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Oscillatory Mode of Darcy-Rayleigh Convection in a Viscoelastic Double Diffusive Binary Fluid Layer Saturated Anisotropic Porous Layer

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ABSTRACT

The onset of Darcy-Rayleigh convection in a viscoelastic double diffusive binary fluid layer saturated in an anisotropic porous with temperature dependent viscosity is investigated numerically. The system is heated from below and cooled from above. The critical Rayleigh number from the double diffusive binary fluid were obtained by using the Galerkin expansion procedure. The effects of strain retardation and thermal anisotropy parameter slow down the formation of heat transfer when their values are increased and stabilized the system. Meanwhile, the stress relaxation, Darcy-Prandtl, and mechanical anisotropy parameter enhanced the heat transfer mechanism rapidly in the convection when the values are increased thus destabilize the system.

Keywords:
Viscoelastic, Double diffusive, Anisotropic

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1. Introduction

The viscoelastic property in a non-Newtonian fluid has given rise to interest in different fields such as science, engineering, and technology. This property exhibit both viscous and elastic characteristics when thermal motion occurs. The dissipation of the fluid characterizes the viscous nature and the energy characterized the fluids elastic response [1]. Shear stress is not proportional to the rate of deformation in a non-Newtonian fluid and does not follow Newton's viscosity law. It is interesting to study this kind of fluid as it can be applied in medicine where the blood modeling as a non-Newtonian fluid may help to understand the human's body and improved health techniques, in food industry such as in the processing of ketchup and jam or maybe in creating body vest for military use. There are also unusual patterns of instability in viscoelastic fluids that are not predicted or observed in Newtonian flow.

Double diffusive convection was studied in a porous medium due to the importance in geophysics where groundwater usually contains salts in solution and hence both thermal expansion and solute

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concentration variations can produce variations in density. This phenomenon explored by Horton and Rogers [2], proved experimentally by Morrison *et al.* [3] and explained thoroughly by Parts [4]. Hirata *et al.* [5] also did an experimental study on the small scaler process occurring in the global climate system where a natural double diffusive was considered under-ice melt.

Later, research was done in a more complex system where the system is saturated by an anisotropic fluid. Anisotropic is found in numerous systems in the industry and in nature. The properties of an isotropic material are uniform in all directions but in anisotropy, the properties of the material have dependent direction. Anisotropy can also be a characteristic of synthetic porous materials such as pellets used in chemical engineering as well as insulating fiber materials. Recent research into convective flow through anisotropic porous media has been documented by Filippi E and T [6], Swamy *et al.* [7] and Rees and Storesletten [8]. The current trends on understanding the anisotropic show the relevance of understanding the characteristics of this porous layer. Nield [9] studied on a constant viscosity in a two-component fluid saturating a porous medium taking only Darcy resistance into consideration while Patil and Rudraiah [10] use the Brinkman model. The Darcy Law's were introduced by Darcy [11] and then the extension of the law is the Brinkman model introduced by Brinkman [12]. Later, Patil and Vaidyanathan [13] extended the study by to a two-component fluid saturating a porous medium where the viscosity varies with temperature, using Darcy and Brinkman models. Both models were compared and they conclude that Darcy model is more stable than the Brinkman Model, since the effects of concentration and temperature dependent viscosity on the critical Rayleigh number is small. The Darcy's Law replaced the nonlinear Navier-Stokes equations, which describe the velocities in the bulk fluid experiment with a linear set of velocity equations. The addition of a porous medium would eliminate a nonlinearity inherent in the binary fluid bulk system and thus provide a step forward to understand more complicated, previous binary fluid bulk results.

The viscoelastic flows through porous media have been the subject of theoretical interest in the industry. Kim *et al.* [14] conducts a theoretical thermal instability analysis in a porous layer saturated with viscoelastic fluid. They found that overstability is a preferred mode for a certain range of parameters, and when the elastic parameters are used, a supercritical and stable bifurcation takes the form of the onset of convection. The initiation of thermal convection in a horizontal porous layer saturated with viscoelastic fluid using linear theory was studied by Yoon *et al.* [15]. To examine the effects of relaxation times, a simple constitutive model is used and it is shown that oscillatory instabilities can be set before stationary modes. Bertola and Cafaro [16] used a dynamic system approach to study the instability theory of the viscoelastic fluid saturating a horizontal porous layer. Equilibrium points and their stability are expressed as a function of the relaxation and retardation parameters. Laroze *et al.* [17] study the convection in a rotating binary viscoelastic which contribute to DNA withdrawal due to the problem complexity. Changes of parameter values can induce a large threshold and affect the frequency variations. Malashetty *et al.* [18] also study the binary fluid where the fluid is saturated by an isotropic porous layer. When analytically derived the stationary, oscillatory, and finite amplitude convection, it is shown that competition exists in the oscillatory case rather than stationary between the processes of thermal, solute diffusion, and viscoelasticity.

The physical configuration viscoelastic double diffusive binary fluid layer saturated in an anisotropic porous layer is considered. In this chapter, the influence of the temperature dependent viscosity to the dynamical system is investigated. Due to the viscoelastic flows, the stress relaxation, λ_1 and the strain retardation, λ_2 were introduced. When only the λ_1 is taken into account, the modal is known as the Maxwell model as done by Wang and Tan [1] where the Maxwell model is applied to double diffusive convection in an isotropic porous medium.

2. Mathematical Formulation

Viscoelastic and anisotropy parameters affect the onset criterion for oscillatory convection. The continuity equation from the mass conservation is

$$\nabla \cdot u = 0 \tag{1}$$

The momentum equation is using the amended Darcy law for the viscoelastic fluid of the Oldroyd type by Malashetty *et al.* [18] which is

$$\left(1+\bar{\lambda}_1 \frac{\partial}{\partial \mathrm{t}}\right)\left\{\frac{\rho_0}{\varepsilon}\frac{\partial \mathrm{u}}{\partial \mathrm{t}}+\nabla p\text{-}\nabla \cdot \left[\mu\left(\nabla u+\nabla u^{\mathrm{T}}\right)-\rho \mathrm{g}\right]\right\}=\frac{\mu}{K}\left(1+\bar{\lambda}_2 \frac{\partial}{\partial \mathrm{t}}\right)\cdot u \tag{2}$$

where *t* is the dimensionless time, ρ is the density, ε is the porosity, p is the pressure, μ is the dynamic viscosity, g is the gravity, and K is the permeability tensor. Meanwhile, $\bar{\lambda}_1$ is the relaxation time depending on viscoelasticity and the $\bar{\lambda}_2$ is the retardation time due to the action of the porous matrix. The relaxation parameter, $\bar{\lambda}_1$ is a dimensionless number used in rheology to characterize the properties of fluid and material. A smaller value of $\bar{\lambda}_1$ define the more fluidity of the material. A diluted polymeric solution is confined between the range of [0.1,2].

The energy equation follows Nield and Kuznetsov [19]

$$\rho_0 c\left[\frac{\partial T}{\partial t}+\left(u \cdot \nabla\right)T\right]=\kappa_t \nabla^2 T+\rho c D_{TS}\nabla^2 S \tag{3}$$

and for solute concentration equation is in the form

$$\frac{\partial S}{\partial t}+\left(u\cdot\nabla\right)S=\kappa_s \nabla^2 S+D_{ST}\nabla^2 T \tag{4}$$

where c is the fluid specific heat (at constant pressure), κ_t is the thermal diffusivity, κ_s is the mass diffusivity, D_{TS} is the Soret diffusivity, D_{ST} is the Dufour diffusivity, *S* is the solute concentration and T is the temperature. Soret effect (thermo-diffusion) is the mass diffusion induced by the temperature gradient and Dufour effect (diffusion-thermo) is the heat transfer induced by the concentration gradient.

The basic state of the fluid is quiescent and is given by

$$u_b=\left(u,v,w\right)=\left(0,0,0\right), T=T_b\left(z\right), \mu_b=\mu_0 f\left(z\right), p=p_b\left(z\right), \rho=\rho_b\left(z\right), S=S_b\left(z\right) \tag{5}$$

where the subscript b denotes the basic state. Then, the system is perturbed with the following form

$$\left(u,p,\rho,T,S\right)=\left[u_b\left(z\right)+u', p_b\left(z\right)+p', \rho_b\left(z\right)+\rho', T_b\left(z\right)+T', S_b\left(z\right)+S'\right] \tag{6}$$

where the primes quantities indicate the perturbed variables and are assumed to be small. Substituting equation (6) into equations (1)-(4) with the basic state solution, we obtained

$$\rho' = \rho_0 \left[\alpha_t T' - \alpha_s S' \right] \tag{7}$$

By operating the curl twice on equation (2) and eliminating *p*', we obtained the nondimensional equation

$$\begin{aligned} &\left(1+\overline{\lambda_1}\frac{\partial}{\partial \mathrm{t}}\right)\left[\frac{\mathrm{Da}}{\varepsilon \mathrm{Pr}_d}\frac{\partial}{\partial \mathrm{t}}+\nabla^2 w\text{-}\mathrm{f}(z)\nabla^4 w\text{-}2\frac{\partial \mathrm{f}(\mathrm{z})}{\partial \mathrm{z}}\nabla^2\left(\frac{\partial w}{\partial \mathrm{z}}\right)-\frac{\partial^2 f(z)}{\partial z^2}\times\left(\nabla^2 w+2\nabla_{\mathrm{h}}^2 w\right)\right.\\ &\left.-\left(1+\overline{\lambda_1}\frac{\partial}{\partial \mathrm{t}}\right)Ra_d\nabla_1^2 T+Rs_d\nabla_1^2 S\right]+\left(1+\overline{\lambda}_2\frac{\partial}{\partial \mathrm{t}}\right)\left(\nabla_1^2+\frac{1}{\varepsilon}\frac{\partial^2}{\partial z^2}\right)w=0 \end{aligned} \tag{8}$$

where $\mathrm{Pr}_d = \dfrac{\varepsilon\,\mathrm{Pr}}{Da}$ is the Darcy-Prandtl number, $Ra_d = \dfrac{RaK_z}{d^2}$ is the Darcy-Rayleigh number, and $Rs_d = \dfrac{RsK_z}{d^2}$ is the solutal Darcy-Rayleigh number.

Substitute the normal mode of equation (9) into (3), (4) and (8),

$$\left(w', T', S'\right) = \left[W(z), \Theta(z), \Phi(z)\right] e^{i\left(\alpha_x x+\alpha_y y\right)+i\omega t} \tag{9}$$

we have

$$\begin{aligned} &(1+\lambda_1\,\omega)\left[\frac{\omega}{\mathrm{Pr}_d}\left(D^2-a^2\right)W\text{-}\overline{f}\left(D^2-\alpha^2\right)^2 W\text{-}D^2\overline{f}\left(D^2-\alpha^2\right)W-2D\overline{f}\left(D^2-\alpha^2\right)\mathrm{D}W\right.\\ &\left.-Ra_d\alpha^2\Theta+Rs_d\alpha^2\Phi\right]+\left(1+\lambda_2\omega\right)\left(\frac{1}{\xi}D^2-\alpha^2\right)W=0 \end{aligned} \tag{10}$$

$$W+\left(D^2-\eta\alpha^2\right)\Theta+Df\left(D^2-\alpha^2\right)\Phi=0 \tag{11}$$

$$W+Sr\left(D^2-\alpha^2\right)\Theta+Le\left(D^2-\alpha^2\right)\Phi=0 \tag{12}$$

where $\lambda_1 = \dfrac{Dz}{d^2}\overline{\lambda}_1$ is the relaxation parameter, $\lambda_2 = \dfrac{Dz}{d^2}\overline{\lambda}_2$ is the retardation parameter. Both boundaries were set to be isosolutal, isothermal and free-free representing the lower-upper boundaries.

$$W = D^2 W = \Theta = \Phi = 0 \tag{13}$$

Then, by using the boundary conditions (13), we solved equations (10), (11) and (12) respectively. By adopting the Galerkin techniques to find an approximate eigenvalue solution, we extract the Darcy-Rayleigh number, Ra_d, from

$$\begin{vmatrix} a_{11} & b_{11} & c_{11} \\ d_{11} & e_{11} & f_{11} \\ g_{11} & h_{11} & i_{11} \end{vmatrix} = 0 \tag{14}$$

where

$$a_{11} = (1+\lambda_1\omega)\frac{\omega}{\Pr}\left\langle (DW)^2 \right\rangle - \alpha^2 \left\langle W^2 \right\rangle - \left\langle \left(D^2W\right)^2 \right\rangle - 2\alpha^2 \left\langle (DW)^2 \right\rangle + \alpha^4 \left\langle W^2 \right\rangle - 2B\left\langle \left(D^2W\right)DW \right\rangle$$
$$-\alpha^2 \left\langle DW^2 \right\rangle + B^2 \left\langle (DW)^2 \right\rangle + \alpha^2 \left\langle W^2 \right\rangle + (1+\lambda_1\omega)\frac{1}{\xi}\left\langle (DW)^2 \right\rangle - \alpha^2 \left\langle W^2 \right\rangle,$$
$$b_{11} = -\alpha^2 Ra_d \left\langle W\Theta \right\rangle,\ c_{11} = \alpha^2 \frac{Rs}{Le}\left\langle W\Phi \right\rangle,\ d_{11} = \left\langle W\Theta^2 \right\rangle,\ e_{11} = -\omega\left\langle \Theta^2 \right\rangle + \left\langle (D\Theta)^2 \right\rangle - \eta\alpha^2 \left\langle \Theta^2 \right\rangle,$$
$$f_{11} = Df\left(\left\langle D\Theta \right\rangle\left\langle D\Phi \right\rangle - \alpha^2 \left\langle \Theta\Phi \right\rangle\right),\ g_{11} = \left\langle W\,\Phi \right\rangle,\ h_{11} = Sr\left(\left\langle D\Theta \right\rangle\left\langle D\Phi \right\rangle - \alpha^2 \left\langle \Theta\Phi \right\rangle\right),$$
$$i_{11} = -\omega\left\langle \Phi^2 \right\rangle + \frac{1}{Le}\left[\left\langle (D\Phi)^2 \right\rangle - \alpha^2 \left\langle \Phi^2 \right\rangle\right],$$

and < …> represent the integration from z = 0 to z = 1.

In the case when both boundaries are isosolutal, isothermal and the lower-upper boundaries are set to be free-free, the respective chosen trial functions to get the critical Darcy-Rayleigh number, Ra_{dc} are

$$W = \Theta = \Phi = \sin(z\pi) \tag{15}$$

Oscillatory stability mode is obtained by separating the eigenvalue equations into the real and imaginary parts. Therefore, we obtained the following expression for the thermal Darcy-Rayleigh number, Ra_d in the form

$$Ra_d = \Delta_1 + i\omega_i\Delta_2 \tag{16}$$

For oscillatory onset $\Delta_2 = 0(\omega_i \neq 0)$and this gives a dispersion relation of the form (on dropping the subscript *i*)

$$b_1\left(\omega^2\right)^2 + b_2\left(\omega^2\right)^2 + b_3 = 0. \tag{17}$$

Then,

$$Ra_d^{osc} = \alpha_0\left(\alpha_1 + \omega^2\alpha^2\right). \tag{18}$$

To obtain the oscillatory onset neutral solutions, the steps are as follows: first, the number of positive equation solutions (17) should be calculated. If there is none, then there can be no oscillating instability. If there are two, then the minimum (over α^2) of equation (18) with ω^2 given by equation (17) gives the oscillatory Darcy-Rayleigh number, Ra_d^{osc}. Since equation (17) is quadratic in ω^2, it can result in more than one positive value of ω^2 for fixed values of the parameters B, Sr, Df, Le, Rs, ξ ,η, λ_1 and λ_2. The numerical solution obtained in this study gives only one positive value of ω^2 which signify that only one positive oscillatory solution exists. To evaluate the influence of oscillatory convection on the onset of convection, various values of physical parameters is substitute for $\omega^2(>0)$from (18).

3. Results and Discussion

The onset of Darcy-Rayleigh convection in a viscoelastic double diffusive binary fluid layer saturated in an anisotropic porous with temperature-dependent viscosity is investigated. Only the oscillatory (over-stability) curves presented in this research. Hence, we used Ra_d instead of Ra_{osc} to represent the Darcy-Rayleigh number obtained in this chapter. Figure 1 and Figure 2 shows the additional parameters involved in the case of viscoelastic fluid which are the stress relaxation, λ_1 and the strain retardation, λ_2 parameters. It is shown that as the stress relaxation, λ_1 increased, the critical Darcy-Rayleigh number decreased. In this figure, other parameters were set to be Le = 2, Rs = 100, ξ = 0.5, η = 0.3, B = 0.3, Sr = Df = 0.005, λ_2 = 0.1 and Pr_d = 10. As for the strain retardation, λ_2, the onset of convection increased as the value increased. Figure 3 shows the effect of Darcy-Prandtl number, Pr_d when the Prandtl number is increased. We use Pr_d = 10 as the fixed value in other figures as this is the typical value in a dilute DNA suspension as stated by Kolodner (1998). In these three figures, besides the anisotropic case, we also represent the isotropic case where η = ξ = 1.

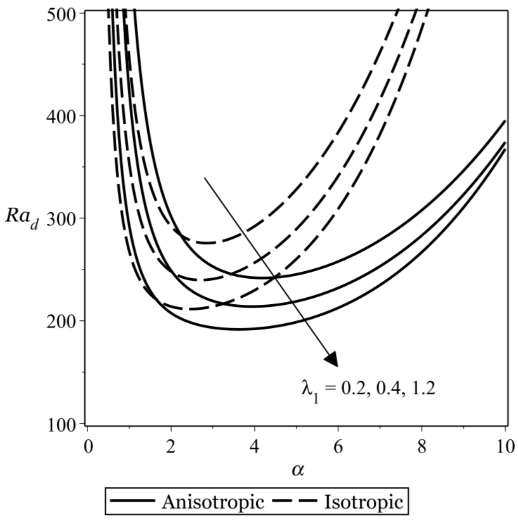

Fig. 1. Variation of Ra_d with α for different values of λ_1

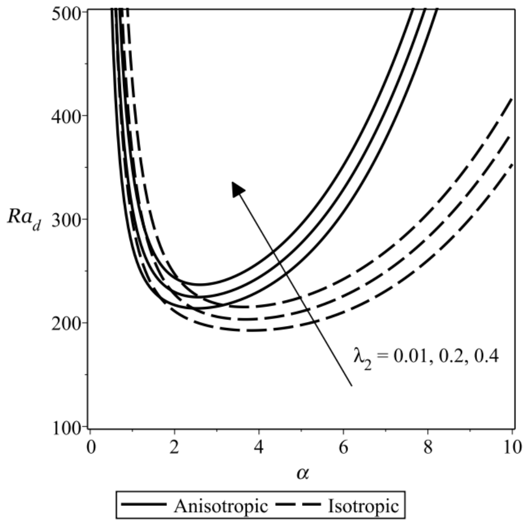

Fig. 2. Variation of Ra_d with α for different values of λ_2

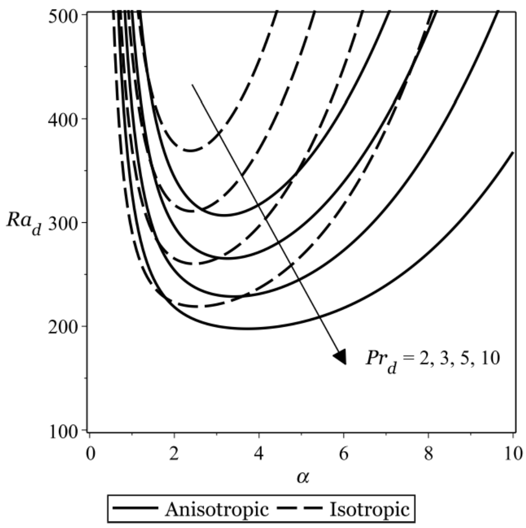

Fig. 3. Variation of Ra_d with α for different values of Pr_d

Figure 4 and Figure 5 show the effect of the anisotropic porous medium parameter where the characteristics of the convection is similar as shown in the stationary case. In other words, in the exchange of the equilibrium regime, the heat transfer characteristics are similar to those of the Newtonian case. As the mechanical anisotropy, ξ increased, the marginal stability curves shift downward and as the thermal anisotropy, η increased, the marginal stability curves shift upward. In Figure 4, η = 0.3 and in Figure 5, ξ = 0.5 while the other paramaters are Le = 2, Rs = 100, B = 0.3, Sr = Df = 0.005, λ_1 = 0.8 and Pr_d = 10. Figure 6 shows the variarion of Ra_d with α for different values of B with Le = 2, Rs = 100, ξ = 0.5, η = 0.3, Sr = Df = 0.005, λ_1 = 0.8, λ_2 = 0.1 and Pr_d = 10. As B increases, the values of Radc decrease showing that B has the effect of destabilizing the system.

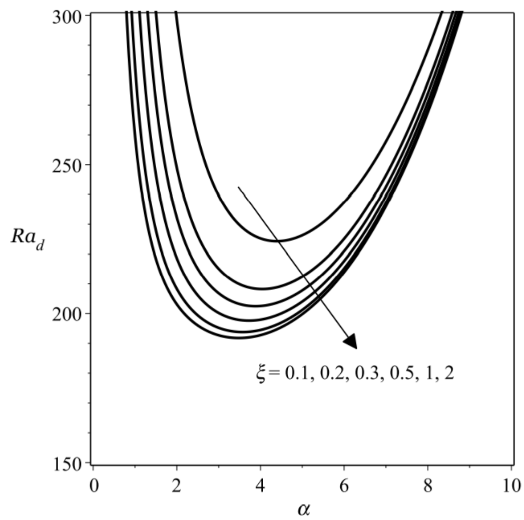

Fig. 4. Variation of Ra_d with α for different values of ξ

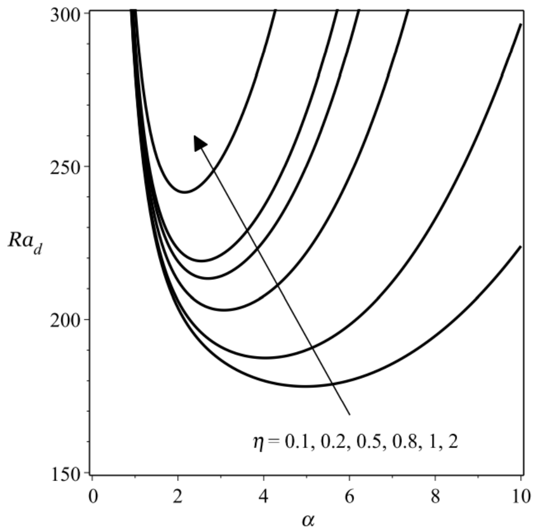

Fig. 5. Variation of Ra_d with α for different values of η

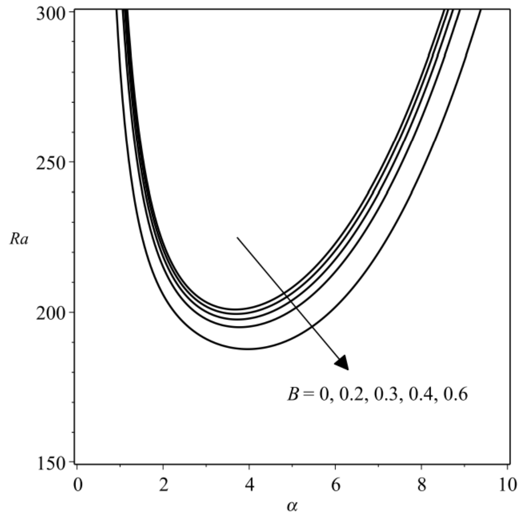

Fig. 6. Variation of Ra_d with α for different values of B

4. Conclusions

The obtained results from the investigation can be concluded as the effects of strain retardation, λ_2, thermal anisotropic, η and Dufour number, Df slow down the formation of heat transfer when their values are increased and stabilized the system. Meanwhile the stress relaxation, λ_1, Darcy-Prandtl, Pr_d , mechanical anisotropy, ξ, temperature dependent viscosity, B and Soret parameter, Sr

enhanced the heat transfer mechanism rapidly in the convection when the values are increased thus destabilize the system.

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