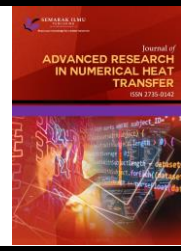




Journal of Advanced Research in Numerical Heat Transfer

Journal homepage:
<https://semarakilmu.com.my/journals/index.php/arnht/index>
ISSN: 2735-0142



Solute Dispersion in an Unsteady Herschel-Bulkley Flow through an Inclined Stenosed Arter

Intan Diyana Munir^{1,*}, Nurul Aini Jaafar¹, Sharidan Shafie¹

¹ Faculty of Science, Universiti Teknologi Malaysia (UTM), Malaysia

ARTICLE INFO

Article history:

Received 9 June 2023
Received in revised form 12 July 2023
Accepted 15 August 2023
Available online 30 September 2023

Keywords:

Unsteady Blood Flow; Unsteady Solute Dispersion; Herschel-Bulkley Model; Perturbation Method; Generalized Dispersion Model

ABSTRACT

Motivated by the concept of blood flow in a stenosed artery, this present research investigates the influence of stenosis shape in terms of height and arterial inclination on the blood flow and solute dispersion behaviour through an inclined stenosed artery. The blood rheology is depicted using the Herschel-Bulkley model in a laminar, axisymmetric and incompressible unsteady flow through the stenosed artery. The effect of stenosis is focused on the stenosis height for both sine and cosine stenosis. Parameters of arterial inclination are also investigated to observe the effect of inclination on the blood velocity and dispersion function. Perturbation method is adopted in solving for the blood flow velocity under the effect of stenosis height and arterial inclination. The dispersion function of solute dispersion is solved using the obtained blood velocity by adopting the Generalized Dispersion Model (GDM) in obtaining steady dispersion functions. This present study shows that the increase in stenosis height decreases both blood velocity and dispersion function. Meanwhile, the increase in arterial inclination increases the blood velocity and dispersion function. The effect of stenosis height also affects blood velocity and dispersion function for the sine stenosis more than the cosine stenosis.

1. Introduction

The research of hemodynamics in stenosed artery is a significant contribution to the biomedical field related to the application in treatment of atherosclerosis, angina, heart attacks and many more. The presence of stenosis at the arterial could cause the narrowing of the artery and total blockage if left untreated. Treatments of stenosed artery include oral medication, drug injection and angiogram which considers the aspect of solute dispersion in determining the success of those treatments. For instance, the effectiveness of drug injection to treat the stenosed artery is influenced by the blood flow and solute dispersion behaviour within the artery. Therefore, it is important to consider the stenosis size and arterial inclination to decide the appropriate artery location and drug dosage to ensure optimal treatment with minimised complication.

* Corresponding author.

E-mail address: intandiyana1995@graduate.utm.my (Intan Diyana Munir)

<https://doi.org/10.37934/arnht.14.1.2938>

In depicting the blood rheology for theoretical study, Newtonian and many other non-Newtonian models have been used by researchers depending on the boundary layer problem studied. This present study focuses on a very narrow artery problem due to the presence of stenosis at the arterial wall. Certain models could not describe the blood flow in a very narrow artery where the yield stress is high and shear rate is low. However, the Herschel-Bulkley model containing an additional parameter of power-law index can explain the different blood physiological behaviours [2]. Chaturani et al. [9] stated that blood behaves like Herschel–Bulkley model rather than Power Law and Bingham models for a tube with 0.095 mm diameter. Iida [7] adopted the Herschel-Bulkley and Casson model in their study of blood flow in arterioles with diameters less than 0.065 mm and concluded that the velocity profiles can be explained using the Herschel-Bulkley model; yet does not obey the Casson model. Additionally, the Herschel-Bulkley model can also be reduced to other models such as Newtonian, Power Law and Bingham models by assigning a certain value to the power-law index parameter. It is clear that Herschel-Bulkley model have advantages compared to other models in terms of its capability to explain blood flow in a very narrow artery and be reduced to other models.

There are numerous studies that investigate the unsteady blood flow through a stenosed artery using the Herschel-Bulkley model. Not to mention, the aspect of arterial inclination should be considered as many ducts in a physiological system have some inclination rather than being horizontal [8]. Priyadharshini and Ponalagusamy [1] studied the blood flow through an inclined, tapered stenosed artery with the presence of body acceleration using the Herschel-Bulkley nanofluid model. They stated that the resistance experienced by blood flow increases as the stenosis height increases. However, increase in the inclination parameter decreases the flow resistance. A researcher [6] adopted the Herschel-Bulkley model in their study of unsteady blood flow through an overlapping stenosed artery and concluded that the plug core radius decreases as the stenosis size increases. Few other studies that utilized the Herschel-Bulkley model in their boundary layer problems of stenosed artery can also be seen in these researches [3-5]. However, most of the studies largely focus on investigating the effect of stenosis or inclination on the blood flow behaviour while neglecting the aspect of solute dispersion such as the dispersion function solution. An extensive study on solute dispersion inside an inclined stenosed artery helps doctors and pharmacists in deciding the dose and distribution rate of medication to patients with less risk of causing toxicity.

In previous studies, the Herschel-Bulkley model is widely used in depicting blood rheology for solving problems related to solute dispersion in blood flow, mainly in a very narrow artery. However, the study on solute dispersion of dispersion function in an unsteady blood flow using Herschel-Bulkley model in an inclined stenosed artery has not yet been explored. Therefore, this present study focuses on the effect of stenosis height and artery inclination on the behaviour of blood flow and solute dispersion using the perturbation and generalized dispersion model (GDM) methods to extend the study of previous researches.

2. Methodology

2.1 Mathematical Formulation

Consider the unsteady, axisymmetric, laminar and fully-developed unidirectional blood flow represented by the Herschel-Bulkley model through an inclined artery with the presence of stenosis at the arterial wall as shown in Figure 1 where $\bar{w}(\bar{z}, \bar{t})$ is the blood velocity in axial direction, \bar{r} is the artery radius, \bar{t} is time, \bar{g} is the gravitational acceleration and θ is the degree of the arterial inclination. Therefore, the gravitational acceleration in the \bar{z} direction is given as $\bar{g} \sin \theta$. The blood flow region within the artery is separated into the plug flow region of $0 \leq \bar{r} \leq \bar{r}_c$ and outer flow region

of $\bar{r}_c < \bar{r} \leq \bar{R}$ where \bar{r}_c is the plug flow radius and \bar{R} is the full artery radius which can be replaced by the formula of cosine or sine stenosis. The presence of stenosis at the arterial wall is indicated by $\bar{R}(\bar{z})$ where $\bar{\delta}$ is the stenosis height and \bar{l}_0 is the stenosis length. Cylindrical polar coordinates are used in the formulation and computation of this present study.

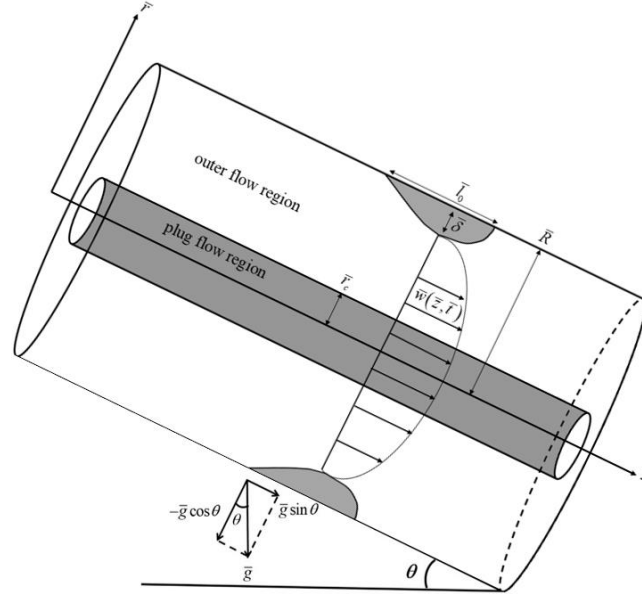


Fig. 1. Geometric depiction of blood flow in an inclined stenosed artery.

Since the blood flow is a unidirectional flow in the \bar{z} direction, the governing equations of continuity and momentum in cylindrical coordinate are reduced into the following form of:

$$\frac{\partial \bar{\rho}}{\partial \bar{t}} + \frac{\partial(\bar{\rho}\bar{w})}{\partial \bar{z}} = 0, \quad (1)$$

$$\bar{\rho} \frac{\partial \bar{w}}{\partial \bar{t}} = -\frac{d\bar{p}}{d\bar{z}} - \frac{1}{r} \frac{\partial}{\partial r}(\bar{r}\bar{\tau}) + \bar{\rho}\bar{g} \sin \theta \quad \text{for } 0 \leq \bar{r} \leq \bar{R}(\bar{z}), \quad (2)$$

where $\bar{\rho}$ is the fluid density, \bar{p} is the pressure and $\bar{\tau}$ is the yield stress. The stenosed artery radius $\bar{R}(\bar{z})$ for cosine stenosis is defined as:

$$\bar{R}(\bar{z}) = \begin{cases} \bar{R} & \text{otherwise,} \\ \bar{R} - \frac{\bar{\delta}}{2} \left[1 + \cos \left(\frac{2\pi}{\bar{l}_0} \left(\bar{z} - \bar{d} - \frac{\bar{l}_0}{2} \right) \right) \right] & \text{when } \bar{d} \leq \bar{z} \leq \bar{l}_0 + \bar{d}, \end{cases} \quad (3)$$

and for sine stenosis is defined as:

$$\bar{R}(\bar{z}) = \begin{cases} \bar{R} & \text{otherwise,} \\ \bar{R} - \bar{\delta} \sin \left[\frac{\pi(\bar{z}-\bar{d})}{\bar{l}_0} \right] & \text{when } \bar{d} \leq \bar{z} \leq \bar{l}_0 + \bar{d}. \end{cases} \quad (4)$$

where \bar{d} is the stenosis location. The momentum equation in Eq. (2) has the boundary condition of:

$$\bar{\tau} \text{ is finite at } \bar{r} = 0 \quad (5)$$

The constitutive equation of the Herschel-Bulkley model is defined as:

$$\bar{\mu}_H \left(\frac{\partial \bar{w}}{\partial \bar{r}} \right) = -(|\bar{\tau}| - \bar{\tau}_y)^n \text{ if } |\bar{\tau}| \geq \bar{\tau}_y, \quad (6)$$

where $\bar{\tau}_y$ is the yield stress, $\bar{\mu}_H$ is the Herschel-Bulkley viscosity, and n is the power-law index. The boundary conditions of Eq. (6) are:

$$\begin{aligned} \bar{w} &= 0 \text{ at } \bar{r} = 0, \\ \bar{w} &= 0 \text{ at } \bar{r} = \bar{R}(\bar{z}) \end{aligned} \quad (7)$$

The governed unsteady convective-diffusion equation for the dispersion of solute is given as:

$$\frac{\partial \bar{C}}{\partial \bar{t}} + \bar{w} \frac{\partial \bar{C}}{\partial \bar{z}} = \bar{D}_m \left(\frac{1}{\bar{r}} \frac{\partial}{\partial \bar{r}} \left(\bar{r} \frac{\partial}{\partial \bar{r}} \right) + \frac{\partial^2}{\partial \bar{z}^2} \right) \bar{C}_m, \quad (8)$$

where \bar{C} is the solute concentration and \bar{D}_m is the molecular diffusivity. Eq. (8) has the initial and boundary conditions of

$$\begin{aligned} \bar{C}(\bar{r}, \bar{z}, 0) &= \bar{C}_0 \text{ if } |\bar{z}| \leq \frac{\bar{z}_s}{2}, \\ \bar{C}(\bar{r}, \bar{z}, 0) &= 0 \text{ if } |\bar{z}| > \frac{\bar{z}_s}{2}, \end{aligned} \quad (9)$$

$$\bar{C}(\bar{r}, \infty, \bar{t}) = 0, \quad (10)$$

$$\frac{\partial \bar{C}}{\partial \bar{r}}(0, \bar{z}, \bar{t}) = \frac{\partial \bar{C}}{\partial \bar{r}}(\bar{R}(\bar{z}), \bar{z}, \bar{t}) = 0, \quad (11)$$

where \bar{C}_0 is the reference solute concentration and \bar{z}_s is the solute length.

2.2 Method of Solution

Consider the non-dimensional variables as below:

$$\begin{aligned} r &= \frac{\bar{r}}{\bar{R}}, \quad w_c = \frac{\bar{w}_c}{\bar{U}}, \quad w_o = \frac{\bar{w}_o}{\bar{U}}, \quad t = \frac{\bar{t} \bar{w}_m}{\bar{R}}, \quad p = \frac{\bar{p} \bar{R}}{\bar{\mu} \bar{U}}, \quad \tau_y = \frac{\bar{\tau}_y \bar{R}}{\bar{\mu} \bar{U}}, \quad z = \frac{\bar{z}}{\bar{R}}, \\ \rho &= \frac{\bar{\rho} \bar{R} \bar{U}}{\bar{\mu}}, \quad g = \frac{\bar{g} \bar{R}}{\bar{U}^2}, \quad \alpha = \frac{\bar{R} \bar{w}_m \bar{\rho}}{\bar{\mu}}, \quad C = \frac{\bar{C}}{\bar{C}_0}, \quad D_m = \frac{\bar{D}_m}{\bar{U} \bar{R}}, \quad R(z) = \frac{\bar{R}(\bar{z})}{\bar{R}}, \end{aligned} \quad (12)$$

where $\bar{\mu} = \bar{\mu}_H \left(\frac{\bar{R}}{\bar{U}} \bar{\mu} \right)^{n-1}$ is in the dimension of the viscosity of Newtonian fluid. Substituting the non-dimensional variable into the momentum equation in Eq. (2) and constitutive equation in Eq. (6), the non-dimensionalized momentum and constitutive equations are respectively obtained as:

$$\alpha \frac{\partial w}{\partial t} = -\frac{dp}{dz} - \frac{1}{r} \frac{\partial}{\partial r} (r\tau) + \rho g \sin \theta, \quad (13)$$

where α is the Reynolds number defined by $\alpha = \bar{R}\bar{w}_m\bar{\rho}/\bar{\mu}$ and

$$\tau = \left(-\frac{\partial w}{\partial r}\right)^{1/m} + \tau_y. \quad (14)$$

The series expansion of the perturbation method is obtained using the Reynolds number α as the small parameter (where $\alpha \ll 1$). Velocity w and shear stress τ are expanded in perturbation series as follows:

$$w(r, z, t) = w_0(r, z, t) + \alpha w_1(r, z, t) + \dots \quad (15)$$

$$\tau(r, z, t) = \tau_0(r, z, t) + \alpha \tau_1(r, z, t) + \dots \quad (16)$$

Substituting w in Eq. (15) and τ in Eq. (16) into Eq. (13) and (14) respectively and equating the coefficient of constant and α term in the left-hand side (LHS) to the right-hand side (RHS), the equations obtained are:

$$\frac{\partial}{\partial r} r\tau_0 = r \left(-\frac{dp}{dz} + \rho g \sin \theta\right) \quad (17)$$

$$\frac{\partial w_0}{\partial t} = -\frac{1}{r} \frac{\partial}{\partial r} r\tau_1 \quad (18)$$

$$\frac{\partial w_0}{\partial r} = -(\tau_0^m - m\tau_0^{m-1}\tau_y) \quad (19)$$

$$\frac{\partial w_1}{\partial r} = -(m\tau_0^{m-1}\tau_1 - m(m-1)\tau_0^{m-2}\tau_1\tau_y) \quad (20)$$

where $r_c = -2\tau_y / ((dp/dz) - \rho g \sin \theta)$. The non-dimensionalized boundary conditions are:

$$\tau_0 \text{ and } \tau_1 \text{ are finite at } r = 0 \quad (21)$$

for Eq. (17) and (18) and:

$$w_0 \text{ and } w_1 = 0 \text{ at } r = R(z) \quad (22)$$

For Eq. (19) and (20). Solving Eq. (17) and (18) subject to boundary conditions in Eq. (21) and Eq. (19) and (20) subject to boundary condition in Eq. (22), the solutions for velocity at the outer flow and plug flow region are obtained respectively as:

$$w_0 = -\left(-\frac{1}{2}\left(\frac{dp}{dz} - \rho g \sin \theta\right)\right)^m \left[\frac{r^{m+1}-1}{m+1} - r_c(r^m - 1)\right]. \quad (23)$$

$$\begin{aligned}
 w_1 = \frac{1}{t} \left(-\frac{1}{2} \left(\frac{dp}{dz} - \rho g \sin \theta \right) \right)^{2m-1} & \left[-\frac{m(m+2)}{2(m+1)^2(m+3)} + \frac{m}{2(m+1)^2} r^{m+1} \right. \\
 & - \frac{m}{(m+1)(m+3)(2m+2)} r^{2m+2} + \frac{(2m+3)(2m^3+5m^2-m-2)}{2(m+1)(m+2)(m+3)(2m+1)} r_c \\
 & - \frac{(m-1)}{2(m+1)} r_c r^m - \frac{m}{2(m+1)} r_c r^{m+1} + \frac{m(2m^2+5m+1)}{(m+1)(m+2)(m+3)(2m+1)} r_c r^{2m+1} \\
 & \left. - \frac{(m-1)(m+1)}{2(m+2)} r_c^2 + \frac{(m-1)}{2} r_c^2 r^m - \frac{(m-1)}{2(m+2)} r_c^2 r^{2m} \right]. \tag{24}
 \end{aligned}$$

The expression of velocity in the core flow region can be obtained by evaluating $r = r_c$ in the w_o to obtain w_c . The velocities obtained are then utilized to solve for the dispersion function by adopting the GDM method. According to the GDM method, the steady dispersion function f_{1s} can be solved from Eq. (8) in the form of:

$$\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial f_{1s}}{\partial r} \right) = (w - w_m), \tag{25}$$

where w_m is the mean velocity obtained using the formula:

$$w_m = \frac{\int_0^{2\pi} \int_0^{R(z)} wr \, dr d\psi}{\int_0^{2\pi} \int_0^{R(z)} r \, dr d\psi}. \tag{26}$$

The boundary condition for Eq. (25) is:

$$\frac{\partial f_{1s}}{\partial r} (r = 0) = \frac{\partial f_{1s}}{\partial r} (r = R(z)) = 0, \quad \text{for } j = 1, 2, 3, \dots \tag{27}$$

Eq. (25) is solved using integration with respect to r subject to the boundary conditions in Eq. (27) to obtain the solution of the steady function f_{1s} .

3. Results

The main objective of this study is to analyse the effects of the stenosis height and arterial inclination on the blood velocity and dispersion function of the Herschel-Bulkley flow through an inclined stenosed artery. Hence, other parameters such as the power-law index, pressure gradient, gravitational acceleration and plug core radius are given a constant value of $n = 0.95$, $p_s = 1$, $g = 10$ and $r_c = 0.04$ respectively throughout the data plotting for the purpose of discussion on the effects of stenosis height and arterial inclination. As for the stenosis height, the other variables affecting the stenosis size are given constant values of $l_0 = 3$, $d = 2$ and $z = 4$. Meanwhile, the stenosis height is

mainly analysed at $\delta = 0, 0.1, 0.2, 0.3$. The plotted results exhibited by all these variations of parameters were analysed at an angle of inclination in the range of $0^\circ \leq \theta \leq 90^\circ$ where the numerical value is arbitrary as long as it shows an increase in arterial inclination for the purpose of observation.

The influence of stenosis height and and arterial inclination on the blood flow velocity are investigated in Figure 2 (a) to (b). Figure 2 (a) shows the blood velocity at arterial inclination $\theta = 0^\circ$ and stenosis height $\delta = 0, 0.1, 0.2, 0.3$ for both cosine and sine stenosis (dashed line). The graph shows a decrease in blood velocity as the stenosis height increases. However, it can be observed that the decrease in velocity affects the sine stenosis more as the stenosis height increases. Increase in stenosis height reduces the flow region within the artery. Hence, the blood has less space to flow efficiently. The fatty substance deposited at the arterial wall increases the resistance flow; hence the decrease in blood velocity. Not to mention, decrease in the flow region causes the yield stress to increase which also increases the blood viscosity. Similar trend can be observed when the arterial inclination is increased to $\theta = 90^\circ$ in Figure 2 (b). Increase in stenosis height reduces the blood velocity and the blood flow through the sine stenosed artery is affected more by the increase in stenosis height. However, it can also be observed that increasing the arterial inclination from $\theta = 0^\circ$ to $\theta = 90^\circ$ increases the overall blood velocity. This is due to the blood flow velocity being accelerated by the gravity as the artery inclined towards vertical position. It can be said that inclining the artery helps the blood flow faster if the stenosis slows down the flow.

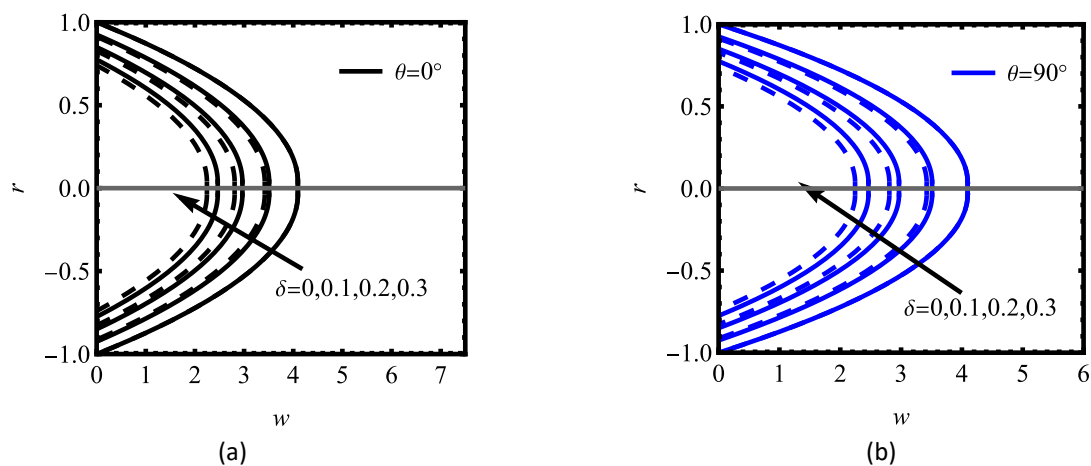


Fig. 2. Variation of non-dimensionalized velocity of unsteady Herschel-Bulkley fluid with fixed values of $m = 1.05$, $g = 10$, $t = 1$, $r_c = 0.04$ for $\delta = 0, 0.1, 0.2, 0.3$ at (a) $\theta = 0^\circ$ and (b) $\theta = 90^\circ$

The impact of stenosis height and and arterial inclination on the blood flow velocity are investigated in Figure 3 (a) to (b). The blood velocity at arterial inclination $\theta = 0^\circ$ and stenosis height $\delta = 0.1, 0.3$ for both cosine and sine stenosis for increasing time parameter of $t = 0.1, 0.5, 1$ is shown in Figure 3 (a). Observation shows that the blood velocity increases as the time increases. It can be noted that the increase in velocity is high at the starting point and slows down as the time increases. After a certain amount of time, the blood flow reaches a steady state velocity flow. However, the increase in stenosis height reduces the increase in velocity as the time increases. Graphical plotting shows that the the increase in stenosis height for sine stenosis amplify the reduction of the blood velocity. Therefore, it can be said that the sine stenosis affects the changes in blood velocity more compared to the cosine stenosis. Similar pattern behaviour is observed when the artery is inclined to $\theta = 90^\circ$ as shown in Figure 3 (b). Increase in time parameter increases the blood velocity for both sine and cosine stenosed artery. Nevertheless, graphical plotting shows that increasing the arterial

inclination from $\theta = 0^\circ$ to $\theta = 90^\circ$ increases the overall blood velocity. The gravitational acceleration helps amplify the blood flow as the artery inclined towards vertical position. From this theoretical result, inclining the artery can help counter the reduction of blood velocity due to the presence of stenosis; for a situation where a high blood velocity is preferred.

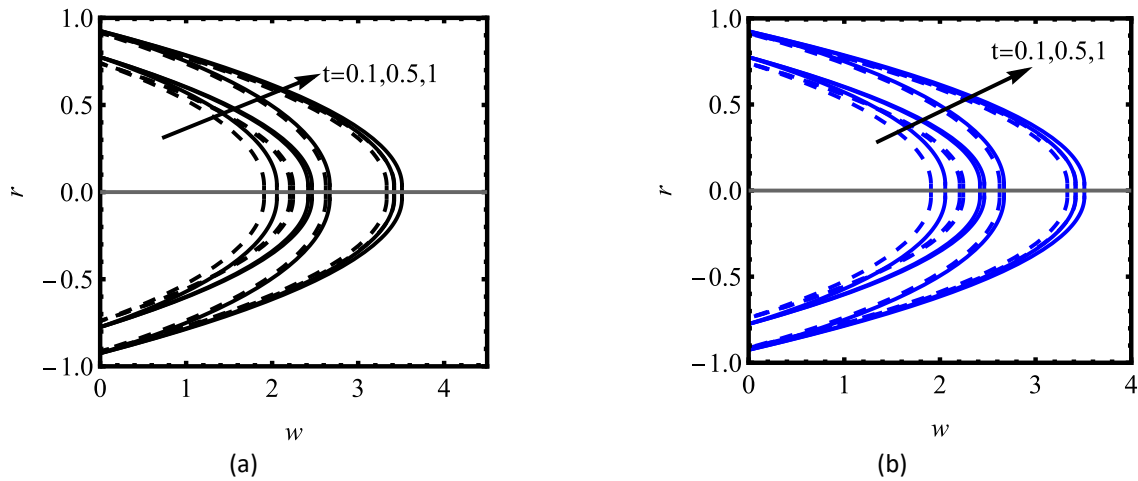


Fig. 3. Variation of non-dimensionalized velocity of unsteady Herschel-Bulkley fluid with fixed values of $m = 1.05$, $g = 10$, $r_c = 0.04$ at $\delta = 0.1, 0.3$ for $t = 0.1, 0.5, 1$ when (a) $\theta = 0^\circ$ and (b) $\theta = 90^\circ$

The effect of stenosis height and arterial inclination on the dispersion function are investigated in Figure 4 (a) to (b). Figure 4 (a) illustrates the dispersion function of solute dispersion at arterial inclination $\theta = 45^\circ$ and stenosis height $\delta = 0, 0.1, 0.2, 0.3$ for both cosine and sine stenosis (dashed line). It can be seen that the dispersion function decreases as the stenosis height increases. This is due to the solutes having less space to disperse smoothly. The blood cells and all the other materials suspended in the blood fluid are cramped and having difficulties in diffusing efficiently. Nevertheless, the decrease in dispersion function due to the increase in stenosis height affects the sine stenosis more compared to the cosine stenosis. Additionally, flow region decrease causes increase in yield stress which in turn increases the blood viscosity. Similar trend can be observed when the arterial inclination is increased to $\theta = 90^\circ$ in Figure 4 (b). As the stenosis height increases, the dispersion function decreases and the dispersion through the sine stenosed artery is affected more by the increase in stenosis height. Nevertheless, the increase of arterial inclination from $\theta = 45^\circ$ to $\theta = 90^\circ$ increases the overall dispersion function. The gravitational acceleration helps the dispersion of solute along the artery as the artery inclines toward vertical position.

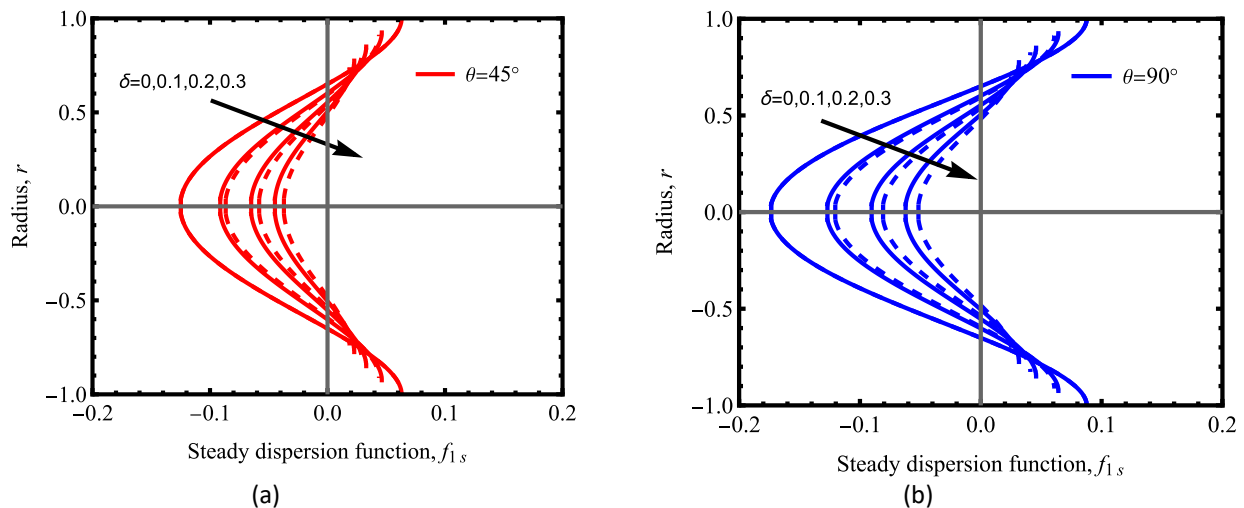


Fig. 4. Variation of steady dispersion function with fixed values of $m = 1.05$, $g = 10$, $r_c = 0.04$ for $\delta = 0, 0.1, 0.2, 0.3$ at (a) $\theta = 45^\circ$ and (b) $\theta = 90^\circ$

4. Conclusions

This present research investigates the influence of stenosis height and arterial inclination on the blood velocity and dispersion function through an inclined stenosed artery. The effects are observed on both artery with sine and cosine stenosis and a comparison is made. This present study concluded that:

- i. Increase in stenosis height decreases the blood velocity and dispersion function.
- ii. Increase in arterial inclination decreases the blood flow and dispersion function.
- iii. The blood velocity increases and reaches a steady state as the time increases for both cosine and sine stenosed artery.
- iv. The decrease in blood velocity and dispersion function due to increase in stenosis height and arterial inclination affects the artery with sine stenosis more compared to the cosine stenosis.

Conflict of interest

The authors declare that there is no conflict of interest regarding the publication of this paper.

Acknowledgements

This research was supported by Ministry of Education (MOE) through Fundamental Research Grant Scheme (FRGS/1/2020/STG06/UTM/02/15).

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