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Effect of Chemically Reactive Nanofluid Flowing Across Horizontal Cylinder: Numerical Solution

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ABSTRACT

The objectives of this work are to build a mathematical model for the nanofluid flow problem in the present first-order chemical reaction. The flow over an isothermal horizontal circular cylinder is selected due to many engineering applications for this geometry. A set of nonlinear partial differential equations (PDE) coupled into system of ODE's that describes the physical behavior of the problem under study was derived. In numerical ways, finite element techniques (FET) were used to treat the mathematical model. The most important quantities that have a good effect on the physical matter that appeared in the mathematical definition are the dimensionless chemical reaction parameter, the thermophoresis parameter and the Brownian motion parameter. The influence of these dimensionless quantities on the rate, temperature, and concentration profiles is drawn graphically. Furthermore, the impact on the engineering measures that have a physical implementation, such as the local Skin-friction coefficient, Nusselt number and Sherwood number are also discussed. It is noticed that increasing chemical reaction Λ parameter tends to decrease velocity and concentration values but to slightly enhance temperatures in the flow field; a rise in Λ also enhances values of the local Sherwood number but strongly reduces the local Nusselt number at the cylinder surface as so as the local surface shear stress function.

1. Introduction

Heat must be added, removed, or moved from one process stream to another throughout any industrial facility, and it has become a significant task for the industrial requirement. These processes provide a means of recovering energy and heating/cooling process fluids. Improving heating or cooling in an industrial process may save energy, reduce process time, increase thermal rating, and extend equipment working life. The action of enhanced heat transfer has a qualitative effect on some techniques. Creating high-performance thermal systems for heat transfer enhancement is becoming increasingly popular. Several studies have been conducted to understand better heat transfer

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performance for practical applications such as heat transfer enhancement. A wide variety of industrial processes involve the transfer of heat energy.

Convective heat transfer of nanofluid has been studied extensively. Choi [1] proposed the nanofluid to indicate colloids due to the existence of nanoparticles that detached in a base fluid. A Nanofluid is a conventional liquid like water, oil, or ethylene glycol containing a suspension of solid nanoparticles up to 100 nm in diameter. Nanofluids are heat transfer fluids with better thermal properties, as described by Masuda *et al.*, [2]. The improved property of nanofluids is thermal conductivity such improvement referred to the solid nanoparticle size, shape, concentration, and thermal properties. The higher relative surface area of nanoparticles significantly improves the heat transfer potential and increases the suspension's stability. The existence of nanoparticles in the fluid can also improve abrasion resistance properties over conventional solid-fluid mixtures. Using nanofluids will hold up the design of smaller and lighter heat transfer structures, Keblinski *et al.* [3]. Many researchers studied the mechanisms behind the enhancement of heat transfer characteristics using nanofluids. Das *et al.*, [4] and Khanafer *et al.*, [5] collected a lot of papers on this matter. The study of transfer phenomena of nanofluid flow is of enormous meaning in many divisions of applied science applications such as cooling and heating, medical field and safe surgery, and different applications manufacturing electronic devices. Buongiorno [6] made a complete review of the convective flow of nanofluids.

Furthermore, He looked at seven slip conditions that could result in a comparable velocity concerning the nanoparticles and the base fluid. The only essential mechanisms were found to be Brownian diffusion and thermophoresis. Many studies also discussed the enhancement in nanofluids' thermal properties and conductivity. See Bachok *et al.*, [7], Makinde and Aziz [8]. Khanafer *et al.*, [9] and Yu *et al.*, [10] show experimentally that the characteristic improvements in thermal conductivity are noticed over the ordinary fluid, and the improvements in the heat transfer coefficient will be up to 40%. The thermal conductivity increases twice when little quantity of nanoparticles is added to the usual fluids, Choi *et al.*, [11]. The convection flow over a horizontal circular cylinder was described by Sparrow and Lee [12], Ingham [13], Sadeghipour and Hannani [14]. Additional investigations of flow over a cylindrical geometry have been reported by Kaminski *et al.*, [15]. The convective mass transfer problem has been presented by Sigwalt *et al.*, [16]. Cesini *et al.*, [17] and Karniadakis [18] have studied the force convection case. Damseh [19] also studied the appearance and inclusion of dust particles as a viscous, incompressible fluid flows across an isothermal, horizontal cylinder.

Studying the problem of heat and mass transfer in a moving chemically reactive fluid undergoing exothermic and endothermic chemical reactions is of significant in many physical applications. A chemical reaction, in any chemical engineering processes, occurs between a strange mass and the occupation fluid. The chemical reaction order depends on a number of means. The straightforward chemical reactions are the first order reaction where the rate of reaction is in direct proportion to the species' attentiveness. Furthermore, Chamkha [20] and Damseh *et al.*, [21] addressed the heat and mass transfer problem of the steady and unsteady flowing fluid of an electrically conducting liquid and the visco-elastic fluid in the presence of a first-order chemical reaction. They found that the friction coefficient decreased as the values of the chemical reaction parameter were reduced. Other articles dealing with chemical reactions and thermophoresis effect on heat and mass transfer problems can be construct in Refs. [22–28].

This model explores the heat exchange of the flow within the nanofluid flow problem in the present first-order chemical reaction. A set of nonlinear (PDE) partial differential equations coupled into system of ODE's that describes the physical behavior of the problem under study was derived. In numerical ways, finite element techniques (FET) were used to treat the mathematical model. The effects are presented graphically.

2. Mathematical Formation of the Problem

Let us use the case of steady, incompressible, laminar, two-dimensional, natural-driven convection heat and mass transfer problem. The fluid with the existence of nano-species past flows over an isothermal horizontal circular cylinder of r_{be} the radius, in the presence of a first-order chemical reaction, as exposed in Figure 1. The coordination system for tangential x is portion along the circumference of the horizontal cylinder, and the radial y -axis is considered to be perpendicular to the cylinder surface.

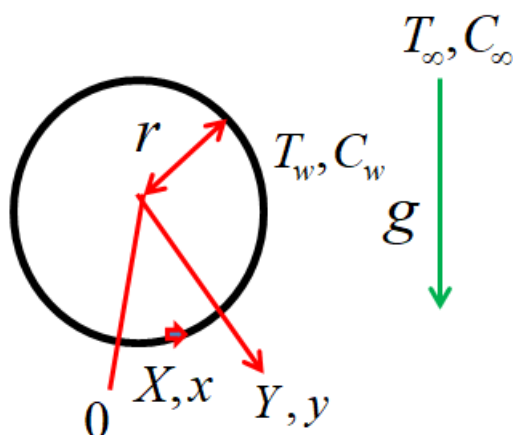


Fig. 1. Configuration of the model

Little concentration of reactive soluble particles are diffuses in the fluid so that thermal energy created through the chemical reaction may be discarded. Initially, the nanofluid is kept at the similar temperature and concentration as the horizontal cylinder. Immediately, the temperature of the nanofluid is higher to a temperature T_w . The concentration of the reactive species is continued at a constant scale. The concentration at $y=0$ cylinder surface equal C_w . The reaction of the species at bulk nanofluid is considered as a first-order homogenous chemical reaction with a concentration equal to C_∞ . In the convective flow, the direction of gravity g acts downward. When applying the Boussinesq-approximation and constant thermo-physical properties, the leading boundary layer equations, continuity, momentum, heat and mass conservation, can be drawn as below

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0 \quad (1)$$

$$U \frac{\partial U}{\partial X} + V \frac{\partial V}{\partial Y} = \beta_T (T - T_\infty) g \sin\left(\frac{X}{r}\right) + \beta_C (C - C_\infty) g \sin\left(\frac{X}{r}\right) + \nu \frac{\partial^2 U}{\partial Y^2} \quad (2)$$

$$U \frac{\partial T}{\partial X} + V \frac{\partial T}{\partial Y} = \frac{k_n}{(\rho c_p)_f} \left(\frac{\partial^2 T}{\partial Y^2} \right) + \frac{(\rho c_p)_p}{(\rho c_p)_f} \left[D_B \frac{\partial T}{\partial Y} \frac{\partial C}{\partial Y} + \left(\frac{D_T}{T_\infty} \right) \left(\frac{\partial T}{\partial Y} \right)^2 \right] \quad (3)$$

$$U \frac{\partial C}{\partial X} + V \frac{\partial C}{\partial Y} = -\gamma C + D_B \left(\frac{\partial^2 C}{\partial Y^2} \right) + \left(\frac{D_T}{T_\infty} \right) \frac{\partial^2 T}{\partial Y^2} \quad (4)$$

where U and V be the velocity components along the X and Y direction, D_B be the Brownian diffusion coefficient and D_T be the thermophoretic diffusion coefficient, T be the temperature and C are the concentration, g be the acceleration due to gravity, ρ be the fluid density, β_T and β_C be the coefficient of expansion and concentration, ν be the kinematic viscosity. The no-slip boundary situations are given by

$$\begin{aligned}
 Y=0, U=0, V=0, T=T_w, C=C_w \\
 Y \rightarrow \infty, T=T_\infty, C=C_\infty,
 \end{aligned}
 \tag{5}$$

2.1 Mathematical Model for the Thermal Physical Property of a Nanofluid

The viscosity, density, heat capacitance and the effective thermal conductivity of the nanofluid are defined as given by Brikman [29] and Hamilton *et al.*, [30] respectively (see Table 1):

Table 1
 Mathematical model for the thermal physical property of a nanofluid

Physical Quantity	Mathematical model
Influential Dynamic viscosity of the nanofluid	$\mu_{nf} = \mu_f (1 - \phi)^{-2.5}$
The influential Density of the nanofluids	$\rho_{nf} = \phi \rho_s + (1 - \phi) \rho_f$
The Heat capacitance of nanofluid	$(\rho C_p)_{nf} = \phi (\rho C_p)_s + (1 - \phi) (\rho C_p)_f$
Thermal conductivity of spherical nanoparticles approximated	$\frac{k_{nf}}{k_f} = \left[\frac{2k_f + k_s - 2\phi(k_f - k_s)}{2k_f + k_s + \phi(k_f - k_s)} \right]$

Where n the nanoparticle is shape factor and ϕ is the nanoparticle volume fraction.

Solve Eq. (1) to (5) the dimensionless variables were originated

$$\begin{aligned}
 x = \frac{X}{r}, y = \frac{Y}{r} Gr^{1/4}, u = \frac{rU}{\nu Gr^{1/2}}, v = \frac{rV}{\nu Gr^{1/4}}, \\
 \theta = \frac{T - T_\infty}{T_w - T_\infty}, \phi = \frac{C - C_\infty}{C_w - C_\infty} Gr^{1/4}, Gr = \frac{g \beta_T (T_w - T_\infty) r^3}{\nu Gr^{1/2}}, \\
 N = \frac{\beta_C (C_w - C_\infty)}{\beta_T (T_w - T_\infty)}, Pr = \frac{\nu}{\alpha_m}, Sc = \frac{\nu}{D_B}, \Lambda = \frac{\gamma r^2}{\nu Gr^{1/2}} \\
 N_T = \frac{\tau D_T (T_w - T_\infty)}{T_\infty \alpha_n}, N_B = \frac{\tau D_B (C_w - C_\infty)}{\alpha}
 \end{aligned}
 \tag{6}$$

Here x and y be the dimensionless coordinates. u and v be the dimensionless velocities. Pr Prandtl number, Gr Grashof number, and Sc Schmidt number respectively. θ be the imensionless temperature and C are the concentration. Λ be the dimensionless chemical reaction parameter, N_T is the thermophoresis parameter, N be the buoyancy ratio parameter, and N_B is the Brownian motion parameter.

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (7)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial Y} = [\theta + \varphi N] \sin(x) + \frac{\partial^2 u}{\partial y^2} \quad (8)$$

$$u \frac{\partial \theta}{\partial x} + v \frac{\partial \theta}{\partial Y} = \frac{1}{Pr} \left(N_r \left(\frac{\partial \theta}{\partial y} \right)^2 + N_B \frac{\partial \phi}{\partial y} \frac{\partial \theta}{\partial y} + \frac{\partial^2 \theta}{\partial y^2} \right) \quad (9)$$

$$u \frac{\partial \phi}{\partial x} + v \frac{\partial \phi}{\partial Y} = \frac{1}{Sc} \left(\frac{\partial^2 \phi}{\partial y^2} + \frac{N_r}{N_B} \frac{\partial^2 \phi}{\partial y^2} \right) - \Lambda \phi \quad (10)$$

The transformed boundary situations for the Eq. (7)-(10) are

$$\begin{aligned} y=0: & \quad u=0, \quad v=0, \quad \theta=1, \quad \varphi=1 \\ y \rightarrow \infty: & \quad u=0, \quad \theta=0, \quad \varphi=0 \end{aligned} \quad (11)$$

In order to solve equations (7-10), subject to boundary conditions (11). Partial differential equations that are non-linear and condensed addicted to non-linear ODE's deliberated for that purpose the stream ψ function where $\psi = \psi(x, y)$ routinely satisfy continuity equation, we use the subsequent variables $\psi = xf(x, y)$

$$u = \frac{\partial \psi}{\partial y}, \quad v = \frac{\partial \psi}{\partial x} \quad (12)$$

Substituting Eq. (12) into Eq. (7)-(12), we can finally write the transformed equations as:

$$f''' + f''' f - f'^2 + (\theta + N\varphi) \frac{\sin x}{x} = x \left(f' \frac{\partial f'}{\partial x} - f'' \frac{\partial f}{\partial x} \right) \quad (13)$$

$$\theta'' + \left(N_B \theta' \varphi' + N_T (\theta')^2 \right) + Pr f \theta' = Pr x \left(f' \frac{\partial \theta}{\partial x} - \theta' \frac{\partial f}{\partial x} \right) \quad (14)$$

$$\varphi'' + \frac{N_T}{N_B} \theta'' + Sc f \varphi' + -Sc \Lambda \varphi = Sc x \left(f' \frac{\partial \varphi}{\partial x} - \varphi' \frac{\partial f}{\partial x} \right) \quad (15)$$

Eq. (13) to (15) are subjected to the following generated

$$\begin{aligned} f'(x, 0) &= 0, \quad f(x, 0) = 1, \quad \theta(x, 0) = 1, \quad \varphi(x, 0) = 1 \\ f'(x, \infty) &= 0, \quad \theta(x, \infty) = 0, \quad \varphi(x, \infty) = 0 \end{aligned} \quad (16)$$

The engineering quantities of physical attention are the local Nusselt number, Skin-friction coefficient, and Sherwood number, and they are definite as

$$Nu_x = -(Gr)^{1/4} \theta^1(x, 0) = -\frac{\partial \theta(x, 0)}{\partial y} \quad (17)$$

$$Sh_x = -(Gr)^{1/4} \phi^1(x, 0) = -\frac{\partial \phi(x, 0)}{\partial y} \quad (18)$$

$$Cf_x = -2(Gr)^{1/4} xf^1(x, 0) = x \frac{\partial^2 f(x, 0)}{\partial y^2} \quad (19)$$

3. Numerical Solution

Eq. (13)-(15) are coupled, nonlinear partial differential equations (PDE's), and no analytical solution can be defined. The local similarity solutions for the set of governing equations can be achieved when assigning zero value to x , which is the lower stagnation point. Nazar *et al.*, [31] simplify the present model conservation equations to simpler (ODE's) ordinary equations. Here, at the upper stagnation point ($x = \pi$), the local Nusselt number, local Sherwood number, and local skin friction coefficient at the cylinder surface are defined by Eq. (17)-(19).

In the present analysis, the derived Eq. (13)-(15) are nonlinear partial differential equations (PDE's), and may be solved using the implicit finite difference approach discussed by Cebeci and Bradshaw [32, 33]. This technique is the most powerful and is used extensively in many viscous fluid simulations. More newsworthy studies in nanofluid dynamics are given in Refs. [34–36].

The computations were carried out for uniform grid of $\Delta y = 0.05$, $\Delta x = 0.05$. The criterion of convergence is employed between any two successive iterations. At the boundary's edge, the grid size and the thickness of the boundary layer y_∞ has been examined. The results at hand are independent of the grid size, and the thickness of the boundary layer is considered up to the fourth decimal point.

4. Results and Discussion

In this paper, we have examined the effect of chemically reactive nanofluid flowing across horizontal cylinder. The rate and temperature profiles (2) to (16) the results are generated graphically in Figures 2-15. The partial governing differential equations are numerically solved, and the variation of relevant physical parameters is analyzed and discussed in detail using tables and figures. The graphical representation of the numerical results are present for the results of Brownian motion parameter (N_B), thermophoresis number (N_T), and chemical reaction parameter (Λ) on temperatures profiles (θ), concentration profiles (ϕ) and velocity profiles (f') with y coordinate which is normal to the cylinder surface, at the stagnation point ($x = \pi$). Extensive computations are presented for the water case ($Pr = 7$). A set of numerical results are carried out for different values of the physical parameters that describe the flow behavior. The local skin friction coefficient (Cf_x), Nusselt number (Nu_x) and Sherwood number (Sh_x) are calculated with curvature parameter x .

In all results a constant values is assigned for the buoyancy ratio parameter, Sherwood number and Prandtl number are $N = 0.5$, $Pr = 7.0$, $Sc = 0.7$. Since the effect of such parameters were addressed in the literature. The values of the other parameters are set as $Pr = 7.0$, $Sc = 0.7$, $N = 0.5$, $N_T = 0.1$, $\Lambda = 1.0$ and different N_B values. From figures 2–3 illustrate the upshot of the Brownian motion parameter (N_B) on the temperature profiles (θ), and concentration profiles (ϕ) with (y) coordinate at the stagnation point ($x = \pi$). The Brownian motion's physical reasons come from the random motion of suspended nanoparticles in the base

fluid. This motion's power results from speedy moving atoms or molecules in the main fluid. It is clear that the Brownian motion significantly affects temperature and concentration shape. The boundary layer velocity decreases with increasing Brownian motion number N_B . The temperature is somewhat increased, and the concentration decreases. Moreover, the nanoparticle volume fraction tends to decrease with an increase in the Brownian motion number N_B , which may also contribute to the enhancement in the temperature gradient. The Brownian motion number also works to elevate thermal conductivity. This may be caused directly by the nanoparticles that carry the thermal energy or by a roundabout contribution due to the micro convection of the fluid surrounding the individual nanoparticles. The Brownian motion is physically powerful for small particles, and the opposite will be relevant for large particles. The values of the other parameters are set as $Pr = 7.0$, $Sc = 0.7$, $N = 0.5$, $N_B = 0.2$, $\Lambda = 1.0$, and different N_T values. Figures 4–6 show the effect of thermophoresis number (N_T) on velocity profiles (f'), temperature profiles (θ), and concentration profiles (ϕ) with distance normal y at the stagnation point ($x = \pi$). It is noticed from these figures that increasing of thermophoresis number (N_T) decreases rates inside the boundary layer; this is explained by the small positive buoyancy forces that exist. The temperature increases inside the thermal boundary layers and the thicknesses of the boundary layer are broadened. To make this, the particles next to the hot surface generate a thermophoretic force, and as well these forces bear the particle decomposition away from the fluid at the cylinder surface. This is the reason for the augmentation in the temperature boundary layer thickness.

The skin friction coefficient (Cf_x) and Nusselt Number (Nu_x) at the cylinder surface are presented in Figure 7 and Figure 8. Both are increased very slightly with increasing the thermophoresis number (N_T) due to extra thermophoretic forces. Finally, increasing of thermophoresis number cause to decrease in the concentration inside boundary layers and, as a result, decreases mass flux. Thermophoresis account for the elevated concentration (nanoparticle) magnitudes. Figure 9 shows that the mass transfer rate (Sh_x) decreases by increasing the thermophoresis number. The values of the other parameters are set as $Pr = 7.0$, $Sc = 0.7$, $N = 0.5$, $N_B = 0.2$, $N_T = 0.2$. variation of velocity (f'), temperatures (θ), and concentration (ϕ) distributions for different values of dimensionless chemical reaction parameter (Λ) is shown in Figure 10–12.

From Figure 13-15, one can observe that a significant increase in chemical reaction parameter (Λ) accounts for the reduction in the nanofluid velocity and the increase in temperature distributions. We see also that the concentration profiles are affected and also decelerated with the elevated values of the chemical reaction parameter. The utilization of chemical particles results in a drop in the concentration fields, which in turn reduces the gravitational effects due to concentration gradients. In theory, the nanoparticle volume fraction decreases as the chemical reaction parameter increases. When the chemical reaction is more potent than the thermophoresis particle deposition, the nanoparticle volume fraction of the fluid regularly shifts from a higher value to a lower value.

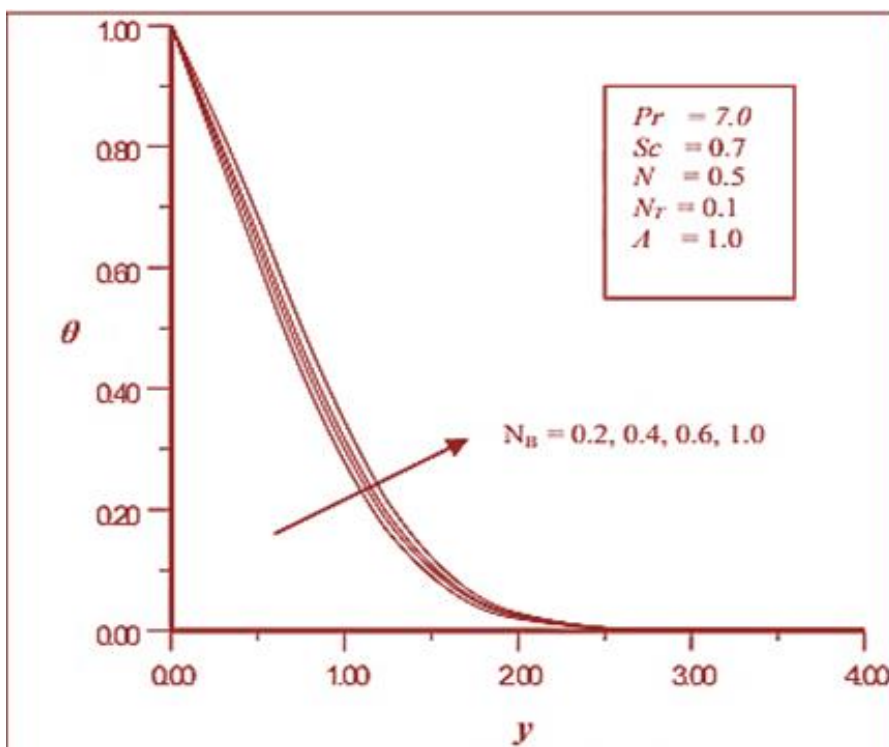


Fig. 2. Influence of Brownian motion parameter on temperature profile at upper stagnation point

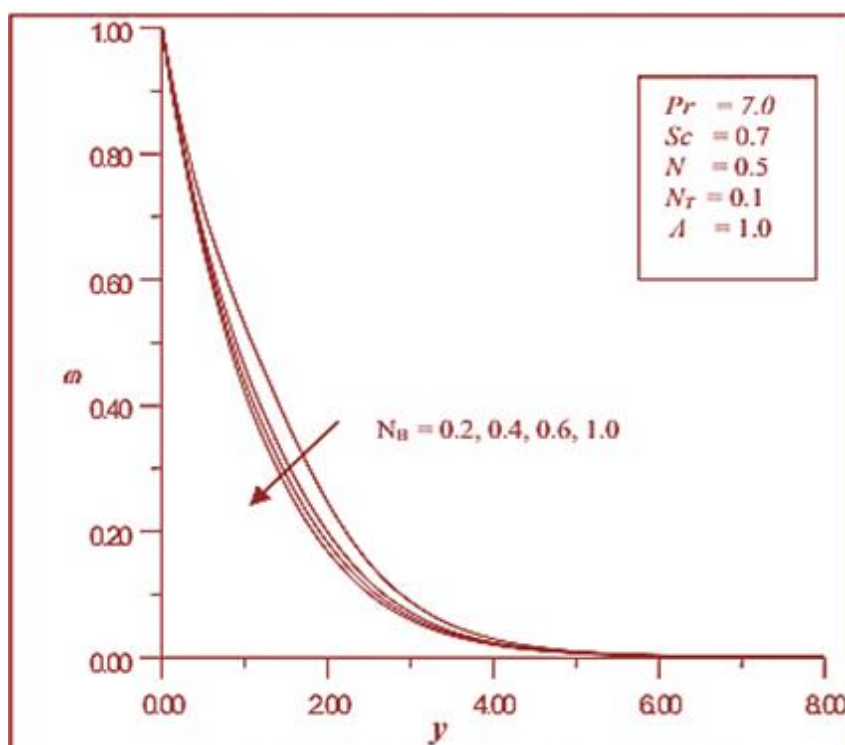


Fig. 3. Influence of Brownian motion parameter on concentration profile at upper stagnation point

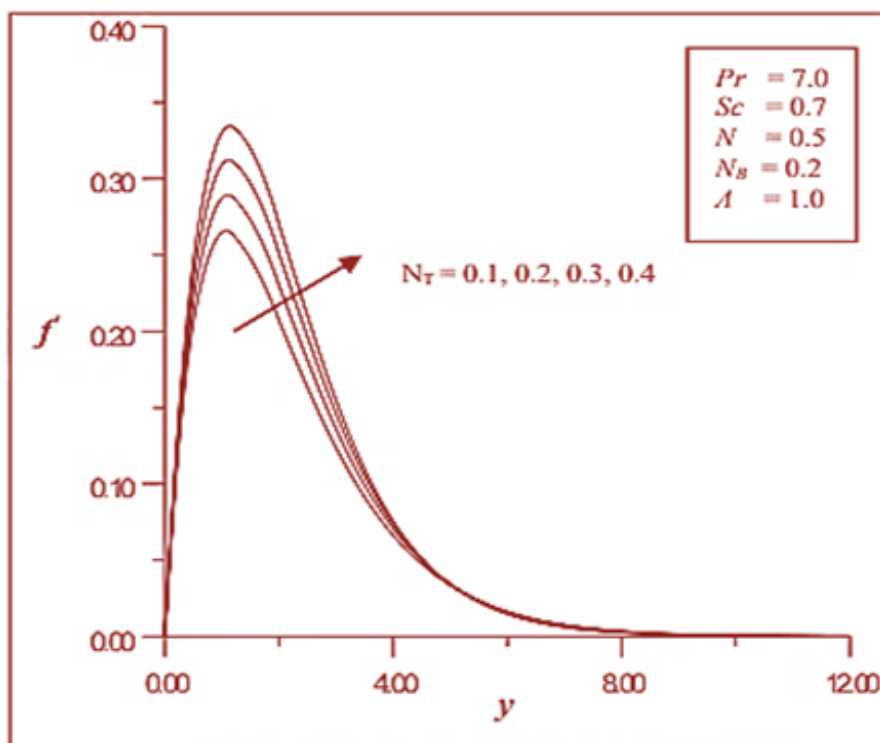


Fig. 4. Influence of thermophoresis number on velocity profile at upper stagnation point

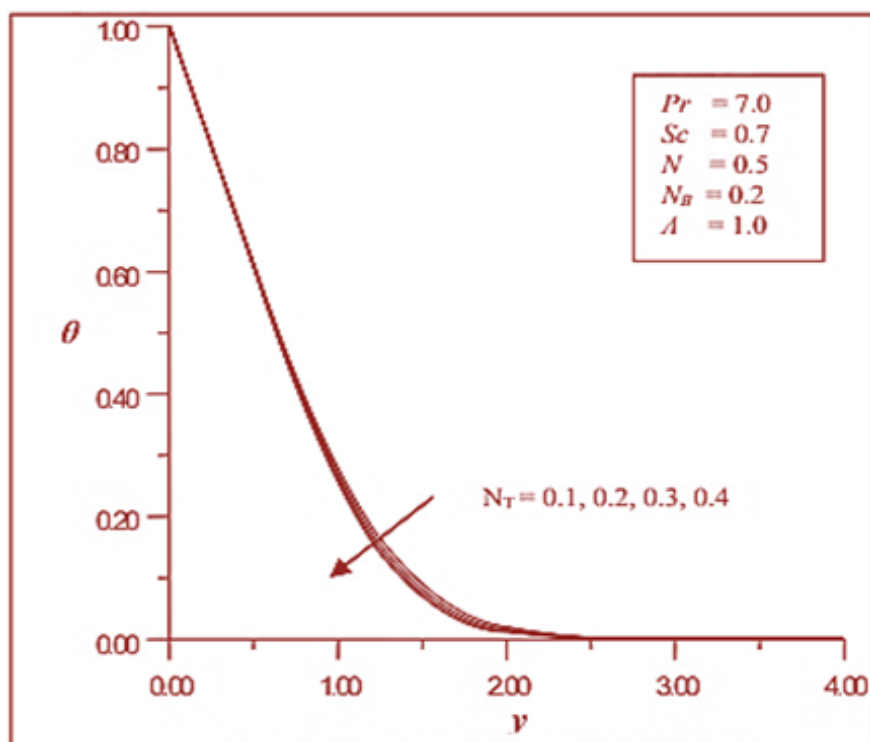


Fig. 5. Influence of thermophoresis number on temperature profile at upper stagnation point

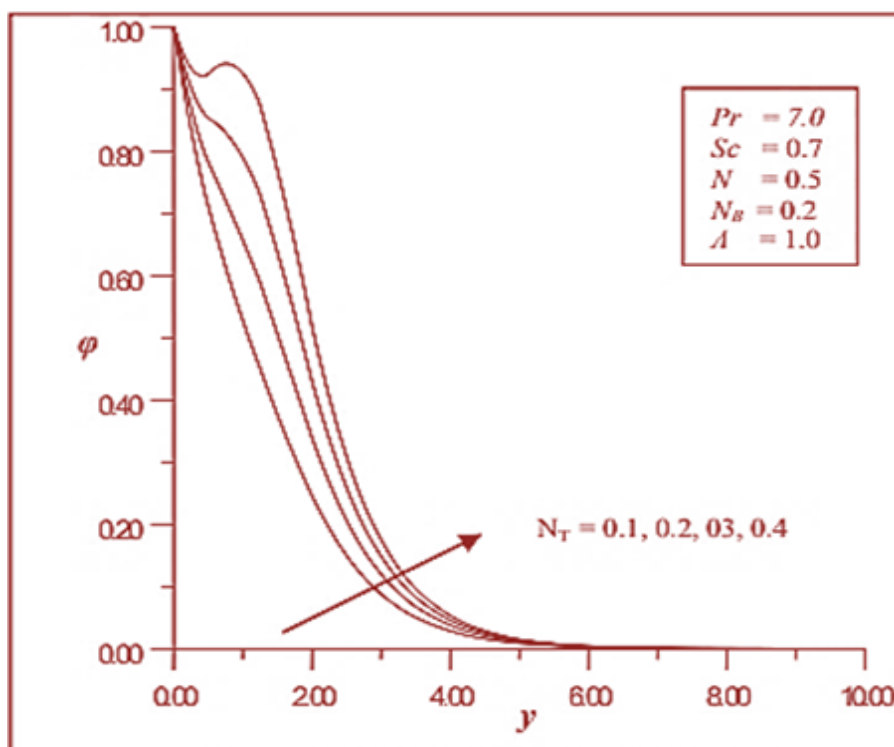


Fig. 6. Influence of thermophoresis number on concentration profile at upper stagnation point

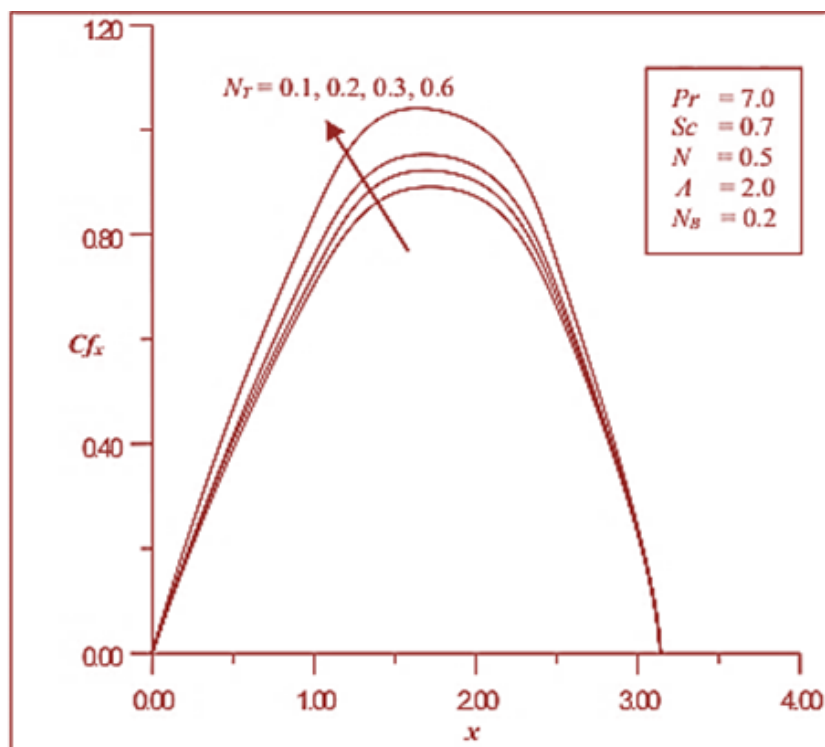


Fig. 7. Local skin friction coefficient versus x for various values of thermophoresis number

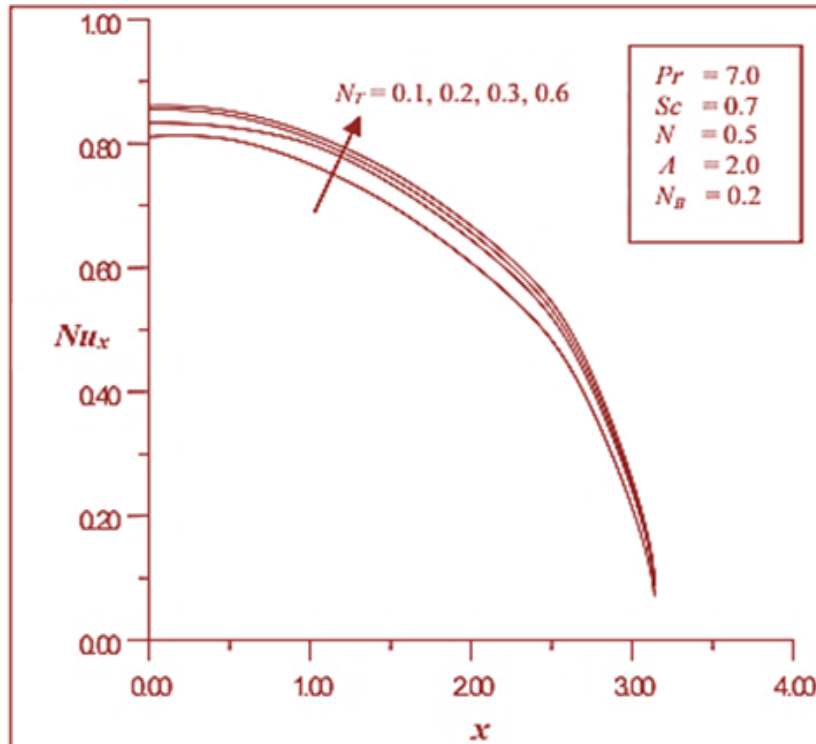


Fig. 8. Local Nusselt number versus x for various values of thermophoresis parameter

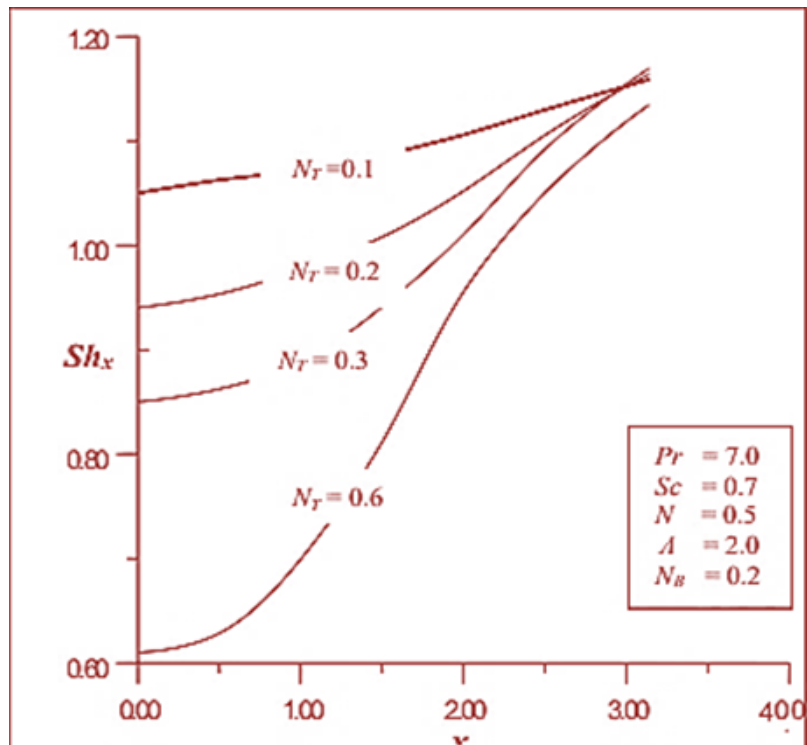


Fig. 9. Local Sherwood number versus x for various values of thermophoresis parameter

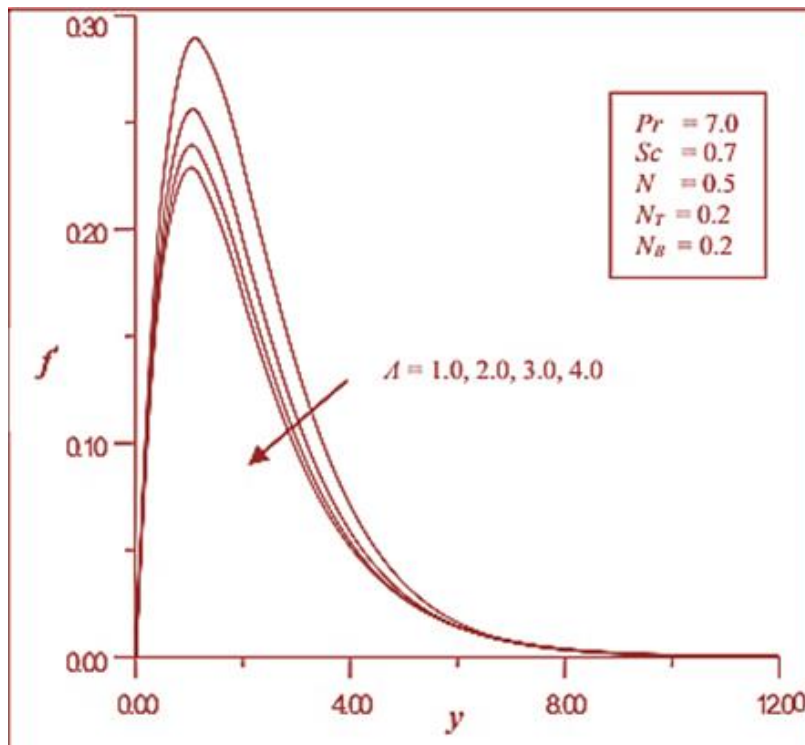


Fig. 10. Influence of chemical reaction parameter on velocity profile at upper stagnation point

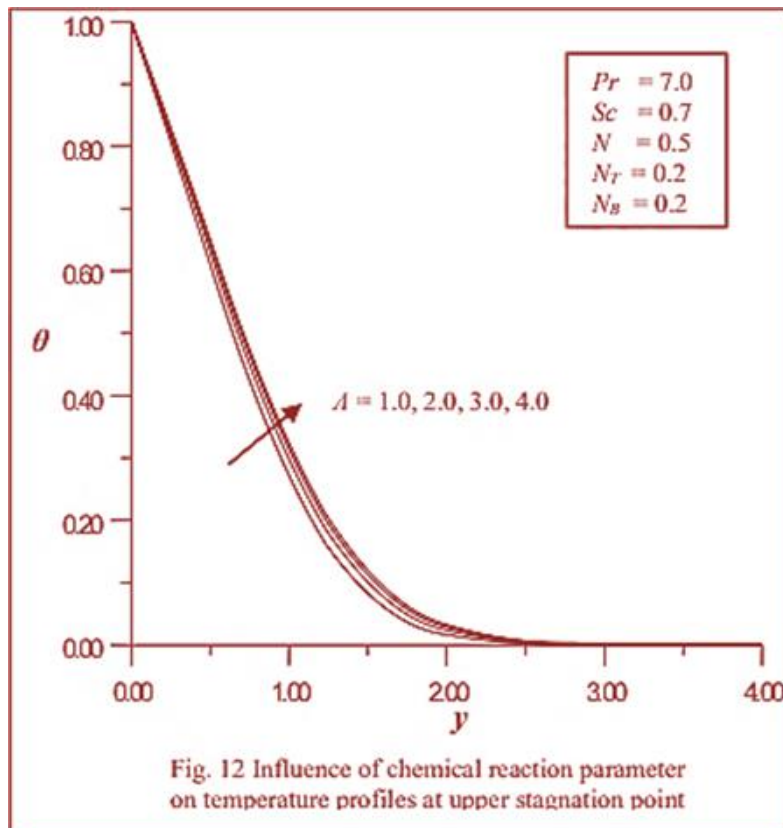


Fig. 11. Influence of chemical reaction parameter on temperature profile at upper stagnation point

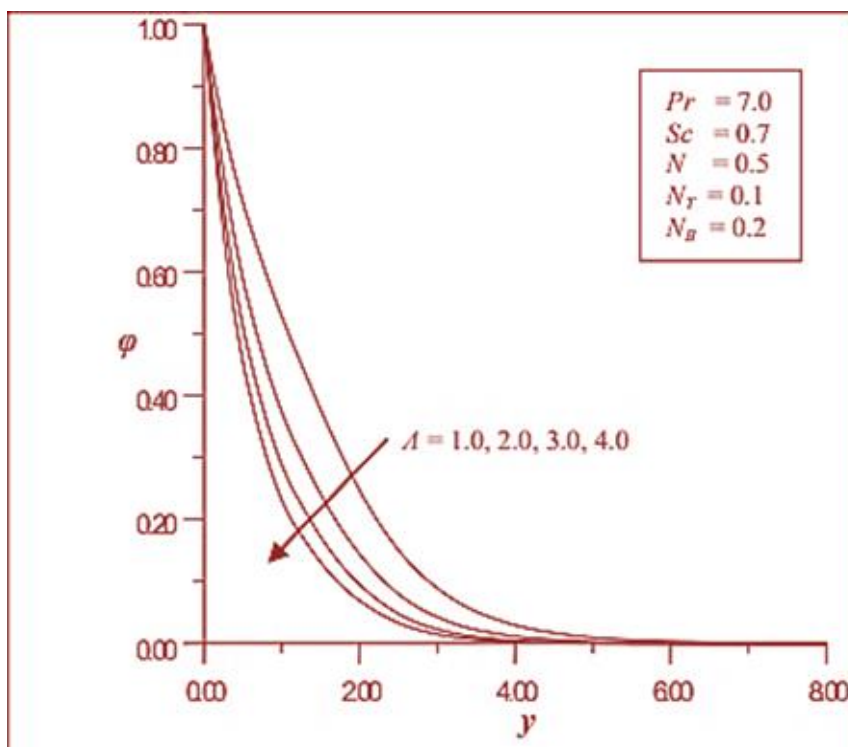


Fig. 12. Influence of chemical reaction parameter on concentration profile at upper stagnation point

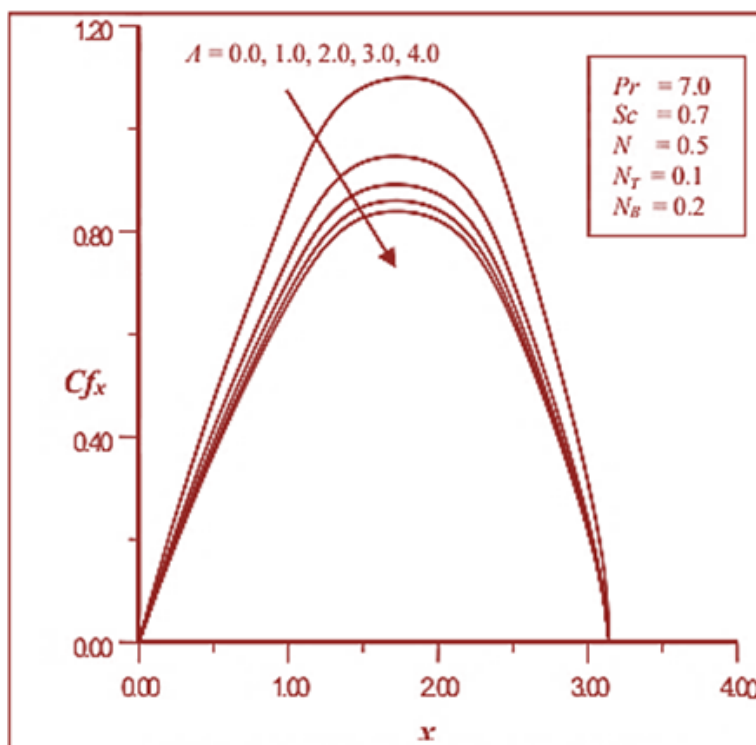


Fig. 13. Local skin friction coefficient versus x for various values of chemical reaction parameter

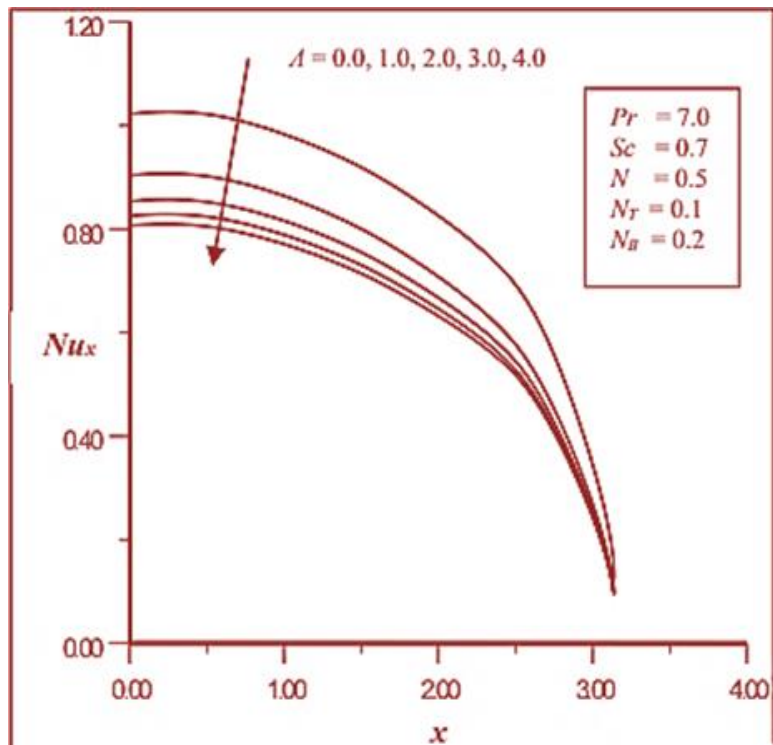


Fig. 14. Local Nusselt number versus x for various values of chemical reaction parameter

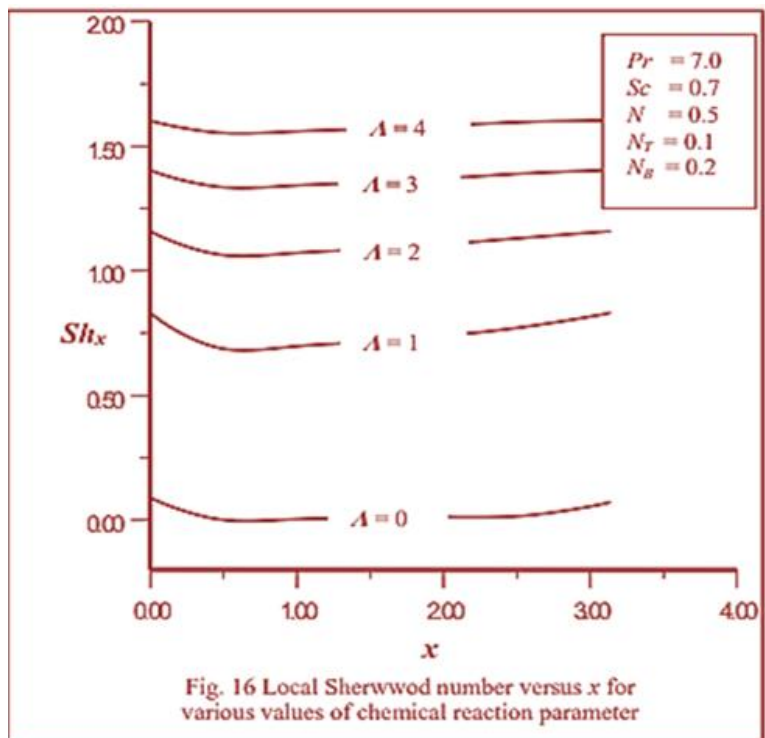


Fig. 15. Local Sherwood number versus x for various values of chemical reaction parameter

5. Validation

To square the accuracy of the current computations, the results have been verified and related with the outcomes reported by Merkin and Pop [37] for the non-reactive, purely fluid, i.e., with $A = N_B = N_T = 0$. It is observed that there is a strong connection between both outcomes up to the acceptable level of accuracy, and that validates the convergence criteria of the present methodology adopted. The outcomes obtained by Merkin and Pop [37] and also the present solutions are all shown in Table 2 below.

Table 2

Calculations come to the local skin-friction coefficient Cf_x for $Pr = 0.72$, $A = N_B = N_T = 0$ for selected values of x

x	Merkin and Pop[37]	Present work
0	0.000	0.000
0.4	0.609	0.603
0.8	1.602	1.158
1.2	1.602	1.600
1.6	1.855	1.867
2	1.971	1.968
2.4	1.824	1.821
2.8	1.403	1.401
π	0.738	0.722

6. Conclusion

In this article, the buoyancy-driven boundary layer problem is formulated mathematically using the continuity, momentum, and heat & mass transfer principles. The addressed geometry considers the fluid flow of a nanofluid past an isothermal horizontal cylinder in the existence of a chemical reaction effect. The produced partial differential equations (PDE's) are nonlinear, and the numerical methods are used to find an approximate solution. The influence of the various parameters that appear in the problem are discussed and analyzed. The results are generated and represented by graphs. Following the results and discussion section, we draw the following conclusions:

- I. Due to the Brownian motion number, thermophoresis number, and chemical reaction parameter, the temperature profiles accelerate, whereas the concentration decelerates in the boundary layer.
- II. A rise in the (N_B) Brownian motion parameter decreases the velocity, and the heat transfer rate is decelerated.
- III. An increase in thermophoresis number (N_T) will accelerate the flow and concentration and also tends to decelerate the mass transfer rate.
- IV. Fluid motions as well as concentration of the nanofluid are retarded due to existence of chemical reaction. Increasing the (Λ) chemical reaction parameter increases the dimensionless rates of mass transfer and decrease the heat transfer rate due to the decrease in flow resistance.

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