparticle (MWCNT).

It should be emphasized that the thermal conductivity k_{nf}, for the classical Hamilton-Crosser model [16] is given as

$$k_{nf} = \frac{k_s + (n-1)k_f - (n-1)\big(k_f - k_s\big)\phi}{k_s + (n-1)k_f + \big(k_f - k_s\big)\phi} k_f, \tag{7}$$

where the shape factor of nanoparticle n, is considered 6 for cylindrical nanoparticles (MWCNT). While for the modified Hamilton-Crosser model, a modified shape factor of nanoparticle $n = 6 + \gamma L/D$ which was invented by Farbod and Ahangarpour [22] is used in this study. Here γ refers to the correction coefficient. Meanwhile, L and D are the length and diameter of the MWCNT. Therefore, the thermal conductivity k_{nf} of the modified Hamilton-Crosser model can be written as

$$k_{nf} = \frac{k_s + (5 + \gamma L/D)k_f - (5 + \gamma L/D)\left(k_f - k_s\right)\phi}{k_s + (5 + \gamma L/D)k_f + \left(k_f - k_s\right)\phi} k_f. \tag{8}$$

The two-dimensional linear type similarity transformation by following Idris *et al.,* [23] was initiated to transform Eqs. (1)–(3) into ODE. The concept of linear similarity transformations can be observed as follows

$$u = Uf'(\eta), \qquad v = \sqrt{\frac{U\nu_f}{2x}}\left(\eta f'(\eta) - f(\eta)\right), \qquad \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}, \qquad \eta = y\sqrt{\frac{U}{2x\nu_f}}. \tag{9}$$

The similarity transformations (9) are applied to transformed Eqs. (2) and (3) and boundary conditions (4) to become

$$\frac{\mu_{nf}}{\mu_f} f''' - \frac{\sigma_{nf}}{\sigma_f} Mf' + \frac{\rho_{nf}}{\rho_f} ff'' = 0, \tag{10}$$

$$\frac{1}{\mathrm{Pr}} \frac{k_{nf}}{k_f} \theta'' + \frac{\left(\rho C_p\right)_{nf}}{\left(\rho C_p\right)_f} f\theta' + \frac{\mu_{nf}}{\mu_f} \mathrm{Ec}(f'')^2 = 0, \tag{11}$$

subject to

$$\begin{aligned} &f(0) = S, \qquad f'(0) = 1 - r, \qquad \theta(0) = 1, \\ &f'(\eta) \to r, \qquad \theta(\eta) \to 0, \qquad \text{as } \eta \to \infty, \end{aligned} \tag{12}$$

where the magnetic parameter, denoted by M, the Prandtl number, denoted by Pr, the Eckert number, denoted by Ec, and the velocity ratio parameter, denoted by r, are defined as follows:

$$M = \frac{\sigma_f B_0^2}{\rho_f U}, \qquad \mathrm{Pr} = \frac{\left(\mu C_p\right)_f}{k_f}, \qquad \mathrm{Ec} = \frac{U^2}{\left(C_p\right)_f (T_w - T_\infty)}, \qquad r = \frac{U_\infty}{U}. \tag{13}$$

The velocity ratio parameter, satisfying the condition $0 < r < 1$, represents the sheet moving in parallel with the direction of the free stream. When the value of r is less than 0 and greater than 1, it indicates that the sheet is moving in the opposite direction.

The primary focus of this subject revolves around the practical consideration of physical quantities, namely the heat transfer rate. The heat transfer rate is precisely described as:

$$\mathrm{Re}_x^{-1/2}\mathrm{Nu}_x = -\frac{1}{\sqrt{2}} \frac{k_{nf}}{k_f} \theta'(0), \tag{14}$$

where $\mathrm{Re}_x = Ux/\nu_f$ is the local Reynolds number.

3. Results and Discussion

This study presents a comparison between modified and classical Hamilton-Crosser models for predicting heat transfer rates for MWCNT water-based nanofluids. The Eqs. (10)–(11), which were subjected to the boundary conditions (12), were numerically solved using the bvp4c. Meanwhile, the equation for predicting the heat transfer rate can be found in Eq. (14). The thermophysical properties of MWCNT and water are shown in Table 1, as provided by Shoaib *et al.,* [25]. The investigation explores the impact of various parameters, including the magnetic parameter *M*, Eckert number Ec, nanoparticle volume fraction ϕ, and surface transpiration rate *S*, on the heat transfer rates predicted by both models. The investigation also explores the impact of the length of MWCNT *L*, and the diameter of MWCNT *D*, on the heat transfer rates predicted by the modified model.

Throughout this paper, the Prandtl number was set to 6.2, the correction coefficient was set to 1, the magnetic parameter and Eckert number were varied from 0 to 0.05, the nanoparticle volume fraction varied from 0 to 0.005, the surface transpiration rate varied from –0.5 to 0.5. The length of MWCNT varied from 10 µm to 30 µm, and the diameter of MWCNT varied from 10 nm to 30 nm as mentioned by Patel *et al.,* [3] and Rudyak *et al.,* [26]. Meanwhile, the velocity ratio parameter was set to 0.5, which corresponds to the sheet moving parallel according to the route of the free stream. We have compared our results with those of Idris *et al.,* [23] and seen a good agreement with our numerical scheme, as shown in Table 2.

Table 1
Thermophysical properties of MWCNT and water

	C_p	ρ	k	σ
MWCNT	796	1600	3000	10^6
Water	4179	997.1	0.613	5.5×10^{-6}

Table 2
A comparison of $f''(0)$ for $\phi = 0$, $S = 0$ and $r = 1$

Idris *et al.,* [23]	Present
0.469600	0.469600

Figure 1 demonstrates that the heat transfer rate exhibits a decreasing trend with minimal variation as the magnetic parameter increases, for both the modified and classical Hamilton-Crosser models. At *M* = 0, the modified model yields a rate of 1.300077, while the classical model predicts a higher rate of 2.544279. As the magnetic parameter increases to 0.05, the modified model consistently predicts lower rates compared to the classical model. The observed decrease in heat transfer rates with increasing magnetic parameters can be attributed to the influence of the magnetic field on the behaviour of MWCNT water-based nanofluids. Magnetic fields are known to affect the movement and distribution of nanoparticles within the fluid. In this case, the magnetic field is impeding the movement of MWCNTs, resulting in reduced heat transfer rates. The alignment or clustering of MWCNTs under the magnetic field may alter the overall thermal conductivity of the nanofluid, leading to the observed trend. The significant difference in heat transfer rates between modified and classical Hamilton-Crosser models highlights the importance of considering model modifications. The modifications made to the classical model appear to better capture the nuanced behaviour of MWCNT nanofluids, especially under the influence of a magnetic field. This discrepancy

underscores the need for tailored models that account for specific characteristics of nanofluids, providing more accurate predictions.

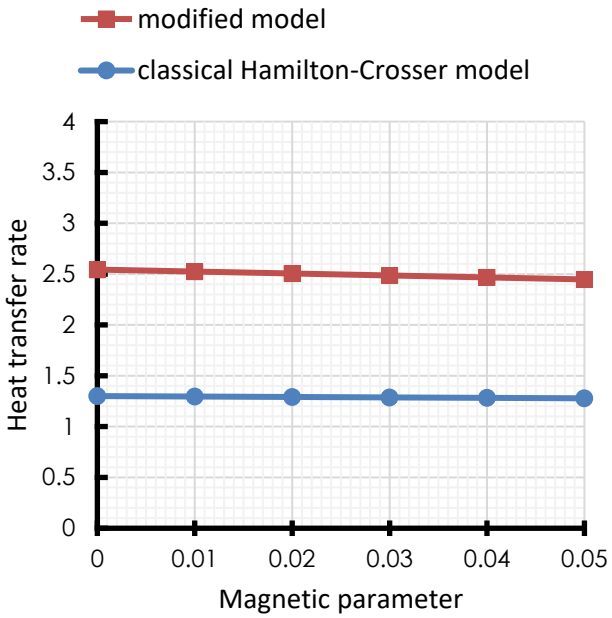

Fig. 1. Comparison of modified and classical Hamilton-Crosser models for MWCNT-water nanofluid heat transfer rates with different magnetic parameters when Ec = 0.01, ϕ = 0.005, S = 0.1, L = 10 µm, and D = 10 nm

Figure 2 rightly points out that the heat transfer rates exhibit minimal variations as the Eckert number increases from 0 to 0.05 for both modified and classical Hamilton-Crosser models. At Ec = 0, the modified model predicts a rate of 1.295694, while the classical model yields a higher rate of 2.525192. The fact that heat transfer rates stayed the same even when Eckert numbers changed suggests that the viscous dissipation parameter does not have a big effect on the heat transfer properties of the MWCNT water-based nanofluids in this study. The Eckert number accounts for the ratio of kinetic energy to enthalpy difference and, in this context, represents the degree of viscous dissipation. The minimal influence of Ec on heat transfer rates implies that the viscous effects within the nanofluid are not predominant under the specified conditions. The fact that the heat transfer rates stay the same for all Eckert numbers shows that the energy lost because of viscous effects does not really change how the nanofluid transfers heat. This finding provides valuable insights for practical applications where the viscous dissipation parameter may not be a decisive factor in determining heat transfer rates.

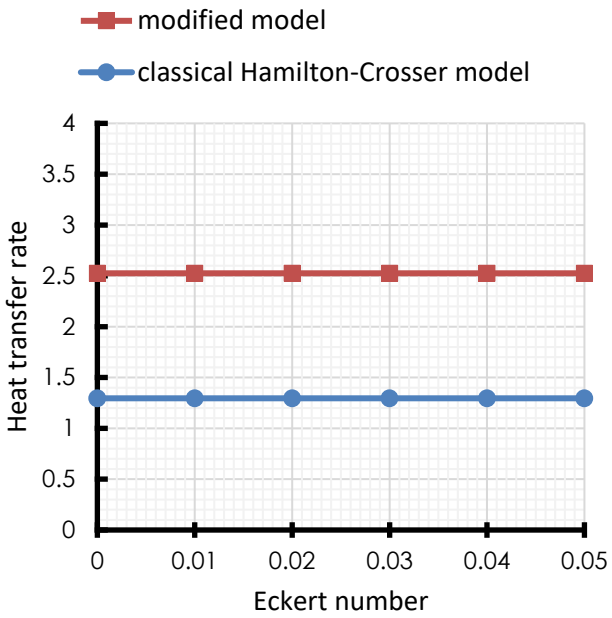

Fig. 2. Comparison of modified and classical Hamilton-Crosser models for MWCNT-water nanofluid heat transfer rates with different Eckert numbers when M = 0.01, ϕ = 0.005, S = 0.1, L = 10 µm, and D = 10 nm

The heat transfer rates show a significant increase with an increment in the volume fraction of nanoparticles ϕ for both modified and classical Hamilton-Crosser models, as shown in Figure 3. At $\phi = 0$ (water), the heat transfer rates are equal for both models, with a value of 1.284011. As the nanoparticle volume fraction increases, the modified Hamilton-Crosser model consistently predicts higher heat transfer rates compared to the classical model. The observed increase in heat transfer rates with a higher nanoparticle volume fraction can be attributed to the enhanced thermal conductivity of the nanofluid. As the volume fraction of MWCNTs increases, more nanoparticles are dispersed in the base fluid, leading to improved heat transfer capabilities. The unique thermal properties of MWCNTs, such as high thermal conductivity, contribute to the overall enhancement of heat transfer rates in the nanofluid. The notable improvement in heat transfer rates with an increasing volume fraction of nanoparticles underscores the positive impact of incorporating MWCNTs into the nanofluid. The modified model, designed to better capture the behaviour of MWCNT nanofluids, consistently outperforms the classical model in predicting the enhanced heat transfer rates.

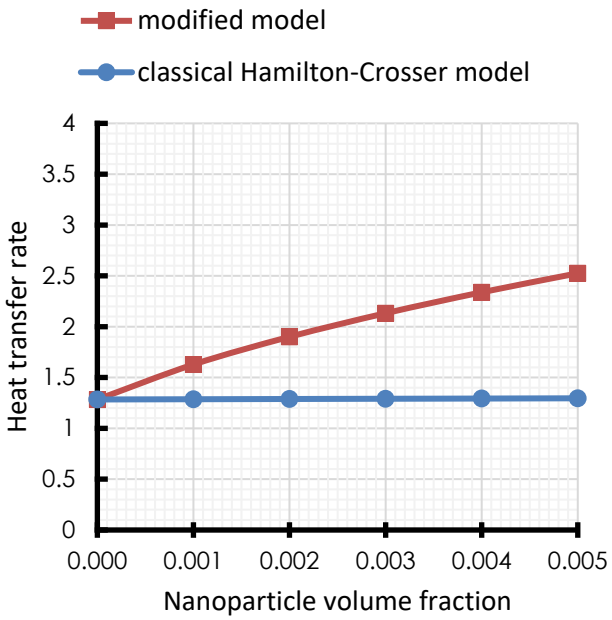

Fig. 3. Comparison of modified and classical Hamilton-Crosser models for MWCNT-water nanofluid heat transfer rates with different nanoparticle volume fractions when M = 0.01, Ec = 0.01, S = 0.1, L = 10 μm, and D = 10 nm

The heat transfer rates exhibit a marked variation with changes in the surface transpiration rate S for both modified and classical Hamilton-Crosser models, as given in Figures 4 and 5. A positive S represents the suction of fluid from the surface, resulting in enhanced heat transfer. As the surface transpiration rate S increases from 0 to 0.5, both models predict an escalation in heat transfer rates, as shown in Figure 4. The observed increase in heat transfer rates with an increase in the surface transpiration rate S can be attributed to the suction effect. The greater the suction, the more efficiently the fluid is drawn towards the surface, facilitating heat exchange. The results indicate that both modified and classical models capture the positive impact of suction on heat transfer rates. However, the magnitude of the effect and the nuances in behaviour may differ between the two models. A detailed comparison provides valuable insights into the predictive capabilities of each model under varying suction conditions.

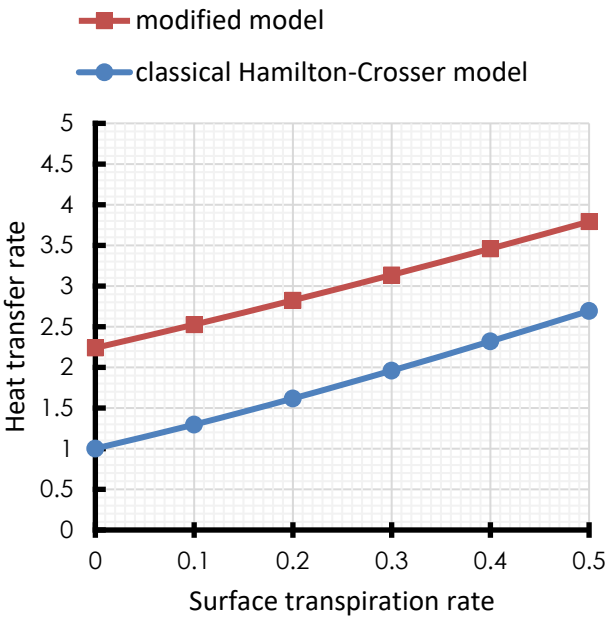

Fig. 4. Comparison of modified and classical Hamilton-Crosser models for MWCNT-water nanofluid heat transfer rates with different surface transpiration rate (suction effect) when M = 0.01, Ec = 0.01, ϕ = 0.005, L = 10 µm, and D = 10 nm

Conversely, negative values of the surface transpiration rate S indicate an injection effect, where fluid is injected into the boundary layer. As the surface transpiration rate S becomes more negative, both the modified and classical Hamilton-Crosser models show a decrease in heat transfer rates, as seen in Figure 5. This injection results in a reduction in heat transfer rates. The more negative the injection parameter, the greater the impact on hindering heat transfer. The fact that heat transfer rates slowed down as injection effects went up suggests that changes in the surface transpiration rate S can affect both modified and classical Hamilton-Crosser models. This aligns with the physical intuition that the injection of fluid tends to impede heat transfer across the boundary layer.

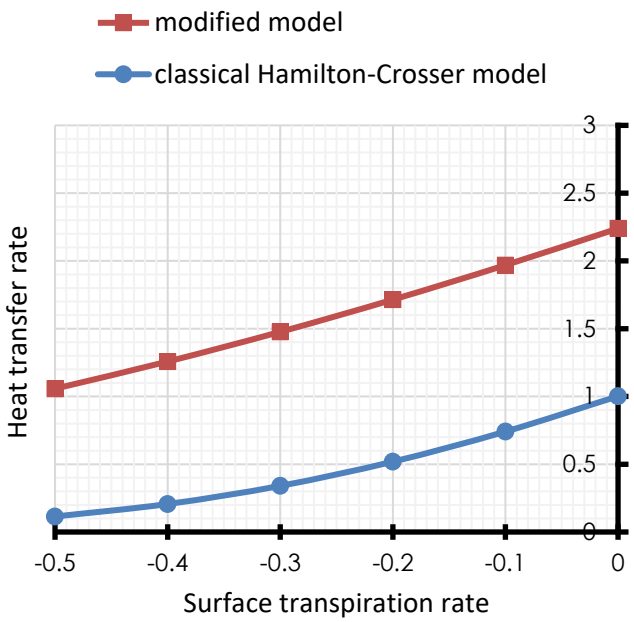

Fig. 5. Comparison of modified and classical Hamilton-Crosser models for MWCNT-water nanofluid heat transfer rates with different surface transpiration rate (injection effect) when M = 0.01, Ec = 0.01, ϕ = 0.005, L = 10 µm, and D = 10 nm

As shown in Figure 6, heat transfer rates exhibit an increasing trend with longer MWCNT lengths. As the length increases from 10 µm to 30 µm, heat transfer rates show an even rise. The observed increase in heat transfer rates with longer MWCNT lengths in the modified Hamilton-Crosser model can be attributed to an enhanced surface area available for interaction with the surrounding fluid. Longer nanotubes provide more surface contact points, facilitating improved heat transfer. The extended length allows for greater interaction with the fluid, promoting efficient heat exchange. Therefore, the important role that MWCNT length plays in heat transfer rates shows that longer nanotubes are related to better heat transfer efficiency. This insight is crucial for optimizing the design and application of MWCNT nanofluids in various heat transfer systems. Furthermore, a modified model can assist in identifying the ideal nanotube length for enhanced heat transfer.

Finally, Figure 7 provides an insight into the impact of the diameter of MWCNT D on the heat transfer rates. Heat transfer rates exhibit a decreasing trend with increasing MWCNT diameter. As the diameter increases from 10 nm to 30 nm, heat transfer rates decline progressively. The observed decrease in heat transfer rates with larger MWCNT diameters in the modified Hamilton-Crosser model can be attributed to the reduced surface area available for interaction with the surrounding fluid. Smaller diameter nanotubes likely offer more surface contact points, facilitating better heat transfer. As the diameter increases, the available surface area decreases, leading to a reduction in heat transfer efficiency. The findings underscore the importance of considering MWCNT diameter as a critical parameter influencing heat transfer rates. According to the diminishing trend, larger nanotubes may be less effective at promoting efficient heat exchange after a certain diameter, which could offset the advantages of increased surface area. The analysis implies that there exists an optimal MWCNT diameter for maximising heat transfer efficiency. Thus, the present study suggests that a modified model can aid in identifying the ideal nanotube diameter for enhanced heat transfer.

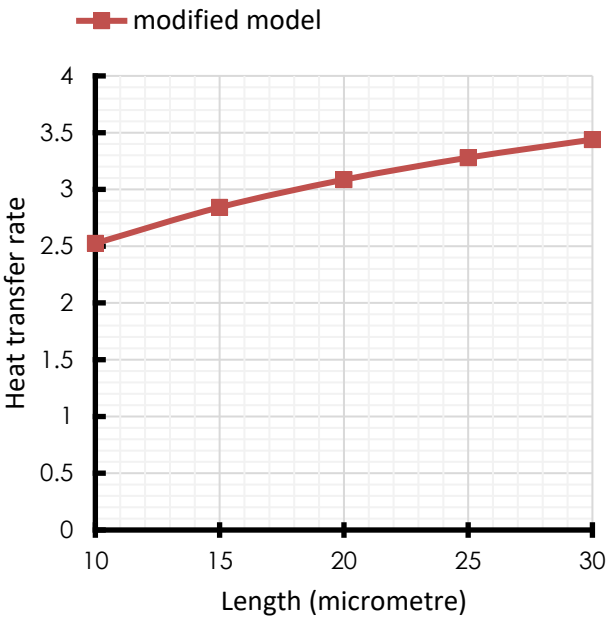

Fig. 6. Heat transfer rates of MWCNT-water nanofluid using a modified model for different MWCNT lengths when M = 0.01, Ec = 0.01, ϕ = 0.005, S = 0.1, and D = 10 nm

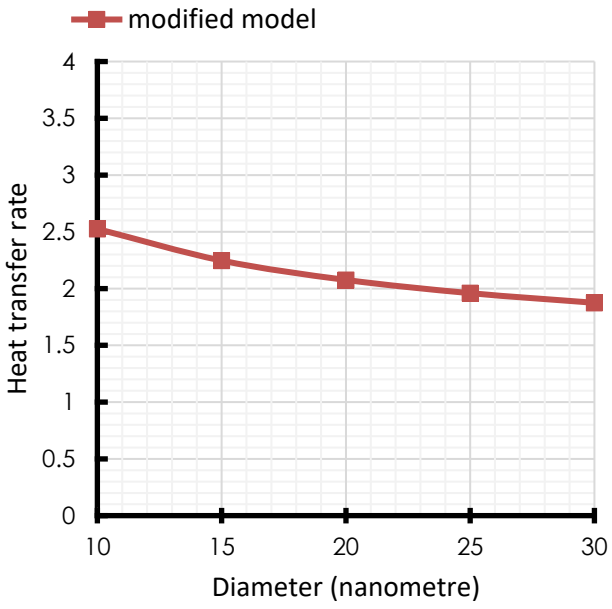

Fig. 7. Heat transfer rates of MWCNT-water nanofluid using a modified model for different MWCNT diameters when M = 0.01, Ec = 0.01, ϕ = 0.005, S = 0.1, and L = 10 µm

4. Conclusions

This study provides a comparative analysis of modified and classical Hamilton-Crosser models for predicting heat transfer rates for water-based nanofluids containing MWCNT. The study examines the impact of different factors, such as magnetic parameter *M*, Eckert number Ec, nanoparticle volume fraction ϕ, and surface transpiration rate *S* on the heat transfer rates predicted by both models. The study also examines the impact of the length of MWCNT *L* and the diameter of MWCNT *D* on the modified model. The modified model demonstrates a decrease in heat transfer rates as the magnetic parameter increases, which may be attributed to the magnetic field's impact on the behaviour of MWCNTs. Conversely, the heat transfer rates exhibit slight changes as the Eckert number rises for both the modified and classical Hamilton-Crosser models. These findings indicate that the viscous dissipation parameter does not exert a significant impact on the heat transfer characteristics of the MWCNT water-based nanofluids. With an increasing nanoparticle volume fraction, the modified Hamilton-Crosser model reliably forecasts larger rates of heat transfer in comparison to the classical model. The rate of transpiration on the surface has an impact on the rates of heat transfer in both modified and classical Hamilton-Crosser models. Besides, increased lengths of MWCNTs lead to higher rates of heat transfer as a result of an augmented surface area that facilitates more contact with the fluid. Consequently, the modified model can aid in determining the optimal nanotube length to improve heat transmission. However, the diameter of the MWCNT also affects heat transfer rates. As the diameter of nanotubes expands, there is a gradual decrease in heat transfer rates. This suggests that larger nanotubes may become less effective at facilitating efficient heat transfer after they reach a particular diameter. Hence, this study proposes that the modified model for the thermal conductivity of nanofluids can facilitate the identification of optimal nanotube length and diameter to enhance heat transfer efficiency. It is crucial to consider the length and diameter of MWCNTs as critical variables that significantly impact the heat transfer rate of nanofluids.

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