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Heat Transfer Flow of Ternary Hybrid Nanofluid (Al_2o_3-Zro_2-Mgo) Over A Sinusoidal Wavy Surface

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ABSTRACT

This work investigates how a ternary hybrid nanofluid can improve heat transmission over a vertically wavy surface. Darcy's law is used in the study to characterise the permeability and resistance properties of the porous material as it pertains to fluid flow. The vertical wall's non-uniformity is represented by a sine wave, which reflects the surface's wavelike quality. To commence, the system's governing equations are derived, with particular attention paid to the energy conservation equation and the conservation of mass and momentum in both the horizontal (x) and vertical (y) directions. These equations combine to create a set of partial differential equations (PDEs) that characterise the system's fluid flow and heat transfer dynamics. These governing equations are non-dimensionalized by applying the proper transformations, simplifying the analysis. Non-dimensionalization is a critical step in lowering the number of parameters and simplifying the equations. A similarity transformation transforms the PDEs into ordinary differential equations (ODEs). This transformation further reduces the mathematical complexity by taking advantage of the problem's symmetry. The resulting ordinary differential equations (ODEs) are then solved using the shooting method, a reliable numerical technique that is well-known for its great precision in solving differential equations. By using this technique, the boundary value problem is changed into an initial value problem, which is frequently simpler to handle. The study presents solutions to these ODEs, showing how various thermo physical factors of the nanoparticles affect the heat transfer performance. These factors, essential in assessing the hybrid nanofluid's effectiveness in improving heat transmission; include viscosity, thermal conductivity, and specific heat capacity. The findings offer significant perspectives on the ideal layout and implementation of hybrid nanofluids in systems featuring porous media and wavy surfaces.

1. Introduction

In recent years, Nanofluids have been significantly heard in research due to their thermo physical and heat transfer features. Nanofluid is a fluid in which the nanoparticles whose size varies between 1 to 100 nanometers in diameter are dispersed. There are different types of nanoparticles based on various physical and chemical properties. They are Carbon-based nanoparticles (Fullerenes, Carbon

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nanotubes), Metal nanoparticles (Copper, Silver, Gold), Ceramic nanoparticles (Calcium oxide, Titanium oxide, Silicon oxide), Semiconductor nanoparticles (GaN, GaP, InP), Polymeric nanoparticles (Chitosan, alginate, cellulose) and Lipid-based (Cisplatin, Irinotecan, paclitaxel) nanoparticles. When two or more nanoparticles are dispersed in the base fluid, it is termed a hybrid nanofluid. A unique fluid in which three different kinds of nanoparticles are evenly distributed throughout a base fluid is called a ternary hybrid nanofluid. Because of their special qualities and improved performance, these fluids—known as THFs—are widely used in a range of technical, scientific, and industrial domains. TNFs are widely used in a variety of contexts. They are utilised in refrigeration systems, for example, to increase cooling efficiency. They are also used in solar energy research to improve heat absorption and transfer. Due to their excellent thermal conductivity, they are also essential components of heat exchangers, heat pipes, and air conditioning systems, where they improve heat management. TNFs are essential for cooling components in the electronics sector, reducing overheating and guaranteeing peak performance. They support safe operation and effective heat transmission in nuclear reactors. TNFs provide cooling benefits to broadcast equipment and are used by spacecraft to regulate temperature in a hostile space environment. TNFs are also used in various chemical reactions to increase reaction efficiency and in air purifiers, which aid in heat management. Because of its potential for targeted drug delivery and hyperthermia treatment, TNFs are being investigated in the medical field for cancer therapy. TNFs' improved heat transfer characteristics are advantageous to engineering operations. They are also used in the manufacture of electrical insulators. TNFs are commonly included in dentistry and hair care products, which may enhance formulas' efficacy. The automotive sector uses TNFs in vehicle parts and green tyre technologies to increase performance and energy efficiency. TNFs' special qualities can improve sensitivity, efficiency, and overall performance in fuel cells, optical chemical sensors, solar cells, and biosensors. One of TNFs' primary thermal features is their capacity to maintain stability and effectiveness at extremely high temperatures, which qualifies them for applications requiring dependable performance in harsh environments. This wide range of uses demonstrates the adaptability and importance of ternary hybrid nanofluids in contemporary industry and technology.

Mohamed *et al.,* [1] 1investigated the thermohydraulic performance and energy output in microchannel1 heat sinks treated with ternary/binary 1hybrid nanofluids. Rectangular heat exchanger sinks fitted with the hybrid nanofluids CuO/MgO/TiO2/water ternary, and MgO/TiO2/water binary was selected to cool devices. The performance of the ternary nanoparticles in the base fluid through a convergent-divergent nozzle was examined by Zahan *et al.,* [2]. This study uses silver (Ag), cobalt (Co), and zinc (Zn) nanoparticles together with the attributes of standard fluid mixes of distilled water (DW) and ethylene glycol (EG) in the ratios of (100:0), (60:40), (50:50), and (0:100). Muhammad *et al.,* [3] showed how to improve heat transit by inserting ternary hybrid-nanofluids into the Prandtl fluid model using the "Galerkin finite element" model. They found that tri-hybrid nanoparticles, as opposed to pure fluid, nanofluid, and hybrid nanomaterials, show more significant fluid motion. They also discovered that attaining ultimate thermal performance is possible with tri-hybrid nanomaterials. Al_2O_3, TiO_2, and SiO_2 nanoparticles were employed in ethylene glycol base fluid. Ebrahem *et al.,* [4] examined the transfer of mass and energy via ternary hybrid nanofluid across a stretched permeable surface. They study the trihybrid flow of nanofluid (TiO_2 + MgO + $CoFe_2O_4$ / EG) along a horizontal stretching sheet that is stable and incompressible. The heat and mass transmission properties of an incompressible electroconductive ternary nanofluid over a stretching cylinder were investigated by Khalid *et al.,* [5]. A computer model has been developed to enhance the mass and energy conveyance rate. The performance of a ternary hybrid nanomaterial's thermal conductivity in relation to entropy generation was examined by Sohail *et al.,* [6]. For the numerical answer, they employed Newton's gunshot approach. In this case, the base fluid—a water and ethylene glycol solution—is used to dissolve ferric oxide, titanium dioxide, and silicon dioxide nanoparticles. Muhammad *et al.,*'s study [7] examined the mass and heat transfer properties of a trihybrid nanofluid flowing in a 3D MHD flow through a porous medium and towards a stretched surface. The thermal transmission and flow characteristics of a ternary nanofluid past a stretched sheet were examined by Manjunatha *et al.,* [8]. The Runge-Kutta-Fehlberg (RKF45) approach was employed to derive the numerical outcomes. The titanium dioxide, aluminum dioxide, and silicon dioxide nanoparticles were dissolved in the water-based base fluid for this study. Haider *et al.,* [9] used the non-similarity method to compute the solutions of the governing equations of heat transfer due to the effect of ternary hybrid nanofluids. Nihal *et al.,* [10] studied the 3D convection of binary, ternary hybrid nanofluid flow through a porous media-filled horizontal annular duct with the effect of the magnetic field. Riaz *et al.,* [11] investigated heat transfer flow due to the ternary hybrid nanofluid past a stretching surface. Kabeir *et al.,* [12] studied magnetoternary hybrid nanofluid flow over a stretching cylinder with Gyrotactic microorganisms. Rashmi *et al.,* [13,14] investigated the development of a new correlation between nanoparticle properties of the ternary hybrid nanofluid flows. Animasaun *et al.,* [15-17] studied on boundary layer flow of ternary hybrid nanofluids over a stretching surface with multiple forces and used MATLAB bvp4c to compute the solutions of the governing equations.

A substance having pores is called a porous medium (voids). The term "matrix" or "frame" is frequently used to refer to the material's skeleton. Darcy's law is commonly used to characterize fluid flow through porous material. Liquids or gases can be introduced into the pores to facilitate their passage. Porous media are essential to many technical applications in the modern world. Porous media, for example, increase heat transmission efficiency in packed bed heat exchangers by offering a high surface area for fluid interaction. They aid in heat dissipation from electronic components, preventing overheating and guaranteeing dependable functioning. Porous structures are crucial in efficiently transporting heat through capillary action in heat pipes. Porous media are essential for tissue engineering in biological applications because they facilitate nutrient exchange and cell proliferation. They help remove moisture from materials in drying technologies, increasing drying speeds and effectiveness. The high surface area of porous media, which increases chemical reactions by offering more active sites, is advantageous for catalytic reactors. By utilizing porous materials to decrease heat transmission, thermal insulation engineering increases the energy efficiency of industrial operations as well as buildings. By utilising a porous medium, enhanced oil recovery processes maximize resource usage by extracting more petroleum from reservoirs. Porous barriers are used to control the spread and containment of chemical waste. Grains like sorghum are affected by agriculture's irrigation and soil management practices facilitated by porous media. These many uses demonstrate the importance of porous media in developing contemporary technology and show why they are the focus of much research. Darcy porous media is defined as a medium where fluid flow complies with Darcy's law. Significant progress has been made in understanding Darcy's law in fluid flow and heat transfer through porous media. Numerous uses for such porous media exist in petroleum engineering, hydrogeology, coffee brewing, etc. In the context of a Darcy porous media with a vertically wavy surface, Srinivasacharya *et al.,* [9] examined the impact of "Soret-Dufour parameters" on mixed convection. They obtained the numerical answer by using the shooting approach in conjunction with the Runge Kutta procedure. The literature corroborates the findings. Mallikarjuna *et al.,* [18] have investigated the effects of1the thermophoresis phenomenon on1the mixed convection flow in conjunction with the vertical1wavy surface in Darcy porous media with varying thermo physical parameters. They computed the results using a numerical method, and the literature verified the results. The numerical results of convective heat transfer flow in the presence of a sinusoidal wavy surface in a fluid-soaked, sparsely packed Darcy porous media with variable porosity, permeability, and thermal conductivity were presented by Srinivasacharya *et al.,* [19]. The "local non-similarity method" was employed to ascertain the solution. The spontaneous convection caused by a vertical sinusoidal surface in a Darcy porous medium with homogeneous heat flux was studied by Rees and Pop [20]. To get the numerical results, they applied the "Keller-box-method." in a Darcy porous medium saturated with a Newtonian fluid. Narayana and Sibanda [21] studied the vertical wavy surface and the natural convection process. The mass and heat1transfer characteristics through a wavy surface put in a porous medium under variable viscosity and varying thermal conductivity in the presence of 1cross diffusion were presented by Srinivasacharya *et al.,* [22] using natural convection. The heat transfer properties in Darcy free convective, forced convective and mixed convective processes caused by a vertical plate1maintained at isothermal conditions in a porous medium soaked in an elastic fluid with a constant viscosity were examined by Shenoy [23]. He got a similar solution by applying the approximate integral method to solve the flow's governing equations.

In essence, vertically wavy surfaces improve the convective process's impact on heat transfer. Grain storage bins, heat transfer devices, refrigerator condensers, solar collectors, electrical and nuclear cooling components, building energy system design, chemical catalytic reactors, compact heat exchangers, and many other engineering and industrial applications depend on a similar process. For the first time, the free convection in a porous medium—which consisted of a wavy surface maintained under isothermal heating—was examined by Rees and Pop [24]. They solved the boundary-layer equations using generalized similarity transformations. Kumari and Pop [25] examined the streamlined natural convection flow of when the fluid atop a vertically wavy surface is isothermal. This non-Newtonian fluid obeys the power law. In a rotating system, Bhuvanavijaya and Mallikarjuna [26] investigated the effects of altering thermal conductivity on flow with convective mass and heat transfer in conjunction with a vertical plate. Using FEM, Rathish [19] conducted a numerical analysis of the free convection in a porous enclosure caused by a wavy surface with uniform heat flux. The magnetohydrodynamic natural convection flow caused by a wavy surface under the viscous dissipation effect with heat generation was investigated by Kabir et al., [27]. To get the numerical solution, they employed the "Keller-box scheme". Siddiqa et al., [28] investigated the bio-convective flow of a nanofluid via a wavy surface containing gyrotactic microorganisms, using water as the base fluid. With the help of the implicit FDM system, they obtained the numerical results. Mahdy and Ahmed [29] quantitatively studied the streamline natural convection in a saturated porous media. The study of the transient heat transfer in free convection of a nanofluid via a wavy surface was conducted by Ahmed and Aziz [30]. The thermal non-equilibrium method is applied to investigate the impact of heat movement. By considering the possibility of temperature differences between the fluid and solid matrix inside a porous media, this method makes it possible to depict heat transfer processes in these kinds of settings accurately. To accurately depict heat transfer processes in these kinds of settings, a wavy surface makes fluid flow more complex. It improves the fluid-surface interaction by generating changes in the flow patterns. This enhanced contact may result in higher heat transfer rates. The writers' objectives in a base fluid make up a ternary hybrid nanofluid. The particular nanoparticles utilized in this investigation are magnesium oxide (MgO), zirconium dioxide (ZrO2), and aluminium oxide (Al2O3). These particular nanoparticles were chosen because of their distinct thermal characteristics, which should work in concert to enhance the nanofluid's total heat transfer efficiency. The wavy surface makes the description more complex, improving the fluid-surface interaction by generating changes in the flow patterns. This enhanced contact may result in higher heat transfer rates. The writers' objective study uses the thermal non-equilibrium technique to account not only to close knowledge gaps regarding the behaviour of ternary hybrid nanofluids in intricate geometries and at different temperatures but also to offer significant perspectives for enhancing heat transfer procedures in numerous practical applications. These include heat exchangers, cooling systems, and other thermal control technologies. The anticipated results of this research hold the potential to significantly contribute to the development of more effective thermal management solutions. The study is to close knowledge gaps regarding the behaviour of ternary hybrid nanofluids in intricate geometries and at different temperatures. The results are anticipated to offer significant perspectives for enhancing heat transfer procedures in numerous uses, including heat exchangers, cooling systems, and other thermal control technologies.

In conclusion, the study explores the potential for improving heat transfer through the use of a wavy surface, which makes fluid flow more complex and improves the fluid-surface interaction by generating changes in the flow patterns. This, in turn, may result in higher heat transfer rates. The study also investigates the use of a ternary hybrid nanofluid composed of Al2O3, ZrO2, and MgO nanoparticles suspended in a base fluid. These nanoparticles were chosen for their distinct thermal characteristics, which are expected to enhance the nanofluid's total heat transfer efficiency. The research contributes to developing more effective thermal management solutions by offering a thorough and accurate understanding of the heat transfer characteristics through a thermal non-equilibrium method.

2. Formulation of the Problem

Examine the laminar, two-dimensional, continuous, incompressible flow of a nanofluid across the vertically wavy surface. The wavy surface's wavyness is denoted as $\bar{y} = \bar{a}\sin\left(\frac{\pi \bar{x}}{l}\right)$, and it is regarded along the x-axis as shown in Figure 1. In a porous material, the geometry is imbedded. The constant ambient temperature T_∞ is supposed to be far from the surface, while the surface is maintained at temperature T_w. The porous medium is described by Darcy's law. The governing equations are (see [31,32]):

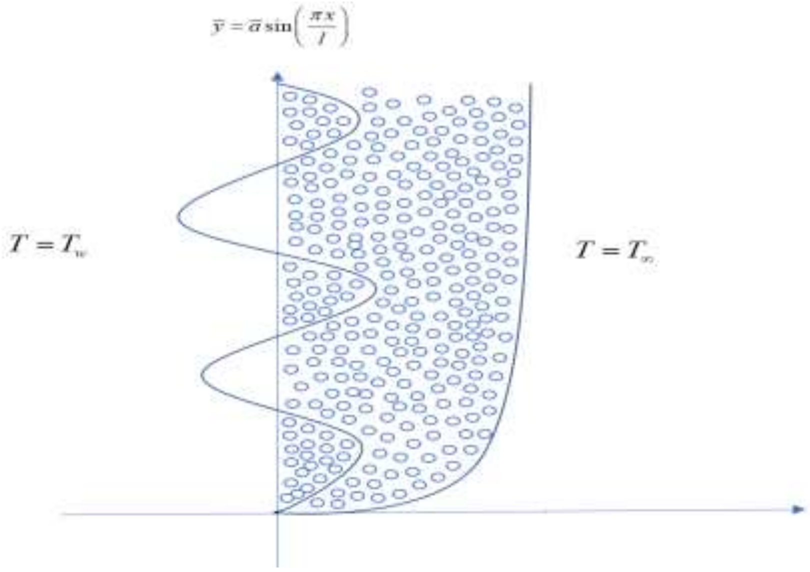


Fig. 1. Geometry of the problem

$$\frac{\mu_{Tnf}}{\rho_{Tnf} K}\bar{u} = -\frac{1}{\rho_{Tnf}}\frac{\partial \bar{p}}{\partial \bar{x}} + g\beta_f (T - T_\infty)$$
$$\frac{\mu_{Tnf}}{K}\bar{v} = -\frac{1}{\rho_{Tnf}}\frac{\partial \bar{p}}{\partial \bar{y}} \tag{1}$$

Energy Equation:

$$\bar{u}\frac{\partial T}{\partial \bar{x}} + \bar{v}\frac{\partial T}{\partial \bar{y}} = \alpha_{Tnf}\left(\frac{\partial^2 T}{\partial \bar{x}^2} + \frac{\partial^2 T}{\partial \bar{y}^2}\right) \tag{2}$$

with the boundary conditions

$$\bar{u} = 0; \bar{v} = 0; T = T_w \quad at \quad \bar{y} = \bar{\sigma}(\bar{x}) = \bar{a}\sin\left(\frac{\pi x}{l}\right)$$
$$\bar{u} \to 0; T \to T_\infty \quad as \quad \bar{y} \to \infty \tag{3}$$

Thermo-physical properties:

$$\alpha_{Tnf} = \frac{K_{Tnf}}{\left(\rho C_p\right)_{Tnf}},$$
$$\rho_{Tnf} = (1-\phi_1)\left\{(1-\phi_2)\left[(1-\phi_3)\rho_f + \phi_3\rho_{np3}\right] + \phi_2\rho_{np2}\right\} + \phi_1\rho_{np1},$$
$$(\rho\beta)_{Tnf} = (1-\phi_1)\left\{(1-\phi_2)\left[(1-\phi_3)(\rho\beta)_f + \phi_3(\rho\beta)_{np3}\right] + \phi_2(\rho\beta)_{np2}\right\} + \phi_1(\rho\beta)_{np1},$$
$$(\rho C_p)_{Tnf} = (1-\phi_1)\left\{(1-\phi_2)\left[(1-\phi_3)(\rho C_p)_f + \phi_3(\rho C_p)_{np3}\right] + \phi_2(\rho C_p)_{np2}\right\} + \phi_1(\rho C_p)_{np1}$$
$$\mu_{Tnf} = \frac{\mu_f}{(1-\phi_1)^{2.5}(1-\phi_2)^{2.5}(1-\phi_3)^{2.5}}$$
$$K_{Tnf} = \left[\frac{K_{np3} + 2K_{hnf} - 2\phi_3(K_{hnf} - K_{np3})}{K_{np3} + 2K_{hnf} + \phi_3(K_{hnf} - K_{np3})}\right]\left[\frac{K_{np2} + 2K_{nf} - 2\phi_2(K_{nf} - K_{np2})}{K_{np2} + 2K_{nf} + \phi_2(K_{nf} - K_{np2})}\right]\times$$
$$\left[\frac{K_{np1} + 2K_f - 2\phi_1(K_f - K_{np1})}{K_{np1} + 2K_f + \phi_1(K_f - K_{np1})}\right]K_f$$

From the following non-dimensional variables,

$$x = \frac{\bar{x}}{l}; y = \frac{\bar{y}}{l}; \psi = \frac{\bar{\psi}}{\alpha_f}; a = \frac{\bar{a}}{l}; \theta = \frac{T - T_\infty}{T_w - T_\infty}; \frac{\partial \bar{\psi}}{\partial \bar{y}} = \bar{u}; -\frac{\partial \bar{\psi}}{\partial \bar{x}} = \bar{v} \tag{4}$$

We obtain the dimensionless equations,

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} = Ra\frac{B}{A}\frac{\partial \theta}{\partial y} \tag{5}$$

$$\frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2} = \frac{D}{C}\left(\frac{\partial \psi}{\partial y}\frac{\partial \theta}{\partial x} - \frac{\partial \psi}{\partial x}\frac{\partial \theta}{\partial y}\right) \tag{6}$$

where $A=\frac{\mu_{Tnf}}{\mu_f}; B=\frac{(\rho\beta)_{Tnf}}{(\rho\beta)_f}; C=\frac{K_{Tnf}}{K_f}; D=\frac{(\rho C_p)_{Tnf}}{(\rho C_p)_f}$.

With the boundary conditions:

$$\psi=0,\ \theta=1 \text{ on } y=a\sin x \qquad (7)$$
$$\psi_y\to 0,\ \theta\to 0 \text{ as } y\to\infty$$

Applying the following similarity transformation,

$$x=\xi;\quad y=\xi^{1/2}Ra^{-1/2}\eta+a\sin x;\quad \psi=Ra^{1/2}\hat{\psi} \qquad (8)$$

Equations (5) and (6) will be transformed to

$$(1+a^2\cos^2\xi)\frac{\partial^2\hat{\psi}}{\partial\eta^2}=\xi^{1/2}\frac{B}{A}\frac{\partial\theta}{\partial\eta} \qquad (9)$$

$$(1+a^2\cos^2\xi)\frac{\partial^2\theta}{\partial\eta^2}=\xi^{1/2}\frac{D}{C}\left(\frac{\partial\hat{\psi}}{\partial\eta}\frac{\partial\theta}{\partial\xi}-\frac{\partial\theta}{\partial\eta}\frac{\partial\hat{\psi}}{\partial\xi}\right) \qquad (10)$$

and the boundary conditions are

$$\hat{\psi}=0,\theta=1 \ on\ \eta=0 \qquad (11)$$
$$\hat{\psi}_y\to 0,\theta\to 0\ as\ \eta\to\infty$$

Performing similarity transformations using the stream function and temperature $\hat{\psi}=\xi^{1/2}f(\hat{\eta});\ \theta=\theta(\hat{\eta})$, where the similarity variable $\hat{\eta}=\frac{\eta}{1+a^2\cos^2\xi}$.

Equations (9) and (10) will get the form

$$\frac{\partial^2 f}{\partial\hat{\eta}^2}=\frac{B}{A}\frac{\partial\theta}{\partial\hat{\eta}} \qquad (12)$$

$$\frac{\partial^2\theta}{\partial\hat{\eta}^2}=-\frac{D}{2C}f(\hat{\eta})\frac{\partial\theta}{\partial\hat{\eta}} \qquad (13)$$

with the boundary conditions

$$f(0)=0;\ \theta(0)=1;\ f'(\infty)=0,\ \ \theta(\infty)=0 \qquad (14)$$

The physical quantity of interest in this problem is to investigate on local Nusselt number. Local Nusselt number is given by

$Nu_x=\frac{qx}{K_f(T_w-T_\infty)}$, where $q=-K_{Tnf}\left[\partial T/\partial y\right]_{y=0}$

3. Numerical procedure

One iterative method for solving ordinary differential equations (ODEs) is the RK4 method. The integration process produces a set of results by selecting presumptive beginning circumstances. These outcomes are contrasted with the beginning conditions to confirm that the assumed values are accurate. The accepted values are regarded as precise if no difference exists between the computed outcomes and the beginning conditions. If disparities are discovered, the procedure entails redoing the RK4 integration and giving new values to the variables not initially supplied. This iterative procedure can be computationally and time-intensive, mainly if it is difficult to determine the proper initial conditions. The Newton-Raphson technique comes to the rescue in this situation. It is a powerful method that iteratively improves estimates until they converge to the correct values, thereby enabling the determination of precise initial conditions. This approach significantly reduces the time and computational effort required compared to the manual trial-and-error method of adjusting initial conditions. Following the Newton-Raphson method's accurate initial condition determination, the RK4 integration is performed with a step size 0.001. This short step size ensures high accuracy in the numerical solutions assumed starting conditions and fine-tuning these circumstances using the Newton-Raphson method. The final integration with a narrow step size ensures that the high-precision solutions procedure entails turning the boundary conditions and governing equations into an initial value problem, integrating the result using the RK4 approach, confirming the starting conditions that are assumed, and fine-tuning these circumstances using the Newton-Raphson method. The final integration with a narrow step size ensures high-precision solutions.

4. Results and Discussions

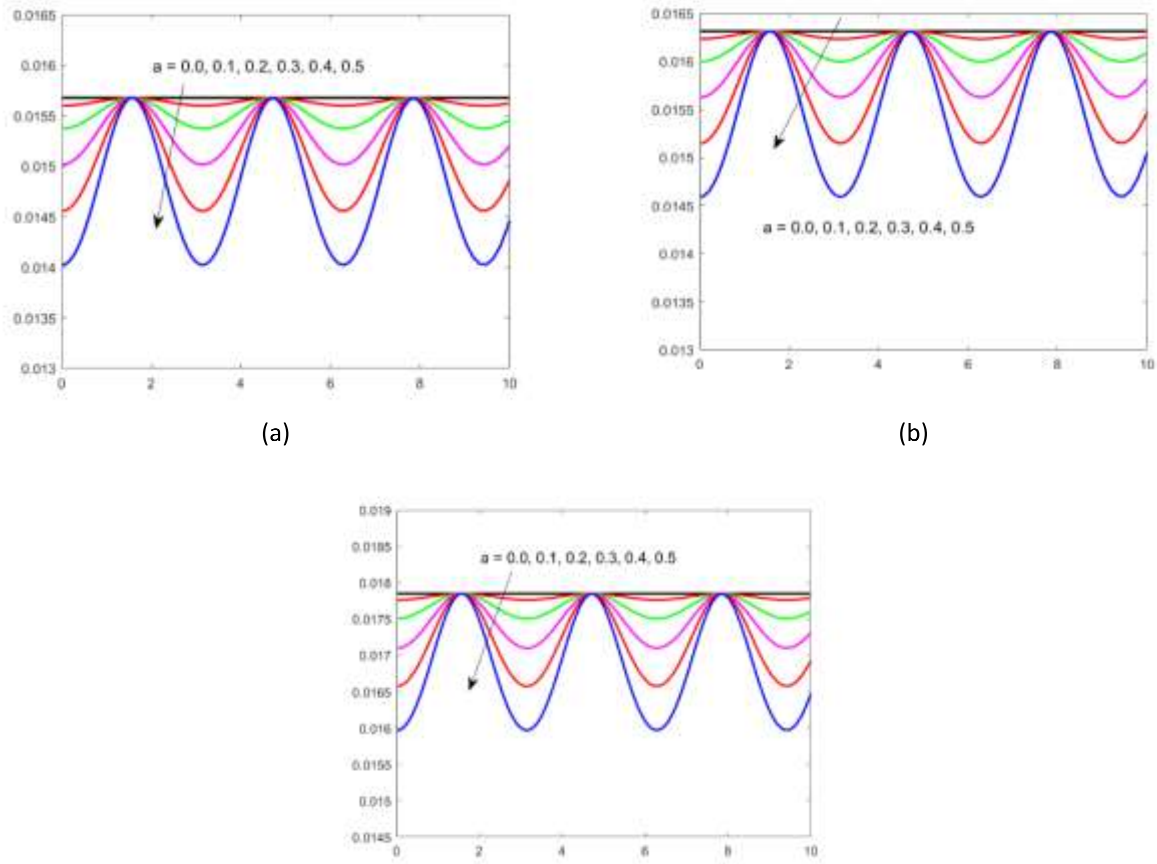
The dimensionless ordinary differential equations were solved numerically using Runge-Kutta fourth order method with the help of shooting technique taking the account of the values of nanoparticles as mentioned in table-1, along the boundary conditions given in (20). In the research work, we utilized three different types of nanoparticles Al_2O_3-ZrO_2-MgO in the base fluid H_2O.

Table 1
Physical quantities of the ternary hybrid nanofluid

Quantity	$\rho\left[kg\,m^{-3}\right]$	$Cp\left[J\left(kg\,K\right)^{-1}\right]$	$\kappa\left[W\left(mK\right)^{-1}\right]$	$\beta\left[K^{-1}\right]$
H_2O	997.1	4179	0.6071	21×10^5
Al_2O_3	3970	765	40	0.85×10^5
ZrO_2	5680	502	1.7	0.0005
MgO	3560	955	45	0.0004

The findings of heat transfer rate for different kinds of fluids are explored and provided in this section. For the varied amplitudes of the wavy surface, the Nusselt number behaviour is instantiated in Figure 2 for three distinct cases: "nanofluid (Al_2O_3hybrid-nanofluid (Al_2O_3-ZrO_2), and ternary hybrid-nanofluid (Al_2O_3-ZrO_2-MgO)". It is noted that when the amplitude grows, the Nusselt number drops in all the three circumstances. It is clear that ternary hybrid nanofluids, as opposed to hybrid and nanofluids, greatly improve heat transmission.

Figure 3, 4 and 5 illustrates the thermal transfer amplification for various values of “volume fraction” in different types of the fluids. It is noticed that the Nusselt number increases as the volume fraction of the fluid increases in all the three cases. It is also worth to mention that ternary hybrid nanofluids enhances the heat transfer effects notably than the other two cases.



(a) (b)

(c)

Fig. 2. Nusselt number for amplitude of the wavy surface for (a) Al_2O_3, (b) Al_2O_3-ZrO_2 and (c) Al_2O_3-ZrO_2-MgO

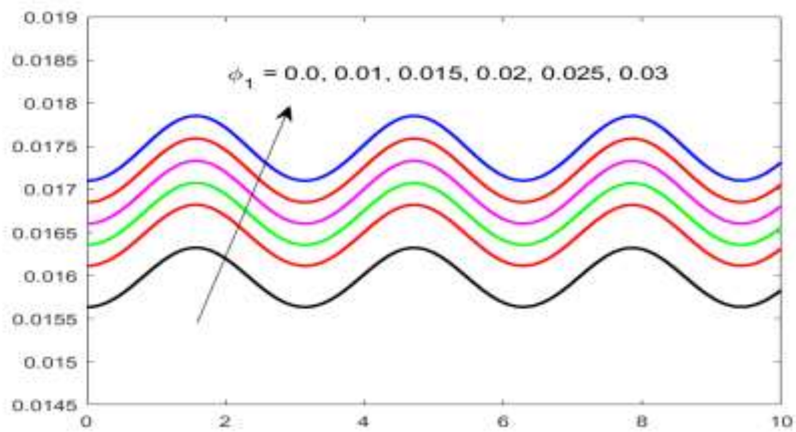


Fig. 3. Nusselt number for the volume fraction ϕ_1

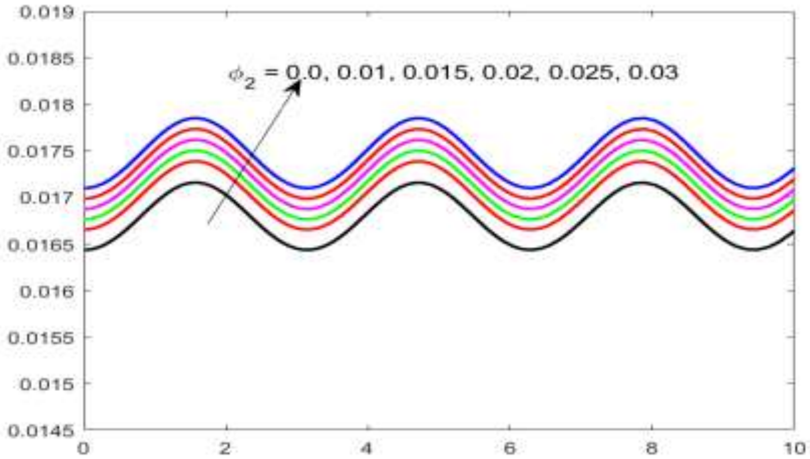


Fig. 4. Nusselt number for the volume fraction ϕ_2

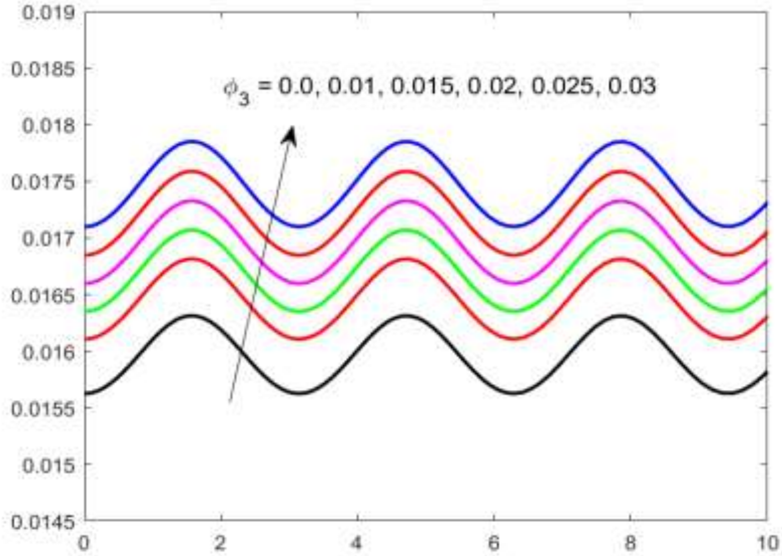


Fig. 5. Nusselt number for the volume fraction ϕ_3

5. Conclusion

This paper introduces a novel mathematical model that investigates heat transmission characteristics for a ternary hybrid nanofluid across a wavy surface. The model's uniqueness lies in its focus on the effects of adding three distinct nanoparticles to a base fluid: magnesium oxide (MgO), zirconium dioxide (ZrO_2) and aluminum oxide (Al_2O_3) Our investigation commences with applying Darcy's law, a fundamental principle governing fluid flow across porous materials. This law is instrumental in understanding the nanofluid's flow through a porous media, a crucial aspect in effectively describing the heat transfer process and its interaction with the porous structure. The governing equations for mass, momentum, and energy conservation are derived to facilitate the analysis. These equations describe the behaviour of the nanofluid under different circumstances. The equations are converted into a non-dimensional form utilizing pertinent mathematical procedures to make them easier to handle. The process of non-dimensionalization renders the equations simpler and more manageable numerically. The partial differential equations (PDEs) must then be transformed into ordinary differential equations (ODEs) using the Similarity Transformation. This modification reduces the mathematical model's complexity and improves its suitability for numerical analysis. The generated ODEs are then solved using the Shooting Technique. This numerical technique first solves boundary value problems by turning them into initial value problems. This method allows for precise computation of the results and guarantees that the solutions meet the initial boundary constraints. The computed results are graphically shown to show the ternary hybrid nanofluid's heat transfer performance clearly. Thermal conductivity, viscosity, and specific heat capacity are only a few of the nanoparticles' thermo physical properties included in these depictions. These characteristics are essential for determining how well the hybrid nanofluid will work overall to improve heat transmission. The results show that adding ZrO_2, MgO and Al_2O_3 nanoparticles greatly enhances the base fluid's heat transfer characteristics. When all three nanoparticles are utilized together, the ternary hybrid nanofluid performs better than fluids with only one or two nanoparticle additions. This suggests a synergistic impact. In conclusion, this study not only enhances our understanding of the characteristics of ternary hybrid nanofluids' heat transmission over wavy surfaces in porous media, but also opens up exciting possibilities for real-world applications. The research suggests that these hybrid nanofluids could be highly beneficial in scenarios that require effective thermal control, promising significant advancements in this technology field.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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