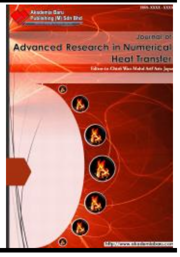
Penerbit Akademia Baru

Journal of Advanced Research in Numerical Heat Transfer

Journal homepage:
http://www.akademiabaru.com/submit/index.php/arnht/index
ISSN: 2735-0142



MHD Mixed Convection of Viscoelastic Nanofluid Flow due to Constant Heat Flux

Rahimah Mahat[1,*], Muhammad Saqib[2], Imran Ulah[3], Sharidan Shafie[2], Sharena Mohamad Isa[4]

[1] Advanced Facilities Engineering Technology Research Cluster (AFET), Universiti Kuala Lumpur Malaysian Institute of Industrial Technology, Persiaran Sinaran Ilmu, Bandar Seri Alam, 81750 Masai, Johor, Malaysia
[2] Department of Mathematical Sciences, Faculty of Science, Universiti Teknologi Malaysia, 81310 Johor Bahru, Malaysia
[3] College of Civil Engineering, National University of Sciences and Technology, 44000, Islamabad, Pakistan
[4] Manufacturing Engineering Technology Section, Universiti Kuala Lumpur Malaysia Italy Design Institute (UniKL MIDI), Kuala Lumpur, 56100, Malaysia

ABSTRACT

A mathematical model for magnetohydrodynamics (MHD) viscoelastic nanofluid flow approaching a linear horizontal circular cylinder has been constructed. The cylinder with constant heat flux is illustrated in the analysis wit magnetic field. We used the Tiwari and Das Nanofluid model in this analysis to learn more about nanofluid effects, where sodium carboxymethyl cellulose with nanoparticles of copper (Cu) was selected as a based fuid. Suitable transformations are used to turn dimensional linear equations into dimensionless expressions. To address the governing dimensionless issues, the Keller box method approach is implemented and executed. The effects of a few chosen factors on flow and heat transmission are investigated. The coefficients of skin friction and heat transmission are provided and analysed. The acquired findings are compared to the existing data in the limiting scenario, and there is a lot of consistency. It was found that velocity, temperature, skin friction and heat transfer coefficients of viscoelastic nanofluid depend strongly on viscosity and thermal conductivity together with magnetic field.

Keywords:
Magnetohydrodynamics (MHD); Viscoelastic; Nanofluid; Constant heat flux

Received: 18 January 2022 **Revised: 26 March 2022** **Accepted: 28 April 2022** **Published: 25 May 2022**

1. Introduction

Heat transfer enhancement, augmentation, or intensification is a term used by researchers to describe a variety of strategies for improving heat transfer performance. Therefore, the addition of nanofluid in fluid flow is to enhance the heat transfer. The study on nanofluids have been studied widely by Waini *et al.*, Mahat *et al.*, Patil *et al.*, Murad *et al.*, and Shorbagy *et al.* [1-7]. The study of the flow of an electrically conducting fluid in a magnetic field is known as magnetohydrodynamics (MHD). Fundamentally, magnetic fields produce currents in a flowing conductive fluid, which tend to form Lorentz force drag forces on the fluid flow. The fluid flow will be affected by the Lorentz force. MHD is being used in astronomy and geophysics, nuclear fission and fusion, metallurgy, and direct energy conversion, among other fields. Many researchers, such as Jamel *et al.*, Hussain *et al.*, Tlili *et al.*, Wakif *et al.*, Hatami *et al.*, Cao *et al.*, Liaqat *et al.*, El-Shorbagy *et al.* and Jamel *et al.* [8-14], have undertaken extensive research on MHD of nanofluids. The benefits of MHD's influence on mixed

* *Corresponding author.*
E-mail address: rahimahm@unikl.edu.my

convection flow via a horizontal circular cylinder are evident in the technology's growth in industrial and technical applications. In the current research, the effects of MHD mixed convection of viscoelastic nanofluid past a horizontal circular cylinder is theoretically investigated. The effects of magnetic field, nanoparticle volume fraction and how these affect the thermal characteristics of the system are of particular interest.

2. Methodology

The Cartesian coordinate (x, y) is used and the dimensional gravitational acceleration is $g_x = g\sin(\bar{x}/a)$, where $\bar{x}$ is the distance from the lower stagnation point. The dimensional velocity outside the boundary layer is $\bar{u}_e(\bar{x}) = U_\infty \sin(\bar{x}/a)$ and the constant free stream velocity is $(1/2)U_\infty$ as mentioned by Merkin [15]. Tiwari and Das model [16] has been chosen in this study and the model is defined as a single-phase model that use Brickman viscosity model. Under the above assumptions and by considering the nanofluid model, the dimensional governing equations of momentum equation and energy equation can be expressed as

$$\frac{\partial \bar{u}}{\partial \bar{x}} + \frac{\partial \bar{v}}{\partial \bar{y}} = 0, \tag{1}$$

$$\begin{aligned} \bar{u}\frac{\partial \bar{u}}{\partial \bar{x}} + \bar{v}\frac{\partial \bar{u}}{\partial \bar{y}} &= \bar{u}_e\frac{\partial \bar{u}_e}{\partial \bar{x}} + \frac{\mu_{nf}}{\rho_{nf}}\frac{\partial^2 \bar{u}}{\partial \bar{y}^2} - \frac{k_o}{\rho_{nf}}\left[\frac{\partial}{\partial \bar{x}}\left(\bar{u}\frac{\partial^2 \bar{u}}{\partial \bar{y}^2}\right) + \bar{v}\frac{\partial^3 \bar{u}}{\partial \bar{y}^3} - \frac{\partial \bar{u}}{\partial \bar{y}}\frac{\partial^2 \bar{u}}{\partial \bar{x}\partial \bar{y}}\right] \\ &+ g\beta_{nf}(T - T_\infty)\sin\left(\frac{\bar{x}}{a}\right) - \sigma B_0^2 \bar{u}, \end{aligned} \tag{2}$$

$$\bar{u}\frac{\partial T}{\partial \bar{x}} + \bar{v}\frac{\partial T}{\partial \bar{y}} = \frac{k}{\rho C_p}\frac{\partial^2 T}{\partial \bar{y}^2}, \tag{3}$$

with the boundary conditions

$$\begin{aligned} &\bar{u} = 0,\ \bar{v} = 0,\ T = -\frac{q_w}{k_{nf}} && \text{at} \quad \bar{y} = 0,\ \bar{x} \geq 0, \\ &\bar{u} = \bar{u}_e(\bar{x}),\ \frac{\partial \bar{u}}{\partial \bar{y}} = 0,\ T = T_\infty && \text{at} \quad \bar{y} \to \infty,\ \bar{x} \geq 0, \end{aligned} \tag{4}$$

where k_{nf} is the thermal conductivity of nanofluid, *T* is the fluid temperature and q_w is the constant heat flux. The numerical values of the thermophysical properties of base fluid and nanoparticles are given in Table 1.

Table 1
Thermophysical properties of nanoparticles and base fluid

Physical Properties	$\rho\left(\text{kg m}^{-3}\right)$	$C_p\left(\text{J kg}^{-1}\text{K}^{-1}\right)$	$k\left(\text{Wm}^{-1}\text{K}^{-1}\right)$	$\beta\times10^5\left(\text{K}^{-1}\right)$
Base Fluid (CMC)	997.1	4179	0.613	21
Nanoparticle (Cu)	8933	385	401	1.67

The dimensionless variables are introduced to simplify the complexity of the governing equations. Based on Anwar *et al.* [17], the dimensionless variables are defined as

$$x=\bar{x}/a,\ \ y=\mathrm{Re}^{1/2}\left(\bar{y}/a\right),\ \ u=\bar{u}/U_\infty,\ \ v=\mathrm{Re}^{1/2}\left(\bar{v}/U_\infty\right),$$
$$u_e\left(x\right)=\bar{u}_e\left(\bar{x}\right)/U_\infty,\ \ \theta=\left(T-T_\infty\right)/\left(T_f-T_\infty\right), \tag{5}$$

where *Re* is Reynolds number. By substituting Eq. (5) into Eqs (1) - (3), the dimensionless system below is yielded

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{6}$$

$$\left[(1-\phi)+\phi\frac{\rho_s}{\rho_f}\right]\left[u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right]=\left[(1-\phi)+\phi\frac{\rho_s}{\rho_f}\right]\sin x\cos x+\frac{1}{(1+\phi)^{2.5}}\frac{\partial^2 u}{\partial y^2}$$
$$-K\left[\frac{\partial}{\partial x}\left(u\frac{\partial^2 u}{\partial y^2}\right)+v\frac{\partial^3 u}{\partial y^3}-\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial x\partial y}\right]+\left[(1-\phi)+\phi\frac{(\rho\beta)_s}{(\rho\beta)_f}\right]\lambda\theta\sin(x)-\frac{\sigma B_0^2 a}{\rho_f U_\infty}u, \tag{7}$$

$$\left[(1-\phi)+\phi\frac{(\rho C_p)_s}{(\rho C_p)_f}\right]\left[u\frac{\partial\theta}{\partial x}+v\frac{\partial\theta}{\partial y}\right]=\frac{(k_s+2k_f)-2\phi(k_f-k_s)}{(k_s+2k_f)+\phi(k_f-k_s)}\frac{1}{\mathrm{Pr}}\frac{\partial^2\theta}{\partial y^2}, \tag{8}$$

with the new boundary conditions as

$$u=0,\qquad v=0,\qquad \frac{\partial\theta}{\partial y}=-1,\qquad \text{at}\qquad y=0, x\geq 0,$$
$$u=u_e(x),\qquad \frac{\partial u}{\partial y}=0,\quad \theta=0,\qquad \text{as}\qquad y\to\infty, x\geq 0, \tag{9}$$

where $\mathrm{Pr}=\mu_f C_p/k_f$ is Prandtl number, $M=\sigma B_o^2 a/\rho_f U_\infty$ is magnetic field, $K=k_o U_\infty/\mu_f a$ is viscoelastic parameter, and λ is mixed convection parameter.

3. Mathematical Solution

In order to solve Eqs. (6) to (8), subject to the boundary conditions (9), the following variables have been considered

$$\psi=xF(x,y),\qquad\qquad \theta=\theta(x,y), \tag{10}$$

are introduced where ψ is the stream function defined as

$$u=\frac{\partial\psi}{\partial y},\quad v=-\frac{\partial\psi}{\partial x}. \tag{11}$$

By substituting Eqs. (10) and (11) into Eqs. (6) to (8), obtained

$$\left[(1-\phi)+\phi\frac{\rho_s}{\rho_f}\right]\left[\left(\frac{\partial F}{\partial y}\right)^2+x\frac{\partial F}{\partial y}\left(\frac{\partial^2 F}{\partial x\partial y}\right)-x\frac{\partial F}{\partial x}\frac{\partial^2 F}{\partial y^2}-F\frac{\partial^2 F}{\partial y^2}\right]$$
$$=\left[(1-\phi)+\phi\frac{\rho_s}{\rho_f}\right]\frac{\sin x\cos x}{x}+\frac{1}{(1+\phi)^{2.5}}\frac{\partial^3 F}{\partial y^3}+\left[(1-\phi)+\phi\frac{(\rho\beta)_s}{(\rho\beta)_f}\right]\lambda\theta\frac{\sin x}{x}-M\frac{\partial F}{\partial y} \tag{12}$$
$$+K\left[2\frac{\partial F}{\partial y}\frac{\partial^3 F}{\partial y^3}-F\frac{\partial^4 F}{\partial y^4}-\left(\frac{\partial^2 F}{\partial y^2}\right)^2+x\left(\frac{\partial^2 F}{\partial x\partial y}\frac{\partial^3 F}{\partial y^3}-\frac{\partial F}{\partial x}\frac{\partial^4 F}{\partial y^4}+\frac{\partial F}{\partial y}\frac{\partial^4 F}{\partial x\partial y^3}-\frac{\partial^2 F}{\partial y^2}\frac{\partial^3 F}{\partial x\partial y^2}\right)\right],$$

$$\left[(1-\phi)+\phi\frac{(\rho C_p)_s}{(\rho C_p)_f}\right]\left[x\frac{\partial F}{\partial y}\frac{\partial \theta}{\partial x}-x\frac{\partial F}{\partial x}\frac{\partial \theta}{\partial y}-F\frac{\partial \theta}{\partial y}\right]=\frac{k_{nf}}{k_f}\frac{1}{\Pr}\frac{\partial^2\theta}{\partial y^2}, \tag{13}$$

which are subject to the following boundary conditions

$$\begin{aligned}&F=0,\quad \frac{\partial F}{\partial y}=0,\quad \frac{\partial \theta}{\partial y}=-1,\quad \text{at}\quad y=0, x\ge 0,\\ &\frac{\partial F}{\partial y}=\frac{\sin x}{x},\quad \frac{\partial^2 F}{\partial y^2}=0,\quad \theta=0,\quad \text{as}\quad y\to\infty, x\ge 0,\end{aligned} \tag{14}$$

When $x\approx 0$, Eqs. (12) and (13) reduce to the following ordinary differential equations:

$$\begin{aligned}&\frac{1}{(1+\phi)^{2.5}}f'''-\left[(1-\phi)+\phi\frac{\rho_s}{\rho_f}\right]\left[f'^2-ff''\right]+K\left(2ff'''-ff^{iv}-f''^2\right)\\ &+\left[(1-\phi)+\phi\frac{(\rho\beta)_s}{(\rho\beta)_f}\right]\lambda\theta-Mf'=0,\end{aligned} \tag{15}$$

$$\frac{(k_s+2k_f)-2\phi(k_f-k_s)}{(k_s+2k_f)+\phi(k_f-k_s)}\frac{1}{\Pr}\theta''+\left[(1-\phi)+\phi\frac{(\rho C_p)_s}{(\rho C_p)_f}\right]f\theta'=0, \tag{16}$$

with the boundary conditions

$$\begin{aligned}&f(0)=0,\quad f'(0)=0,\quad \theta'(0)=-1,\\ &f'(\infty)=1,\quad f''(\infty)=0,\quad \theta(\infty)=0,\end{aligned} \tag{17}$$

4. Results

The behavior of fluid flow for viscoelastic nanofluid affected by magnetic field *M* is analyzed. The validation for numerical solutions is done by comparing the skin friction and heat transfer coefficients with the results from Nazar *et al.* [18]. Good agreements have been obtained from the results as shown in Table 2.

Table 2
Comparison values of heat transfer coefficient when $K=0$, $\Pr=1$, $\phi=0$, $M=0$, and different values of λ
.

λ	Nazar *et al.* [18]	Present $f''(0)$	Nazar *et al.* [18]	Present $\theta(0)$
-0.2	1.0340	1.033028	1.8157	1.816890
0.4	1.5747	1.573759	1.7018	1.702823
3.0	2.4913	2.489892	1.4015	1.402232
10.0	5.7730	5.777805	1.1770	1.178456

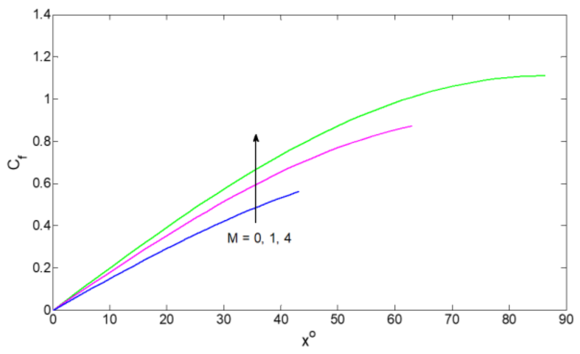


Fig. 1. Variation values of *M* for skin friction coefficient

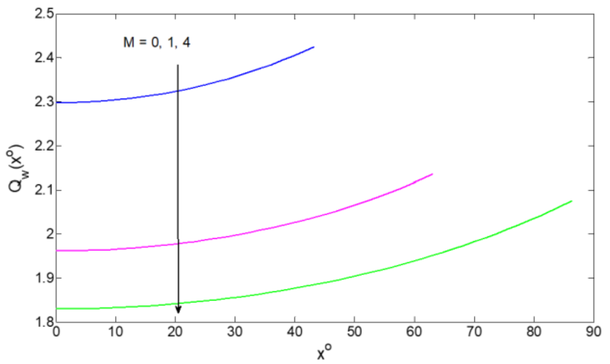


Fig. 2. Variation values of *M* for heat transfer coefficient

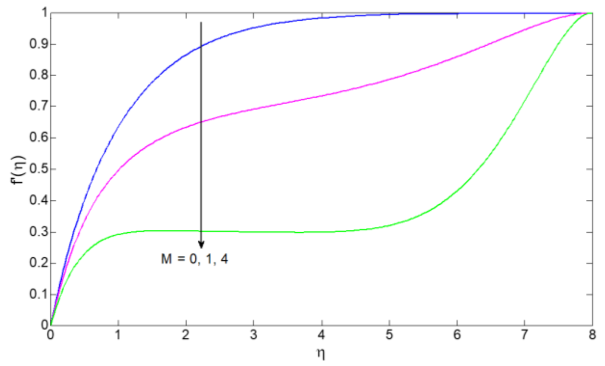


Fig. 3. Variation values of *M* for velocity profile

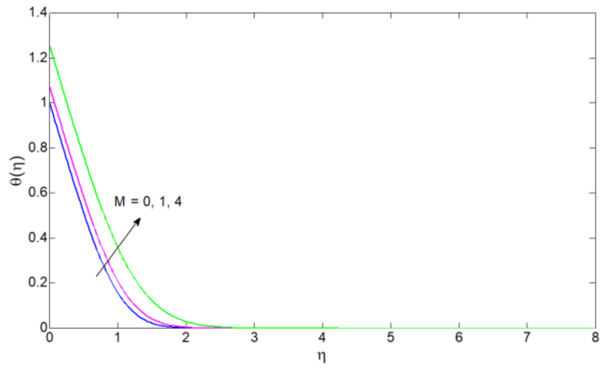


Fig. 4. Variation values of *M* for temperature profile

Figures 1–2 demonstrate how skin friction and heat transfer coefficients vary with M. Skin friction rises as M increases, as shown in Figure 1. Figure 2 shows, however, that heat transport reduces as M increases. Figures 3 and 4 depict the effects of M on velocity and temperature curves. As the value of M increases, the fluid velocity decreases (see Figure 3). Lorentz force is created by the existence of a transverse magnetic field that acts parallel to the flow. The flow is then slowed due to the resistance caused by this force. It's worth noting that this force has an unusual effect. It's worth noting that this force causes the fluid to suffer resistance by increasing friction between its layers, which causes the flow's temperature to rise, as seen in Figure 4. The temperature profile climbs significantly as the M value rises.

4. Conclusions

The conclusions drawn from this modern research are as follows: Velocity slows down and increase with augmentation in magnetic parameters. In the meantime, skin friction increases while heat transfer coefficient decreases for the increasing values of *M*.

Acknowledgement
The authors would like to acknowledge the Ministry of Higher Education (MOHE) for the financial support through vote numbers, FRGS/1/2019/STG06/UNIKL/03/1.

References

[1] Iskandar, Waini, Anuar Ishak, Grosan, T. and Pop, I. "Mixed convection of a hybrid nanofluid flow along a vertical surface embedded in a porous medium." International Communications in Heat and Mass Transfer 114, no. 104565 (2020): 1-5.
https://doi.org/10.1016/j.icheatmasstransfer.2020.104565

[2] Mahat, Rahimah, Sharidan Shafie, and Fatihhi Januddi. 2021. "Numerical Analysis of Mixed Convection Flow Past a Symmetric Cylinder With Viscous Dissipation in Viscoelastic Nanofluid". CFD Letters 13 (2):12-28. https://doi.org/10.37934/cfdl.13.2.1228

[3] Mahat, Rahimah, Noraihan Afiqah Rawi, Abdul Rahman Mohd Kasim and Sharidan Shafie. "Mixed Convection Flow of Viscoelastic Nanofluid Past a Horizontal Circular Cylinder with Viscous Dissipation." *Sains Malaysiana* 47, no. 7 (2018): 1617–1623.
http://dx.doi.org/10.17576/jsm-2018-4707-33

[4] Patil, P.M., Shashikant, A., and Hiremath, P.S. "Diffusion of liquid hydrogen and oxygen in nonlinear mixed convection nanofluid flow over vertical cone." *International Journal of Hydrogen Energy* 44, no. 31 (2019): 17061-17071.
https://doi.org/10.1016/j.ijhydene.2019.04.193

[5] Nurul Musfirah Murad, Noraihan Afiqah Rawi, Sharidan Shafie, Yeou Jiann Lim and Rahimah Mahat. "Unsteady Falkner-Skan flow of Hybrid Nanofluid over a Nonlinear Moving Wedge." *Malaysian Journal of Fundamental and Applied Sciences* 18, no. 1 (2022): 116-123.
https://doi.org/10.11113/mjfas.v18n1.2421

[6] Patil, P.M., Benawadi, S., and Shanker, B. " Influence of mixed convection nanofluid flow over a rotating sphere in the presence of diffusion of liquid hydrogen and ammonia." *Mathematics and Computers in Simulation* 194, April (2022): 764-781.
https://doi.org/10.1016/j.matcom.2021.12.022

[7] El-Shorbagy, M.A., Farshad Eslami, Muhammad Ibrahim, Pouya Barnoon, Wei-Feng Xia and Davood Toghrai. " Numerical investigation of mixed convection of nanofluid flow in a trapezoidal channel with different aspect ratios in the presence of porous medium." *Case Studies in Thermal Engineering* 25, no. 100977 (2021): 1-14.
https://doi.org/10.1016/j.csite.2021.100977

[8] Jamel Bouslimi, Abdelhafez, M.A., Abd-Alla, A.M., Abo-Dahab, S.M. and Mahmoud, K.H. " MHD Mixed Convection Nanofluid Flow over Convectively Heated Nonlinear due to an Extending Surface with Soret Effect." *Dynamic Analysis, Learning, and Robust Control of Complex Systems* 2021, Article ID 5592024 (2021): 1-20.
https://doi.org/10.1155/2021/5592024

[9] Hussain, S., Mehmood, K. and Sagheer, M. "MHD mixed convection and entropy generation of water–alumina nanofluid flow in a double lid driven cavity with discrete heating." Journal of Magnetism and Magnetic Materials 419, (2016): 140-155.
https://doi.org/10.1016/j.jmmm.2016.06.006

[10] Tlili, I., Khan, W.A., and Ramadan, K. "MHD Flow of Nanofluid Flow Across Horizontal Circular Cylinder: Steady Forced Convection." *Journal of Nanofluids* 8, no.1 (2019): 179-186(8).
https://doi.org/10.37934/cfdl.13.2.1228

[11] Abderrahim Wakif, Zoubair Boulahia, Farhad Ali, Mohamed R. Eid and Rachid Sehaqui. "Numerical Analysis of the Unsteady Natural Convection MHD Couette Nanofluid Flow in the Presence of Thermal Radiation Using Single and Two-Phase Nanofluid Models for Cu–Water Nanofluids." *International Journal of Applied and Computational Mathematics* 4, no. 81 (2018): 1–27.
https://doi.org/10.1007/s40819-018-0513-y

[12] Hatami, M., Nouri, R. and Ganji, D.D. "Forced convection analysis for MHD Al2O3–water nanofluid flow over a horizontal plate." *Journal of Molecular Liquids* 187, (2013): 294-301.
https://doi.org/10.1016/j.molliq.2013.08.008

[13] Yan Cao, Hamdi Ayed, Fahd Jarad, Hussein Togun, Hajar Alias, AlibekIssakhov, Mahidzal Dahari, Makatar Wae-hayee and Mohamed Hechmi El Ouni. "MHD natural convection nanofluid flow in a heat exchanger: Effects of Brownian motion and thermophoresis for nanoparticles distribution." *Case Studies in Thermal Engineering* 28, no. 101394 (2021): 1-12.
https://doi.org/10.1016/j.csite.2021.101394

[14] Liaqat Ali, Xiaomin Liu, Bagh Ali, Saima Mujeed and Sohaib Abdal "Finite Element Simulation of Multi-Slip Effects on Unsteady MHD Bioconvective Micropolar Nanofluid Flow Over a Sheet with Solutal and Thermal Convective Boundary Conditions." *Coatings* 9, no.12 (2019): 1-22.
https://doi.org/10.3390/coatings9120842

[15] Merkin, J. H. "Mixed convection from a horizontal circular cylinder." International Journal of Heat and Mass Transfer 20, no. 1 (1977): 73-77.
https://doi.org/10.1016/0017-9310(77)90086-2

[16] Tiwari, Raj Kamal, and Manab Kumar Das. "Heat transfer augmentation in a two-sided lid-driven differentiallyheated square cavity utilizing nanofluids." International Journal of Heat and Mass Transfer 50, no. 9-10 (2007):2002-2018..
https://doi.org/10.1016/j.ijheatmasstransfer.2006.09.034

[17] Anwar, Ilyana, Norsarahaida Amin, and Ioan Pop. "Mixed convection boundary layer flow of a viscoelastic fluid over a horizontal circular cylinder." International Journal of Non-Linear Mechanics 43, no. 9 (2008): 814-821.
https://doi.org/10.1016/j.ijnonlinmec.2008.04.008

[18] Roslinda Nazar, Amin, N. and Pop, I. " Mixed convection boundary-layer flow from a horizontal circular cylinder with a constant surface heat flux." Heat and Mass Transfer 40, (2004):219-227.
https://doi.org/10.1007/s00231-002-0367-y