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Maxwell Hybrid Nanofluid Flow Towards a Stagnation Point on a Stretching/Shrinking Inclined Plate with Radiation and Nanoparticles Shapes Effect

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ABSTRACT

The current study explored the Maxwell hybrid nanofluid on mixed convective radiative over a stretching/ shrinking inclined plate with nanoparticles shapes effect. Copper and aluminum oxide were introduced to sodium alginate as a base fluid to formulate the problem and the effect of shape factor is examined by considering spherical, bricks, cylindrical and platelet nanoparticles. Using similarity transformation, the governing nonlinear partial differential equations of the Maxwell hybrid nanofluid are converted to nonlinear ordinary differential equations. Then, they are solved numerically using the Keller Box method and the system is solved by using Fortran software. The physical behavior of controlling factors on velocity and temperature profiles as well as skin friction and local Nusselt number are depicted graphically and tabulated. The various shapes of nanoparticles produce considerable differences in the Maxwell hybrid nanofluid's velocity and temperature functions. For all parameters, nanoparticles shape with the highest Nusselt number is platelet, followed by cylindrical, bricks and spherical. The findings of this study will provide information and knowledge in mathematics for mathematicians who interested in future research on Maxwell hybrid nanofluid.

1. Introduction

Nanotechnology has become an excessive field over the past few decades. According to Rivas-Cruz *et al*., [1], nanotechnology is described as the study of matter at scales of 1 to 100 nanometers and usually composed of oxides, nanotubes, and metal. Nanotechnology creates many new instruments and materials for a large range of applications, for instance, biosensors, nanomedicine, chemical industry, tissue engineering, agriculture and, energy productions [2,3]. Numerous researchers are currently interested in nanotechnology because it lowers production costs, conserves

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energy and time, enhances the properties of materials, and ultimately raises human quality of life. [4]. One of the key components of nanotechnology is nanofluid, which is effectively used to solve problems with heat transfer.

Nanofluid was initially introduced by [5] and considered as a next generation medium of heat transfer and it is widely used in high technology for heating or cooling process [6]. It is an innovative concept in which it has offered solutions to many problems including nuclear system cooling, electronics cooling, solar water heating, heat exchanger, fuel cells and transportation [7-8]. Even though nanofluids meet the demands of engineers and scientists to obtain higher thermal performance, an enhanced type of fluid is still sought after today. In order to meet these demands, researchers have proposed that the latest class of nanofluids is recognized as hybrid nanofluid. In general, hybrid nanofluids are created by combining two or more different types of nanoparticles (metal oxides, metals, and carbon materials) into base fluids (water, ethylene glycol, a mixture of ethylene glycol and water) [9]. The primary goal of adopting hybrid nanofluid is to obtain a promising improvement in thermophysical hydrodynamic and heat transfer properties [10]. The studies on hybrid nanofluids by some researchers was referred on [11-14]

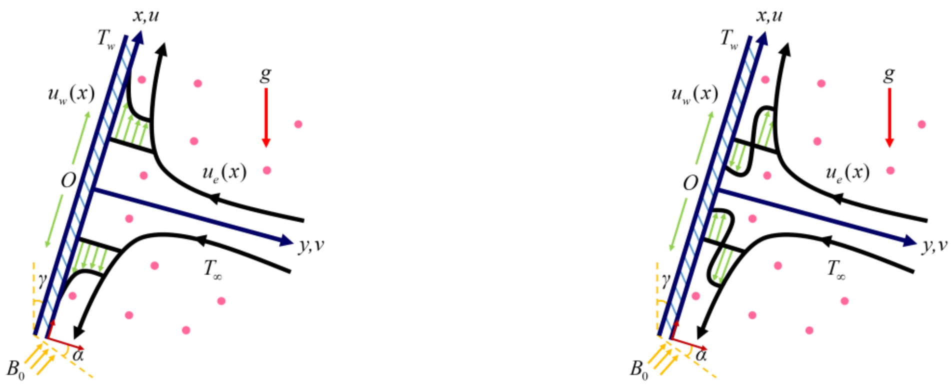
Mathematical models have three categories which are differential, rate, and integral type models. Maxwell fluid model, a subclass rate type model is used to predict both elastic and memory effects simultaneously and determine the relaxation time effect [15]. Lok *et al.,* [16] heat transfer of an upper convected Maxwell fluid near a stagnation-point of a permeable shrinking sheet found that the effects of shrinking and suction are direct and obvious as the flow near the surface is seen to suck through the permeable sheet and drag to the origin of the sheet. However, aligned but reverse flow occurs for the case of lower branch solutions. Studied the Maxwell nanofluid on a stretching sheet surface by Zainal *et al.,* [17] found that Maxwell parameter in hybrid nanofluids embarks on a substantial increment of the heat transfer rate contrast to traditional fluids. In magnetohydrodynamics (MHD) flow, magnetic force is applied to a fluid that conducts electricity to create the currents needed to generate the opposing Lorentz force in the field. The interaction between the magnetic and electrical fields, affects the industrial equipment especially for MHD generators [18]. Currently, the research related to MHD has developed quickly by focusing on various problems [19-26]. Due to the fundamental differences in the energy-exchange processes for radiation, conduction and convection, thermal radiation impacts on the boundary layer flow of fluid becomes noticeable when mechanical procedures take place at high temperature [27]. Thermal radiation effect is applicable in fields such as biomedical fields. Waini *et al.,* [28] investigates a hybrid nanofluid flow towards a stagnation region of a vertical plate with radiation effects found that heat transfer rate is intensified with the thermal radiation by 49.63% and the hybrid nanoparticles by 32.37%. The studied on various problems related in thermal radiation has been done by some researchers in [29-31].

Mixed convection is a combination of forced and free convection. Forced convection and free convection is a mixing motion produced by an external source and difference in density, respectively [32]. The fluid flow characteristics and heat transfer over a stretching surface can be used in a variety of industries. For example, in the ultimate quality of thinning and annealing some metallic wires [33]. Nanoparticles shapes are crucial as they have a great effect on the nanofluid properties. To identify the improvement of nanofluid thermal conductivity, different shapes of nanoparticles can be used in different host liquids [34]. Bosli *et al.,* [35] using five different shapes of nanoparticles including sphere, platelet, cylinder, lamina, and brick to study the effects of nanoparticles shape towards the behavior of aligned MHD natural convection Casson nanofluid passing a vertical plate with convective boundary condition. Research by Rawi *et al.,* [36] examined the influence of different shapes of copper nanoparticles (sphere, needle and disk-shaped) and the effect of material parameter, solid nanoparticles volume fraction, amplitude of modulation and frequency of oscillation on the mixed convection flow of second grade nanofluid past an inclined stretching sheet.

It can be concluded that Maxwell hybrid nanofluid plays a significant role in enhancing the thermal conductivity of fluids. Therefore, the study on Maxwell hybrid nanofluid on mixed convective radiative flow over a stretching/ shrinking inclined plate with nanoparticles shapes effect cannot be ignored and needs further investigation. Copper (Cu) and aluminum oxide (Al_2O_3) are used as nanoparticles within the base fluid, sodium alginate (NaAlg). Conventional boundary condition of constant wall temperature is applied. There are four nanoparticles' shapes considered, which are spherical, bricks, cylindrical and platelet shapes. The effects of parameters such as aligned angle of magnetic field, interaction of magnetic field, plate inclination, Maxwell parameter, mixed convection parameter, stretching parameter, radiation parameter, and volume fraction of nanoparticles towards the velocity and temperature profiles as well as skin friction and Nusselt number are examined.

2. Methodology

This study considers a continuous, incompressible two-dimensional stagnation point flow of magnetohydrodynamic Maxwell hybrid nanofluid over a stretching/ shrinking inclined plate as shown in Figure 1, where x and y are the Cartesian coordinates such that x-axis is parallel to the inclined plate while y-axis is normal to the inclined plate with γ as the inclination angle under the gravitational force g. The ambient velocity of the fluid is $u_e(x)$ and the stretching/shrinking velocity of the inclined plate is $u_w(x) = cx$, such that when $c > 0$ and $c < 0$, the inclined plate are stretching and shrinking, respectively, while the inclined plate is static when $c = 0$. The temperature of the ambient fluid and at the wall are T_∞ and T_w, respectively. The magnetic field that is assumed to be from the origin with an inclination of acute angle α is defined as $B(x) = B_0$ where $B_0 (\neq 0)$ is the magnetic field strength. In addition, the base fluid, and the suspended nanoparticles of the hybrid nanofluid are assumed to be in thermal equilibrium.



(a) (b)

Fig. 1. Physical model of (a) stretching inclined plate and (b) shrinking inclined plate

Based on the abovementioned assumptions as well as referring to the work by Zainal *et al.,* [17], Waini *et al.,* [28] and Ilias *et al.,* [25], the governing partial differential equations are given as:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=u_e\frac{\partial u_e}{\partial x}+\frac{\mu_{hnf}}{\rho_{hnf}}\frac{\partial^2 u}{\partial y^2}-k_0\left(u^2\frac{\partial^2 u}{\partial x^2}+v^2\frac{\partial^2 u}{\partial y^2}+2uv\frac{\partial^2 u}{\partial x\partial y}\right)+\frac{(\rho\beta)_{hnf}}{\rho_{hnf}}g\cos\gamma\left(T-T_\infty\right)-\frac{\sigma B_0{}^2}{\rho_{hnf}}\sin^2\alpha\left(u-u_e\right) \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{k_{hnf}}{\left(\rho C_p\right)_{hnf}}\frac{\partial^2 T}{\partial y^2}-\frac{1}{\left(\rho C_p\right)_{hnf}}\frac{\partial q_r}{\partial y} \tag{3}$$

while the boundary conditions used in this study are as follows:

$$\begin{aligned} &u=u_w(x) \quad v=0 \quad T=T_w \quad \text{on} \quad y=0 \\ &u\to u_e(x) \quad T\to T_\infty \quad \text{as} \quad y\to\infty \end{aligned} \tag{4}$$

where u and v are the velocities in x and y- directions, respectively, σ is the electrical conductivity, μ_{hnf}, ρ_{hnf}, β_{hnf}, k_{hnf}, and $\left(\rho C_p\right)_{hnf}$ refer to the dynamic viscosity, density, thermal expansion, thermal conductivity, and heat capacity for the hybrid nanofluid, respectively.

For the thermal radiation, the Rosseland approximation is utilized to obtain the following radiative heat flux:

$$q_r=-\frac{4\sigma *}{3k *}\frac{\partial T^4}{\partial y} \tag{5}$$

where $\sigma *$ is the Stefan Boltzmann whereas $k*$ is the mean absorption coefficients. Taylor's series is considered to extend to the temperature difference in the flow. By extending T^4 over T_∞ with the higher-order terms is being neglected, we get $T^4 \cong 4T_\infty{}^3T-3T_\infty{}^4$. So, that

$$\frac{\partial q_r}{\partial y}=-\frac{16\sigma * T_\infty^3}{3k *}\frac{\partial^2 T}{\partial y^2} \tag{6}$$

Therefore, the energy equation Eq. (3) becomes:

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{k_{hnf}}{\left(\rho C_p\right)_{hnf}}\frac{\partial^2 T}{\partial y^2}+\frac{1}{\left(\rho C_p\right)_{hnf}}\frac{16\sigma * T_\infty^3}{3k *}\frac{\partial q_r}{\partial y} \tag{7}$$

For the hybrid nanofluid, the thermophysical properties of the base fluid and hybrid nanoparticles are displayed in Table 1, taken from research by Zainal *et al.,* [17] and Alwawi *et al.,* [26]. Table 2 demonstrates the thermophysical relation of hybrid nanofluids [14] where ϕ_1 and ϕ_2 are the nanoparticle volume fraction for copper and aluminum oxide, respectively. The subscripts of s_1, s_2, hnf, bf and f refer to copper nanoparticle, aluminum oxide nanoparticle, hybrid nanofluid, nanofluid and base fluid, respectively. The shape factor and its numerical shape factor with sphericity values for different kinds of shapes is shown in Table 3, referred from research by Liu *et at.,* [37].

Table 1
Thermophysical Properties of Base Fluid and Hybrid Nanofluid

Properties	NaAlg	Cu	Al_2O_3
$C_P(\frac{J}{kgK})$	4175	385	765
$K(\frac{W}{mK})$	0.6376	400	40
$\rho(\frac{kg}{m^3})$	989	8933	3970
$\beta \times 10^{-5}$	99	1.67	0.85
Pr	6.45		

Table 2
Thermophysical Relation of Hybrid Nanofluid

Properties	Hybrid Nanofluid	
Density	$\rho_{hnf} = (1-\phi_2)[(1-\phi_1)\rho_f + \phi_1\rho_{s_1}] + \phi_2\rho_{s_2}$	(8)
Heat Capacity	$(\rho C_p)_{hnf} = (1-\phi_2)[(1-\phi_1)(\rho C_p)_f + \phi_1(\rho C_p)_{s_1}] + \phi_2(\rho C_p)_{s_2}$	(9)
Viscosity	$\mu_{hnf} = \frac{\mu_f}{(1-\phi_1)^{2.5}(1-\phi_2)^{2.5}}$	(10)
Thermal Conductivity	$\frac{k_{hnf}}{k_{bf}} = \frac{k_{s_2} + (m-1)k_{bf} - (m-1)\phi_2(k_{bf} - k_{s_2})}{k_{s_2} + (m-1)k_{bf} + \phi_2(k_{bf} - k_{s_2})}$	(11)
	$\frac{k_{bf}}{k_f} = \frac{k_{s_1} + (m-1)k_f - (m-1)\phi_1(k_f - k_{s1})}{k_{s_1} + (m-1)k_f + \phi_1(k_f - k_{s_1})}$	(12)
Thermal Expansion Coefficient	$\beta_{hnf} = [(1-\phi_2)((1-\phi_1)\beta_f + \phi_1(\rho\beta)_{s_1})] + \phi_2\beta_{s_2}$	(13)
	$(\rho\beta)_{hnf} = [(1-\phi_2)((1-\phi_1)(\rho\beta)_f + \phi_1(\rho\beta)_{s_1})] + \phi_2(\rho\beta)_{s_2}$	(14)
Thermal Diffusivity	$\alpha_{hnf} = \frac{k_{hnf}}{(\rho C_p)_{hnf}}$	(15)

Table 3
The Nanoparticles Shape Factors (m) and Sphericity (ε)

Nanoparticles Shape	Shape Factor (m)	Sphericity (ε)
Spherical	3.0	1.00
Bricks	3.7	0.81
Cylindrical	4.8	0.62
Platelets	5.7	0.52

As per usual, to simplify the governing partial differential equation, the following similarity variables are introduced:

$$\eta = y\sqrt{\frac{a}{\upsilon_f}}, \psi = \sqrt{a\upsilon_f}\, x\, f(\eta), \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty} \tag{16}$$

$$u = ax\, f'(\eta), v = -(a\upsilon_f)^{\frac{1}{2}} f(\eta) \tag{17}$$

Hence, by substituting the similarity variables into Eq. (2) and Eq. (7), the ordinary differential equations of the mathematical model obtained are

$$\begin{aligned} & f'''(\eta) - A_1 A_2 \left[f'(\eta)^2 - A_1 A_2 \;\; f(\eta) f''(\eta) - 1 + K\left(f(\eta)^2 f'''(\eta) - 2 f(\eta) f'(\eta) f''(\eta) \right) \right] \\ & - \; A_1 M \sin^2\alpha \left(f'(\eta) - 1 \right) + \; A_1 A_3 \lambda_T \, \theta(\eta) \cos\gamma = 0 \end{aligned} \tag{18}$$

$$\frac{1}{\Pr} A_4 \left[A_5 + \frac{4}{3} Rd \right] \theta''(\eta) + f(\eta)\, \theta'(\eta) = 0 \tag{19}$$

while Eq. (4) is transformed to the following boundary conditions

$$\begin{aligned} & f'(0) = \lambda \qquad f(0) = 0 \qquad \theta(0) = 1 \qquad \text{at} \quad y = 0 \\ & f'(\eta) \to 1 \qquad \qquad \qquad \theta(\eta) \to 0 \qquad \text{as} \quad y \to \infty \end{aligned} \tag{20}$$

In this formulation, the related parameters are defined as follows:

$$\begin{aligned} & A_1 = \left[(1-\phi_1)^{2.5} (1-\phi_2)^{2.5} \right], \quad A_2 = \left[(1-\phi_2) \left((1-\phi_1) + \phi_1 \frac{\rho_{s1}}{\rho_f} \right) + \phi_2 \frac{\rho_{s2}}{\rho_f} \right], \\ & A_3 = \left[(1-\phi_2) \left((1-\phi_1) + \phi_1 \frac{(\rho\beta)_{s1}}{(\rho\beta)_f} \right) + \phi_2 \frac{(\rho\beta)_{s2}}{(\rho\beta)_f} \right], \quad A_4 = \frac{(\rho C_\rho)_f}{(\rho C_\rho)_{hnf}}, \\ & A_5 = \frac{k_{hnf}}{k_f}, \quad \lambda_T = \frac{Gr_x}{{Re_x}^2}, \quad Gr_x = \frac{g\, \beta_f \left(T_w - T_\infty \right) x^3}{{\upsilon_f}^2}, \quad Re_x = \frac{a x^2}{\upsilon_f}, \\ & M = \frac{\sigma {B_0}^2}{\rho_f a}, \quad K = k_0\, a, \quad \Pr = \frac{\upsilon_f}{\alpha_f}, \quad Rd = \frac{4{T_\infty}^3 \sigma *}{k_f k *}, \; \lambda = \frac{c}{a} \end{aligned} \tag{21}$$

where A_1, A_2, A_3, A_4, A_5 are the related relations for the hybrid nanofluid thermophysical properties, λ_T is the mixed convection parameter, Gr_x is the local Grashof number, Re_x is the local Reynolds number, M is the magnetic parameter, K is the Maxwell parameter, $\Pr$ is Prandtl number, Rd is the thermal radiation parameter and λ is the stretching/shrinking parameter such that $\lambda > 0$ is for stretching, $\lambda < 0$ is for shrinking, and $\lambda = 0$ is for static plate. It also should be noted that, in order to have a true similarity solution, the parameter Gr_x must be a constant and not dependent on x, hence we assume that $\beta_f = ax^{-3}$ so that it will become $Gr = ag\left(T_w - T_\infty\right) / {\upsilon_f}^2$.

The quantities of physical interest in this present work are based on the definition of the skin friction, C_{f_x} at the plate's surface and the local Nusselt number, Nu_x:

$$C_{f_x} = \frac{\mu_{hnf}}{\rho_f u_e^{\,2}}\left(\frac{\partial u}{\partial y}\right)_{y=0} \text{ and } Nu_x = -\frac{x k_{hnf}}{k_f T\left(_f - T_\infty\right)}\left(\frac{\partial T}{\partial y}\right)_{y=0} + \left(q_r\right)_{y=0} \qquad (22)$$

Substituting into Eq. (16) and Eq. (17) into Eq. (22), the reduced skin friction and Nusselt number can be written as

$$Re_x^{\frac{1}{2}} Cf_x = \frac{\mu_{hnf}}{\mu_f} f''(0) \text{ and } \left(Re_x\right)^{-\frac{1}{2}} Nu_x = -\left[\frac{k_{hnf}}{k_f} + \frac{4}{3}Rd\right]\theta'(0) \qquad (23)$$

3. Numerical Solution

The Eq. (18) and Eq. (19) and boundary conditions (20) are numerically solved by using Keller Box method, an efficient implicit finite-difference method, which has been discussed by Cebeci and Bradshaw [38]. The four steps that needed to be followed in obtaining the solution are:

i. Reduce the differential equations to first-order equations.
ii. Write the difference equations using central differences.
iii. Linearize the algebraic equations by Newton's method and write them in the form of matrix-vector.
iv. Solve the linear system by block tri-diagonal elimination technique.

4. Results and Discussion

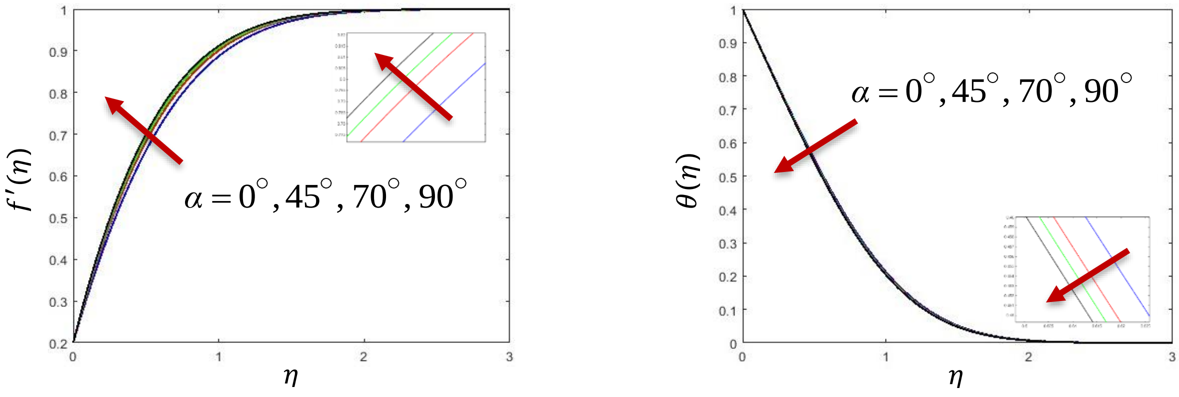
The impact of the parameters will be investigated in relation to the results. The effect of parameters on the Maxwell hybrid nanofluid's velocity and temperature profiles as well as skin friction and Nusselt number will be displayed using graphs and table. Tables will be used to illustrate how the parameters employed in this study affected skin friction and Nusselt number. Indicated in Table 4 below, the numerical values of the skin friction coefficient obtained are compared with those from four previous studies which are Lok *et al.,* [16], Kimiaeifar *et al.,* [39], Wang *et al.,* [40] and Zainal *et al.,* [17]. The comparison is made to ensure the validity and accuracy of this study. It is reported that the present results to be in fair agreement, which then confirms and supports the accuracy of the numerical results obtained.

Table 4
Results of $f''(0)$ for different values of λ when $M = K = \lambda_T = Rd = \phi_1 = \phi_2 = 0$

λ	Lok *et al.,* [16]	Kimiaeifar *et al.,* [39]	Wang *et al.,* [40]	Zainal *et al.,* [17]	Present Result
	$f''(0)$	$f''(0)$	$f''(0)$	$f''(0)$	$f''(0)$
-0.25	1.402240	1.402241	1.402240	1.402241	1.402241
-0.50	1.495670	1.495671	1.495670	1.495670	1.495670
-0.75	1.489300	1.489335	1.489300	1.489299	1.489299
-1.00	1.328820	1.328809	1.328820	1.328820	1.328820
-1.15	1.082230	-	1.082230	1.082245	1.082245
-1.20	0.932470	-	-	0.932508	0.932508
-1.12465	0.584300	-	-	0.586974	0.586974

The effect of parameters on the Maxwell hybrid nanofluid's velocity and temperature profiles as well as skin friction and Nusselt number will be displayed in the corresponding figures and tables. For the Maxwell hybrid nanofluid, the Prandtl number taken is 6.45 and fit the non-dimensional values as follows for the numerical computation $\alpha = 45^{o}, M = 1, \gamma = 45^{\circ}, K = 0.1, \lambda_T = 0.1, \lambda = 0.2,$ $Rd = 1,\ \phi_1 = 0.05$ and $\phi_2 = 0.05$ unless stated otherwise. Figure 2 to figure 9 demonstrate how the velocity and temperature profiles change with the varied values of $\alpha, M, \gamma, K, \lambda_T, \lambda, Rd,\ \phi_1$ and ϕ_2 whereas the numerical value of skin friction coefficient and Nusselt number are shown in Table 5.

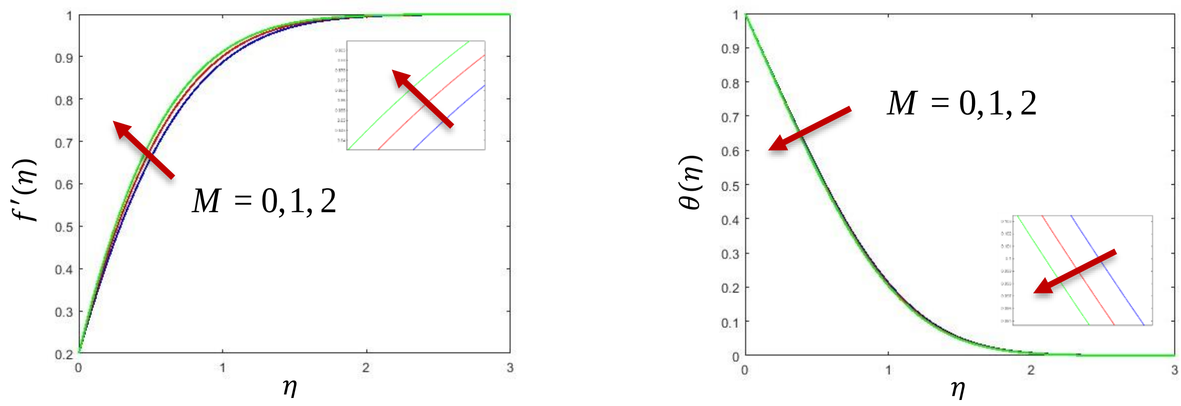
From Figure 2(a), it is observed that the increment in α causes the velocity profiles to increase and the thickness of the momentum boundary layer to decrease. The increase of the aligned angle makes the magnetic field becomes stronger thus pushes the Maxwell hybrid nanofluid towards the plate. When $\alpha = 0^{\circ}$, it indicates that there is no magnetic field but when $\alpha = 90^{\circ}$, the aligned magnetic field behaves like a transverse magnetic field and due to changes in the position of the aligned magnetic field, it attracts the nanoparticles. From Figure 2(b) it is depicted that an increase in α lead to the decrement in temperature profiles. Besides, thermal boundary layer thickness also decreases. The heat transfer is enhanced by an increase in α. As displayed in Table 5, the skin friction and Nusselt number increases as α increases.



(a) (b)

Fig. 2. Effects of α on (a) velocity and (b) temperature profiles of maxwell hybrid nanofluid

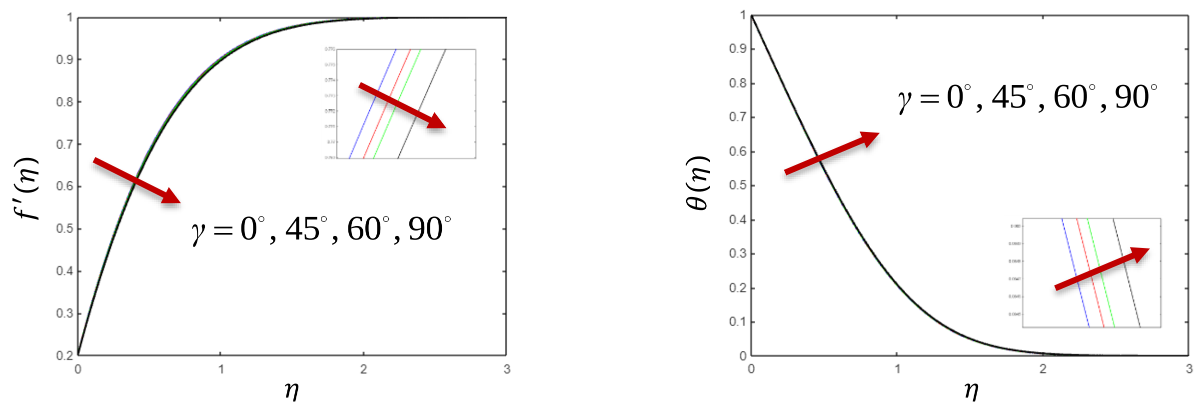
It can be noticed from Figure 3(a), when M increases, the velocity profiles increase while the momentum boundary layer thickness decreases. When the strength of the magnetic field increases, the fluid is pushed towards the plate thus the momentum boundary layer is reduced. The increasing in M leads to the increase in Lorentz force, in which it operates in such a way that causes the flow to accelerate thus producing more resistance to the transport phenomena. The effect of parameter M on the temperature profiles is demonstrated in Figure 3(b). When M rises, the temperature profiles across the plate decrease, along with the thermal boundary layer thickness. According to Table 5, the value for the skin friction and Nusselt number rises as M increases.



(a) (b)

Fig. 3. Effects of M on (a) velocity and (b) temperature profiles of maxwell hybrid nanofluid

In Figure 4(a), the increase in γ causes the velocity profiles to decrease and the momentum boundary layer thickness to rise. The plate is in a vertical and horizontal position when $\gamma = 0^{\circ}$ and $\gamma = 90^{\circ}$, respectively. Meanwhile the plate is in a slanted position when $\gamma = 45^{\circ}$ and $\gamma = 60^{\circ}$. The decrement in the velocity profile is because the drag is experienced along the surface of the plate thus make it harder for the fluid to flow. This is due to the reduction in the buoyancy effect by a factor $\cos\gamma$ and largely owing to the gravitational effects. The gravitational effect is minimal for $\gamma = 90^{\circ}$ and maximal for $\gamma = 0^{\circ}$. It can be observed that from Figure 4(b), an increase in γ increases the temperature profiles and the thermal boundary layer. This is because a stronger force is required by the fluid to flow better thus affects the temperature. Table 5 demonstrates that both skin friction and Nusselt number decrease, as γ increases.



(a) (b)

Fig. 4. Effects of γ on (a) velocity and (b) temperature profiles of maxwell hybrid nanofluid

The growing values of K make the velocity profiles in Figure 5(a) to increase thus reduce the momentum boundary layer thickness. The explanation of this behavior is the momentum equation tends to the momentum equation of Newtonian's fluid. Therefore, the velocity for nanofluid increases as the effective viscous drag forces decrease with the increase in K. In Figure 5(b), it is observed that there is a decreasing trend in the temperature profiles with the increasing value of K. The thermal boundary layer thickness also declines. This is due to the fluid reduction yield stress in the boundary layer as the K rises. Both skin friction and Nusselt number increase as parameter value of K increases as displayed in Table 5.

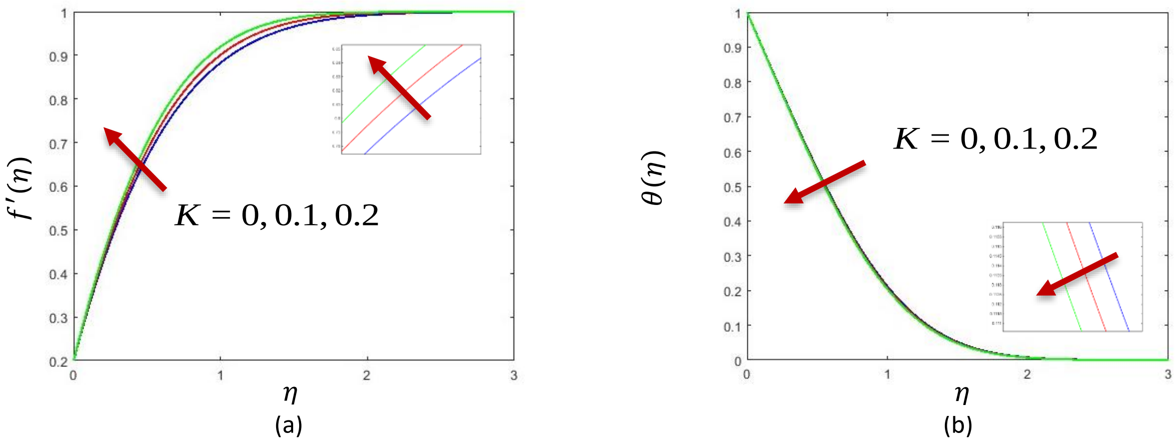


Fig. 5. Effects of K on (a) velocity and (b) temperature profiles of maxwell hybrid nanofluid

As seen in Figure 6(a), the velocity profiles increase with increasing λ_T values while the momentum boundary layer decreases. This happens due to the larger values of buoyancy force. If the value of the mixed convection parameter is magnified, therefore the buoyancy will increase. Hence, the flow velocity increases as the buoyancy grow. On the other hand, it can be seen from Figure 6(b), when there is an increase in λ_T , the temperature profiles and the thermal boundary layer will shrink. This is due to the increasing in the convection cooling effect as λ_T increase thus the temperature is reduced. The skin friction coefficient and Nusselt number rise as λ_T rises, as seen in Table 5.

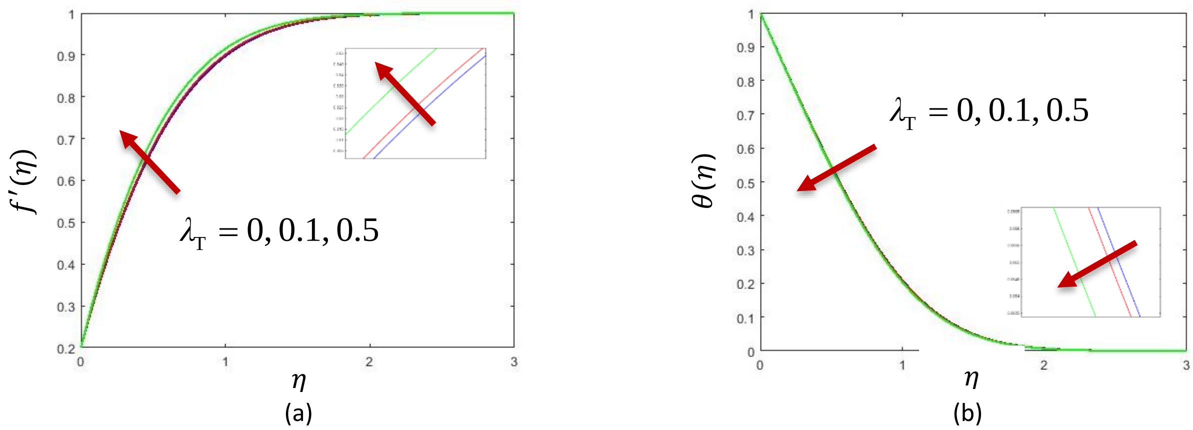
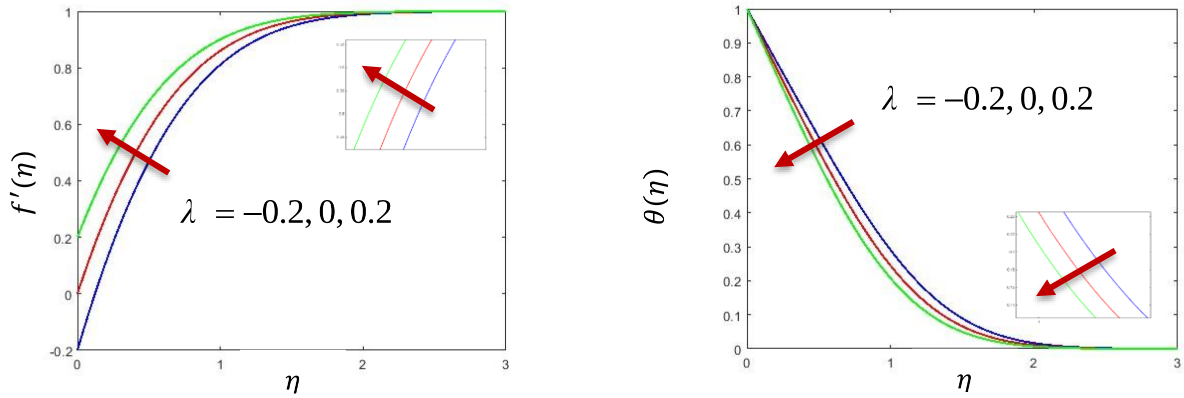


Fig. 6. Effects of λ_T on (a) velocity and (b) temperature profiles of maxwell hybrid nanofluid

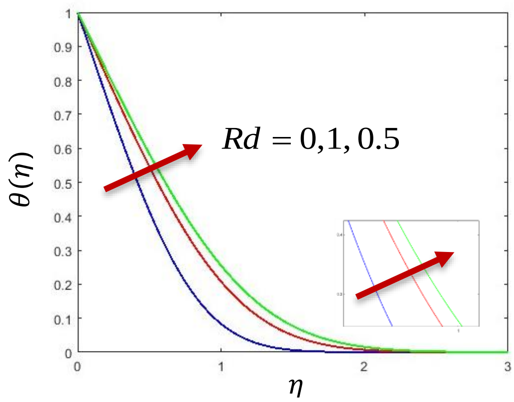
From Figure 7(a), it can be observed that as λ increase, the velocity profiles also increases thus reduces the momentum boundary layer thickness. When λ > 0 denoted as stretching, λ = 0 as static and λ < 0 is shrinking. In terms of physics, negative sign implies that the surface exerts the fluid with a dragging force, meanwhile positive sign implies the opposite. This explains that the velocity of fluid in a stretching surfaces is larger than in the shrinking surfaces. Based on Figure 7(b), temperature profiles decreases along with the thermal boundary layer thickness. Table 5 depicts that skin friction declines whereas Nusselt number rises as λ increases.



(a) (b)

Fig. 7. Effects of λ on (a) velocity and (b) temperature profiles of maxwell hybrid nanofluid

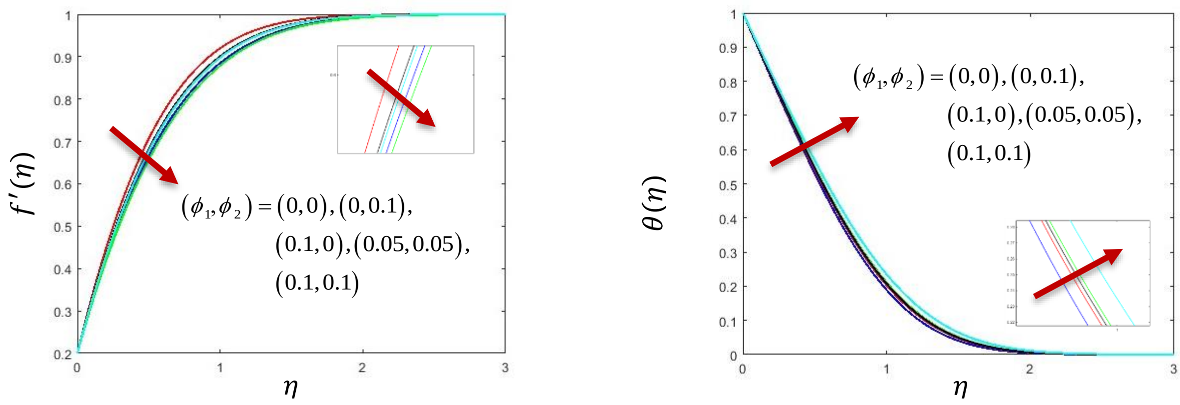
Radiation, Rd parameter gives no effects to the velocity profiles. Meanwhile Figure 8 exhibits the effect of the Rd on the temperature profiles. It shows that the temperature of fluid enhances with the increase in the parameter value of Rd . The reason behind this behavior is that Rossland, which means absorption coefficient is increases when Rd rises. This led to an expansion of tremendous heat quantity to improve the values of Rd . Besides, based on Table 5, Nusselt number increases and skin friction coefficient remain the same whereas despite the increase value of Rd .



(a)

Fig. 8. Effect of Rd on (a) temperature profiles of maxwell hybrid nanofluid

In Figure 9(a), the velocity profile decreases due to the growth of ϕ_1 and ϕ_2, meanwhile the thickness of the momentum boundary layer increases. This coincides with the increase in viscosity, which causes the velocity to decrease. The temperature profiles in Figure 9(b) increase when ϕ_1 and ϕ_2 increase. This is due to the increases in the rate of heat transportation and more energy is physically disperse when the nanoparticles volume fraction is increased, thus raises the temperature consequently. Additionally, the thermal boundary layer thickness is also enhanced by the influence of the increment in ϕ_1 and ϕ_2. As illustrated in Table 5, the magnitude of skin friction and Nusselt number rise with the increase in ϕ_1 and ϕ_2.



(a) (b)

Fig. 9. Effects of ϕ_1 and ϕ_2 on (a) velocity and (b) temperature profiles of maxwell hybrid nanofluid

Table 5
Variation of Skin Friction Coefficient and Nusselt number at different dimensionless parameters

α	M	γ	K	λ_T	λ	Rd	ϕ_1	ϕ_2	Skin friction	Nusselt number
0°									1.558668	2.479022
45°	1	45°	0.1	0.1	0.2	1	0.05	0.05	1.690212	2.506383
60°									1.752373	2.518720
90°									1.812447	2.530301
	0								1.558668	2.479022
45°	1	45°	0.1	0.1	0.2	1	0.05	0.05	1.690212	2.506383
	2								1.812447	2.530301
		0°							1.702994	2.509194
45°	1	45°	0.1	0.1	0.2	1	0.05	0.05	1.690212	2.506383
		60°							1.681216	2.504392
		90°							1.659644	2.499571
			0						1.640519	2.486473
45°	1	45°	0.1	0.1	0.2	1	0.05	0.05	1.690212	2.506383
			0.2						1.746448	2.528158
				0					1.659644	2.499571
45°	1	45°	0.1	0.1	0.2	1	0.05	0.05	1.690212	2.506383
				0.5					1.816552	2.533227
					-0.2				2.242844	2.006022
45°	1	45°	0.1	0.1	0	1	0.05	0.05	1.991202	2.264244
					0.2				1.690212	2.506383
						0			1.690212	1.669303
45°	1	45°	0.1	0.1	0.2	1	0.05	0.05	1.690212	2.506383
						1.5			1.690212	2.854626
							0	0	1.255784	2.280644
							0	0.1	1.823166	2.472139
45°	1	45°	0.1	0.1	0.2	1	0.05	0.05	1.580559	2.506383
							0.1	0	1.690212	2.533128
							0.1	0.1	2.170873	2.744409

5. Conclusions

This study investigated the Maxwell hybrid nanofluid on mixed convective radiative flow over a stretching/ shrinking inclined plate. Similarity transformation is used to transform the nonlinear PDE to a dimensionless ODE and numerically solved by using Keller Box method using a Fortran software. The following are the findings in this study:

i. The velocity increases due to the increasing of α, M, K, λ_T and λ
ii. The temperature profiles decrease to the increasing of α, M, K, λ_T and λ
iii. An increase γ, ϕ_1 and ϕ_2 in exhibit a decrement in velocity profile but a rise in the temperature profiles.
iv. When the value of Rd increase, it gives no effect to the velocity profiles but increase the temperature profiles.
v. The skin friction and Nusselt number increase due to the increase in of $\alpha, M, K, \lambda_T, \phi_1$ and ϕ_2 except for γ, λ and Rd.
vi. The nanoparticles shape with the highest Nusselt number is platelet followed by cylindrical, bricks and spherical shapes.

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