Journal of Advanced Research in Numerical Heat Transfer

Journal homepage:
https://semarakilmu.com.my/journals/index.php/arnht/index
ISSN: 2735-0142

Activation energy and Hall current Effects of Magnetohydrodynamic 3D flow of Non-Newtonian Hybrid Nanofluid over a Stretched Plate

Yeddula Rameswara Reddy[1,*], Donti Ratnam Srinivasan[2]

[1] Department of Mechanical engineering, JNTUA College Of Engineering Pulivendula, Constitute College of Jawaharlal Nehru Technological University Anantapur, Ananthapuramu, Andhra Pradesh, India
[2] Department of Mechanical Engineering, JNTUA College of Engineering Kalikiri, Constitute college of Jawaharlal Nehru Technological University Anantapur, Ananthapuramu, Andhra Pradesh, India

ARTICLE INFO

Article history:
Received 11 February 2024
Received in revised form 13 March 2024
Accepted 12 April 2024
Available online 30 May 2024

Keywords:
Activation energy; Stretching Plate; Rotation Parameter; Hall Current

ABSTRACT

Nanofluids are of great importance to researchers as they have significant uses industrially due to their high heat transfer rates. Recently, a new class of nanofluid, ‘‘hybrid nanofluid” is being used to further enhance the heat transfer rate. This new model in 3D is employed to examine the impact of activation energy, Rotational and hall current on a Non-newtonian hybrid Fe_3O_4/Al_2O_3 nanofluid flow over-stretched plate. Using similarity transformations, the controlling partial differential equations are turned into a set of nonlinear ordinary differential equations. For that system of equations, the shooting method is used to generate numerical solutions. The impact of various entry parameters on transversal and longitudinal velocities, temperature, heat flow and surface shear stress are studied numerically and graphically. A good correlation between the earlier studies is obtained in specific cases showing the convergence criteria of the present procedure. Further, the physical significance of the contributive parameters is presented through graphs and tables. The observation shows that the particle concentration for the hybrid nanofluid augments the fluid velocity. Moreover, the inclusion of dissipative heat favors enhancing the fluid temperature for the involvement of the particle concentration.

1. Introduction

Due to its wide range of applications in biomedicine, heat exchangers, cooling of electrical devices, double pane windows, food, transportation, etc., the concept of nanofluids have become a more widespread area of research for scientists in recent times. To enhance the thermal conductivity of base fluids such as ethylene glycol, water, kerosene, and motor oils, it is necessary to add nanoparticles such as graphene, silica, silver, gold, copper, alumina, carbon annotates, etc. The diameter of such nanoparticles may vary anywhere between 1-100 nm. It was Choi [1] who first observed that nanoparticles suspended in base fluid could enhance the thermal conductivity which in turn improved the heat transfer rate of the fluid. Later, Buongiorno [2] investigated the factors

* *Corresponding author.*
E-mail address: yrameswarareddy.mech@jntua.ac.in (Yeddula Rameswara Reddy)

https://doi.org/10.37934/arnht.20.1.3652

that influenced the thermal conductivity of nanofluids and reported that Brownian motion and thermophoresis effect increases the nanofluids thermal conductivity. As a consequence of the revelation, Nield and Kuznetsov [3] used the Buongiorno's model on the boundary layer stream. Khan and Pop [4] were the first to explore the steady flow of nanofluid on a stretching sheet using convective boundary conditions. Hamid et al. [5] investigated the bio-convection flow of magneto-cross nanofluid including microorganisms by using an effective Prandtl number technique. Varun Kumar et al. [6] explored the Arrhenius activation energy for hybrid nanofluid fluid above a curved stretching surface. Shah et al. [7] used the Prandtl hybrid nanofluid flow with chemical reactions and motile microorganisms to study the bio-convection effects. Faraz et al. [8] explored the multi-slip effect on axisymmetric Casson fluid flow with a chemical reaction. Arshad and Hassan [9] numerically investigated the hybrid nanofluid flow over permeable stretching surface considering magnetic field. Hassan et al. [10] explored viscous dissipation and heat absorption with chemical reactions and heat source/sink. Krishnamurthy et al. [11] explored the chemical reaction effects and melting heat transfer for frontier layer slip flow.

A new type of fluid, called a hybrid nanofluid, is finding widespread technological usage due to its excellent thermophysical properties. Hybrid nanofluids are the advanced fluid composed by adding two or more nanoparticles in a base fluid. Such kinds of nanofluids have more advanced properties than conventional nanofluids. An individual substance may never possess all of the needed traits; hence, the substance may be missing or deficient in some properties. Customizable hybrid nanoparticles can process important information more effectively than other nanofluids. Hybrid nanofluids ordinary outperform nanofluids in a variety of heat transfer applications, making them ideal for use in industries as diverse as refrigeration, electronics cooling, drug reduction, generator cooling, machining coolant, cooling for nuclear systems, cooling for transformers, biomedicine, and many more. Recently, Sundar [12] has suggested detailed procedure for creating hybrid nanofluids, including their benefits and drawbacks. Waini et al. [13] have studied the unsteady flow of a hybrid nanofluid made by adding Cu nanoparticles in $Al2O3$ /water nanofluid due to a stretching/shrinking sheet. Liaquat et al. [14] numerically computed the MHD hybrid nanofluid flow toward the shrinking surface for stability analysis and dual solutions and their problem consists of Cu − Al2O3 − water the base fluid. They found that for increasing values of suction and radiation parameters temperature is enhanced (decrease) for both solutions. Shoaib et al. [15] scrutinized the phenomena of viscous dissipation in 3D MHD hybrid nanofluid flow via rotating disk. In this research work, it is noted that magnetic field reduced both the radial and azimuthal skin frictions coefficients. Ahmad et al. [16] pointed out the study of Go + Silver (Ag) in Maxwell hybrid nanoliquid for the improvement of thermal performance. Their concluding remarks illustrated that the heat transition is improved through the addition of silver volume fraction with graphene oxide. Alhussain et al. [17] inspected the influence of variable viscosity in a blood-based two-dimensional Casson hybrid nanofluid through the stretching sheet by introducing a magnetic field perpendicular in a flow field. From this analysis, the authors have demonstrated that with the increment of nanomaterials concentration in the base fluid, the thermal expansion rate is increased but the specific heat capacity decreased.

Thermal radiation's effect on natural convection has risen in prominence due to its many practical applications in engineering and physics, especially in the development of tools and machinery, aerospace engineering, and gas turbines. Since it does not need a medium, thermal radiation is the preferred mode of heat transmission above conduction and convection. Because of these characteristics, thermal radiation plays a crucial role in the heat transmission of MHD nanofluids, minimising heat loss. Among the most important control elements for the flux of liquid and heat in a high-temperature heat system are radiation parameters. Thermal radiation has an essential impact on the development of steady kit, satellites, nuclear energy plants, turbines of gas, assortment of advanced transformation systems and missiles. Essam and Khalid [18] have analyzed Impact of Hall Current and Joule Heating on a Rotating Hybrid Nanofluid Over a Stretched Plate with Nonlinear Thermal Radiation. In the early stage, the impact of heat radiation on Air and CO_2 of laminar flow through the vertical plate was studied by England and Emery [19]. Essam and Abdel-wahed [20] have investigated MHD mixed convection Ferro Fe3O4/Cu-hybrid-nanofluid runs in a vertical channel. Kumar et al. [21] developed a model to simulate the flow and heat transfer of a nanofluid across an infinite vertical plate subject to a magnetic field and viscous dissipation. Subsequent research by Ali et al. [22] studied how thermal radiation made an impact on the MHD hybrid nanofluid flow along the stretching cylinder. In a situation where the bottom plate was permeable and stretchy, Lv et al. [23] have investigated the impact of thermal radiation, hall current, and uneven heat source/sink on the flow of nanofluid between two horizontal flat plates. Rao and Deka [24] have considered the effect of thermal radiation and chemical reaction on MHD Casson nanofluid flow caused due to a stretching sheet. Very recently, Rao and Deka [25] made a numerical investigation on the heat and mass transfer phenomena of a nanofluid under the impact of solar radiation. Consider a rotating fluid that is steady–laminar–incompressible and has a constant angular velocity and contains electrically conducting Fe_3O_4/Al_2O_3 hybrid nanoparticles traveling at a uniform velocity $U_w=b_x$ over a stretched plate.

Heat and mass transfer from different geometries embedded in porous media has many engineering and geophysical applications such as drying of porous solids, thermal insulations, cooling of nuclear reactors, crude oil extraction, underground energy transport, etc. Micropolar fluids are those consisting of randomly oriented particles suspended in a viscous medium, which can undergo a rotation that can affects the hydrodynamics of the flow, making it a distinctly non-Newtonian fluid. They constitute an important branch of non-Newtonian fluid dynamics where microrotation effects as well as microinertia are exhibited. Raghunath et al. [26] have studied Hall current and thermal radiation effects of 3D rotating hybrid nanofluid reactive flow via stretched plate with internal heat absorption. Raghunath et al. [27] have studied unsteady magneto-hydro-dynamics flow of Jeffrey fluid through porous media with thermal radiation, Hall current and Soret effects. Raju et al. [28] have studied Chemical Radiation and Soret Effects on Unsteady MHD Convective Flow of Jeffrey Nanofluid Past an Inclined Semi-Infinite Vertical Permeable Moving Plate. Ramachandra et al. [29] have reviewed Effects of Hall Current, Activation Energy and Diffusion Thermo of MHD Darcy-Forchheimer Casson Nanofluid Flow in the Presence of Brownian motion and Thermophoresis. Raghunath et al. [30] have possessed processing to pass unsteady MHD flow of a second-grade fluid through a porous medium in the presence of radiation absorption exhibits Diffusion thermo, hall and ion slip effects. Very Recently Li et al. [31] have expressed Effects of activation energy and chemical reaction on unsteady MHD dissipative Darcy–Forchheimer squeezed flow of Casson fluid over horizontal channel. Suresh Kumar et al. [32] have observed Numerical analysis of magnetohydrodynamics Casson nanofluid flow with activation energy, Hall current and thermal radiation. Raghunath et al. [33] have analyzed Diffusion Thermo and Chemical Reaction Effects on Magnetohydrodynamic Jeffrey Nanofluid over an Inclined Vertical Plate in the Presence of Radiation Absorption and Constant Heat Source. Maatoug et al. [34] have possessed Variable chemical species and thermo-diffusion Darcy–Forchheimer squeezed flow of Jeffrey nanofluid in horizontal channel with viscous dissipation effects. Omar et al. [35] have reviewed Hall Current and Soret Effects on Unsteady MHD Rotating Flow of Second-Grade Fluid through Porous Media under the Influences of Thermal Radiation and Chemical Reactions. Deepthi et al. [36] have studied Recent Development of Heat and Mass Transport in the Presence of Hall, Ion Slip and Thermo Diffusion in Radiative Second Grade Material: Application of Micromachines. Aruna et al. [37] have examined An unsteady MHD flow of a second-grade fluid passing through a porous medium in the presence of radiation absorption exhibits Hall and ion slip effects. Raghunath et al. [38] reviewed Hall, Soret, and rotational effects on unsteady MHD rotating flow of a second-grade fluid through a porous medium in the presence of chemical reaction and aligned magnetic field. Raghunath et al. [39] have examined Hall and ion slip radiative flow of chemically reactive second grade through porous saturated space via perturbation approach. Ramachandra et al. [40] have studied Characteristics of MHD Casson fluid flow past an inclined vertical porous plate. Raghunath et al. [41] have analyzed Effects of Radiation Absorption and Aligned Magnetic Field on MHD Cassion Fluid Past an Inclined Vertical Porous Plate in Porous Media.

The main objective of this research is to examine the steady flow of laminar, incompressible, Non-Newtonian hybrid nanofluid above a stretchy, rotating plate with an activation energy and hall current, using hybrid nanoparticles Fe_3O_4/Al2O_3 under the influence of a magnetic field. The fluid and plates are rotating simultaneously with constant speed about the axis of rotation. The governing equations of momentum, energy and concentration are transformed into ODEs by a similarity transformation and tackled at MATLAB using the boundary value problem technique. The influence of different constraints is discussed in the form of graphs and tables.

2. Flow Governing Equations

Consider an activation energy, rotating and hall current of Non-Newtonian fluid that is steady–laminar– incompressible and has a constant angular velocity and contains electrically conducting Fe_3O_4/Al_2O_3 hybrid nanoparticles traveling at a uniform velocity $U_w = b_x$ over a stretched plate. A uniform magnetic fields of intensity B_0 and hall current are applied to the plate in a normal direction. The temperature on the surface is T_w, whereas T_∞ is the temperature away from the surface. The equations describing the model are Refs. [18]:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \tag{1}$$

$$\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} - 2\omega v \right) = \frac{\mu_{hnf}}{\sigma_{hnf}}\left(1+\frac{1}{\lambda}\right)\frac{\partial^2 u}{\partial z^2} - \frac{\sigma_{hnf}}{\sigma_{hnf}}\frac{B_0^2}{(1+m^2)}(u - mv) \tag{2}$$

$$\left(u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} + 2\omega u \right) = \frac{\mu_{hnf}}{\rho_{hnf}}\left(1+\frac{1}{\lambda}\right)\frac{\partial^2 v}{\partial z^2} - \frac{\sigma_{hnf}}{\rho_{hnf}}\frac{B_0^2}{(1+m^2)}(v + mu) \tag{3}$$

$$\left(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} + w\frac{\partial T}{\partial z} \right) = \frac{k_{hnf}}{(\rho c_p)_{hnf}}\left(\frac{\partial^2 T}{\partial z^2}\right) + \frac{\sigma_{hnf}}{(\rho c_p)_{hnf}} B_0^2 (u^2 + v^2) + \frac{Q_0}{(\rho c_p)_{hnf}}(T - T_\infty) \tag{4}$$

$$u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} + w\frac{\partial C}{\partial z} = \beta_{hnf}\left(\frac{\partial^2 C}{\partial z^2}\right) - k_r^2 (C - C_\infty)\left(\frac{T}{T_\infty}\right)^m \exp\left(\frac{-E_a}{K^* T}\right) \tag{5}$$

For this flow, corresponding boundary conditions are [18]

$$u = U_w, \quad v = 0, \quad w = 0, \quad T = T_w, \quad C = C_w \quad \text{at } z = 0$$
$$u \to 0, \quad v \to 0, \quad w \to 0, \quad T \to T_\infty \quad C \to C_\infty \quad \text{as} \quad z \to \infty \tag{6}$$

The effective properties of hybrid nanofluid are given by Refs. [18, 20]:

$$\frac{\mu_{hnf}}{\mu_f} = \left(\frac{1}{(1-\phi_1)^{2.5}\,(1-\phi_2)^{2.5}} \right), \ \rho_{hnf} = (1-\phi_2)\left[(1-\phi_2)\rho_f + \phi\rho_{s1}\right] + \phi_2\rho_{s2},$$

$$(\rho C_p)_{hnf} = (1-\phi_2)\left[(1-\phi_2)(\rho C_p)_f + \phi_1(\rho C_p)_{s1}\right] + \phi_2(\rho C_p)_{s2}, \quad k_{nf} = k_f\left(\frac{k_{s1} + 2k_{nf} - 2\phi_1(k_{nf} - 2k_{s1})}{k_{s1} + 2k_{nf} + 2\phi_1(k_{nf} - 2k_{s1})} \right),$$

$$k_{hnf} = k_{nf}\left(\frac{k_{s2} + 2k_{nf} - 2\phi_2(k_{nf} - 2k_{s2})}{k_{s2} + 2k_{nf} + 2\phi_2(k_{nf} - 2k_{s2})} \right), \quad \sigma_{nf} = \sigma_f\left(\frac{\sigma_{s1} + 2\sigma_{nf} - 2\phi_1(\sigma_{nf} - 2\sigma_{s1})}{\sigma_{s1} + 2\sigma_{nf} + 2\phi_1(\sigma_{nf} - 2\sigma_{s1})} \right),$$

$$\sigma_{hnf} = \sigma_{nf}\left(\frac{\sigma_{s2} + 2\sigma_{nf} - 2\phi_2(\sigma_{nf} - 2\sigma_{s2})}{\sigma_{s2} + 2\sigma_{nf} + 2\phi_2(\sigma_{nf} - 2\sigma_{s2})} \right),$$

Here φ_1, $\varphi2$ are the solid volume fractions of Fe_3O_4 and Al_2O_3 respectively, subscript s_1, s_2, f, and h_{nf} are for nano-solid-particles, base fluid, and hybrid nanofluid respectively. As shown in Table 1.

Table 1
Thermo-physical properties of H_2O, Fe_3O_4, and Al_2O_3 followed by [18, 20]

Physical properties	Water	Fe_3O_4	Al_2O_3
C_p (J/KgK)	4179	670	765
ρ (Kg/m^3)	997.1	5180	3970
k (w/mK)	0.613	9.7	40
σ(Ω/m)	25*10^{-2}	25*10^3	35*10^6

Dimensionless quantities are introduced to simplify the mathematical analysis of the problem by introducing the following similarity transformation used to transform the PDEs to dimensionless ODEs

$$u = bx\,f'(\eta),\ v = by\,g(\eta),\ w = -\sqrt{b\nu_f}\,\left(f(\eta) + g(\eta)\right),\ T = T_\infty + \Delta T\,\theta(\eta),\ \eta = \sqrt{\frac{b}{\nu_f}}z, \quad \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty},$$
$$\phi(\eta) = \frac{C - C_\infty}{C_w - C_\infty} \tag{7}$$

Substituting Eq. (7) into Equations (2), (3) and (4), we get the following system of non-linear ordinary differential equations

$$\left(1 + \frac{1}{\lambda}\right)\left(\frac{l_4}{l_1}\right) f''' - f'^2 + ff'' + 2\lambda g - \left(\frac{l_5}{l_1}\right)\frac{M}{(1+m^2)}\left(f' - mg\right) = 0 \tag{8}$$

$$\left(1+\frac{1}{\lambda}\right)\left(\frac{l_4}{l_1}\right)g'' + fg' - f'g - 2\lambda f' - \left(\frac{l_5}{l_1}\right)\frac{M}{\left(1+m^2\right)}\left(g+mf'\right)=0 \tag{9}$$

$$\theta'' + \left(\frac{R_d}{l_3}\right)\left[\left(1+\left(\theta_w-1\right)\theta\right)\right]^3\theta'' + \Pr\left(\frac{l_2}{l_3}\right)f\theta' + \Pr Ec\, M\left(\frac{l_5}{l_3}\right)\times\left(f'^2+g'^2\right) + \Pr Q\,\theta = 0 \tag{10}$$

$$\phi'' + \frac{Sc}{\left(1-\phi_1\right)\left(1-\phi_2\right)}\left(f+g\right)\phi' - K_E\left(1+\theta\right)^m\varphi\exp\left(\frac{-E}{1+\theta}\right)=0 \tag{11}$$

where the prime signifies differentiation with respect to (η) and it is given by φ_2

$$l_1=\frac{\rho_{hnf}}{\rho_f},\ l_2=\frac{\left(\rho C_p\right)_{hnf}}{\left(\rho C_p\right)_f},\ l_3=\frac{k_{hnf}}{k_f},\ l_4=\frac{\mu_{hnf}}{\mu_f},\text{ and } l_5=\frac{\sigma_{hnf}}{\sigma_f}.$$

The transformed corresponding boundary conditions (5) become

$$\begin{aligned} &f(\eta)=0,\ \ g(\eta)=0,\ \ f'(\eta)=1,\ \ \theta(\eta)=0,\ \ \phi(\eta)=1 \qquad at \quad \eta=0\\ &f'(\eta)\to 0,\ \ g(\eta)\to 0,\ \ \theta(\eta)\to 0,\ \ \phi(\eta)\to 0 \qquad as \quad \eta\to\infty \end{aligned} \tag{12}$$

where prime denotes differentiation with respect to η, and the significant thermophysical parameters indicating the flow dynamics are defined by

$$\begin{cases} \lambda=\dfrac{\omega}{b}, M=\dfrac{\sigma_f B_0^2 x}{\rho_f\, b},\ \Pr=\dfrac{\nu_f}{k_f}=\dfrac{\left(\nu\rho C_p\right)_f}{k_f},\ \theta_w=\dfrac{T_w}{T_\infty}, E_c=\dfrac{u^2\rho_f}{\left(\rho C_p\right)_f\left(T_w-T_\infty\right)}, S_c=\dfrac{\nu_f}{\beta_f}\\ R_d=\dfrac{16\sigma^* T_\infty^3}{3k_f\alpha^*},\ K_r=\dfrac{\xi_1 k_T\left(C-C_\infty\right)^{n-1}}{b}, Q=\dfrac{Q_0}{\left(\rho Cp\right)_f k_f}, K_E=\dfrac{k_r^2}{c}, E=\dfrac{E_a}{K^* T_\infty}. \end{cases} \tag{13}$$

3. Physical Quantities Of Interests

The physical quantities of the given problem are, the ‘‘Skin friction” along x and y axis Cf_x; Cf_y, the ‘‘local Nusselt number” Nu_x and the ‘‘Sherwood number” Sh_x, defined by

$$\begin{aligned} &Cf_x=\frac{\mu_{hnf}}{\rho_f\,(bx)^2}\left(\frac{\partial u}{\partial z}\right)_{z=0},\ \ Cf=\frac{\mu_{hnf}}{\rho_f\,(bx)^2}\left(\frac{\partial v}{\partial z}\right)_{z=0},\\ &Nu_x=-\frac{xk_{hnf}}{k_f\left(T_w-T_\infty\right)}\left(\frac{\partial T}{\partial z}\right)_{z=0},\ Sh_x=-\frac{xk_{hnf}}{k_f\left(C_w-C_\infty\right)}\left(\frac{\partial C}{\partial z}\right)_{z=0} \end{aligned} \tag{14}$$

The coefficient of skin friction, the Nusselt number, and the Sherwood number are all expressed in their non-dimensional versions in terms of the similarity variable as follows:

$$\mathrm{Re}_x^{1/2} Cf_x = \frac{\mu_{hnf}}{\mu_f} f''(0),\ \mathrm{Re}_x^{1/2} Cf_y = \frac{\mu_{hnf}}{\mu_f} g''(0),$$
$$\mathrm{Re}_x^{-1/2} Nu_x = -\left(\frac{k_{hnf}}{k_f}\right)\theta'(0),\ \mathrm{Re}_x^{-1/2} Sh_x = -\frac{k_{hnf}}{k_f}\phi'(0) \quad (15)$$

4. Method of Solution

The set of non-linear coupled ordinary differential equations (8)-(11) have been solved numerically by RungeKutta-4th order method with shooting technique using MATLAB software with step size of $\nabla\eta$=0.01 and error bound 10^{-6} in all cases. Advantages of this method are that the coupled nonlinear ODEs are transformed to a set of linear first order ODEs with the introduction of the new variables. Secondly, the boundary value problem gets transformed to initial value problem by providing guess values to unknown initial values as required by the problem to be solved. The guess values are corrected by the shooting method to tally with the specified boundary conditions at the other boundary. Once the guess values are corrected with required number of iterations, then the forward integration is carried out to give the numerical solutions of the desired points comprising interval. The limitations are: Not all the PDEs, representing the governing equations, do not admit similarity transformations and cannot be transformed to ODEs. Only specific types of flow problems admit similarity transformations and hence similar solutions. There might be dual solutions to a specific problem, if exists, then which one is stable or unstable that is to be decided upon and discussed. To assess the accuracy of the present code and validity check, the numerical values of $f''(0)$, $g'(0)$ and $Nu\,R_e^{-1/2}$ are presented in Table 2 and Table 3. Finally, the numerical simulations for the rate coefficients such as the shear rate, and heat transfer rate is obtained and exhibited in Table 4 for the variation of several contributing parameters. It is detected that the augmented values of the nanoparticle concentration, magnetic parameter, and thermal buoyancy enrich the rate of shear stress significantly. Further, the nanoparticle concentrations of both the Fe_3O_4 and Al_2O_3 increase the rate of heat transfer in magnitude whereas the other contributing parameters have retarding effect on the profile of the rate of heat transfer.

Table 2
Comparison of $f''(0)$, $g'(0)$ with a previous studied when M=m=Ec=φ_1 =φ_2=Sc=Rc=0, E=0, Q=0

Abdel-wahed [39]			Current Study	
Λ	$f''(0)$	$g'(0)$	$f''(0)$	$g'(0)$
0.50	-1.13837	-0.51276	-1.1484746	-0.5086767
1.00	-1.32505	-0.83710	-1.3285756	-0.8385757
2.00	-1.62235	-1.28726	-1.6485757	-1.2485757

Table 3
Comparison of $Nu\,R_e^{-1/2}$ with a previous studied when M=1, Ec=E=Q=0, φ_1 =0.1, φ_2=0, λ =0.5, Sc=R_C=0

R_d	θ_w	Abdel-wahed [39]	Current Study
0	1	1.85750	1.8576464
1	1	2.23679	2.2474464
1	1.1	2.30471	2.3484747
1	1.5	2.60886	2.6384747

Table 4
Significant behaviour of the parameters on the rate coefficients

Φ_1	Φ_2	M	m	Ec	Q	C_{fx}	C_{fy}	Nu_x
0	0.01	0	0	0		-0.986863	1.231415	1.276154
0.1						-0.991919	1.741317	1.431213
0.2						-1.019282	1.975624	1.761514
	0.1					-1.252422	1.625242	1.872526
	0.2					-0.987126	1.242314	1.623324
		0.1				-0.126359	1.652426	1.736356
		0.2				-0.981734	1.782673	1.523637
			0.2			-0.827533	1.283636	1.653436
			0.4			-0.998353	1.567273	1.435346
				0.2		-0.274543	1.537378	1.756343
				0.4		-0.756373	1.673836	1.253536
					0.2	-0.984654	1.746579	0.567697
					0.4	-0.716346	1.956768	0.746588

5. Discussion of the Result

Over a stretched plate, a boundary layer is modeled over a Non-Newtonian hybrid nanofluid with water as a base fluid and $Fe3O4/Al2O3$ nanoparticles. Through a series of Figures 1–5, effects magnetic field, hall current, rotation parameter, joule heating and activation energy on the boundary layer was investigated. Longitudinal velocity is measured along the x-axis, while transversal velocity is measured along the y-axis. In addition, Table IV shows the influence of all employed parameters on heat flow and skin friction. The results will be discussed in the following section.

In Figure 1, the impact of magnetic field is indicated by the Hartman number (M), and it can be seen that rising the Hartman number reduces the velocities in x & y directions. Physically, introducing a magnetic field thins the boundary layer, reducing the fluid’s ability to move in each direction. The existence of hall current alongside the magnetic field, furthermore, is indicated by a dashed line in each illustration. Certainly, the Hall current had no discernible impact on longitudinal velocity; however the transversal velocity increased when the Hall current was present. Furthermore, as seen in Figure 2, the increase in transversal velocity is more visible when the magnetic strength is high. From a physical point of view, the existence of hall current under the influence of a magnetic field causes a drag perpendicular force known as a Lorentz force. As a result, one can see that this force causes the fluid molecules to shift their travel direction, resulting in an increase in transversal velocity. The impact of a magnetic field on temperature profiles is demonstrated in Figure 3, where the existence of a magnetic field causes fluid molecules near the surface to move, raising the boundary layer temperature. In addition, Due to influence of joule heating caused by the powerful magnetic field, the existence of hall current raises the thermal boundary layer thickness and the temperature of boundary layer.

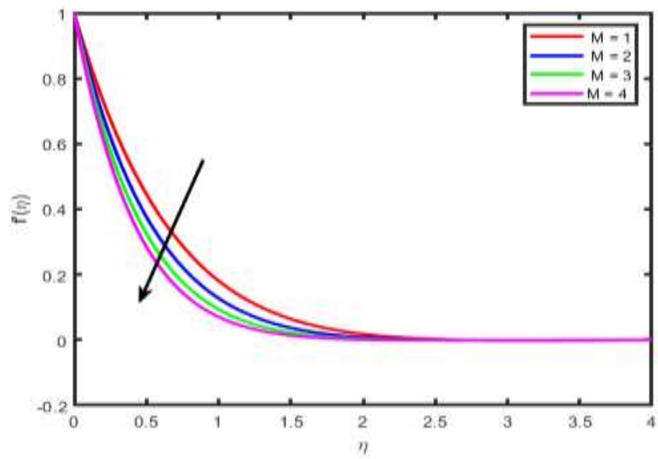

Fig. 1. Effect of M (weak magnetic strength) on $f'(\eta)$

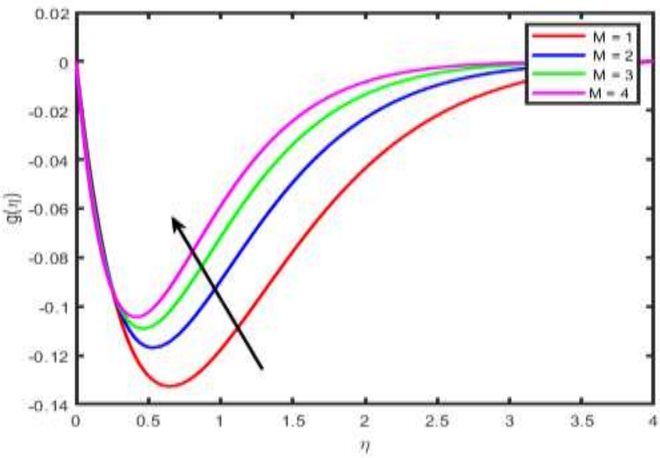

Fig. 2. Effect of M (weak magnetic strength) on $g(\eta)$

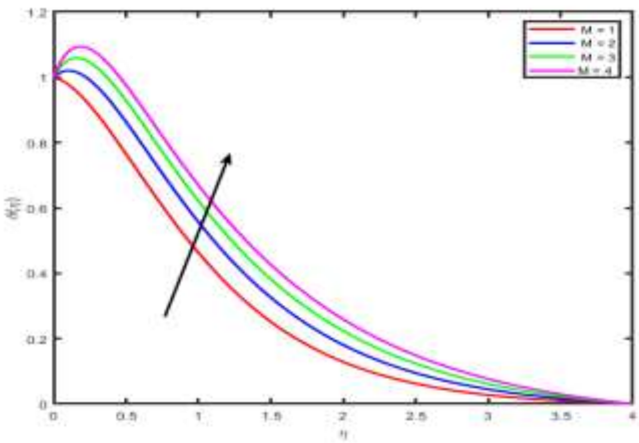

Fig. 3. Effect of M (weak magnetic strength) on $\theta(\eta)$

Figure 4–5 shows the effects of Hall current parameter m on both longitudinal and transversal velocities profiles. Figure 4 illustrates that longitudinal velocity $f'(\eta)$ in fluid temperature enlarges due to increase of Hall current parameter m. Figure 5 shows the features of Hall parameter m on transversal velocity $g(\eta)$. It is observed that transversal velocity is a decreasing function of Hall parameter m.

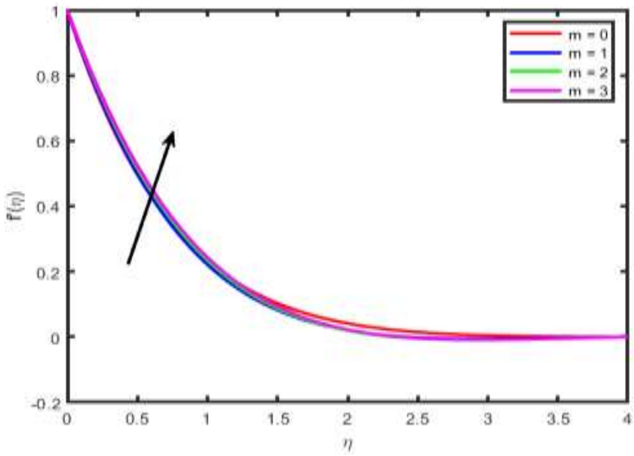

Fig. 4. Effect of m Hall parameter on $f'(\eta)$

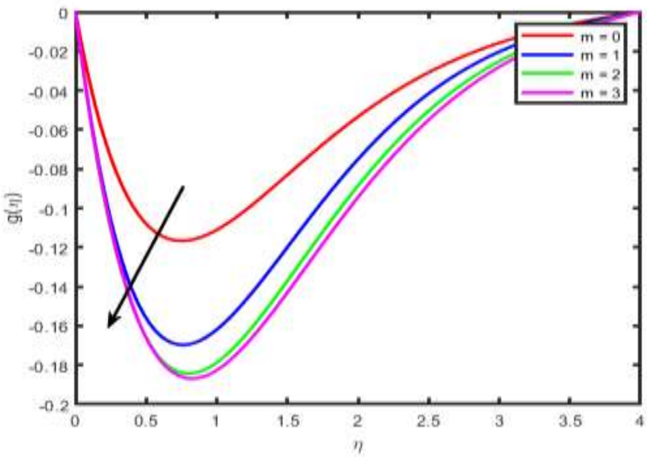

Fig. 5. Effect of m Hall parameter on $g(\eta)$

Figure 6 displays the effects of rotation parameter λ on longitudinal velocity $f'(\eta)$. The rotation parameter is defined as Ω\b. Thus, it clearly shows that increasing value of longitudinal velocity implies the strong rotation rate and therefore the rotation rate becomes higher as compared to the stretching rate. Physically, it is noted that an increase in rotation parameter implies boosting of the centrifugal force which in turns exerts pressure on the fluid to accelerate the fluid particles more rapidly in the radial direction. Similarly, the effects of rotation parameter λ on longitudinal velocity $g(\eta)$ as shown in Figure 7 display that an increase in the rotation parameter λ causes to enhance the transversal velocity. Figure 8-9 depicts the effects of rotation parameter λ on temperature and concentration field respectively. It is illustrated that the temperature of hybrid nanofluid decreases by enhancing the rotation rate. The similar behavior has observed in concentration field.

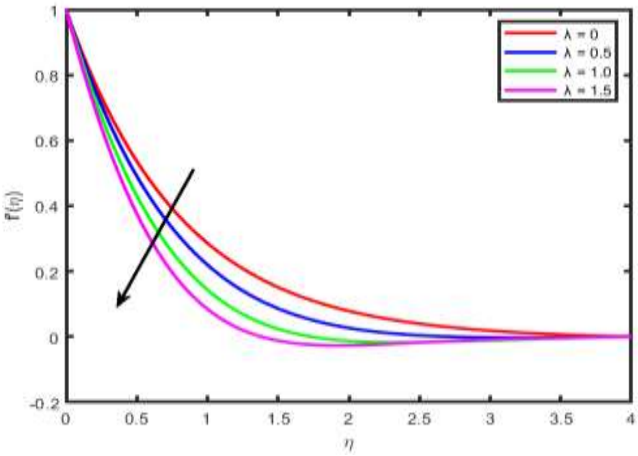

Fig. 6. Effect of λ Rotation parameter on $f'(\eta)$

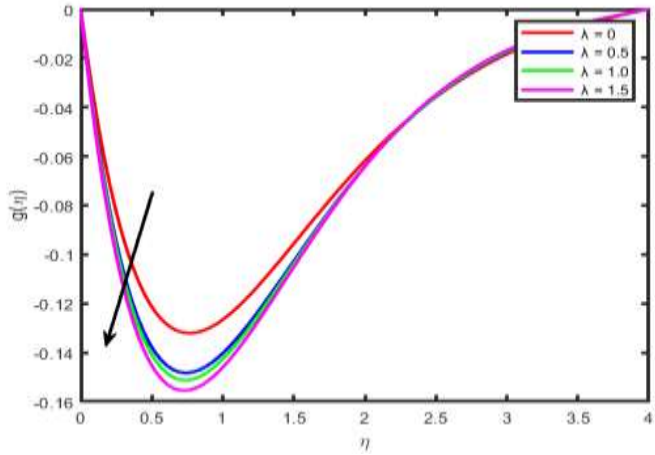

Fig. 7. Effect of λ Rotation parameter on $g(\eta)$

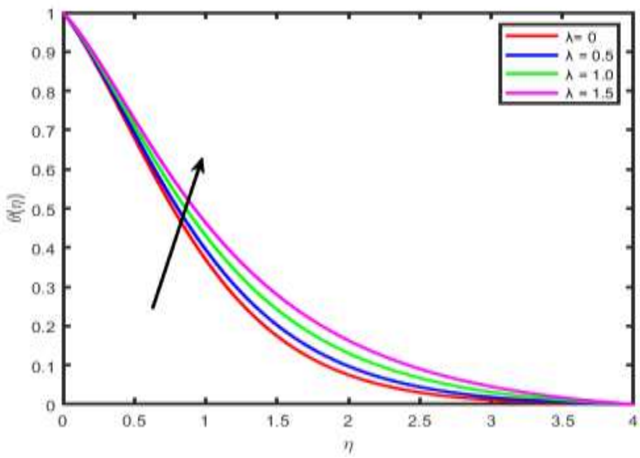

Fig. 8. Effect of λ Rotation parameter on $\theta(\eta)$

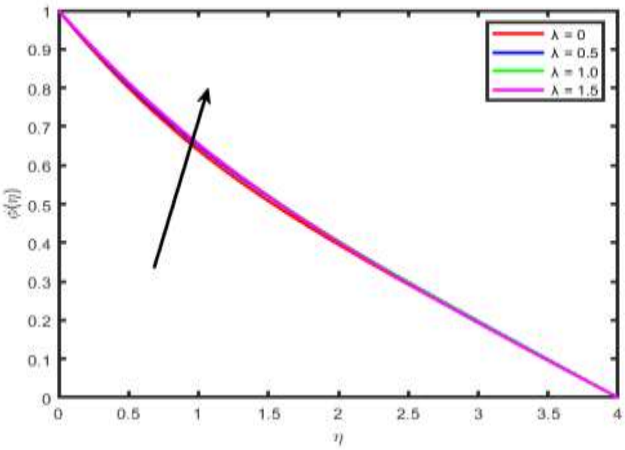

Fig. 9. Effect of λ Rotation parameter on $\phi(\eta)$

Figure 10 displays the effects of Eckert number Ec on the temperature field. The temperature field rises by increasing the amount of Eckert number. The reason behind is that the conversion of mechanical energy into thermal energy. This effect rises only due to heat dissipation. The impact of chemical response R*c* for the concentration profile is presented in Figure 11. The concentration boundary layer diminishes by increasing the chemical reaction parameter R*c* because it has a direct relation with the chemical reaction parameter R*c*. Figure 12 shows the effect of Schmidt number on concentration profile. It is a relation of momentum diffusivity and mass diffusivity. The concentration profile decays when the Schmidt number increases and a lower concentration boundary layer are noted for mixed nanoparticles hybrid nanofluid.

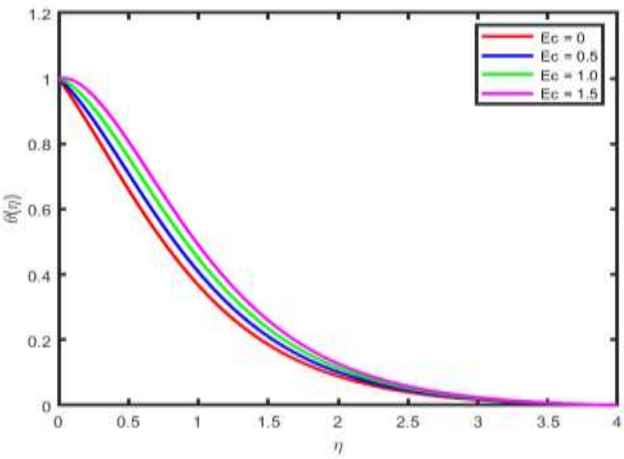

Fig. 10. Effect of E_c Eckert number on $\theta(\eta)$

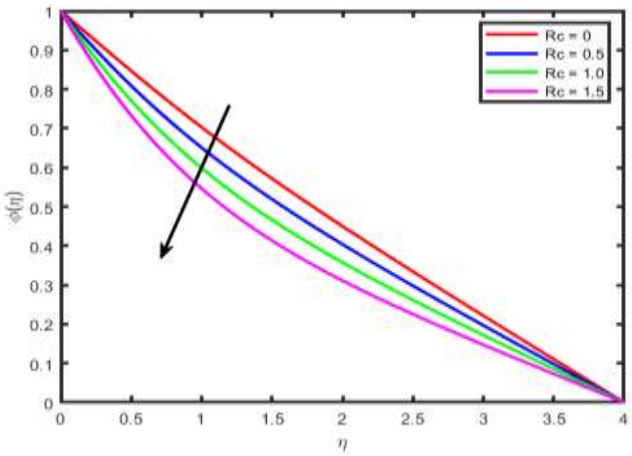

Fig. 11. Effect of R_C chemical reaction on $\phi(\eta)$

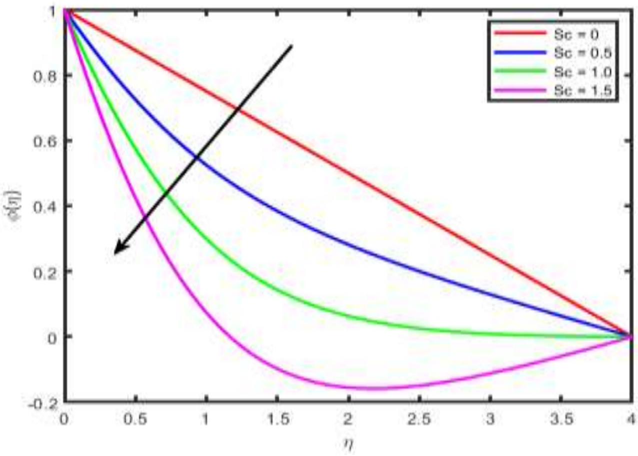

Fig. 12. Effect of S_C Schmidt number on $\phi(\eta)$

Figure 13 shows that the temperature θ(η) increases with an increase in the resistance of the heat source / sink, due to an increase in the resistance of the heat generation, the temperature rises. The opposite behavior is observed in the case of concentration shown in Figure 14. Figure 16 envisages the activation energy (*E*) impact on concentration field. Graph elucidate that concentration profile increases for large value of *E*. The Arrhenius function deteriorations by snow balling the value of the activation energy, which outcomes in the promotion of the generative chemical reaction causing an improvement in the concentration field. Within the occurrence of low temperature and higher activation energy leads to a smaller reaction rate constant which slow down the chemical reaction. In this manner concentration profile boost up. Figure 15 shows that when chemical reaction rate (σ) increases, concentration profile strongly reduces because of high chemical reaction rate which fallouts solute boundary layer becomes thicker. When σ increases steadily, the factor $(1+\Gamma\theta)\, e^{-E/(1+\Gamma\theta)}$ is enriches because of increase in values σ.

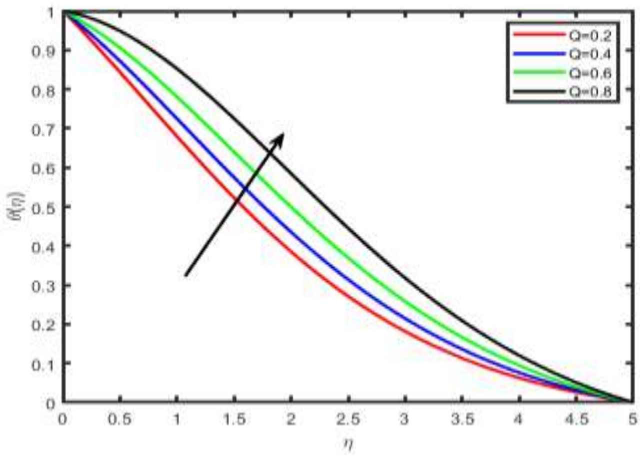

Fig. 13. Effect of heat source parameter (Q) on θ(η)

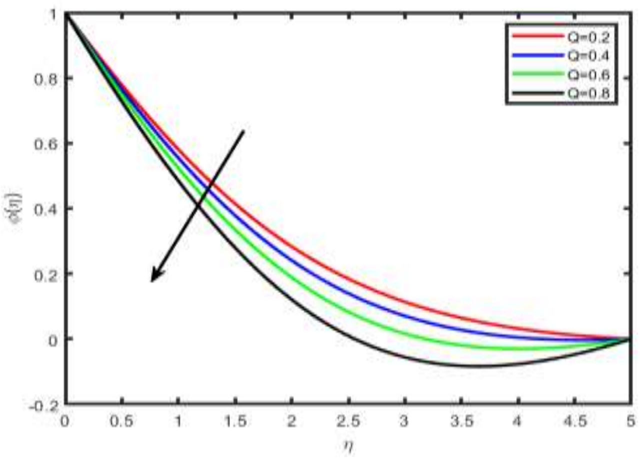

Fig. 14. Effect of heat source parameter (Q) on φ(η)

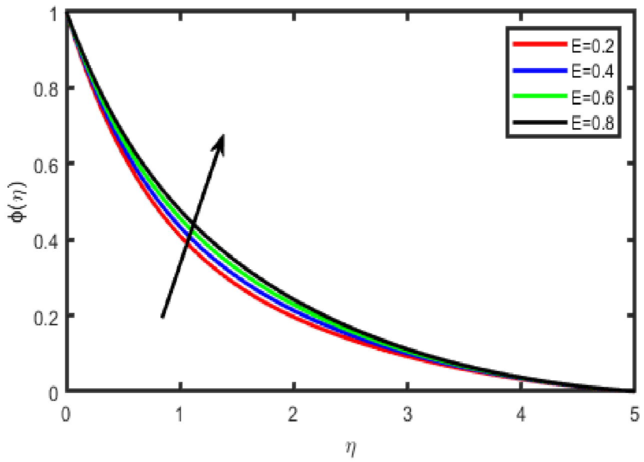

Fig. 15. Effect of Activation energy (E) on φ(η)

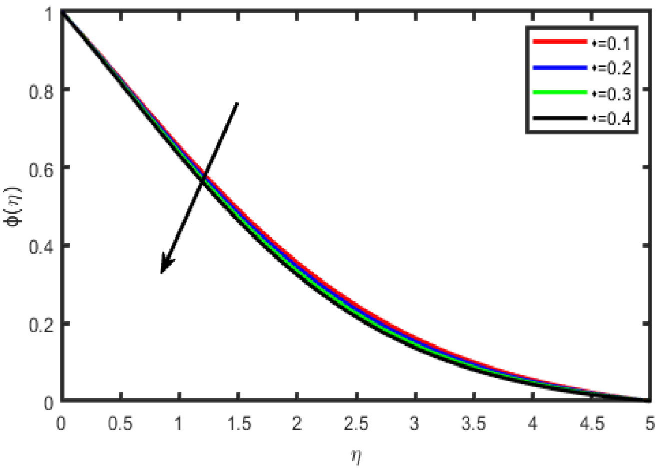

Fig. 16. Effect of chemical reaction rate (σ) on φ(η)

6. Conclusion

The following are the most important aspects of this research:

i. An increment in the magnetic parameter M diminishes the velocity of the hybrid nanofluid, but at the same time, causing enhances in the profile of (η), temperature and concentration of the hybrid nanofluid.
ii. When the rotation parameter is present the longitudinal velocity decreases while the transverse velocity increases and the temperature of a boundary layer rise as the kinetic energy of the fluid increases. When a boundary layer is exposed to thermal radiation, it has a higher temperature than when it isn't.
iii. Because the fluid viscosity rises when a dual-type of the nanoparticle is used, the velocity decreases more rapidly.

References

[1] Choi, S. US, and Jeffrey A. Eastman. *Enhancing thermal conductivity of fluids with nanoparticles*. No. ANL/MSD/CP-84938; CONF-951135-29. Argonne National Lab.(ANL), Argonne, IL (United States), 1995.

[2] Buongiorno, Jacopo. "Convective transport in nanofluids." (2006): 240-250. https://doi.org/10.1115/1.2150834

[3] Nield, D. A., and Andrey V. Kuznetsov. "Thermal instability in a porous medium layer saturated by a nanofluid." *International Journal of Heat and Mass Transfer* 52, no. 25-26 (2009): 5796-5801. https://doi.org/10.1016/j.ijheatmasstransfer.2009.07.023

[4] Khan, W. A., and I. Pop. "Boundary-layer flow of a nanofluid past a stretching sheet." *International journal of heat and mass transfer* 53, no. 11-12 (2010): 2477-2483. https://doi.org/10.1016/j.ijheatmasstransfer.2010.01.032

[5] Hamid, Aamir, M. Ijaz Khan, R. Naveen Kumar, RJ Punith Gowda, and B. C. Prasannakumara. "Numerical study of bio-convection flow of magneto-cross nanofluid containing gyrotactic microorganisms with effective prandtl number approach." (2021). https://doi.org/10.1038/s41598-021-95587-2.

[6] Varun Kumar, R. S., A. Alhadhrami, R. J. Punith Gowda, R. Naveen Kumar, and B. C. Prasannakumara. "Exploration of Arrhenius activation energy on hybrid nanofluid flow over a curved stretchable surface." *ZAMM-Journal of Applied Mathematics and Mechanics/Zeitschrift für Angewandte Mathematik und Mechanik* 101, no. 12 (2021): e202100035. https://doi.org/10.1002/zamm.202100035

[7] Shah, Syed Asif Ali, N. Ameer Ahammad, ElSayed M. Tag El Din, Fehmi Gamaoun, Aziz Ullah Awan, and Bagh Ali. "Bio-convection effects on prandtl hybrid nanofluid flow with chemical reaction and motile microorganism over a stretching sheet." *Nanomaterials* 12, no. 13 (2022): 2174. https://doi.org/10.3390/nano12132174

[8] Faraz, Faraz, Syed Muhammad Imran, Bagh Ali, and Sajjad Haider. "Thermo-diffusion and multi-slip effect on an axisymmetric Casson flow over a unsteady radially stretching sheet in the presence of chemical reaction." *Processes* 7, no. 11 (2019): 851. https://doi.org/10.3390/pr7110851

[9] Arshad, Mubashar, and Ali Hassan. "A numerical study on the hybrid nanofluid flow between a permeable rotating system." *The European Physical Journal Plus* 137, no. 10 (2022): 1126. https://doi.org/10.1140/epjp/s13360-022-03313-2
[10] Hassan, Ali, Azad Hussain, Mubashar Arshad, Soumaya Gouadria, Jan Awrejcewicz, Ahmed M. Galal, Fahad M. Alharbi, and S. Eswaramoorthi. "Insight into the significance of viscous dissipation and heat generation/absorption in magneto-hydrodynamic radiative casson fluid flow with first-order chemical reaction." *Frontiers in Physics* 10 (2022): 920372. https://doi.org/10.3389/fphy.2022.920372.
[11] Krishnamurthy, M. R., B. J. Gireesha, B. C. Prasannakumara, and Rama Subba Reddy Gorla. "Thermal radiation and chemical reaction effects on boundary layer slip flow and melting heat transfer of nanofluid induced by a nonlinear stretching sheet." *Nonlinear Engineering* 5, no. 3 (2016): 147-159. https://doi.org/10.1515/nleng-2016-0013
[12] Sundar, L. Syam, Korada Viswanatha Sharma, Manoj K. Singh, and A. C. M. Sousa. "Hybrid nanofluids preparation, thermal properties, heat transfer and friction factor–a review." *Renewable and Sustainable Energy Reviews* 68 (2017): 185-198. https://doi.org/10.1016/j.rser.2016.09.108
[13] Waini, Iskandar, Anuar Ishak, and Ioan Pop. "Unsteady flow and heat transfer past a stretching/shrinking sheet in a hybrid nanofluid." *International Journal of Heat and Mass Transfer* 136 (2019): 288-297. https://doi.org/10.1016/j.ijheatmasstransfer.2019.02.101
[14] Lund, Liaquat Ali, Zurni Omar, Ilyas Khan, and El-Sayed M. Sherif. "Dual solutions and stability analysis of a hybrid nanofluid over a stretching/shrinking sheet executing MHD flow." *Symmetry* 12, no. 2 (2020): 276. https://doi.org/10.3390/sym12020276
[15] Shoaib, Muhammad, Muhammad Asif Zahoor Raja, Muhammad Touseef Sabir, Muhammad Awais, Saeed Islam, Zahir Shah, and Poom Kumam. "Numerical analysis of 3-D MHD hybrid nanofluid over a rotational disk in presence of thermal radiation with Joule heating and viscous dissipation effects using Lobatto IIIA technique." *Alexandria Engineering Journal* 60, no. 4 (2021): 3605-3619. https://doi.org/10.1016/j.aej.2021.02.015
[16] Ahmad, Farooq, Sohaib Abdal, Hela Ayed, Sajjad Hussain, Suleman Salim, and A. Othman Almatroud. "The improved thermal efficiency of Maxwell hybrid nanofluid comprising of graphene oxide plus silver/kerosene oil over stretching sheet." *Case Studies in Thermal Engineering* 27 (2021): 101257. https://doi.org/10.1016/j.csite.2021.101257
[17] Alhussain, Ziyad A., and Asifa Tassaddiq. "Thin film blood based Casson hybrid nanofluid flow with variable viscosity." *Arabian journal for science and Engineering* 47, no. 1 (2022): 1087-1094. https://doi.org/10.1007/s13369-021-06067-8
[18] Elsaid, Essam M., and Khalid S. AlShurafat. "Impact of hall current and joule heating on a rotating hybrid nanofluid over a stretched plate with nonlinear thermal radiation." *Journal of Nanofluids* 12, no. 2 (2023): 548-556. https://doi.org/10.1166/jon.2023.1927
[19] England, W. G., and A. F. Emery. "Thermal radiation effects on the laminar free convection boundary layer of an absorbing gas." (1969): 37-44. https://doi.org/10.1115/1.3580116
[20] Elsaid, Essam M., and Mohamed S. Abdel-wahed. "MHD mixed convection Ferro Fe3O4/Cu-hybrid-nanofluid runs in a vertical channel." *Chinese Journal of Physics* 76 (2022): 269-282. https://doi.org/10.1016/j.cjph.2021.12.016.
[21] Kumar, M. Anil, Y. Dharmendar Reddy, V. Srinivasa Rao, and B. Shankar Goud. "Thermal radiation impact on MHD heat transfer natural convective nano fluid flow over an impulsively started vertical plate." *Case studies in thermal engineering* 24 (2021): 100826. https://doi.org/10.1016/j.csite.2020.100826
[22] Ali, Aamir, Tasmia Kanwal, Muhammad Awais, Zahir Shah, Poom Kumam, and Phatiphat Thounthong. "Impact of thermal radiation and non-uniform heat flux on MHD hybrid nanofluid along a stretching cylinder." *Scientific reports* 11, no. 1 (2021): 20262. https://doi.org/10.1038/s41598-021-99800-0
[23] Lv, Yu-Pei, Naila Shaheen, Muhammad Ramzan, M. Mursaleen, Kottakkaran Sooppy Nisar, and M. Y. Malik. "Chemical reaction and thermal radiation impact on a nanofluid flow in a rotating channel with Hall current." *Scientific Reports* 11, no. 1 (2021): 19747. https://doi.org/10.1038/s41598-021-99214-y
[24] Rao, Shiva, and Paramananda Deka. "A numerical solution using EFDM for unsteady MHD radiative Casson nanofluid flow over a porous stretching sheet with stability analysis." *Heat Transfer* 51, no. 8 (2022): 8020-8042. https://doi.org/10.1002/htj.22679
[25] Rao, Shiva, and P. N. Deka. "A numerical investigation on transport phenomena in a nanofluid under the transverse magnetic field over a stretching plate associated with solar radiation." In *Nonlinear Dynamics and Applications: Proceedings of the ICNDA 2022*, pp. 473-492. Cham: Springer International Publishing, 2022. https://doi.org/10.1007/978-3-030-99792-2_39
[26] Kodi, Raghunath, Mohana Ramana Ravuri, V. Veeranna, M. Ijaz Khan, Sherzod Abdullaev, and Nissren Tamam. "Hall current and thermal radiation effects of 3D rotating hybrid nanofluid reactive flow via stretched plate with internal heat absorption." *Results in Physics* 53 (2023): 106915. https://doi.org/10.1016/j.rinp.2023.106915.

[27] Kodi, Raghunath, Ramachandra Reddy Vaddemani, M. Ijaz Khan, Sherzod Shukhratovich Abdullaev, Attia Boudjemline, Mohamed Boujelbene, and Yassine Bouazzi. "Unsteady magneto-hydro-dynamics flow of Jeffrey fluid through porous media with thermal radiation, Hall current and Soret effects." *Journal of Magnetism and Magnetic Materials* 582 (2023): 171033. https://doi.org/10.1016/j.jmmm.2023.171033.

[28] Raju, Konduru Venkateswara, Ravuri Mohanaramana, S. Sudhakar Reddy, and Kodi Raghunath. "Chemical Radiation and Soret Effects on Unsteady MHD Convective Flow of Jeffrey Nanofluid Past an Inclined Semi-Infinite Vertical Permeable Moving Plate." *Communications in Mathematics and Applications* 14, no. 1 (2023): 237. https://doi.org/10.26713/cma.v14i1.1867

[29] Vaddemani, Ramachandra Reddy, Sreedhar Ganta, and Raghunath Kodi. "Effects of hall current, activation energy and diffusion thermo of MHD Darcy-Forchheimer Casson nanofluid flow in the presence of Brownian motion and thermophoresis." *Journal of Advanced Research in Fluid Mechanics and Thermal Sciences* 105, no. 2 (2023): 129-145. https://doi.org/10.37934/arfmts.105.2.129145

[30] Raghunath, Kodi, Ravuri Mohana Ramana, Charankumar Ganteda, Prem Kumar Chaurasiya, Damodar Tiwari, Rajan Kumar, Dharam Buddhi, and Kuldeep Kumar Saxena. "Processing to pass unsteady MHD flow of a second-grade fluid through a porous medium in the presence of radiation absorption exhibits Diffusion thermo, hall and ion slip effects." *Advances in Materials and Processing Technologies* (2023): 1-18. 10.1080/2374068X.2023.2191450

[31] Li, Shuguang, Kodi Raghunath, Ayman Alfaleh, Farhan Ali, A. Zaib, M. Ijaz Khan, Sayed M. ElDin, and V. Puneeth. "Effects of activation energy and chemical reaction on unsteady MHD dissipative Darcy–Forchheimer squeezed flow of Casson fluid over horizontal channel." *Scientific Reports* 13, no. 1 (2023): 2666. https://doi.org/10.1038/s41598-023-29702-w

[32] Suresh Kumar, Y., Shaik Hussain, K. Raghunath, Farhan Ali, Kamel Guedri, Sayed M. Eldin, and M. Ijaz Khan. "Numerical analysis of magnetohydrodynamics Casson nanofluid flow with activation energy, Hall current and thermal radiation." *Scientific Reports* 13, no. 1 (2023): 4021. https://doi.org/10.1038/s41598-023-28379-5

[33] Raghunath, K., R. Mohana Ramana, V. Ramachandra Reddy, and M. Obulesu. "Diffusion Thermo and Chemical Reaction Effects on Magnetohydrodynamic Jeffrey Nanofluid Over an Inclined Vertical Plate in the Presence of Radiation Absorption and Constant Heat Source." *Journal of Nanofluids* 12, no. 1 (2023): 147-156. https://doi.org/10.1166/jon.2023.1923

[34] Maatoug, Samah, K. Hari Babu, V. V. L. Deepthi, Kaouther Ghachem, Kodi Raghunath, Charankumar Ganteda, and Sami Ullah Khan. "Variable chemical species and thermo-diffusion Darcy–Forchheimer squeezed flow of Jeffrey nanofluid in horizontal channel with viscous dissipation effects." *Journal of the Indian Chemical Society* 100, no. 1 (2023): 100831. https://doi.org/10.1016/j.jics.2022.100831

[35] Bafakeeh, Omar T., Kodi Raghunath, Farhan Ali, Muhammad Khalid, El Sayed Mohamed Tag-ElDin, Mowffaq Oreijah, Kamel Guedri, Nidhal Ben Khedher, and Muhammad Ijaz Khan. "Hall current and Soret effects on unsteady MHD rotating flow of second-grade fluid through porous media under the influences of thermal radiation and chemical reactions." *Catalysts* 12, no. 10 (2022): 1233. https://doi.org/10.3390/catal12101233

[36] Deepthi, V. V. L., Maha MA Lashin, N. Ravi Kumar, Kodi Raghunath, Farhan Ali, Mowffaq Oreijah, Kamel Guedri, El Sayed Mohamed Tag-ElDin, M. Ijaz Khan, and Ahmed M. Galal. "Recent development of heat and mass transport in the presence of Hall, ion slip and thermo diffusion in radiative second grade material: application of micromachines." *Micromachines* 13, no. 10 (2022): 1566. https://doi.org/10.3390/mi13101566.

[37] Ganjikunta, Aruna, Hari Babu Kommaddi, Venkateswarlu Bhajanthri, and Raghunath Kodi. "An unsteady MHD flow of a second-grade fluid passing through a porous medium in the presence of radiation absorption exhibits Hall and ion slip effects." *Heat Transfer* 52, no. 1 (2023): 780-806. https://doi.org/10.1002/htj.22716

[38] Raghunath, Kodi, and Ravuri Mohanaramana. "Hall, Soret, and rotational effects on unsteady MHD rotating flow of a second-grade fluid through a porous medium in the presence of chemical reaction and aligned magnetic field." *International Communications in Heat and Mass Transfer* 137 (2022): 106287. https://doi.org/10.1016/j.icheatmasstransfer.2022.106287

[39] Kodi, Raghunath, Mohanaramana Ravuri, Nagesh Gulle, Charankumar Ganteda, Sami Ullah Khan, and M. Ijaz Khan. "Hall and ion slip radiative flow of chemically reactive second grade through porous saturated space via perturbation approach." *Waves in Random and Complex Media* (2022): 1-17. https://doi.org/10.1080/17455030.2022.2108555

[40] Vaddemani, Ramachandra Reddy, Raghunath Kodi, and Obulesu Mopuri. "Characteristics of MHD Casson fluid past an inclined vertical porous plate." *Materials Today: Proceedings* 49 (2022): 2136-2142. https://doi.org/10.1016/j.matpr.2021.08.328.

[41] Kodi, Raghunath, Ramachandra Reddy Vaddemani, and Obulesu Mopuri. "Effects of radiation absorption and aligned magnetic field on MHD Cassion fluid past an inclined vertical porous plate in porous media." *Simulation and Analysis of Mathematical Methods in Real-Time Engineering Applications* (2021): 273-291. https://doi.org/10.1002/9781119785521.ch12