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On the Features of Convection in a Compressible Gas

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ABSTRACT

The problem of Rayleigh-Benard convection in a compressible, viscous and heat-conducting gas is considered. By solving the complete nonlinear system of equations of gas dynamics, the neutral curve for air under normal conditions is calculated as function of the domain height. The formula for critical height of domain is derived analytically. It is established that the isobaric convection regime is realized when the height of the region is less than the critical value, and when the height of the region exceeds this critical value, the convection regime is superadiabatic. During convection in the superadiabatic regime, its adiabatic suppression is observed. Within the framework of the isobaric convection regime, the limits of applicability of the Boussinesq approach are determined.

Keywords:
Rayleigh-Bénard convection, compressible media, Boussinesq approach, superadiabatic regime

1. Introduction

Rayleigh-Benard convection is a classical problem which has a mathematical model developed based on the Boussinesq approach for an incompressible fluid and corresponding numerical methods [1,2]. However, in case of gas convection, the Boussinesq approach can be used only in the laboratory, while convection in regions with typical heights of a few tens of centimeters or more requires that we should take into account gas compressibility based on complete equations of gas dynamics due to a relatively big variation of hydrostatic pressure and a corresponding adiabatic suppression of the convection. Thus, gas convection based on full nonlinear equations of the gas dynamic theory needs additional attention [3,4].

The somewhat compromise is simplified gas convection models. Obtained from the equations of gas dynamics while assuming Mach number and hydrostatic compressibility to be small, such systems of equations describe the media where sonic disturbance of the flow propagates at infinitely high velocities. In terms of mathematics, the structure of the systems obtained is in a way similar to that of the equations of a viscous incompressible liquid [5]. As a result, such a system can be used to calculate convection in the regions with low heights and a wide range of temperature and density [6], but convective flows in the regions with large heights, where the effect of adiabatic suppression of the convection is noticeable, should be calculated using full nonlinear equations of gas dynamics.

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In [7], the authors consider gas convection in a horizontal layer with the horizontal boundaries free from shear stresses in linear approximation. Convection is shown to develop at the value of the Schwarzschild number less than 1.

In [8], the area of application of the Navier-Stokes equations to the case of compressible flows was expanded by taking into account the work of compression forces in the heat transfer equations. The condition for the occurrence of convection in this case takes the form:

$$Ra \cdot (1 - Sc) > Ra_{cr}, \qquad (1)$$

where Ra is the Rayleigh number for a compressible medium and Ra_{cr} is its critical value for an incompressible one in the Boussinesq approximation, and Sc is the Schwarzschild number equal to the ratio of the adiabatic temperature gradient to the given one. The proposed relation (1) describes with satisfactory accuracy the dependence of the critical Rayleigh number on the height of the region and predicts adiabatic suppression of convection at its large value.

The same formula is used in [9] and in experimental works [10,11] to correct the obtained data in order to reduce the effect of the adiabatic gradient. We emphasize that in [10,11] a significant role of the adiabatic temperature gradient was shown at a region height of 0.2 m.

Numerous works (all references can be found in [7]) on the numerical study of convection in a compressible gas have shown qualitatively the presence of adiabatic suppression of convection. However, due to technical difficulties, the specified values of the defining dimensionless parameters are far from real, for example, the value of the criterion of hydrostatic compressibility, which characterizes the relative change in pressure and relative change in density, is overestimated by three orders of magnitude. Insufficient information on the gas convection based on full nonlinear equations for gas dynamics is due to some technical restrictions including two main factors, namely a stiff system of equations, which results in calculating using very small steps in time, and a very low relative change in the pressure [4,5].

Nevertheless, new technological capabilities appear that allow us to use CUDA-enabled GPUs and explicit scheme with massive-parallel data treatment. Note that the advantages of such massive-parallel data treatment are even more evident as the volume of processed information increases [12].

It should be noted that if the region has a simple geometry and small supercriticality, convection develops as two-dimensional rolls [13], which allows us to consider the convective flows of compressible and incompressible fluids as two-dimensional.

This work includes two parts. In the first part, a convective flow of a viscous incompressible fluid in the Boussinesq approximation is considered. The purpose of this stage of work is to calculate the critical value of the Rayleigh number, with its subsequent comparison with its value for a compressible gas. In the second part, we analyze the influence of the region height on the stability of the equilibrium regime of the compressible viscous and heat-conducting gas by calculation a neutral curve using full equations of gas dynamics.

2. Methodology

For the system of equations describing the Rayleigh-Benard convection of an incompressible viscous fluid in the Boussinesq approximation, using a numerical method on the basis of the finite-difference representation of the solution and a technique for calculating of the vorticity values at rigid boundaries of the region [2], it was obtained that the critical Rayleigh number is equal to 1971.4

for a relative horizontal extent of flow domain is equal to π. All boundaries of the region were considered rigid and isothermal, calculations were carried out on a grid of (240·80) nodes.

Convective motion in a compressible, viscous and heat-conducting gas within the gravity field can be described with the following system of equations (2) [1,14]:

$$\begin{aligned}
&\rho_t + \rho\, div\vec{u} + u\cdot\rho_x + v\cdot\rho_y = M\,\Delta(\rho-\rho_h),\\
&u_t + u\cdot u_x + v\cdot u_y = -\frac{1}{\gamma\rho}(\rho T)_x + M\,(\frac{4}{3}u_{xx} + u_{yy} + \frac{1}{3}v_{xy}),\\
&v_t + u\cdot v_x + v\cdot v_y = -\frac{1}{\gamma\rho}(\rho T)_y + M\,(v_{xx} + \frac{4}{3}v_{yy} + \frac{1}{3}u_{xy}) - C_F,\\
&T_t + u\cdot T_x + v\cdot T_y = \frac{M}{\Pr}\Delta T - \frac{\gamma-1}{\gamma}T\,div\vec{u},
\end{aligned} \tag{2}$$

where $M = \nu/((\gamma T_0 R)^{0.5}H) = 4.608\cdot 10^{-8}\cdot H^{-1}$ is the convection Mach number, where velocity calculated according to the kinematic viscosity and related to the adiabatic sound velocity, $T_0 = 300^oK$ chosen as characteristic temperature and pressure of 1 atm, R = 287 J/(кg·K), adiabatic index γ = 1.4, kinematic viscosity $\nu = 16\cdot 10^{-6}\ m^2/sec$ and Pr = 0.71 corresponding to air, with
$C_F = gH/(\gamma RT_0) = 8.130\cdot 10^{-5}\cdot H$ as hydrostatic compressibility. The scale of the size is the height of the region H, temperature and density - values at their low horizontal boundaries, adiabatic sound velocity $(\gamma RT_0)^{0.5}$, pressure $R\rho_0 T_0$ and time $H/(\gamma RT_0)^{0.5}$.

We neglect the dependence of viscosity and heat conductivity coefficients from temperature. The geometry of the region was not changed, i.e., the relation between the vertical and horizontal sizes of the region was always equal to π.

By analogy with the Boussinesq approximation, the equation for the density of system (2) includes the mass diffusion term, taking into account the fact that in an ideal gas the coefficients of kinematic viscosity and diffusion are equal.

All the walls of the region were considered rigid with the no-slip condition for velocity; the temperature on horizontal boundaries was considered constant, equal to T_0 and T_0-ΔT at the lower and upper boundaries respectively. On vertical boundaries, the temperature, its initial and equilibrium distribution were considered linear. The density on all the boundaries of the region was equal to its values in the state of hydrostatic equilibrium, which was defined by relation (3):

$$(\rho T_l)_y = -\gamma\rho C_F,\ T_l = 1 - \Delta T\cdot y. \tag{3}$$

Our calculations were done according to explicit time scheme, and as we didn't expect shock waves in flow, we used a non-divergence form for the system of equations, where convective nonlinear and diffusive members were approximated according to the monotonous scheme [2]. Thus, we used the numeric method of the first order approximation on time and second order on space.

In the calculations, we used the mesh with (240·80) knots and the step on dimensionless time of 0.01. By conducting some test calculations on more detailed space and time meshes, our choice was justified.

All the calculations were done in the vicinity of the instability threshold, so the Reynolds number defined by relation (4) and

$$\mathrm{Re} = \frac{1}{M}\sqrt{\frac{2Ek}{\pi}}, \tag{4}$$

calculated according to the mean-square velocity, kinematic viscosity and height of the layer was not greater than 1. Here Ek stands for the total kinematic energy.

Due to the low velocities of the convective flow, the pressure is determined mainly by the hydrostatic component [4].

Our numerical investigation was organized as follows.

We changed ΔT in our calculations and determined the critical temperature difference at which the solution has a zero increment, i.e., it neither increases nor fades with respect to time. Then we used this temperature difference to calculate by relation (5) the critical Rayleigh number:

$$Ra = \Pr C_F \Delta T / M^2 = gH^3 \Delta T / (\chi \nu). \quad (5)$$

Here and below, always the dimensionless temperature difference is used.

It should be noted that this method is accurate at small H, when disturbances of the equilibrium solution develop monotonously with respect to time. However, at relatively large H (about 0.5 m in this case) fluctuations might appear, so corresponding data should be treated as crude data. Note that the appearance of fluctuations is typical for convective flows [2].

3. Results

Figures 1 and 2 show data as functions of the size height in region H, where H is measured in meters for illustration purposes.

The blue solid lines in Figures 1 and 2 defined by relation (6) correspond to the dimensionless adiabatic difference of temperatures [9-11,14]:

$$\Delta T = (\gamma - 1) C_F, \, \Delta T = (\gamma - 1) gH / (\gamma R T_0), \quad (6)$$

while the red lines defined by relation (7) correspond to the critical Rayleigh number 1971.4 for incompressible media at the Boussinesq approximation with

$$\Delta T = 1971.4 \chi \nu / (gH^3), \, Ra_{cr} = gH^3 \Delta T / (\chi \nu) = 1971.4, \quad (7)$$

and the dots represent data obtained by numerical integrating of full nonlinear equations for gas dynamics.

In Figure 1, the yellow curve also shows described by relation (8) the result predicted theoretically [8]

$$\Delta T = 1971.4 + (\gamma - 1) g^2 H^4 / (\gamma R T_0 \chi \nu), \quad (8)$$

which, taking into account the discrepancy (excess) of 25% at H = 0.4 m, corresponds with satisfactory accuracy to the result of this work. We also note that as H increases, the result [8] asymptotically coincides with the adiabatic temperature difference.

Point A (here it corresponds to H = 0.2173 m), where curves meet in Figures 1 and 2, is the place where the convection regime changed, since the damping of the critical temperature difference with an increase in the height of the region is replaced by its increasing. The height of the region corresponding to the position of point A will be called critical.

The area to the left of A has a smaller height and isobaric convection (a relatively small variation of hydrostatic pressure). The position of point A gives the order estimation of the region from above where Boussinesq approximation (the private case of isobaric convection) may be used. The area to the right, with a bigger height and a relatively big variation of hydrostatic pressure, has a superadiabatic regime. The strong adiabatic suppression can be seen in the area.

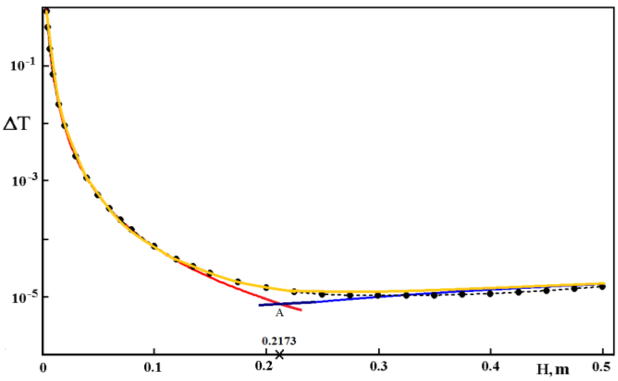

Fig. 1. Critical temperature difference

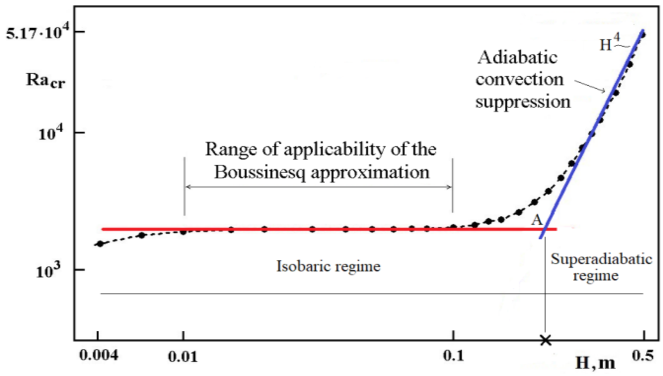

Fig. 2. Critical Rayleigh number

With the increase in the characteristic temperature T_0, point A moves to the right according to the fourth power root law following relation (9):

$$H_{cr} = (1971.4\nu\chi\gamma RT_0 / (g^2(\gamma - 1)))^{0.25} = 11.98(\nu\chi T_0)^{0.25} = 0.05222T_0^{0.25}, \quad (9)$$

where ν and χ are the constant kinematic viscosity and temperature conductivity of the medium expressed in SI system respectively. The dependence of kinematic viscosity and thermal conductivity can be taken into account using the Sutherland's formula [15].

More exactly, according to the constant condition of the calculated critical Rayleigh number in Figure 2, we see that the range of applying Boussinesq approximation varies with the height of the region changing from 1 cm to 10 cm. In the region with lower heights, Boussinesq approximation cannot be used correctly due to big changes in temperature and density, while in the case with bigger heights - due to a relatively big change in hydrostatic pressure and resulting compression.

When the height of the region is greater than the critical one, the convective motion is observed, as a rule, in the superadiabatic regime, when the given temperature gradient exceeds the adiabatic one.

We note, however, that at $H > 0.322$ m, convective motion near the critical point can be observed at a given temperature gradient noticeably, by 14% less than the adiabatic one, and, as a consequence, $Sc > 1$. Thus, the critical temperature gradient here is less than the value of the adiabatic gradient. This seems surprising and may appear due to the action of viscosity and thermal conductivity. The results of our studies show that the statement about the sufficiency and overestimation of the condition $Sc > 1$ for the absence of convection in the linear and nonlinear approximations [7] is too rough and needs to be corrected.

When the height of the region is less than the critical one, the isobaric critical temperature gradient is always greater than the adiabatic one.

4. Conclusions and Discussion

The problem of Rayleigh-Benard convection in a compressible, viscous and heat-conducting gas is considered. By solving the complete nonlinear system of equations of gas dynamics we have obtained:

(1) A neutral curve, where the critical temperature difference and the critical value of the Rayleigh number are presented as functions of the height of the region over a wide range of its variation. The obtained numerically dependences refine the already existing analytical ones;

(2) The critical Schwarzschild number as a function of the height of the region. It is shown that convection can develop at a temperature gradient noticeably less than the adiabatic one and, at the same time, convection may not develop at the temperature gradient greater than the adiabatic one.

(3) An analytical formula for the critical height of the region. The critical height of the region is the boundary of the separation of convection modes. At a lower height of the region, an isobaric convection mode is realized, while at a higher one, the mode is superadiabatic where the adiabatic suppression is observed. A special case of isobaric convection is the convection regime described by the Boussinesq approximation.

Note that the characteristics of the convective flow in the isobaric and superadiabatic regimes should differ significantly, which is due to the activation of adiabatic processes in the superadiabatic convection regime. Therefore, it is of great interest to simulate convection at subcritical and supercritical heights with subsequent comparison and analysis of all integral characteristics.

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