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Influence of MHD Flow on Shrinking Sheet with Partial Slip and Heat Generation at Stagnation Point

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ABSTRACT

Fluid dynamics encompasses the fundamental principles of continuity, momentum, and energy conservation, which are applied through mathematical models like the Navier-Stokes equations. These equations are essential for describing how fluid properties like velocity, pressure, and density change in response to forces and environmental conditions. Thus, this study attempted to explore the characteristics of flow and heat transfer of a shrinking sheet in magnetohydrodynamics (MHD), along with the effect of partial slip and heat generation on the system. We employ a similarity transformation technique for turning the governing partial differential equations into ordinary differential equations. These equations are solved numerically through shooting method in Maple, and the results are compared to the previous research. The analysis shows that the suction parameter and velocity slip parameter have an increasing effect on both the skin friction rate and the heat transfer rate. In the meantime, the heat transfer rate decreases as the parameter increases for the heat generation, magnetic parameter, Eckert number and thermal slip parameter. The bvp4c solver in MATLAB is implemented to conduct a stability analysis and determine the physically feasible solution. According to our research, the stability of the solution occurs only in the first solution.

1. Introduction

Recent research has increasingly focused on the boundary layer flow produced by shrinking sheets, a phenomenon with significant applications in polymer processing [1] and film production [2]. Pioneering work by Miklavčič and Wang [3] introduced the concept of shrinking sheets, sparking further exploration across various contexts [4-12]. Wang [7] notably identified the stagnation flow characterized by vorticity against a shrinking sheet, where maximum pressure, mass deposition, and

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heat transfer occur. Building on this, Uddin *et al.,* [8] investigated magnetohydrodynamic (MHD) stagnation point nanofluid flow toward a permeable stretching and shrinking sheet, expanding our understanding of these complex flows.

Boundary slip, another critical factor in fluid dynamics, plays a crucial role in technological applications like emulsion suspensions [13] and polymeric foams [14]. The foundational work by Bearvers and Joseph [15] on slip flow through permeable walls laid the groundwork for Wang's [16] exact similarity solutions for fluid flow at stagnation points, incorporating velocity slip and heat generation. Subsequent studies, including those by Mahmood *et al.,* [17] and Sarkar and Makinde [18], have shown that velocity slip reduces skin friction while enhancing heat transfer, further illustrating the intricate behaviour of fluids under these conditions. While MHD flow in electrically conducting fluids has been somewhat underexplored [18-23], Gupta [19] contributed valuable insights into free convection in the presence of a magnetic field.

Meanwhile, heat generation, a fundamental process where energy forms convert into sensible heat, remains a critical area of study with numerous contributions in the literature [24-27]. Heat generation is significant in boundary layer flows with heat transfer, as it affects the temperature within the boundary layer, which in turn governs the quality of the final output [27]. The study by Reddy [28] emphasizes the impact of thermal diffusion and viscous dissipation on MHD flows, particularly under conditions of heat generation. The results indicate that heat generation can lead to increased fluid temperatures, which in turn affects the flow behaviour and heat transfer efficiency. Additionally, the research by Hazarika and Gogoi [29] explores the effects of variable viscosity and thermal conductivity on MHD flows with heat generation, highlighting the complex interplay between these factors and their influence on the thermal boundary layer.

The integration of magnetic field terms into momentum and energy equations has practical applications across various fields, particularly in engineering, biomedical applications, and materials science. One significant application is in the design of magnetic bearings and lubrication systems. Lin *et al.,* [30] demonstrated that the use of electrically conducting fluids as lubricants, combined with magnetic fields, can enhance the dynamic characteristics of wide parallel step slider bearings. The Lorentz force generated by the interaction of the conducting fluid with the magnetic field leads to increased pressure and film forces, which can improve the performance and reliability of mechanical systems [30]. In addition, the integration of magnetic field terms into momentum and energy equations not only aligns with existing models but also enhances our understanding of fluid dynamics in magnetized environments. The implications of these findings extend across multiple disciplines, suggesting that future research should focus on leveraging these insights to develop innovative technologies and improve existing systems.

This study aims to advance the field by examining fluid flow at stagnation points and heat transfer to a shrinking sheet, incorporating MHD effects, heat generation, and partial slip conditions. The novelty of current study would be adding the effect of magnetic field terms into the momentum and energy equations with inspired by the work of Pop *et al.,* [31]. Our research offers a fresh perspective on this problem, which has not been previously explored, and compares our findings with those of Mahapatra *et al.,* [32]. Additionally, we will conduct a stability analysis to evaluate the robustness of the solutions. To the best of the authors' knowledge, this problem has not been previously published. The presence of a magnetic field, slip conditions, and heat generation all contribute to the fluid flow and heat transfer characteristics. The integration of these factors is essential for optimizing thermal management in various engineering applications, underscoring the importance of continued research in this area. Future research could expand these models to include non-linear effects and interactions with complex geometries, which are common in real-world applications such as microfluidics and biomedical devices.

2. Formulation

Consider the viscous flow in two dimensions of an incompressible, electrically conducting fluid that is approaching its stagnation point. This fluid is moving at a constant rate towards a sheet that is shrinking. Given that $u_w(x) = cx$ is the velocity of stretching and shrinking with the shrinking rate is $c < 0$, and the stretching rate is $c > 0$. The x-axis, measured in the opposite direction of the motion of the sheet, is parallel to the shrinking sheet. The y-axis, on the other hand, is normal to the sheet. Additionally, the sheet is subjected to a consistent magnetic field B_0 that is applied in a normal direction. Figure 1 shows how the fluid dynamics of this problem are configured. Under these assumptions, this study's governing equations are [31-32].

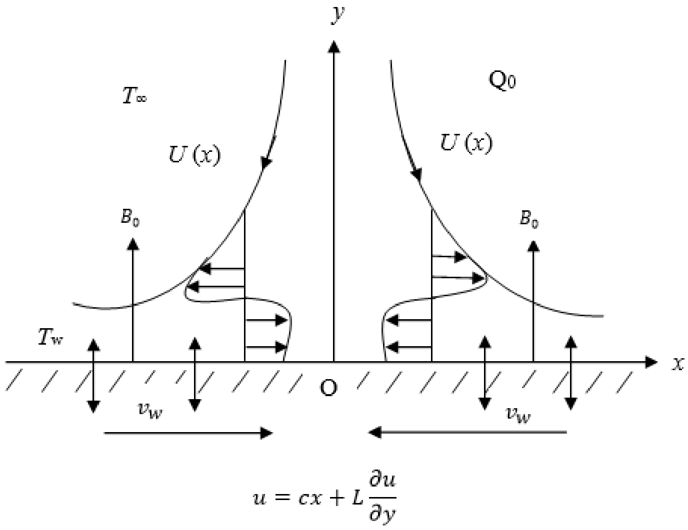

Fig. 1. A sketch of physical model

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = U\frac{dU}{dx} + v\frac{\partial^2 u}{\partial y^2} + \frac{\sigma B_0^2}{\rho}(U - u) \tag{2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \kappa\frac{\partial^2 T}{\partial y^2} + \frac{\mu}{\rho C_\rho}\left(\frac{\partial u}{\partial y}\right)^2 + \frac{Q_0}{\rho C_\rho}(T - T_\infty) + \frac{\sigma B_0^2}{\rho C_\rho}(U - u)^2 \tag{3}$$

and boundary conditions are

$$v = v_w,\ u = cx + L\frac{\partial u}{\partial y},\ T = T_w + S\frac{\partial T}{\partial y} \quad \text{at} \quad y = 0$$
$$u \to U(x) = ax,\ T \to T_\infty \quad \text{as} \quad y \to \infty \tag{4}$$

where u stands for the velocity in the x-direction and v for the velocity in the y-direction. T is the temperature of the fluid, ρ is the density of the fluid, $U(x)$ is the velocity of the external fluid, ν is the kinematic viscosity, and μ is the dynamic viscosity while the strength of the stagnation-point flow is $a > 0$. Assuming that T_∞ is the temperature of the surrounding environment and that b is a constant, we will refer to$T_w = bx^2 + T_\infty$ as the surface temperature, which will change depending on how far away we are from the stagnation point. Subsequently, to obtain a solution for Eqs. (1) – (4), we use the similarity transformation that is presented below.

$$\psi = \sqrt{av}xf(\eta),\ \eta = \sqrt{\frac{a}{v}}y,\ \theta(\eta) = \frac{T-T_\infty}{T_w-T_\infty} \tag{5}$$

where ψ is referred to as a stream function. Subsequently,

$$u = axf'(\eta) \text{ and } v = -\sqrt{va}f(\eta) \tag{6}$$

The following are the ordinary differential equations that we were able to acquire after substituting Eq. (5) and Eq. (6) into Eq. (2) and Eq. (3) as follows:

$$f''' + ff'' - f'^2 + 1 + M^2(1-f') = 0 \tag{7}$$

$$\frac{1}{Pr}\theta'' + f\theta' - 2f'\theta + Ecf''^2 + EcM^2(1-f')^2 + Q\theta = 0 \tag{8}$$

with boundary conditions as follows

$$\begin{gathered} f(0) = s, \qquad f''(0) = \alpha + \delta f''(\eta), \qquad \theta(0) = 1 + \lambda\theta'(\eta) \\ f'(\infty) \to 1,\ \theta(\infty) \to 0 \end{gathered} \tag{9}$$

where $s > 0$ is the dimensionless suction parameter with $v_w = -\sqrt{av}s, M = (\sigma B_0^2/a\rho)^{1/2}$ is the a magnetic parameter that describes the applied magnetic field's strength. Also, $\delta = (a/v)^{1/2}L$ is the parameter of velocity slip, $Ec = a^2/(bC_\rho)$ is the Eckert number, $Pr = \nu/\kappa$ is the Prandtl number and $\lambda = (a/v)^{1/2}S$ is the dimensionless thermal slip parameter. Next, the physical parameters of interest are

$$C_f = \frac{\mu}{\rho U^2}\left(\frac{\partial u}{\partial y}\right)_{y=0},\ Nu_x = \frac{x}{T_w-T_\infty}\left(\frac{\partial T}{\partial y}\right)_{y=0} \tag{10}$$

where C_f is referring to the coefficient of skin friction and Nu_x is referring to the local Nusselt number. Eq. (6) is used to derive the following.

$$Re_x^{1/2}C_f = f''(0),\ Re_x^{-1/2}Nu_x = -\theta'(0) \tag{11}$$

where $Re_x = U(x)x/\nu$ is referring to the local Reynolds number.

3. Stability Analysis

As a starting point for analysing a stability study, let the problem as an unstable situation. Eqs. (2) and (3) are then substituted by

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = U\frac{dU}{dx} + v\frac{\partial^2 u}{\partial y^2} + \frac{\sigma B_0^2}{\rho}(U-u) \tag{12}$$

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \kappa\frac{\partial^2 T}{\partial y^2} + \frac{\mu}{\rho C_\rho}\left(\frac{\partial u}{\partial y}\right)^2 + \frac{Q_0}{\rho C_\rho}(T-T_\infty) + \frac{\sigma B_0^2}{\rho C_\rho}(U-u)^2 \tag{13}$$

where t refers to time. To address the unsteady problem, a new dimensionless variable was introduced, as identified by variable (5),

$$\psi = \sqrt{av}xf(\eta,\tau),\ \eta = \sqrt{\frac{a}{v}}y,\ \theta(\eta,\tau) = \frac{T-T_\infty}{T_w-T_\infty},\ \tau = at \tag{14}$$

so that Eqs. (2) and (3) will become

$$\frac{\partial^3 f}{\partial\eta^3} - \left(\frac{\partial f}{\partial\eta}\right)^2 + f\frac{\partial^2 f}{\partial\eta^2} + 1 + M^2\left(1 - \frac{\partial f}{\partial\eta}\right) - \frac{\partial^2 f}{\partial\eta\partial\tau} = 0 \tag{15}$$

$$\frac{1}{Pr}\frac{\partial^2\theta}{\partial\eta^2} + f\frac{\partial\theta}{\partial\eta} - 2\frac{\partial f}{\partial\eta}\theta + Ec\left(\frac{\partial^2 f}{\partial\eta^2}\right)^2 + EcM^2\left(1 - \frac{\partial f}{\partial\eta}\right)^2 + Q\theta - \frac{\partial\theta}{\partial\tau} = 0 \tag{16}$$

and the conditions at the boundary are:

$$\begin{aligned}&\frac{\partial f}{\partial\eta}(\eta,\tau) = \alpha + \delta f''(0,\tau),\ f(0,\tau) = s,\ \theta(0,\tau) = 1 + \lambda\frac{\partial\theta}{\partial\eta}\\ &\frac{\partial f}{\partial\eta}(\infty,\tau) \to 1,\ \theta(\infty,\tau) \to 0\end{aligned} \tag{17}$$

Next, $f(\eta) = f_0(\eta)$ and $g(\eta) = g_0(\eta)$ be the stable flow solutions that fulfill the boundary-value problem (1) – (4) and are analysed for their stability. We write ([32-37]).

$$\begin{aligned}f(\eta,\tau) &= f_0(\eta) + e^{-\gamma\tau}F(\eta,\tau)\\ g(\eta,\tau) &= g_0(\eta) + e^{-\gamma\tau}G(\eta,\tau)\end{aligned} \tag{18}$$

where γ is an unidentified eigenvalue, $F(\eta,\tau)$ is relatively small to $f_0(\eta)$, $G(\eta,\tau)$ is relatively small to $g_0(\eta)$. When Eqs. (15) and (16) are replaced with Eq. (18), the outcome is

$$\frac{\partial^3 F}{\partial\eta^3} + f_0\frac{\partial^2 F}{\partial\eta^2} + f_0''F(\gamma + M^2 - 2f_0')\frac{\partial F}{\partial\eta} - \frac{\partial^2 F}{\partial\eta\partial\tau} = 0 \tag{19}$$

$$\frac{1}{Pr}\frac{\partial^2 G}{\partial\eta^2} + f_0\frac{\partial G}{\partial\eta} + \theta_0'F + (\gamma + Q - 2f_0')G - 2\theta_0'\frac{\partial F}{\partial\eta} + 2Ecf_0''\frac{\partial^2 F}{\partial\eta^2} + EcM^2\left(2\frac{\partial F}{\partial\eta} - 2f_0'\frac{\partial F}{\partial\eta}\right) - \frac{\partial\theta}{\partial\tau} = 0 \tag{20}$$

and the boundary conditions are

$$\begin{aligned}&\frac{\partial F}{\partial\eta}(0,\tau) = \delta\frac{\partial^2 F}{\partial\eta^2}(0,\tau),\ F(0,\tau) = 0,\ G(0,\tau) = \lambda\frac{\partial G}{\partial\eta}(0,\tau)\\ &\frac{\partial F}{\partial\eta}(\infty,\tau) \to 0,\ G(\infty,\tau) \to 0\end{aligned} \tag{21}$$

The stable solution (7) and solution (8) can be derived by initially setting $\tau = 0$, which subsequently results in the solutions , $f(\eta) = f_0(\eta)$ and $g(\eta) = g_0(\eta)$. Therefore, both , $F = F_0(\eta)$ and $G = G_0(\eta)$ in Eqs. (19) and (20) point to the initial increment of the solution (18). In this situation, we need to address the linear eigenvalue problem.

$$F_0''' + f_0 F_0'' + f_0'' F_0 + (\gamma + M^2 - 2f_0')F_0' = 0 \quad (22)$$

$$\frac{1}{Pr}G_0'' + f_0 G_0' + \theta_0' F_0 + (\gamma + Q - 2f_0')G_0 - 2\theta_0' F_0' + 2Ecf_0'' F_0'' + EcM^2(2F_0' - 2f_0' F_0') = 0 \quad (23)$$

followed by the boundary conditions

$$F_0'(0) = \delta F''(0),\ F_0(0) = 0,\ G_0(0) = \lambda G'(0)$$
$$F_0'(\infty) \to 0,\ G_0(\infty) \to 0 \quad (24)$$

Note that the smallest eigenvalue γ for the specific parameter values, such as M, S, δ, Q, λ and Ec, can be employed to confirm the stability of the steady-state flow solutions for both $f_0(\eta)$and $\theta_0(\eta)$. Relaxing the boundary constraint on both $F_0(\eta)$ and $G_0(\eta)$ allows us finding the range of eigenvalues, as described by Harris *et al.* [37]. Subsequently, this study opted to relax the condition $F_0' \to 0$, and the Eqs. (22) and (23) were solved (23) by an updated boundary condition which is $F_0'' = 1$.

4. Result and Discussions

A numerical solution to the ODEs (7) and (8) with boundary conditions (9) was obtained by applying the shooting method was obtained by using Maple. The thickness of the boundary layer value, denoted by η_∞, was varied between 3 and 8 to achieve the infinity boundary conditions asymptotically. By estimating multiple initially values for $f''(0)$ and – $\theta'(0)$ and verifying that all profiles satisfy the boundary conditions, the dual solutions were discovered (9). Since Eq. (7) and Eq. (8) are uncoupled, the thermal field has no impact on the fluid field. As a result, the flow field is unaffected by the Prandtl number, Pr, thermal slip parameter, λ, the heat generation, Q, or the Eckert number, Ec.

The coefficient of skin friction results from this study were compared with those found by Mahapatra *et al.* [32], regarding the shrinking sheet case ($\lambda < 0$) when $S = \lambda = \delta = Q = Ec = M = 0$ as the problem reduces to the steady stagnation point flow. Table 1 shows that in the case of $S = \lambda = \delta = Q = Ec = M = 0$, $\alpha_c = -1.2465$ is the turning (critical) point. Excellent agreement has been found between the present numerical results and previous ones. Therefore, we believe that our current results are sufficient for us to continue.

Table 1
$f''(0)$ values compared to various α values

	Mahapatra *et al.* [24]		Present results	
λ	Upper Branch	Lower Branch	Upper Branch	Lower Branch
-0.25	1.402242	—	1.402241	—
-0.50	1.495672	—	1.495667	—
-0.75	1.489296	—	1.489298	—
-1.00	1.328819	0	1.328817	0
-1.10	1.186680	0.049229	1.186680	0.049229
-1.15	1.082232	0.116702	1.082231	0.116702
-1.20	0.932470	0.233648	0.932473	0.233650
-1.2465	0.584374	0.554215	0.584282	0.554296

Based on the numerical result, there is one solution if $\alpha > -1$, two solutions if $\alpha_c < \alpha < -1$ and no solution if $\alpha < \alpha_c$. As the suction parameter is heightened, it leads to an increase in both the skin friction rate and the heat transfer rate. As a result, increasing the suction parameter accelerates transverse fluid motion and increases the rate of heat transfer as shown in Figures 2 and 3. The impacts of velocity slip parameter δ on $f''(0)$ and $-\theta'(0)$ are illustrated in Figures 4 and 5 respectively. As demonstrated in Figure 4, the range of α increases as the parameter of velocity slip, δ increases. The reason for this situation is that the generated vorticity by the shrinking velocity is significantly reduced when the slip at the sheet increases. Due to this, the vorticity remains confined to the boundary layer with the same strain for higher values of the shrinkage parameter [19].

As other parameters are put at a constant of 0.2, the critical value for $\delta_c = 0$ is –1.38040, $\delta_c = 0.5$ is –1.93997 and $\delta_c = 1.0$ is –2.70419. The rate of heat transfer $-\theta'(0)$ also increases as δ increases as clearly shown in Figure 5. All these conditions are based on the first solution's numerical results. We can see that the numerical results for the second solution are inconsistent because the second solution is unstable.

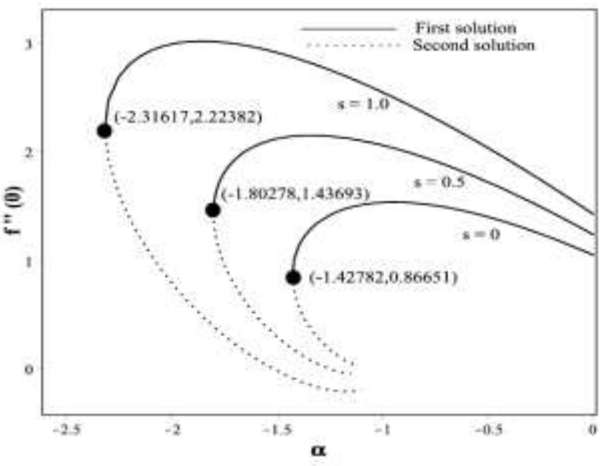

Fig. 2. $f''(0)$ variations for various s values

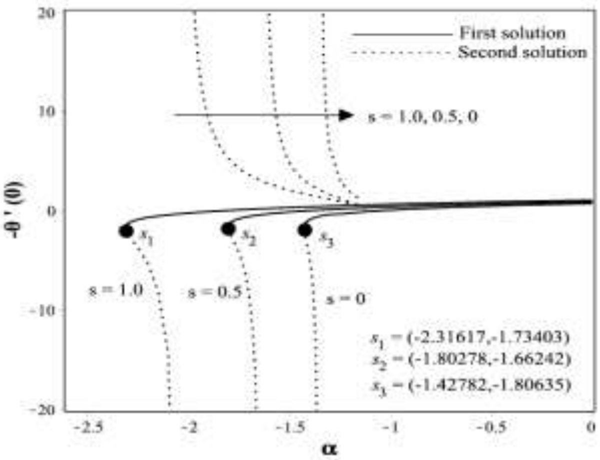

Fig. 3. $-\theta'(0)$ variations for various s values

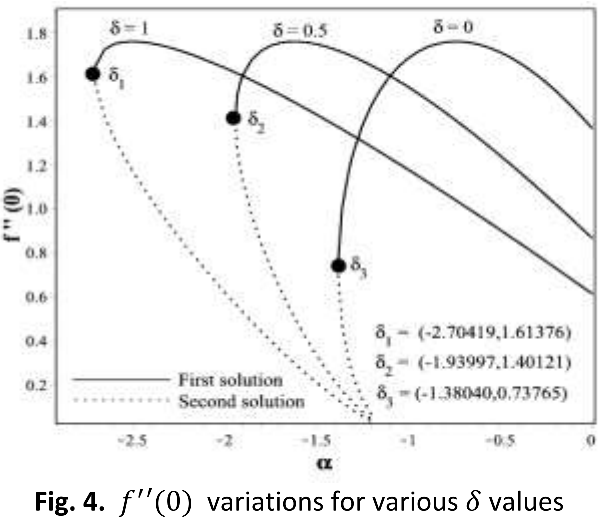

Fig. 4. $f''(0)$ variations for various δ values

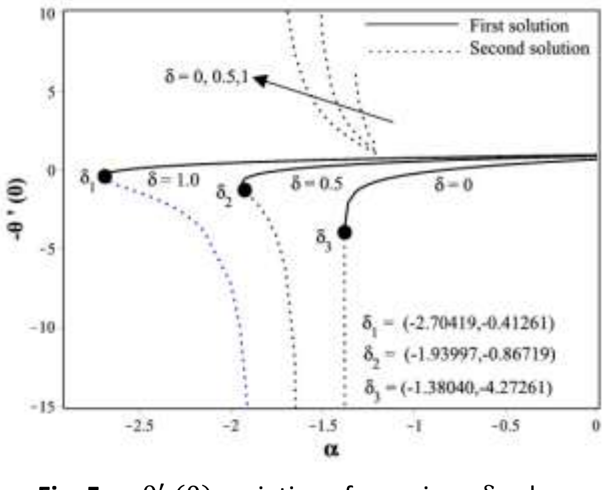

Fig. 5. $-\theta'(0)$ variations for various δ values

A magnetic field fluid contains electrical conductors, so the presence of a magnetic field causes a body force act against the flow of a fluid. When resistive forces like this are present, the fluid motion within the boundary layer tends to decelerate, thereby decreasing the heat transfer rate [25]. This statement is also supported by our result as depicted in Figure 6. When the magnetic parameter M, increases, indicating a rise in values, the heat transfer rate $-\theta'(0)$ exhibits the opposite effect, displaying a decrease in values. Other than that, the effect of the heat transfer rate $-\theta'(0)$ for certain values of parameter Eckert number Ec is illustrated in Figure 7. The observed increase in Eckert number Ec could be attributed to the decreasing rate of transfer. Figures 8 and 9 display the variations of the heat transfer rate $-\theta'(0)$ for various values of the thermal slip parameter λ and heat generation Q. Both graphs have shown a similar pattern: the heat transfer rate, $-\theta'(0)$ declines with increasing values of both parameters.

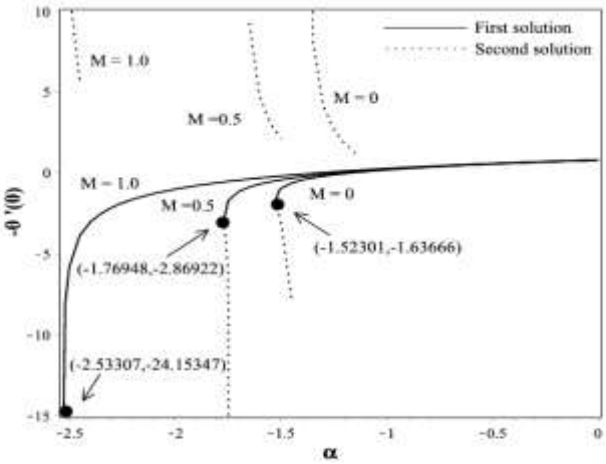

Fig. 6. $-\theta'(0)$ variations for various M values

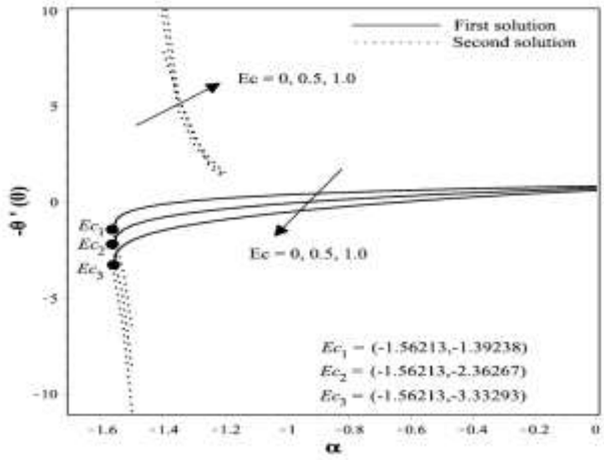

Fig. 7. $-\theta'(0)$ variations for various Ec values

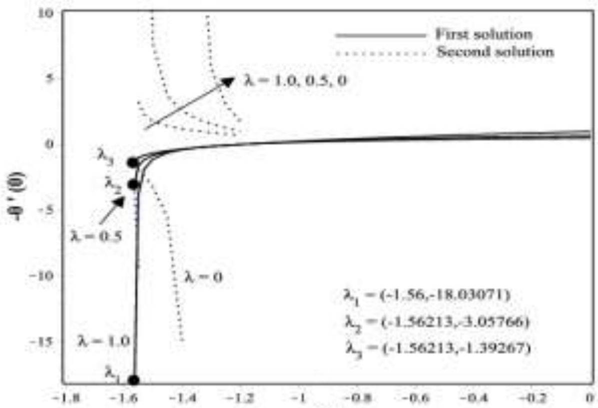

Fig. 8. $-\theta'(0)$ variations for various λ values

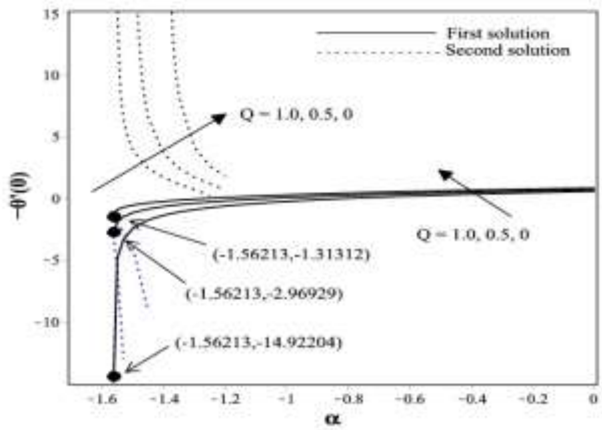

Fig. 9. $-\theta'(0)$ variations for various Q values

Further, Figures 10 – 15 display the variations of velocity profiles $f'(\eta)$ and temperature profiles $\theta(\eta)$ for different values of δ, s, M, Ec, Q and λ when $Pr = 0.71$. These graphs demonstrate that all curves converge asymptotically towards the boundary conditions, $f'(\eta) \rightarrow 1$ and $\theta(\eta) \rightarrow 0$ as $\eta \rightarrow \infty$. In addition, this proves that the first solution has a thinner boundary layer than the second.

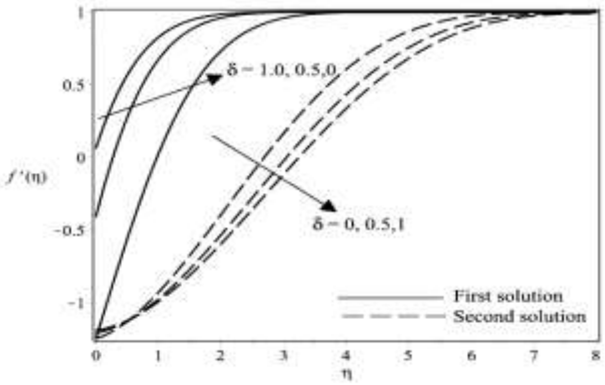

Fig. 10. Some δ values on velocity profiles

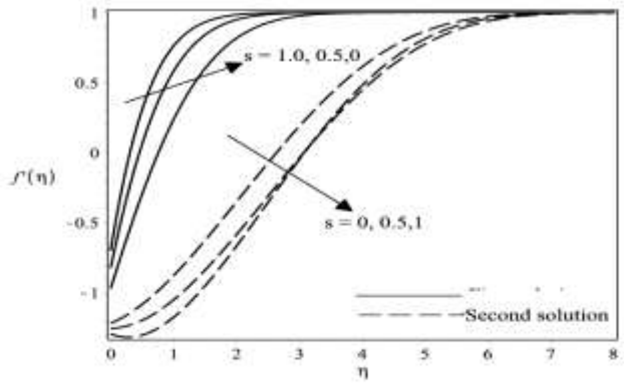

Fig. 11. Some s values on velocity profiles

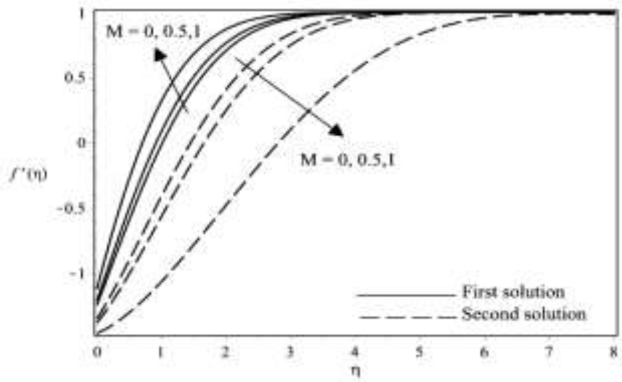

Fig. 12. Some M values on velocity profiles

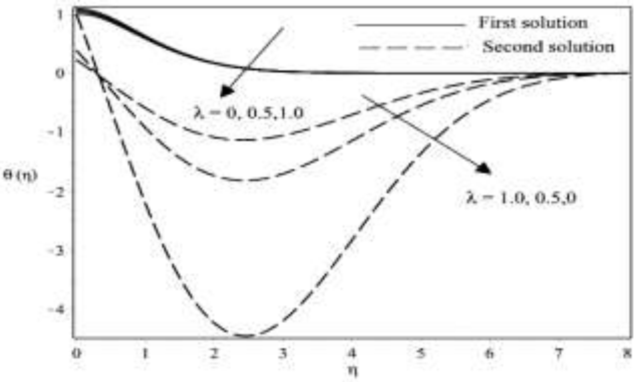

Fig. 13. Some λ values on temperature profiles

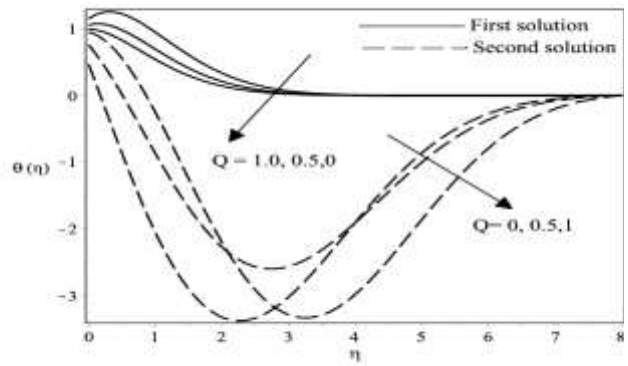

Fig. 14. Some Q values on temperature profiles

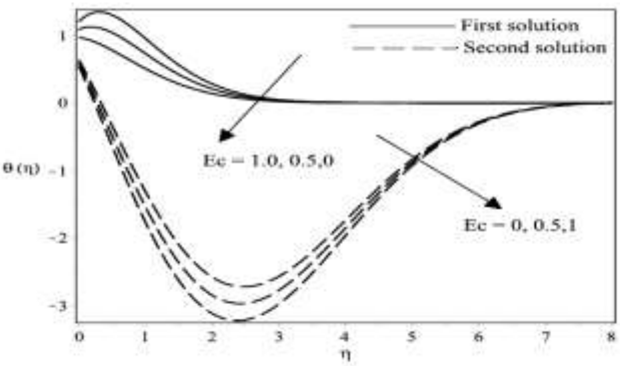

Fig. 15. Some Ec values on temperature profiles

We performed the stability analysis by endorsing the techniques [32–37] and obtained the result as shown in Table 2. To preserve workspace because other results are comparable, Table 2 only displays the smallest eigenvalues γ for the first and second solutions for certain values of suction. As stated previously, the flow is stable if and only if the smallest eigenvalue is positive. As can be seen, γ is a real with positive values for the first solution, while for the second solution, negative values were obtained. Furthermore, system of differential equations for the existence of the second solution, despite being unstable and physically negligible.

Table 2
The smallest eigenvalues of γ for a variety value of α when $M = \lambda = Q = Ec = \delta = 0.2$

s	α	1st solution	2nd solution
0	−1.40	0.42567	- 0.39748
	−1.42	0.22306	- 0.21519
	−1.38	0.56190	- 0.51307
0.5	−1.80	0.13697	- 0.13461
	−1.78	0.39727	- 0.37790
	−1.70	0.85864	- 0.76972
1.0	−2.30	0.34560	- 0.33362
	−2.22	0.52023	- 0.49339
	−2.14	0.94419	- 0.85703

5. Conclusion

Each of the parameters examined in this study has an effect on both the fluid flow and the heat transfer. Hence, we have come to the following conclusions:

i) Dual solutions exist when $\alpha_c < \alpha < -1$, one solution when $\alpha > -1$ and no solution when $\alpha < \alpha_c$.
ii) The suction parameter and velocity slip parameter have an increasing effect on $f''(0)$ and $-\theta'(0)$ as the parameter increases. Increasing the suction parameter accelerates transverse fluid motion and increases the rate of heat transfer.
iii) The heat transfer rate decreases as the parameter increases for the heat generation, magnetic parameter, Eckert number and thermal slip parameter. A magnetic field fluid contains electrical conductors, so the presence of a magnetic field causes a body force act against the flow of a fluid
iv) The first solution is the only one in which the stability of the solution is achieved.

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