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Stagnation Point Flow and Heat transfer of Hybrid Nanofluid over a Stretching/Shrinking Cylinder with Suction/Injection Effects

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ABSTRACT

The focus of this research is to observe how suction/injection affects the stagnation point flow and heat transfer in a hybrid nanofluid over stretching/shrinking cylinder. Silver (Ag) and copper oxide (CuO) nanoparticles are dispersed in pure water to create a hybrid nanofluid. By using similarity transformations, the governing partial differential equations are turned into a ordinary differential equations which are then solved by implementing bvp4c function in MATLAB software. The influence of the nanoparticle volume fraction, magnetic parameter, curvature parameter and suction/injection parameter, on velocity and temperature profiles, local skin friction and local Nusselt number are discussed and presented in graphical forms. The results indicate that all of the problems have dual solutions for a given range of parameters. It is noticed that with the suction effect, energy losses is reduced, thus the heat transfer increases and decreases the boundary layer separation. Furthermore, the presence of curvature parameter and suction effect expand the range of dual solutions. In addition, the rate of heat transfer for hybrid nanofluid was higher than viscous fluid and nanofluid.

1. Introduction

The fluid mechanics community has recognised the importance of geometry in advancing the understanding of fluid flow. Different geometries have different mathematical expressions. The study of flow over a stretching sheet was pioneered by Crane [1], who solved analytically the steady two-dimensional flow past a linearly stretching plate. This problem was later extended to the three-dimensional case by Wang [2]. Since then, many researchers have studied various aspects of this flow. For example, they investigated the effects of fluid flow and heat transfer to stretching sheets considering different types of fluids in the presence of magnetic fields, slip effects, convective boundary conditions, suction/injection, viscous dissipation, radiation effects, and heat generation/absorption. In subsequent studies of flow on stretching sheets, fluid flow on shrinking sheets has become of great interest. Miklavˇciˇc and Wang [3] first studied the viscous flow on a

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shrinking sheet with a suction effect at the boundary. Following this pioneering work, many papers were published on the topic. For this type of problem, the direction of motion of the sheet is opposite to that of the stretching case, so the flow moves towards the fixed point.

In fluid mechanics problems, a stagnation point is the point in a flow field at which the local velocity of a fluid becomes zero. The point where the fluid is at rest due to the force exerted by the object occurs on the surface of the object. Ali *et al.,* [4] investigated steady magnetohydrodynamic (MHD) stagnation point flow problem for incompressible viscous fluids over stretched sheets. Furthermore, Aman *et al.,* [5] extended the work of Ali *et al.,* [4] to investigate magnetohydrodynamic (MHD) stagnation-point flow over a linearly stretching/shrinking sheet in a viscous and incompressible fluid in the presence of a magnetic field. Moreover, Ghadikolaei *et al.,* [6] conducted a study to investigate the induced magnetic field effect on stagnation flow of a titanium-copper/water hybrid nanofluid over a stretching sheet.

On the other hand, Javed and Mustafa [7] investigated the unsteady viscous incompressible stagnation point flow over a stretching/shrinking cylinder with suction and heat transfer. Sulochana and Sandeep [8] analysed the stagnation point flow and heat transfer behavior of copper-water nanofluids into horizontal and exponentially osmotic stretching/shrinking cylinders. Najib *et al.,* [9] implemented an investigation to examine the flow and heat transfer at stagnation points on a cylinder filled with nanofluids that shrinking and stretching exponentially in the presence of slip at the boundary. Apart from that, the effects of thermal radiation and viscous dissipation on the stagnation-point flow of copper-water nanofluids through convective stretching/shrinking cylinders was investigated by Alqahtani *et al.,* [10]. Reddy *et al.,* [12] analysed the effect of hydromagnetic heat transfer properties of incompressible viscous fluids along a stretching cylinder with a porous medium. The findings showed that the increase of the curvature parameter and the porosity factor will help to improve the temperature gradient in the boundary layer region around the cylinder. Makhdoum *et al.,* [13] inspected an unsteady stagnation point flow properties of TiO_2-$C_2H_2O_2$ nanofluids through a shrinking horizontal cylinder with mass suction, nanoparticles aggregation, viscous dissipation, and joule heating. In the calculation software MATHEMATICA, Runge Kutta-IV with shooting method was used to numerically solve the simplified mathematical model. Their efforts showed that the implementation of a heat transfer operation may be improved by increasing suction settings.

Heat transfer is the energy exchanged between materials due to temperature differences. Wang *et al.,* [11] introduced three modes of heat transfer, including conduction, convection, and radiation. Latif *et al.,* [14] explained the differences between conduction, convection, and radiation. Moreover, magnetohydrodynamics (MHD) is the study of the magnetism and behaviour of electrically conductive fluids. Sheikholeslami and Ganji [15] introduced the basic concept behind MHD is that a magnetic field can induce a current in a moving conductive fluid, which in turn creates a force on the fluid and changes the magnetic field itself.

A nanofluid is defined as a fluid that contains nanoparticles, that is, nanometer-sized particles (Ali, [16]). According to Loon and Sidik [17], nanofluids are a new class of heat transfer fluids that have been introduced to enhance the heat transfer performance of various heat exchange systems. Before the invention of nanofluids, water was the most used fluid to extract the heat generated by the system. The application of water has reached a thermal bottleneck as it can only improve heat transfer performance to a certain extent. Nanoparticle suspensions with enhanced thermal conductivity provide better heat transfer performance. Since then, nanofluids have aroused great interest among scientists and researchers due to their significantly improved thermal conductivity. Due to the enhanced thermophysical properties, nanofluids can be incorporated into high heat transfer devices, such as solar thermal conversion systems, heat exchangers, and electronic devices.

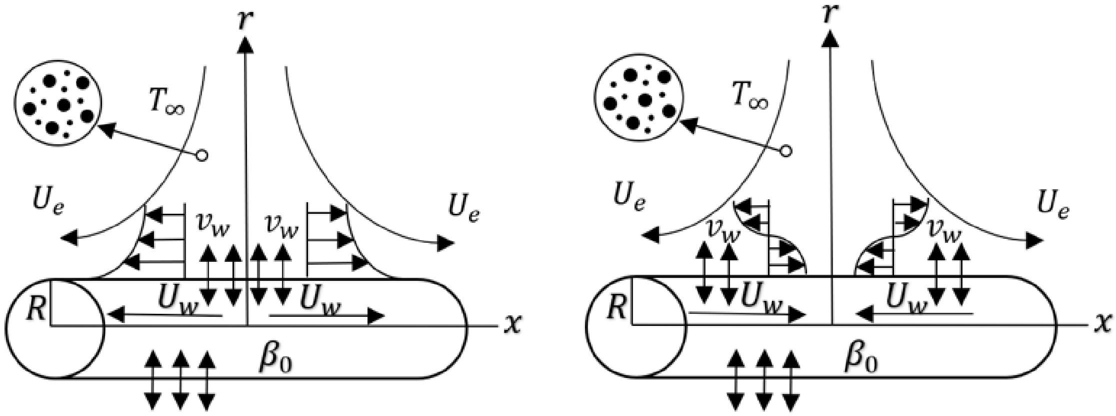
Hybrid nanofluids are formed by displacing two different types of nanofluids in a base fluid (Waqas *et al.,* [18]). According to Khan *et al.,* [19], the introduction of hybrid nanofluids is an important concept in various engineering and industrial applications. It is widely used in various engineering applications such as wider absorption range, low pressure drops, generator cooling, nuclear system cooling, good thermal conductivity and heat exchangers. Aladdin and Bachok [20] investigated the silver-copper oxide/water stagnation point flow in the presence of effects of chemical reaction and magnetic field. Due to the decision to improve thermal efficiency, the exclusive behavior of hybrid nanofluids is actively emphasized. Thus, Zainal *et al.,* [21] proposed a mathematical model to address the stagnation point alumina-copper/water hybrid nanofluid flow, and the effect of Arrhenius dynamics and thermal radiation on stretching or shrinking sheet. In contrast to viscous and nanofluidic flows, hybrid nanofluidic flows have been convincingly demonstrated to enhance surface friction coefficients and heat transfer properties.

Suction/injection is a mechanical effect that controls energy loss in the boundary layer region by reducing surface drag (Yao *et al.,* [22]). According to Epifanov [23], suction is one of the methods of boundary layer control, and its purpose is to reduce the resistance of objects in the external flow or reduce the energy loss in the channel. This method is applied to prevent or delay the separation of the boundary layer. Mansur *et al.,* [24] proposed a mathematical model to investigate the magnetohydrodynamic (MHD) stagnation point flow of nanofluids on permeable stretching or shrinking sheets by considering suction effect. Anuar *et al.,* [25] investigated the effect of suction or injection on stagnation point flow on exponentially shrinking sheets in hybrid nanofluids. Makhdoum *et al.,* [13] inspected an unsteady stagnation point flow properties of TiO_2-$C_2H_2O_2$ nanofluids through a shrinking horizontal cylinder with mass suction.

In this new technological era in the field of fluids, hybrid nanofluids have become a fruitful topic among researchers due to their thermal properties and potential to achieve better results in increasing heat transfer rates than nanofluids. Latest experimental and numerical studies mainly demonstrate that the performance of the hybrid nanofluid in heating exchangers may be improved. Apart from that, it is known that flow through a cylinder is one of the basic concepts of fluid mechanics. Important fields of application for the study of flow through bluff bodies are offshore and marine engineering, marine pipelines. In structural applications such as chimneys, long-span bridges, transport ships, high-rise buildings, towers, columns, heat exchangers and cables, it is important to reduce vibration, drag and lift caused by eddy currents to prevent structural damage (Doreti and Dineshkumar, [26]). As can be seen through the literacy, the current literature does not recognise the problem on stagnation point flow of hybrid nanofluids over a stretching/shrinking cylinder with the presence of suction/injection effects. Hence, this study is performed to explore the stagnation point flow and heat transfer of hybrid nanofluid over a permeable stretching/shrinking cylinder.

2. Mathematical Framework

Consider a boundary layer of Ag-CuO/water hybrid nanofluid over a stretching/shrinking cylinder with radius, R at constant temperature, T_w and ambient temperature, T_∞ where it is noted that T_w > T_∞. Figure 1 displays the physical model of stretching and shrinking cylinder.



(a) Stretching case (b) Shrinking case

Fig. 1. Physical model of stretching and shrinking cylinder

In the above diagram, (x, r) is used as the cylindrical polar coordinates which assigned the axial and radial direction respectively. The surface velocity of the cylinder is given as $U_w = \frac{c}{L}x$ and free stream velocity is $U_e = \frac{a}{L}x$ where a and c are constants while L is the characteristics length. The governing equation to model the fluid flow are (Aladdin and Bachok, [20]):

$$\frac{\partial}{\partial x}(ru) + \frac{\partial}{\partial r}(rv) = 0, \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial r} = U_e\frac{dU_e}{\partial x} + \frac{\mu_{hnf}}{\rho_{hnf}}\left[\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r}\right] - \frac{\sigma_{hnf}\beta_0^2}{\rho_{hnf}}(U_e - u), \tag{2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial r} = \frac{k_{hnf}}{(\rho C_p)_{hnf}}\left[\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r}\right], \tag{3}$$

with assumptions of boundary conditions (BCs):

$$\begin{aligned} &u = U_w(x), \quad v = V_w(x), \quad T = T_w \quad at \quad r = R(x); \\ &u \to U_\infty, \quad T \to T_\infty \quad as \quad r \to \infty \end{aligned} \tag{4}$$

in which u and v are the component of velocity for x axes and r axes, respectively. V_w is indicating the surface mass transfer velocity. Note that the subscript of hnf represents hybrid nanofluids. The thermal diffusivity is denoted as $\frac{k_{hnf}}{(\rho C_p)_{hnf}}$ where k is thermal conductivity, ρ is density, and C_p is specific heat at constant pressure. Moreover, $\frac{\mu_{hnf}}{\rho_{hnf}}$ and $\frac{\sigma_{hnf}\beta_0^2}{\rho_{hnf}}$ are kinematic viscosity and magnetic field where μ is dynamic viscosity, σ is the conductivity of the liquid being pumped and β_0 defined as the ratio of the pressure to the square of the magnetic field. A set of a suitable transformation is introduced as below:

$$\eta = \frac{r^2 - R^2}{2R}\left(\frac{a}{\nu_f L}\right)^{\frac{1}{2}}, \qquad \psi = \left(\frac{\nu_f a}{L}\right)^{\frac{1}{2}} xRf(\eta), \qquad T = T_\infty + (T_w - T_\infty)\theta(\eta) \tag{5}$$

where η is the similarity variable, ψ is the stream function defined as $u = r^{-1}\frac{\partial\psi}{\partial r}$ and $v = -r^{-1}\frac{\partial\psi}{\partial x}$, which identically satisfies Eq. (1) after employing Eq. (5). Also, ν_f is kinematic viscosity of the fluid. The new momentum, energy and concentration equations are in the following form of ODEs.

$$\frac{\mu_{hnf}/\mu_f}{\rho_{hnf}/\rho_f}[(1+2\gamma\eta)f''' + 2\gamma f''] + ff'' - f'^2 - \frac{\sigma_{hnf}/\sigma_f}{\rho_{hnf}/\rho_f}M[1-f'] + 1 = 0 \tag{6}$$

$$\frac{1}{Pr}\frac{k_{hnf}/k_f}{(\rho C_p)_{hnf}/(\rho C_p)_f}[(1+2\gamma\eta)\theta'' + 2\gamma\theta'] + f\theta' = 0 \tag{7}$$

along the BCs,

$$\begin{aligned} &f(0) = S,\ \ f'(0) = \varepsilon,\ \ \theta(0) = 1, \\ &f'(\infty) \to 1,\ \ \theta(\infty) \to 0 \end{aligned} \tag{8}$$

where the parameter $\gamma = \left(\frac{\nu_f L}{R^2 a}\right)^{\frac{1}{2}}$, $M = \frac{\sigma_f \beta_0^2 L}{a\rho_f}$ and $Pr = \frac{\nu_f}{\alpha_f}$ are symbolize for curvature parameter, magnetic field, and Prandtl number respectively . Also, S is the mass transfer parameter defined as $S = -\frac{V_w r}{\left(\frac{\nu_f a}{L}\right)^{\frac{1}{2}}R}$ for suction ($S > 0$) and for injection ($S < 0$). Furthermore, $\varepsilon = \frac{c}{a}$ is the velocity ratio parameter which explain if $\varepsilon < 0$, $\varepsilon = 0$ and $\varepsilon > 0$ correspond to shrinking, static and stretching cases respectively.

The interest of physical quantities involve in this study are skin friction, C_f and Nusselt number, Nu_x,

$$C_f = \frac{\mu_{hnf}}{\rho_f U_e^2}\left(\frac{\partial u}{\partial r}\right)_{r=R},\quad Nu_x = -\frac{x k_{hnf}}{k_f(T_w - T_\infty)}\left(\frac{\partial T}{\partial r}\right)_{r=R} \tag{9}$$

Solving Eq. (9) we acquire,

$$C_f(Re_x)^{\frac{1}{2}} = \frac{\mu_{hnf}}{\mu_f}f''(0),\quad Nu_x(Re_x)^{-\frac{1}{2}} = -\frac{k_{hnf}}{k_f}\theta'(0) \tag{10}$$

where $Re_x = \frac{U_e x}{\nu_f}$.

The study applies the mathematical model hybrid nanofluid given by Oztop and Abu-Nada [27]. Table 1 shows the thermophysical properties of hybrid nanofluid where φ_1 is the Ag (silver) nanoparticle while φ_2 is the addition of nanoparticles which is CuO (copper oxide). Mixing CuO nanoparticles into 0.01 volume of Ag/water forms the formation of hybrid nanofluid. Note that the nanoparticles Ag with 0.01 volume of solid fraction (φ_1 = 0.01) is added constantly to the water throughout most of the problem. Also, CuO (φ_2) is added to produce Ag-CuO/water.

Table 1
Thermophysical properties of hybrid nanofluid (Oztop and Abu-Nada, [27])

Thermophysical Properties	Hybrid Nanofluid
Density	$\rho_{hnf} = (1-\varphi_2)\left[(1-\varphi_1)\rho_f + \varphi_1\rho_{s1}\right] + \varphi_2\rho_{s2}$
Heat capacity	$\left(\rho C_p\right)_{hnf} = \left(1-\varphi_2\right)\left[\left(1-\varphi_1\right)\left(\rho C_p\right)_f + \varphi_1\left(\rho C_p\right)_{s1}\right] + \varphi_2\left(\rho C_p\right)_{s2}$
Viscosity	$\mu_{hnf} = \dfrac{\mu_f}{(1-\varphi_1)^{2.5}(1-\varphi_2)^{2.5}}$
Thermal conductivity	$k_{hnf} = \left[\dfrac{k_{s2} + 2k_{nf} - 2\varphi_2\left(k_{nf} - k_{s2}\right)}{k_{s2} + 2k_{nf} + \varphi_2\left(k_{nf} - k_{s2}\right)}\right] k_{nf}$ where, $k_{nf} = \left[\dfrac{k_{s1} + 2k_f - 2\varphi_1\left(k_f - k_{s1}\right)}{k_{s1} + 2k_f + \varphi_1\left(k_f - k_{s1}\right)}\right] k_f$

Table 2 displays the thermophysical characteristics of Ag-CuO and base fluid (Hayat and Nadeem, 2017). These numbers are used to calculate the numerical solutions.

Table 2
Thermophysical properties of nanoparticles (Hayat and Nadeem, [28])

Properties	Ag	CuO	Base fluid (water)
$\rho(kgm^{-3})$	10500	6320	997.1
$k(Wm^{-1}K^{-1})$	429	76.50	0.613
$C_p(Jkg^{-1}K^{-1})$	235	531.80	4179

3. Numerical Computation

Shampine *et al.,* [29] claimed that the effective techniques to solve boundary value problems for ordinary differential equations (ODEs) is by employing bvp4c. Hence, MATLAB software will be used in this research since it has bvp4c function that capable of solving the problems. Code A is the main code that required to solve the system of ordinary differential equations (ODEs). To initiate the computation, the ODEs is written as first order ODEs system by introducing the variables as follow:

$$y(1) = f, \;\; y(2) = f', \;\; y(3) = f'', \quad y(4) = \theta, \;\; y(5) = \theta' \tag{11}$$

The momentum equation Eq. (6) can be rewrite as:

$$y(3)' = \frac{1}{1+2\gamma\eta}\left[\frac{-y(1)y(3) + \left(y(2)\right)^2 + \frac{\sigma_{hnf}/\sigma_f}{\rho_{hnf}/\rho_f}M[1-y(2)] - 1}{\frac{\mu_{hnf}/\mu_f}{\rho_{hnf}/\rho_f}} - 2\gamma y(3)\right] \tag{12}$$

Equation Eq. (7) can be rewrite as:

$$y(5)' = \frac{1}{1+2\gamma\eta}\left[\frac{-Pry(1)y(5)}{\frac{k_{hnf}/k_f}{(\rho C_p)_{hnf}/(\rho C\_p)_f}} - 2\gamma y(5)\right] \tag{13}$$

with boundary conditions Eq. (8) as follows:

$$ya(1) = S, \ \ ya(2) = \varepsilon, \ \ ya(4) = 1,$$
$$yb(2) \to 1, \ \ yb(4) \to 0 \tag{14}$$

where $a = 0$ and $b = \infty$ on the position of surface $\eta = 0$ and far field $\eta \to \infty$ respectively. Code A requires two sets of initial guesses for the first and second solutions since there are dual solutions for the current problem. As a result, two sets of initial guesses are required, which are chosen through a multiple of trials until the boundary conditions are satisfied and the required answers are found.

4. Results and Discussion

The individual equations such as momentum equation Eq. (11) and energy equation Eq. (12) together with boundary conditions Eq. (13) are solved numerically using the bvp4c solver in the MATLAB software. The numerical results are presented in both table and figures. Table 3 shows the relative values of reduced skin friction coefficient, $f''(0)$ when $\varphi_1 = \varphi_2 = 0$ and $M = 0$ with several values of ε, γ and $Pr = 6.2$, which indicates a good correlation between the current results and the results obtained from the previous studies by Najib et al., [30] and Aladdin and Bachok [20]. Thus, it is assured that the present numerical method can be used confidently to solve the fluid flow and heat transfer of the current problem.

One of the previous studies, as mentioned, is Hayat and Nadeem [28] as the reference for the thermophysical properties of nanoparticles. The range of parameters is taken from the main reference for this study Aladdin and Bachok [20] the required (Ag-CuO/water) hybrid nanofluid with various volume fractions $\varphi_1 = \varphi_2 = 0.1$ is applied to achieve the objectives of this work. Therefore, the nanoparticles volume fraction φ values are in the range of $0 \leq \varphi \leq 0.3$. For the suction value is $S = 0.2$ and injection value is $S = -0.2$. The range of magnetic parameter, M are in the range of $0 \leq M \leq 0.2$, while curvature parameter, γ are in the range of $0 \leq \gamma \leq 0.4$. Prandtl number, Pr is fixed to 6.2, which represent water as stated in the work of Oztop and Abu-Nada [27].

Table 3
Numerical computation for $f''(0)$ when $\varphi_1 = \varphi_2 = M = 0$ with several values of ε, γ and $Pr = 6.2$

		Najib et al., [30]		Aladdin and Bachok [20]		Current Result	
ε	γ	First Solution	Second Solution	First Solution	Second Solution	First Solution	Second Solution
-0.25	0	1.4022408		1.4022408		1.4022408	
	0.2	1.5396153		1.5396153		1.5396153	
	0.4	1.6672783		1.6672781		1.6672811	
-0.5	0	1.4956696		1.4956698		1.4956698	
	0.2	1.6705695		1.6705695		1.6705695	
	0.4	1.8307527		1.8307524		1.8307579	
-1.0	0	1.3288169		1.3288169		1.3288169	
	0.2	1.6297678	0	1.6297671	0	1.6297678	0
	0.4	1.8836199	0	1.8836185	0	1.8836416	0
-1.2	0	0.9324736	0.2336491	0.9324733	0.2336497	0.9324733	0.2336497
	0.2	1.4106126	0.0015206	1.4106102	0.0015206	1.4106125	0.0015211
	0.4	1.7432654	-0.1165759	1.7423618	-0.1165759	1.7424146	-0.1165759
-1.4	0.4	1.3838578	0.1797279	1.3838419	0.1797279	1.3840578	0.1797280

4.1 Effect on Reduced Skin Friction $f''(0)$ and Reduced Heat Transfer $-\theta'(0)$

Figures 2 and 3 show the effect of the volume fraction of nanoparticle, on reduced skin friction $f''(0)$ and reduced heat transfer $-\theta'(0)$. Both figures portray the result for three different types of fluid, which are viscous fluid ($\varphi_1 = \varphi_2 = 0$), Ag/water nanofluid ($\varphi_1 = 0.01, \varphi_2 = 0$) and Ag-CuO/water hybrid nanofluid ($\varphi_1 = \varphi_2 = 0.01$). There are unique solutions exist for $\varepsilon > -0.8$, dual solutions exist for $\varepsilon_c \leq \varepsilon \leq -0.8$ and there is no solution exist when $\varepsilon < \varepsilon_c$. The critical value of each increment of $\varphi_1 = \varphi_2 = 0$, $\varphi_1 = 0.01, \varphi_2 = 0$ and $\varphi_1 = \varphi_2 = 0.01$ is $\varepsilon_c = -1.3900, -1.3898$ and -1.3896 respectively. It is observed that when the volume fraction of nanoparticle increase, the range of solutions reduces. This implies that the increase of the volume fraction of nanoparticle will affect the region in which the solutions can exist. Furthermore, we can see that the increase of volume fraction of nanoparticle will increase the skin friction and heat transfer rate at the surface. This is because an increase in the volume fraction of nanoparticles causes the fluid's viscosity to rise. In addition, the hybrid nanofluid exhibit excellent heat transfer performance, followed by nanofluid and viscous fluid.

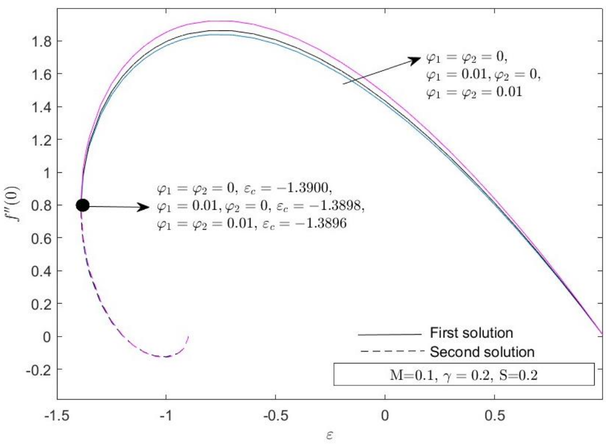


Fig. 2. Effect of the volume fraction of nanoparticle, φ on reduced skin friction $f''(0)$

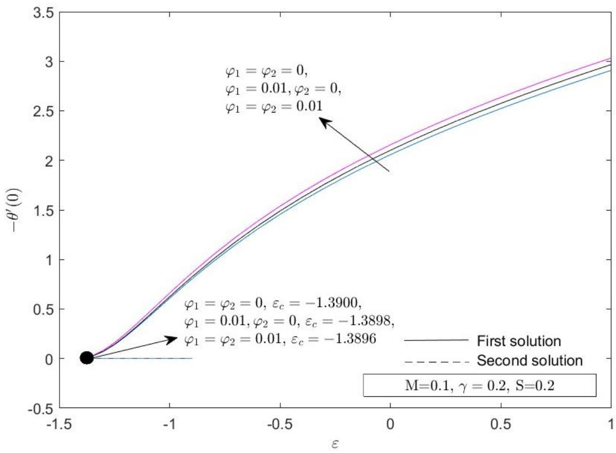


Fig. 3. Effect of the volume fraction of nanoparticle, φ on reduced heat transfer $-\theta'(0)$

The influence of different values of γ on reduced skin friction $f''(0)$ and reduced heat transfer $-\theta'(0)$ are depict in Figures 4 and 5. The findings expose that the addition of γ helps in delaying the boundary layer separation. In fact, the hybrid nanofluid are seemed to accelerate the separation of boundary layer at the flat plat $\gamma = 0$ ($\varepsilon_c = -1.2636$), compared to $\gamma = 0.2$ ($\varepsilon_c = -1.3896$) and $\gamma = 0.4$ ($\varepsilon_c = -1.4946$) which obviously occur in shrinking region, $\varepsilon < 0$. Besides that, the $f''(0)$ and $-\theta'(0)$ increase with the augmentation of γ. These results are seemed to fulfill the fact divulge by the Aladdin and Bachok [20] that the performance of heat transfer improved as the value of γ upsurge, by means lessen the curvature's radius and reduce the surface area of the cylinder which contain hybrid nanofluid particles. Thus, decreases the resistance of the fluid particles and increases the fluid velocity. Consequently, boosts up the rate of heat transfer.

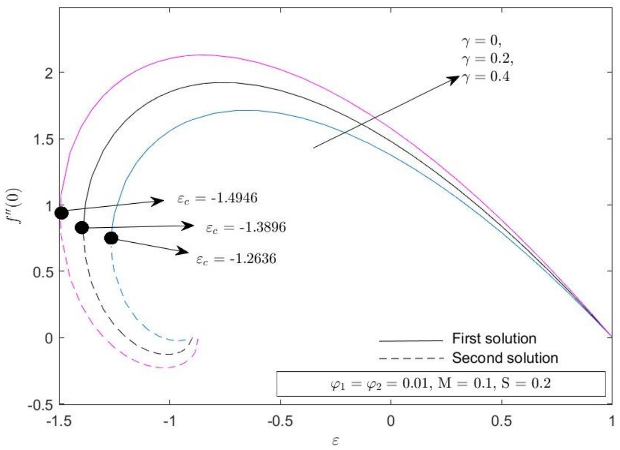


Fig. 4. Effect of curvature parameter, γ on reduced skin friction $f''(0)$

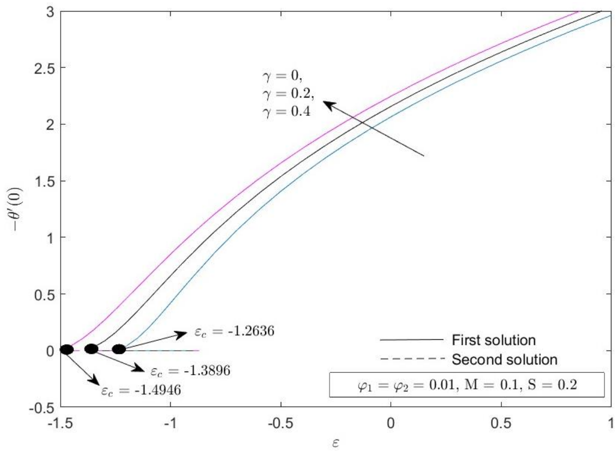


Fig. 5. Effect of curvature parameter, γ on reduced heat transfer $-\theta'(0)$

Then, Figures 6 and 7 illustrate the variation of values of M, where the range is chosen between $0 \leq M \leq 0.2$ on reduced skin friction $f''(0)$ and reduced heat transfer $-\theta'(0)$. Apparently, the range of the solution narrow for $f''(0)$ with additional of M. This incident coincides with the theory of Lorentz force which arise due to the interaction of an electrically magnetic field during the movement of electrically hybrid nanofluid. This force has a powerful rapport to slow down the movement of the fluid which subsequently detaining the separation of the boundary layer from occur rapidly. This implies that when the magnetic parameter increases, the $f''(0)$ decreases, indicating that the presence of magnetic field causes a rapid reduction of velocity in the vicinity of the boundary due to the action of Lorentz force which resists the fluid flow. As a result, the rate of heat transfer decreases. Therefore, the $f''(0)$ and $-\theta'(0)$ decrease with the increases of M.

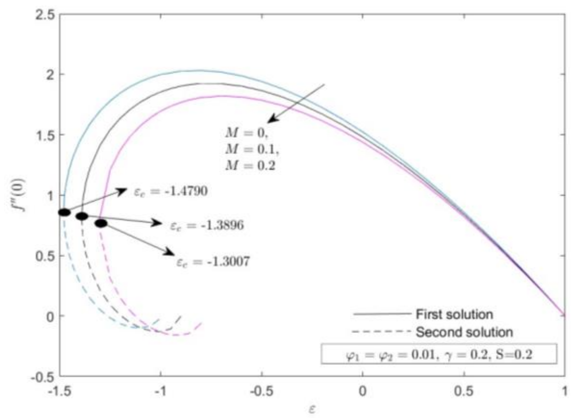


Fig. 6. Effect of magnetic parameter, M on reduced skin friction $f''(0)$

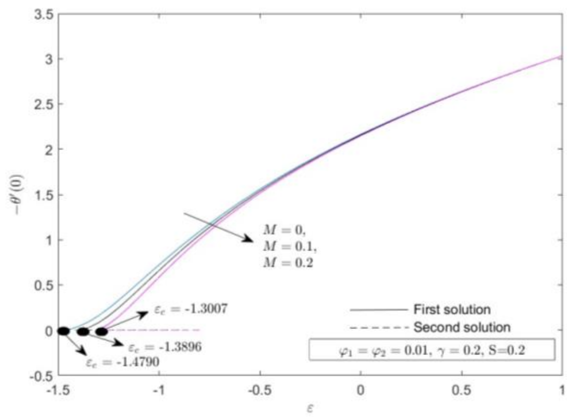


Fig. 7. Effect of magnetic parameter, M on reduced heat transfer $-\theta'(0)$

Moreover, Figures 8 and 9 present the graphical results of reduced skin friction coefficient $f''(0)$ and reduced heat transfer rate $-\theta'(0)$ at the surface with ε for different S. It shows that the existence of unique solution when $\varepsilon > -0.8$ while the dual solution exists when $\varepsilon_c \leq \varepsilon \leq -0.8$. There exists no solution when $\varepsilon < \varepsilon_c$ because the separations of the boundary layer are predictable. The figures of $f''(0)$ and $-\theta'(0)$ are observed to be increasing when the values of S get higher for both solutions. The increasing value of S pushes the fluid into an empty space affecting the surface limit. As a result, more force is used to fluid flow, and more temperature will eventually rise.

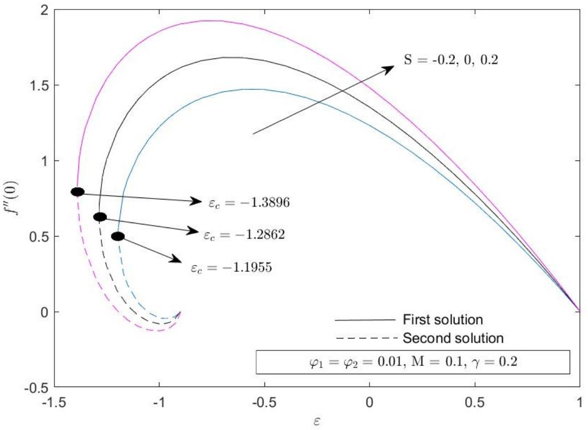


Fig. 8. Effect of suction, S on reduced skin friction $f''(0)$

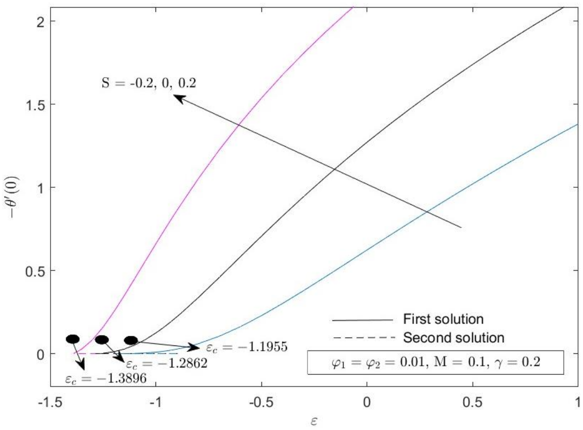


Fig. 9. Effect of suction, S on reduced heat transfer $-\theta'(0)$

Besides that, Figures 10 and 11 illustrate the graphical results of reduced skin friction coefficient $f''(0)$ and reduced heat transfer rate $-\theta'(0)$ with suction effect for different γ. Figure 10 shows that the existence of dual solution when $S_c \leq S \leq 0.5$ for shrinking case ($\varepsilon = -1.0$). The critical value of each increment of $\gamma = 0$, $\gamma = 0.2$ and $\gamma = 0.4$ is $S_c = -0.3979$, -0.7878 and -1.1740 respectively. It is observed that when the curvature parameter increases, the range of solutions expands. On the contrary, Figure 11 present the rise of γ reduces the resistance of the fluid particles and increases the fluid velocity. Therefore, the heat transfer rate is increased.

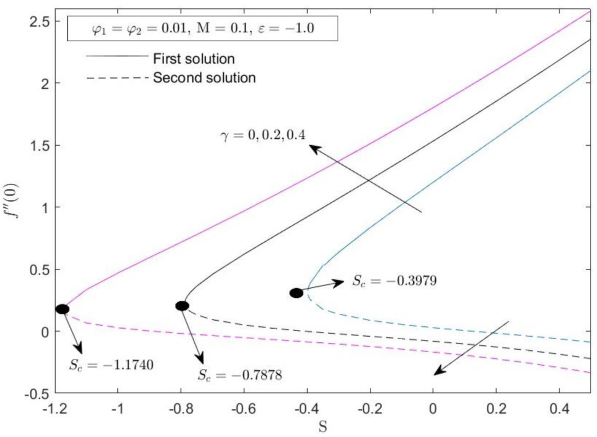


Fig. 10. Effect of curvature parameter, γ with suction, S on reduced skin friction $f''(0)$

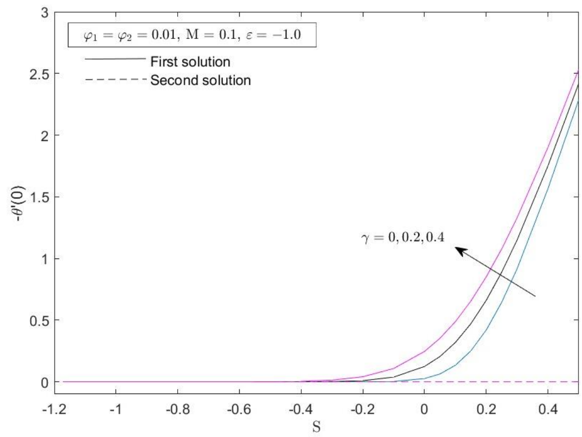


Fig. 11. Effect of curvature parameter, γ with suction, S on reduced heat transfer $-\theta'(0)$

4.2 Effect on Local Skin Friction $C_f(Re_x)^{\frac{1}{2}}$ and Local Nusselt Number $Nu_x(Re_x)^{-\frac{1}{2}}$

Figures 12 and 13 demonstrate the influence of curvature parameter, γ on local skin friction $C_f(Re_x)^{\frac{1}{2}}$ and local Nusselt number $Nu_x(Re_x)^{-\frac{1}{2}}$ with nanoparticle volume fraction (φ_1, φ_2). Figure 12 shows that the value of $C_f(Re_x)^{\frac{1}{2}}$ increases with both increase of γ and increase of φ. On the other hand, the value of $Nu_x(Re_x)^{-\frac{1}{2}}$ increases with the increase of γ and increases with the increase of φ value in Figure 13.

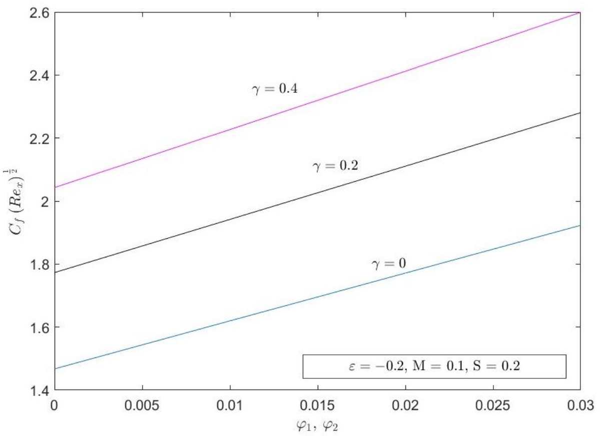


Fig. 12. Effect of curvature parameter, γ on local skin friction $C_f(Re_x)^{\frac{1}{2}}$

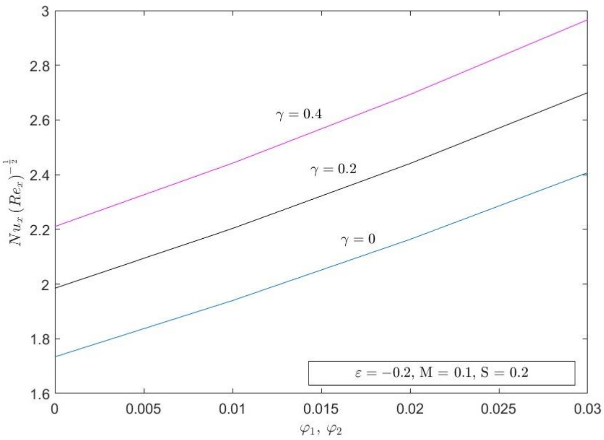


Fig. 13. Effect of curvature parameter, γ on local Nusselt number $Nu_x(Re_x)^{-\frac{1}{2}}$

Apart from that, the results on variation of local skin friction $C_f(Re_x)^{\frac{1}{2}}$ and local Nusselt number $Nu_x(Re_x)^{-\frac{1}{2}}$ with nanoparticle volume fraction (φ_1, φ_2) for various value of suction/injection parameter $S = -0.2, 0$ and 0.2 in shrink ing case $\varepsilon = -0.2$ which are shown in Figures 14 and 15. As the hybrid nanoparticle (φ_1, φ_2) tested are increase in value when suction/injection parameter rise, the $C_f(Re_x)^{\frac{1}{2}}$ and $Nu_x(Re_x)^{-\frac{1}{2}}$ increase as well. This suggests that adding suction/injection effect to the shrinking cylinder in (Ag-CuO/water) hybrid nanofluid give a significant impact. It is observed that as S increases, the suction parameter is widened because suction generally increases skin friction and heat transmission in most cases. This is due to the S effect, which increases the shear stress.

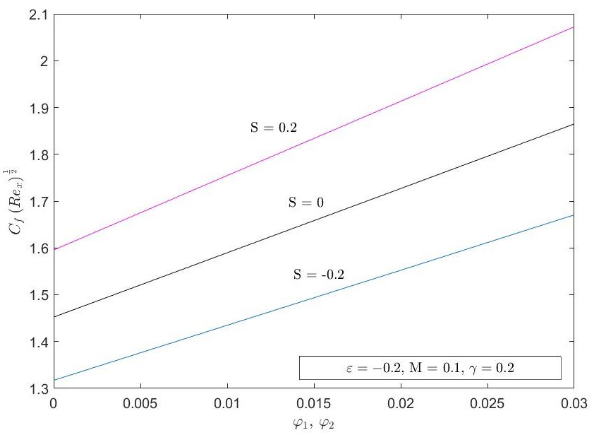


Fig. 14. Effect of suction, S on local skin friction $C_f(Re_x)^{\frac{1}{2}}$

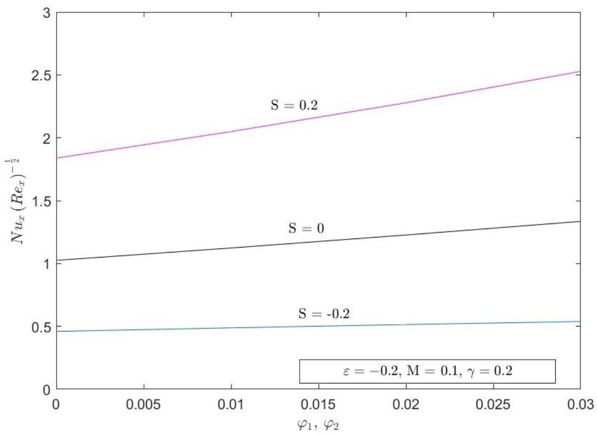


Fig. 15. Effect of suction, S on local Nusselt number $Nu_x(Re_x)^{-\frac{1}{2}}$

4.3 Effect on Velocity Profile $f'(\eta)$ and Temperature Profile $\theta(\eta)$

Next, the graphical results focus on the different volume fractions of CuO nanoparticles in a certain range of $\varphi_2 = 0, 0.01, 0.03$. Therefore, Figures 16 and 17 present the outcome of CuO nanoparticle φ_2 for a stretching/shrinking cylinder case with suction effect $S = 0.1$. To be specific, Figures 16 and 17 show the effect of CuO nanoparticle φ_2 on the velocity profiles $f'(\eta)$ and temperature $\theta(\eta)$ profiles in the shrinking condition $\varepsilon = -1.2$. When CuO nanoparticle φ_2 rises from 0 to 0.03 for the first and second solutions, $f'(\eta)$ increases significantly within the momentum boundary layer but $\theta(\eta)$ decreases. Hence, the Ag-CuO/water hybrid nanofluid ($\varphi_1 = \varphi_2 = 0.01$) have higher velocity profile and lower temperature profile compared to the other fluids.

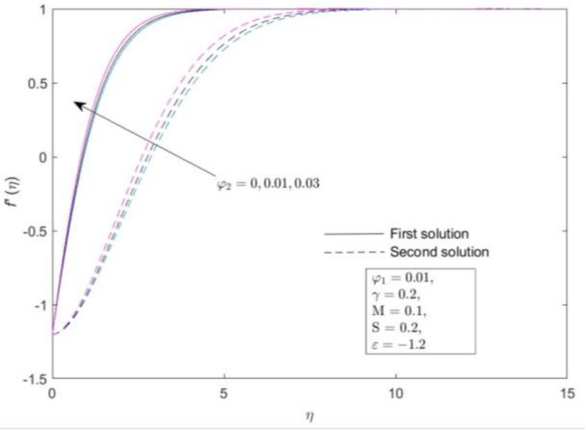


Fig. 16. Effect of nanoparticles, φ_2 on velocity profile, $f'(\eta)$

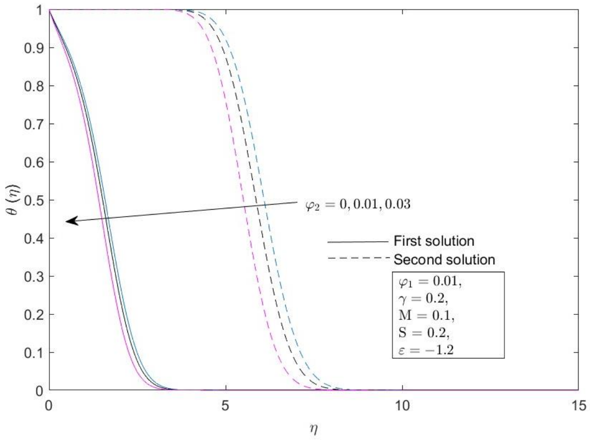


Fig. 17. Effect of nanoparticles, φ_2 on velocity profile, $\theta(\eta)$

Furthermore, Figures 18 and 19 show the effect of curvature parameter, γ on the variations of velocity profiles $f'(\eta)$ and temperature profile $\theta(\eta)$ in the (Ag-CuO/water) hybrid nanofluid for shrinking case ($\varepsilon = -1.2$). When the curvature parameter is increase in the range $0 \leq \gamma \leq 0.4$ in first solution, $f'(\eta)$ increases significantly within the momentum boundary layer while decreases for second solution. Thus, the presence of curvature parameters expands the range of dual solutions. Moreover, the increase of the curvature parameter improves the temperature gradient in the boundary layer region around the cylinder. The boundary layer thickness of the second solution appears to be higher than that of the first solution.

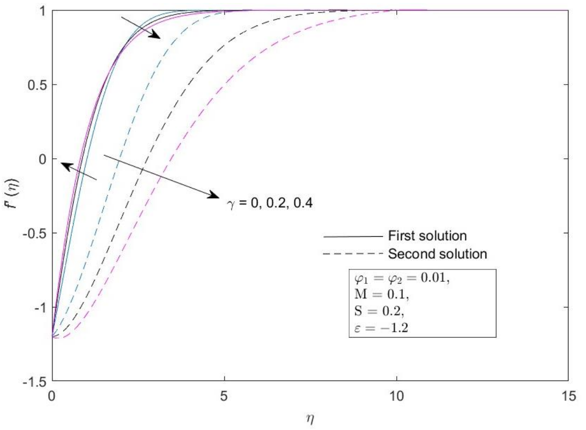


Fig. 18. Effect of curvature parameter, γ on velocity profile, $f'(\eta)$

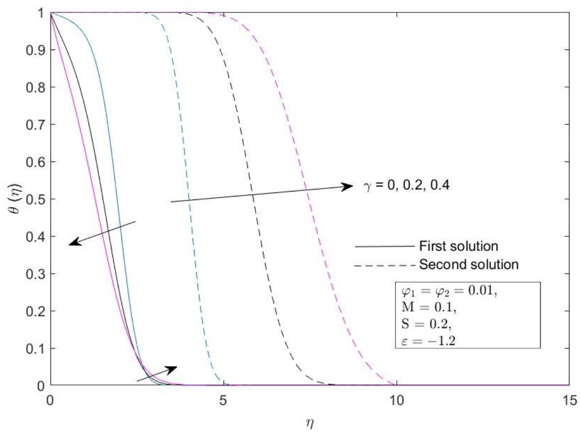


Fig. 19. Effect of curvature parameter, γ on temperature profile, $\theta(\eta)$

Lastly, Figures 20 and 21 show the effect of suction/injection S on the variations of velocity profiles $f'(\eta)$ and temperature profile $\theta(\eta)$ in the (Ag-CuO/water) hybrid nanofluid with nanoparticle volume fractions of $\varphi_1 = \varphi_2 = 0.01$ for shrinking case ($\varepsilon = -1.1$). When the suction/injection effect is applied in the range $-0.2 \leq S \leq 0.2$ in first solution, $f'(\eta)$ increases significantly within the momentum boundary layer while decreases for second solution. This signifies that the presence of suction $S = 0.2$ affects the range of duality to be widened. Therefore, the temperature profile

$\theta(\eta)$ decreases as the thermal boundary layer thickness is thinning at the first solution and increasing at the second solution.

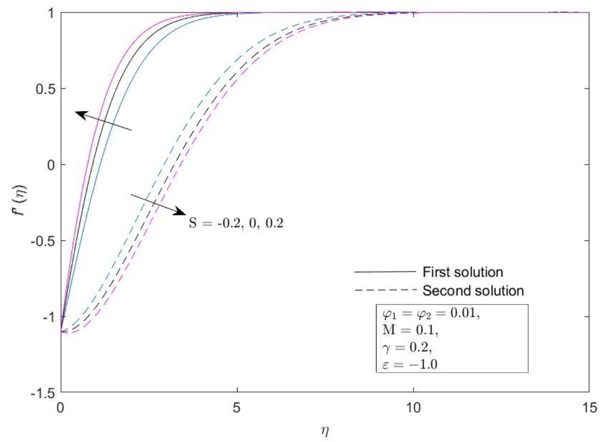


Fig. 20. Effect of suction, S on velocity profile, $f'(\eta)$

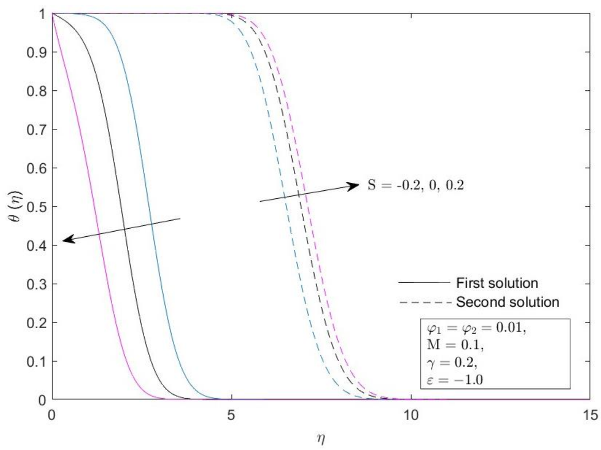


Fig. 21. Effect of suction, S on temperature profile, $\theta(\eta)$

5. Conclusions

In this research, the study of stagnation point flow and heat transfer of hybrid nanofluid over a stretching/shrinking cylinder with suction/injection effects is discussed. To solve this problem, similarity transformation is used to transform partial differential equations (PDEs) to ordinary differential equations (ODEs). The numerical results are presented in specific values parameter graphically by using the bvp4c solver in the MATLAB software. As a result, the findings of this study can be summarized as follows:

- For stretching/shrinking cylinder surface, unique solutions exist for $\varepsilon > -0.8$, dual solutions exist for $\varepsilon_c \leq \varepsilon \leq -0.8$ and there is no solution exist when $\varepsilon < \varepsilon_c$. The presence of curvature parameter and suction effect expands the range of dual solutions.
- As the curvature parameter γ increases, the reduced skin friction $f''(0)$ and reduced heat transfer $-\theta'(0)$ also increase.
- The reduced skin friction $f''(0)$ and reduced heat transfer $-\theta'(0)$ are increase when the values of suction/injection parameter, S increases.
- The local skin friction, $C_f(Re_x)^{\frac{1}{2}}$ and local Nusselt number, $Nu_x(Re_x)^{-\frac{1}{2}}$ decreases as suction/injection parameter, S increases.
- Hybrid nanofluid has a higher velocity profile and lower temperature profile compared to viscous fluid and nanofluid.
- With an increase in the value of S, the velocity profile, $f'(\eta)$ of first solution decreases while the second solution increases and it is opposite to temperature profile, $\theta(\eta)$ for first and second solution.

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