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Significant Effect of Radiation on Combined Convection Vertical Channel with Internal Heat Generation and Boundary Conditions of a Third Kind

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ABSTRACT

Heat transfer process involving combined convection, along with the influence of radiation, within a fully developed vertical channel, holds significant importance in environmental, industrial, and engineering applications. Concerns have arisen regarding the complexity, cost, and time required to understand the heat transfer process, especially when considering radiation effects. This study aims to assess the combined impact of the Robin temperature boundary condition and radiation on flow and heat transfer, to examine the role of viscous dissipation, including its interaction with radiation, in fluid flow and heat transfer and to compare the heat transfer effectiveness under the boundary conditions of Dirichlet, Neumann, and Robin. Various dimensional parameters are systematically tested in this investigation, with particular emphasis on discussing the phenomenon of flow reversal in the presence of radiation. The numerical solution to the Boundary Value Problem (BVP) is achieved using Maple and its built-in routine, *dsolve*. A validation study on a previously published problem is conducted to ensure the accuracy of the computational approach, considering the added complexity of radiation effects and the transformation of the partial differential equation into an ordinary differential equation applying the similarity technique. Graphical representations of the numerical results for flow and temperature profiles, incorporating radiation effects, are presented. Notably, the occurrence of flow reversal is observed in instances where the values of internal heat generation (G), combined convection parameter (λ), and radiation effects (R_d) were substantial. Conversely, an increase in the values of the local heating exponent (p) and Biot numbers (Bi), while accounting for radiation, eliminated the occurrence of flow reversal.

1. Introduction

In the area of heat transfer and fluid dynamics, the investigation of combined convection in vertical channels holds essential importance, particularly when internal heat generation is considered. This study investigates the connection between radiation and convection within a fully

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developed vertical channel, with a focus on the implications of boundary conditions of a third kind. The presence of internal heat generation introduces a dynamic element, influencing the temperature distribution and flow patterns within the channel. The incorporation of radiation as a heat transfer mechanism adds complexity to the system, warranting a comprehensive examination of its effects. The consideration of boundary conditions of a third kind requires a detailed analysis to unravel the synergistic impacts on the overall thermal performance of the vertical channel. This research aims to contribute valuable insights into the behaviour of such systems, shedding light on the combined influences of radiation, convection, internal heat generation, and non-trivial boundary conditions for a more holistic understanding of heat transfer phenomena in vertical channels.

The term "heat transfer" refers to transferring heat energy between two objects with different temperatures. The typical calculations that most researchers are interested in involve determining how long it takes for a material to heat up to its final temperature or finding out the times at which these temperatures are reached. This can be used as a guideline for determining how much insulation is needed to prevent heat loss. In most situations, the rate of heat loss is directly proportional to the temperature gradient, which is known as the driving force or potential. Conduction, convection, and radiation are the three mechanisms through which heat may be transferred (Harding [1]). Heat transfer through simultaneous radiation and convection has applications in various technological problems. On the other hand, it is important to note that the transfer of heat through radiation and convection is significant in the context of space technology and processes that involve extremely high temperatures (Mallikarjun and Kavitha [2]). Radiation is energy that moves from one place to another in a form that can be described as waves or particles. When radiation transfers heat from a source to a sink, no intermediary or direct contact between the two is necessary. In contrast, according to Howell *et al.,* [3], thermal radiation is a type of electromagnetic radiation that an object's surface emits due to the object's temperature. Besides, heat is released as thermal radiation when the motion of charged particles inside atoms is transformed into electromagnetic radiation. In the study conducted by Yaseen *et al.,* [4], it was highlighted that during operating at high temperatures, heat production or absorption and thermal radiation must be addressed. Since many technical operations occur at high temperatures, it is important to keep these two factors in mind while developing suitable tools. They are also used for various industrial purposes, including compressors on ships and rocket thrusters, plasma mechanics in spacecraft, glass production, and many more.

Convection describes the heat transfer process in a fluid that results from the combined effects of molecular-level conduction and the transport of energy via the macroscopic motion of the fluid itself. It is necessary for the fluid to be moving to prevent the heat transfer mechanism from turning into a situation of static conduction. A solid surface is typically present next to the fluid when the word convection is employed. Moreover, convection is a method of heat transfer that takes place when a fluid in motion meets a surface that is not moving. The heat is transported by the flowing fluid, which either deposits it on the surface or takes it away from the surface (Zohuri [5]). Convection can be classified into three distinct types which are natural, forced, and mixed. Various types of convection, including natural convection, forced convection, and mixed convection, have been studied because of their usefulness and importance in different areas. The process of heat transfer in fluids known as "mixed convection" occurs when the flow field is significantly altered from what would prevail under conditions of uniform density due to variations in gravitational body force associated with non-uniformity of density within the system. Effects of buoyancy on heat transfer are a common way to describe the processes involved, as they are typically conceived of in terms of fluid buoyancy (Pizzarelli [6]). The heat transfer process involving forced and natural convection is influenced by the characteristics of the surrounding environment. When pressure and buoyancy forces combine, a phenomenon known as mixed convection occurs. This phenomenon is distinguished by a high Grashof number (Gr) in natural convection and a low Reynolds number (Re) in forced convection. Even when using a combination of natural and forced convection, relying solely on a high-power leading device in forced convection may not be able to provide enough heat on its own (Thanesh Kumar *et al.,* [7]).

Vertical channels are commonly found in various thermal engineering equipment, such as solar energy collectors and cooling devices for electronic and micro-electronic equipment. Given their extensive use, numerous studies have been conducted to explore the characteristics of fully developed mixed convection flow in these vertical channels. Due to its wide applications, a lot of investigations have been done toward the understanding of fully developed mixed convection flow in a vertical channel (Xu and Pop [8]). Grosan and Pop [9] analyzed the influence of thermal radiation on steady, fully developed mixed convection in vertical channels. They examined cases where the channel walls have uniform but different wall temperatures using the Rosseland approximation model. This model resulted in ordinary differential equations for an optically dense viscous incompressible fluid flowing through the channel. Patra *et al.,* [10] investigated the effects of radiative heat transfer on magnetohydrodynamic (MHD) fully developed mixed convective flow in a vertical channel with asymmetric wall heating in the presence of a transverse magnetic field. They observed that radiative heat transfer and buoyancy forces significantly affected the fluid velocity. The induced magnetic field decreased near the cooler wall and increased near the hotter wall with increasing radiation parameters. Radiation also lowered the fluid temperature. More recently, Ojemeri *et al.,* [11] further extended Grosan and Pop [9] study by examining heat transfer analysis in MHD Casson fluid flow with thermal radiation in a vertical porous channel. They discussed the variations of various relevant parameters in detail and found that thermal radiation and rarefaction effects greatly influenced the temperature and velocity profiles, respectively, leading to significant changes.

Radiation transfer was first introduced in astrophysical problems concerning the transmission and reflection of light by planetary atmospheres, the formation of absorption lines in stellar spectra, and the transfer of energy in stars, planets, nebulae, and galaxies. One of the most difficult phenomena to model and predict in engineering is heat transfer in a radiating medium, whether the medium is at rest or in motion (Viskanta [12]). Ashraf *et al.,* [13] studied the effects of heat radiation on the hydromagnetic mixed convection laminar boundary layer flow of a viscous, incompressible, and electrically conducting fluid moving over a magnetized permeable surface with a changing magnetic field applied in the direction of the stream at the surface. It was discovered that as the radiation parameter increased, the skin friction coefficient decreased, and the surface heat transfer coefficient and magnetic intensity increased. Then, natural convection flow along a magnetized vertical permeable plate was studied by Ashraf *et al.,* [14] using two approaches, the primitive variable transformation for the finite difference method (FDM) and the stream function formulation for the asymptotic series solutions near and away from the leading edge of the plate. They discovered that in the downstream regime, the coefficient of the current density rises significantly with an increase in the radiation parameter, while the coefficient of the skin friction drops and the coefficient of the rate of heat transfer increases. Elsaid and Abdel-Wahed [15] investigated the influence of thermal radiative flux on the mixed convection of hybrid-nanofluid in a vertical channel. According to their findings, the velocity and temperature behaviour is significantly affected by hybrid fluids and thermal radiation. In addition, when heat radiation is absent, the Nusselt number value decreases when nanoparticles are added to the base fluid, and when two kinds of nanoparticles are used, the decreases are much greater. Abbas *et al.,* [16] investigated the combined effects of thermal radiation and thermophoretic motion on mixed convection boundary layer flow. They discovered that the mass transfer rate increases by increasing the values of the radiation parameter, mixed convection parameter, and thermophoresis parameter but decreases by increasing the thermophoretic coefficient and Prandtl number. Hossain *et al.,* [17] used finite element analysis to investigate radiation impacts on the unsteady mixed convection of kerosene oil-based CNT nanofluid. Many hydrodynamic behaviors and heat transport processes that may be attributed to radiation have been examined and described in this research. They found that raising the radiation level led to a rise in fluid temperature because it boosted the flow of fluid through the region with higher vorticity. Moreover, as the amount of radiation increases, the bulk temperature of the fluid, the temperature of the fluid itself, and the magnitude of the velocity and drag forces also rise.

Mixed convection plays a significant role in various technical fields due to its occurrence in industrial, technological, and natural settings. The coolants in these systems may be propelled by free convection, forced convection, or mixed convection, depending on the power density of the circuit boards (Jeng et al., [18]). The research conducted by Das *et al.,* [19] focuses on investigating the radiation effect on fully developed mixed convection in a vertical channel filled with nanofluids. The researchers intend to comprehend the effect of radiation on the channel's flux and thermal behavior. This study investigates the behavior of nanofluids in the context of mixed convection heat transfer, thereby advancing the understanding of heat transfer phenomena in complex fluid systems. The result obtained shows that the critical Rayleigh number is strongly dependent on the nanoparticle volume fraction parameter and the thermal radiation parameter. Mixed convection is a type of flow that is affected by both the pressure gradient and thermal buoyancy. Research conducted by Rashevski *et al.,* [20] studied the combined effects of natural and mixed convection along with solar radiation in a vertical water-flow channel with transparent rigid walls. The analysis emphasizes the significant impact of solar radiation and mixed convection on the power outlet, showing a viable opportunity for energy harvesting on transparent building facades using vertical circulating water chambers. The effect of the chemical kinetic exponent on the transient magnetohydrodynamic (MHD) mixed convection of an exothermic fluid in a vertical channel was investigated in a study by Hamza and Shuaibu [21]. The goal of the numerical calculations, which make use of the IFD method, is to comprehend how the fluid would behave in response to different values of parameters like the thermal Biot number, chemical reaction parameter, Navier slip parameter, combined convection parameter, and magnetic field parameter. According to the results, the velocity increases as the thermal Biot number, chemical reaction parameter, Navier slip parameter, and combined convection parameter increase in value. Ajibade *et al.,* [22] investigated the effect of thermal dissipation on mixed convection flow in a partially filled vertical channel with porous materials. To get temperature and velocity fields, the governing equations are approximation analytically solved using the Homotopy Perturbation Method (HPM). The study's main result is that the Brinkman number significantly affects the channel's temperature and velocity profiles. This result suggests that regulating the Brinkman number may be crucial for optimising the fluid flow characteristics in composite channels.

There has been a lot of focus on investigating the effect of radiation on combined convection in vertical channels ever since its discovery. However, this topic does not have much attention to using internal heat generation in their model development. Internal heat generation is generating heat within a solid or fluid medium due to internal sources. Makinde [23] studied the hydromagnetic mixed convection stagnation point flow towards a vertical plate immersed in a very porous medium with radiation and internal heat production. In his paper, it has been visually shown how different embedded factors influence the speed, temperature, and concentration of a fluid which describes the fact that elevated internal heat production leads to a rise in fluid temperature, resulting in enhanced fluid velocity along the plate due to the buoyancy effect. The thermodynamics involved in the forced convective flow of a third-grade fluid via a vertical channel were analysed by Adesanya and Makinde [24]. Since the fluid is reactive, the impact of internal heat production is considered and assumed to be a linear function of temperature. According to their research findings, fluid velocity increased along with the value of the internal heat production parameter. It is because the fluid emits its emissions. Joshi *et al.,* [25] conducted a numerical study on the higher-order chemical reaction and volumetric heat production associated with the free convective flow of magnetic hybrid nanofluid across a bidirectional porous stretchy surface in three dimensions. They discovered that if the order of a chemical reaction is increased, the features of heat transmission in hybrid nanofluids rise, but the rate of mass transfer decreases. Jha and Samaila [26] investigated how nonlinear thermal radiation with suction or injection affected the mixed convection flow from a vertical porous plate heated by convection. The major impact of internal heat production is considered as well. They found that heat transmission might be improved by increasing internal heat production in the presence of weak convection and buoyancy effects but not with nonlinear thermal radiation. The previously cited literature has given interest and motivation to pursue further research in investigating the conjunction of combined convection, internal heat generation, radiation, and boundary conditions of a third kind.

Hence, the interaction of combined convection, radiation, internal heat generation, and boundary conditions of a third kind on a fully developed channel must be carried out. The primary purpose of the present work is to extend the studies of Grosan and Pop [9] in investigating the effect of radiation and the study on internal heat generation by Mealy and Merkin [27], as well as Yaman *et al.,* [28] on boundary conditions of a third kind.

The main purpose of the present work is to develop a mathematical model, find numerical solutions, and perform parametric studies to investigate the current problem. Several different analytical and numerical methods can be used to solve nonlinear systems of ordinary differential equations. In this study, shooting method is applied for solving the flow and temperature distributions using a built-in routine *dsolve* in MAPLE.

2. Mathematical Formulation

In the context of a fully developed parallel plate vertical channel, we analyse the characteristics of a continuous flow of a viscous and incompressible fluid, while also accounting for the influence of radiation. The channel possesses a parameter known as L, which signifies the measurement of the distance between the walls. The x-axis is chosen to be parallel to the vector of gravitational acceleration, g, but oriented in the opposite direction. The points $-\frac{L}{2}$ and $\frac{L}{2}$ correspond to the boundaries of the channel. The temperatures at the colder and hotter walls are represented by T_C and T_H respectively. The isothermal conditions at the boundaries located at $-\frac{L}{2}$ and $\frac{L}{2}$ are assumed with $T_C < T_H$. The flow has a uniform upward vertical velocity, U_0 at the channel entrance. The illustration of the geometry and boundary conditions is shown in Figure 1.

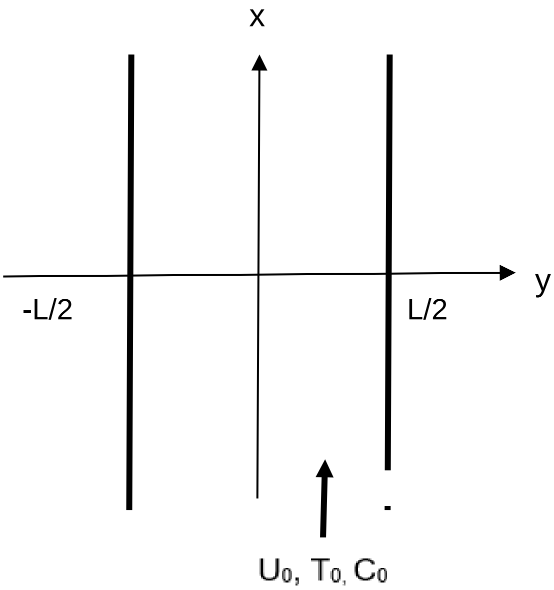


Fig. 1. Physical Configuration

The following relations apply if the fully developed assumption is adopted:

$$v = 0,\ \frac{\partial u}{\partial x} = 0,\ \frac{\partial T}{\partial x} = 0 \tag{1}$$

$$\frac{\partial p}{\partial y} = 0,\ \frac{\partial p}{\partial x} = P_x, \tag{2}$$

in which v is the velocity component in *y* direction and *p* is the pressure. This will lead to the unsteady flow of the viscous incompressible fluid within this channel under the Boussinesq's approximation which is governed by the following system of partial differential equations:

$$v\frac{\partial^2 u}{\partial y^2} + g\beta(T - T_0) - \frac{1}{P}\frac{\partial P}{\partial X} = 0 \tag{3}$$

$$\alpha\frac{\partial^2 T}{\partial y^2} + g(T - T_0)^{\lambda} - \frac{\partial q_r}{\partial y} = 0 \tag{4}$$

with the following boundary conditions:

$$u\left(-\frac{L}{2}\right) = 0 \qquad ;\ u\left(\frac{L}{2}\right) = 0$$

$$-k_{nl}\left.\frac{\partial T}{\partial Y}\right|_{Y=-\frac{L}{2}} = h_C\left[T_C - T\left(X, -\frac{L}{2}\right)\right]$$

$$-k_{nl}\left.\frac{\partial T}{\partial Y}\right|_{Y=\frac{L}{2}} = h_H\left[T\left(X, -\frac{L}{2}\right) - T_H\right] \tag{5}$$

The non – dimensional quantities are shown below:

$$\begin{aligned}
&X = \frac{x}{LRe}\ , \quad Y = \frac{y}{L}\ , \quad U = \frac{u}{u_0}\ , \quad Re = \frac{u_0 L}{v}\ , \quad GR = \frac{Gr}{Re}\ , \quad \gamma = \frac{dP}{dX} \\
&\theta = \frac{T-T_0}{T_c-T_0}\ , \quad \omega = \frac{\lambda}{pu_0^2}\ , \quad G = \frac{gL^2(\Delta T)^p-1}{\alpha}\ , \quad Gr = \frac{g\beta\Delta T L^3}{v^2} \\
&R_d = \frac{4\sigma Tm^3}{k\chi}\ , \quad q_r = -\left(\frac{4\sigma}{3\chi}\right)\frac{\partial T^4}{\partial y}\ , \quad \theta_R = \frac{T_2}{T_m}\ , \quad S = \frac{Bi_C Bi_H}{Bi_C Bi_H + 2Bi_C + 2Bi_H}
\end{aligned} \tag{6}$$

These parameters will be substituted into the equations (2), (3), (4), (5) and (6) to transform into non – dimensional equation.

Hence, these are the dimensionless forms of the governing equations:

$$\frac{d^2U}{dY^2} + GR\theta - \gamma = 0 \tag{7}$$

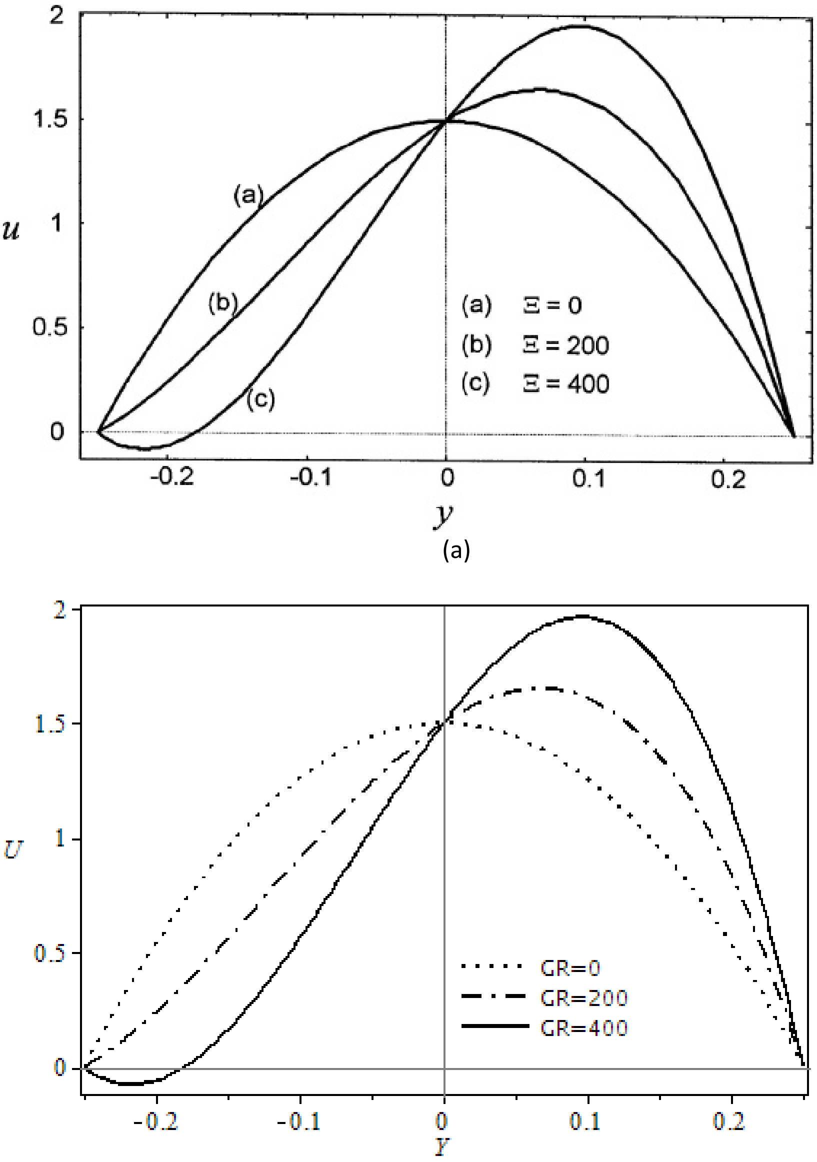
$$\frac{d}{dY}\left\{\left[1 + \frac{4}{3}R_d[1 + (\theta_R - 1)\theta]^3\right]\frac{d\theta}{dY}\right\} + G\theta^{\lambda} = 0 \tag{8}$$

and the corresponding boundary conditions are

$$\begin{aligned}
&U\left(-\frac{1}{4}\right) = 0,\ U\left(\frac{1}{4}\right) = 0 \\
&\left.\frac{d\theta}{dy}\right|_{y=-\frac{1}{4}} = Bi_C\left[\theta\left(-\frac{1}{4}\right) + \frac{R_T S}{2}\left(1 + \frac{4}{Bi_C}\right)\right] \\
&\left.\frac{d\theta}{dy}\right|_{y=\frac{1}{4}} = Bi_H\left[-\theta\left(\frac{1}{4}\right) + \frac{R_T S}{2}\left(1 + \frac{4}{Bi_H}\right)\right]
\end{aligned} \tag{9}$$

where *U* is the dimensionless velocity, ϑ is the dimensionless fluid temperature, *G* is the internal heat generation., GR is the combined convection parameter, G is the internal heating parameter, λ is the local heating generation, R_d mass coefficient transfer with radiation and Bi is the Biot numbers.

In justifying the numerical code, it is important to carry out a similarity test with the previous related studies. The current numerical outputs are compared to the output acquired by Barletta [29] as shown in Figure 2. Barletta [29] did an analytical study on combined, free and forced convection flow in a parallel plate vertical channel in the fully developed region by taking into account the effect of viscous dissipation. The study found that viscous dissipation enhances the effect of flow reversal in the case of downward flow while it lowers this effect in the case of upward flow. Viscous dissipation also increases the buoyancy forces. Obviously, from this observation, the results of the current study agree well with the results by Barletta [29].



(a)

(b)

Fig. 2. Velocity configurations of (a) Barletta [29] and (b) current research for different combined convection parameter *GR*

In this paper, MAPLE, a fourth-generation programming language developed by MathWorks will be used for the numerical computing. Maple is available in many operating systems such as Windows, Linux and Mac OS. The *pdsolve* code in MAPLE solves partial differential equations (PDEs) and systems of PDEs. Finding analytical solution is the main purpose of the *pdsolve* function. *pdsolve* would manage to solve PDEs or PDE systems regardless of the type, differential order, or number of dependent or independent variables of the systems. For now, only certain PDE groups that could be solved with common approach are acknowledged by *pdsolve* command. If the PDE comes from a group which is not acknowledged by *pdsolve* command, a heuristic algorithm which separates the variables according to certain structure of the PDE is applied. *pdsolve* targeted in producing the most common solution of the PDE or simply to get a complete separation of variables.

3. Results and Discussions

The problem investigates radiation on combined convection fully developed vertical channel with intenal heat generation. It is characterized by 6 dimensionless parameters within the given range: combined convection parameter $100 \leq GR \leq 400$, internal heat generation $0 \leq G \leq 20$, local-heating exponent $1 \leq p \leq 9$, Biot numbers $0 \leq Bi \leq \infty$, radiation parameter $0.4 \leq R_d \leq 1.6$, and temperature parameter $1.1 \leq \theta_R \leq 2.0$

3.1 Velocity and temperature profiles

The velocity profile depicted in Figure 3 illustrates the influence of GR, highlighting a more significant impact on the left wall. Increasing GR results in a notable decrease in flow rate, with no signs of reversal flow observed at GR = 100. However, beyond GR > 200, a reversal flow in the velocity profile becomes evident. This phenomenon is attributed to the intensified wall heating at higher GR values, leading to buoyancy-assisted flow reversal. A heightened combined convection parameter induces significant distortions in velocity configurations, marked by the presence of strong reverse flows that are highly unstable and exhibit substantial fluctuations. In contrast, the temperature profile, as depicted in Figure 4, remains unaffected by variations in GR. This observation suggests that the influence of GR on the temperature profile diminishes with increasing GR values.

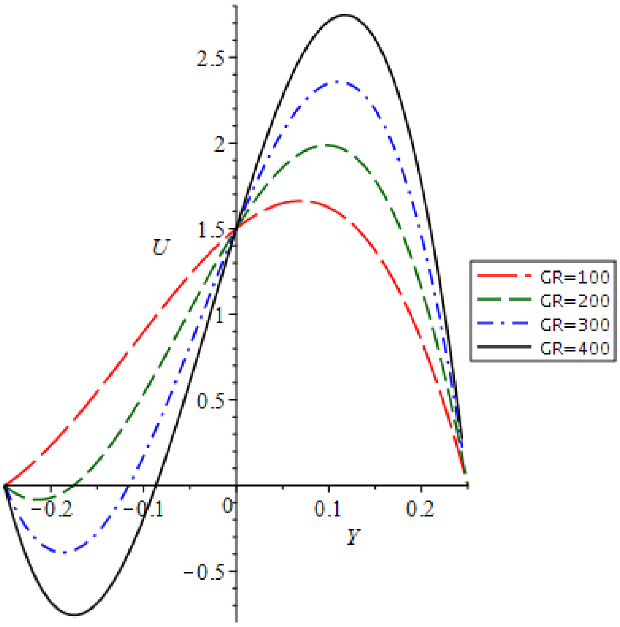


Fig. 3. Velocity profile of varied Combined Convection Parameter (GR)

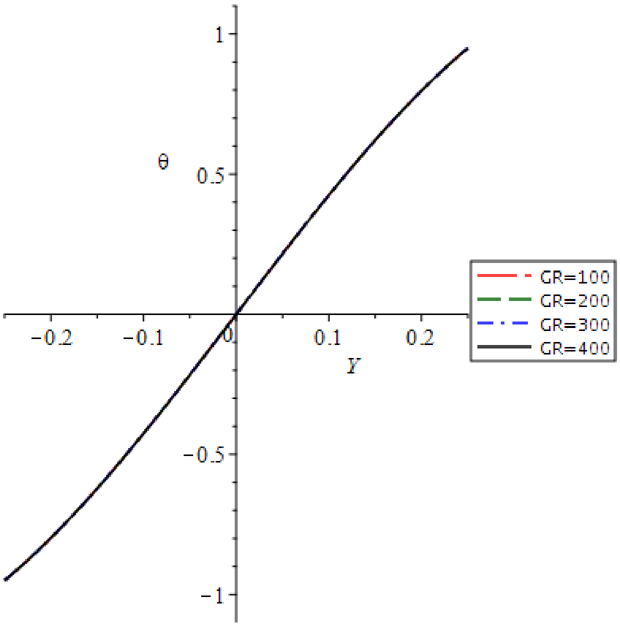


Fig. 4. Temperature profile of varied Combined Convection Parameter (GR)

The influence of internal heat generation, G, on the dimensionless velocity profile, U(Y) and temperature profile, θ(Y) is shown in Figure 5 and Figure 6. The influence of dimensionless velocity and temperature profiles using four (4) different values of G are investigated in this study. It is significant to remember that the negative signs in G figures are only employed to indicate conditions related to heat absorption and transfer. Fluid cooling is linked to a positive value of G, while fluid heating is linked to a negative value. The velocity increases on the right side of the wall and decreases on the left as the value of G rises. The graph in Figure 5 indicates that flow reversal happens at the left wall. The impact of internal heat generation on the temperature profile is seen in Figure 6. The fluid heats up as the value of G increases, moving heat from centre of the channel to the hotter wall of the hotter channel because of the rising temperature on the right side of the wall.

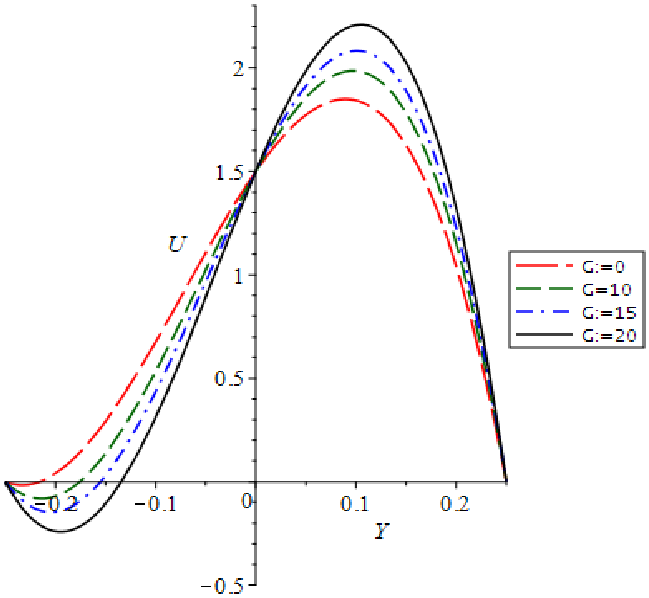


Fig. 5. Velocity profile of varied internal heat generation (G)

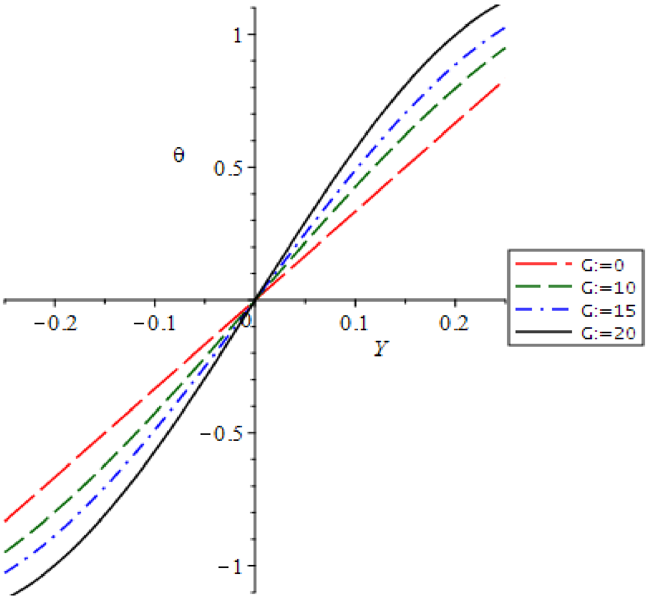


Fig. 6. Temperature profile of varied internal heat generation (G)

In this study, we use four (4) different values of local heating exponent, p. Figure 7 and Figure 8 show, correspondingly, the effect of different values of p on the velocity and temperature profiles. The representation in the figure displays that the dimensionless velocity and dimensionless temperature are increasing functions of p as the colder wall is approached and decreasing functions of p as the hotter wall is approached. When p falls below the value of 1, the flow reversal occurs on the left side of the channel. As p rises, it is observed that the velocity distributions become increasingly non-symmetric with respect to the centre of the channel. Additionally, it can be shown that the effect of p on the design velocity decreases as p increases. As p increases, the velocity profile does not exhibit appreciable changes in flow design because the G value predominates over the p value. As observed in the figure 8, the dimensionless temperature rises from the colder wall to the centre of the channel. When p is changed, the flow pattern appears comparable. It is demonstrated that the impact of p on the temperature profiles decreases with increasing power of p.

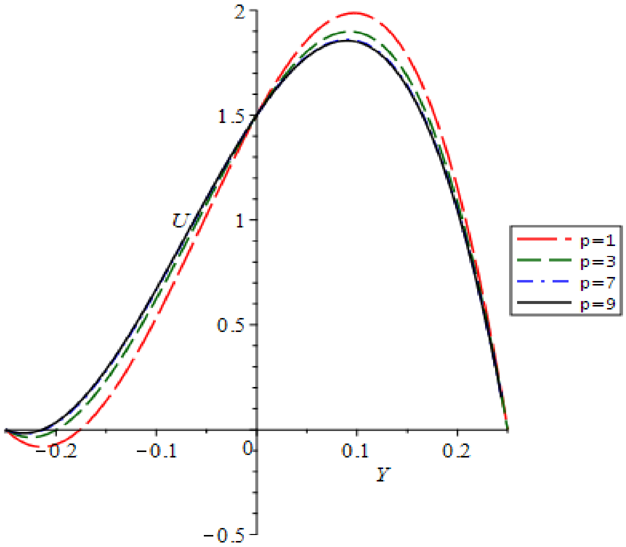


Fig. 7. Velocity profile of varied local heating exponent (p)

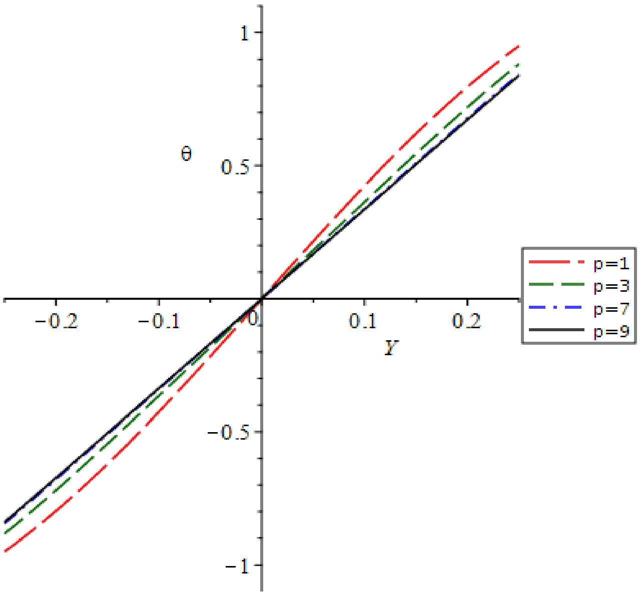


Fig. 8. Temperature profile of varied local heating exponent (p)

In Figure 9 and Figure 10, we can see significant effect on the three different boundary conditions on the velocity and temperature profiles respectively. The graph pattern in both figures shows that the Neuman boundary condition is relevant when Bi = 0, the Robin boundary condition when Bi = 10, and the Dirichlet boundary condition when Bi = ∞. The velocity profiles at the heated wall increase as Bi rises, whereas the channel thermal resistance at the cold wall decreases. The Biot Number (Bi) represents the thermal resistance ratio between the liquid and the channel. In the context of this research, the Biot numbers on the cooler (left) and warmer (right) walls are denoted as Bi_C and Bi_H, respectively. If the Biot Number is large (Dirichlet condition), then the velocity and temperature profile close to the hot wall are large as well.

Based on the data presented in Figure 10, it can be observed that since the hot side of the channel is totally insulated, no convective heat is transferred to the cold side of the wall. Therefore, the temperature profile remains constant. In the meantime, the same trend happens when Bi increase where the channel's thermal resistance at the cool wall goes down, which causes the temperature profiles at the hot wall to rise. This is because when the convective process's strength was increased, the buoyancy forces acting on the channel were also increased, leading to higher heat transfer. Because of this, the amplitude of the velocity and temperature profile at the right wall of the channel increases as the value of Bi in the Dirichlet boundary condition increases. Due to the region of change reaching the border, the outcome would be impractical on the narrower domain of Dirichlet condition. In contrast, the Neumann condition is inferior to the Robin condition.

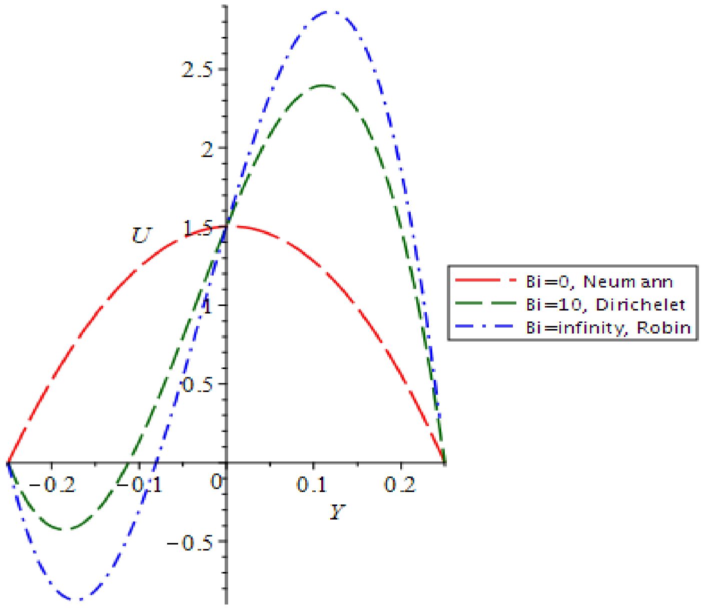


Fig. 9. Velocity profile for three kinds of boundary conditions on temperature

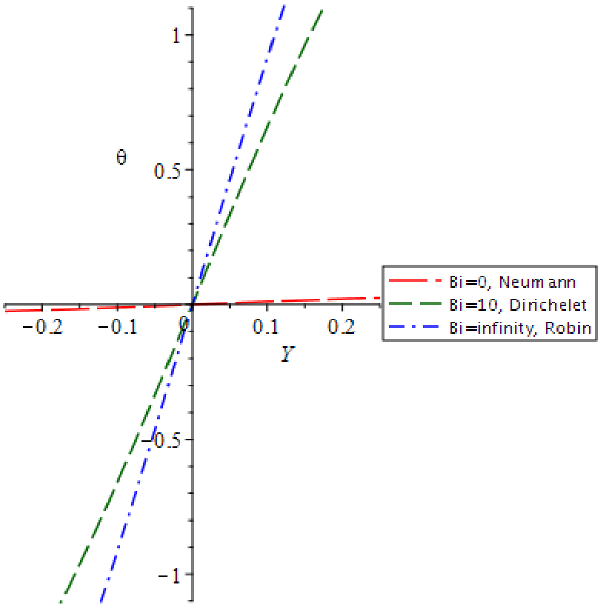


Fig. 10. Temperature profile for three kinds of boundary conditions on temperature

The temperature's temporal variation in relation to changes in the radiation parameter, R_d is depicted in Figures 11 and Figure 12. The relative significance of radiant heat transmission compared to conductive heat transfer is shown by the radiation parameter, R_d. Both the velocity profile and temperature profile in Figure 11 and Figure 12 respectively shows and increasing value with the increase of R_d from the cold wall to mid-channel and later decrease with the increase of R_d from mid-channel to the hotter wall. Flow reversal is seen at the left wall, since $R_d \geq 0.4$. Therefore, a larger value of R_d indicates a larger amount of radiant energy poured into the system. In the case of water, where convective heat transfer is stronger, an increase in R_d also results in an increase in velocity and temperature.

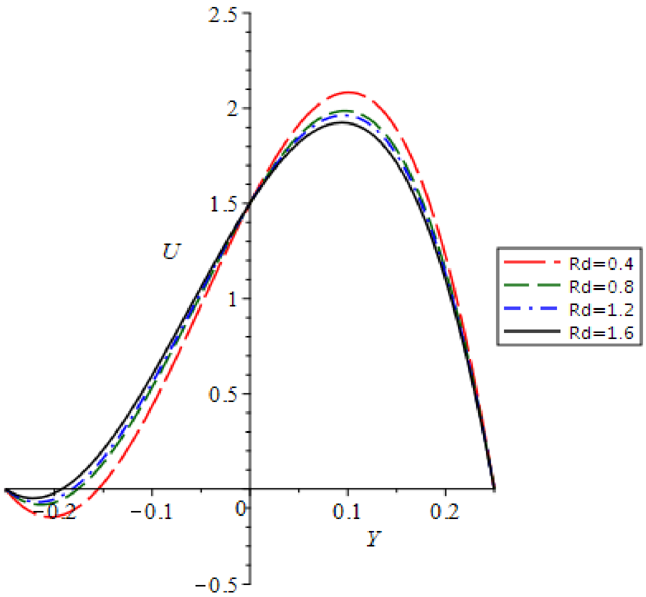


Fig. 11. Velocity profile for Radiation parameter (R_d)

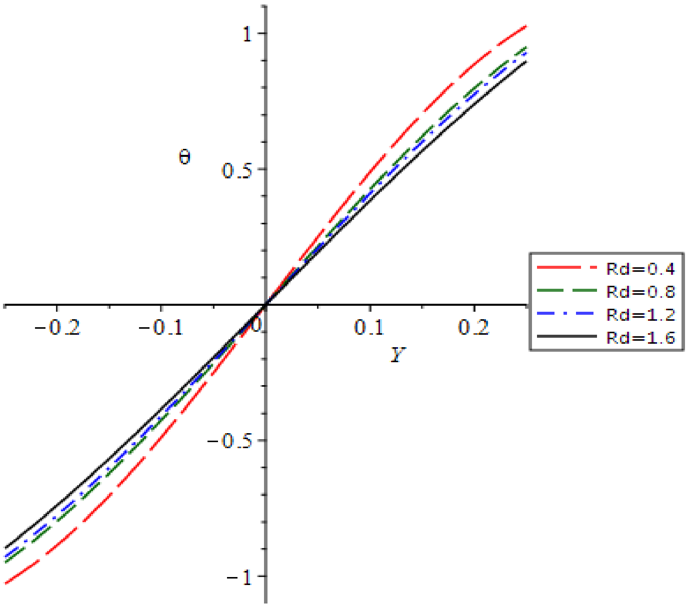


Fig. 12. Temperature profile for Radiation parameter (R_d)

4. Conclusions

This study focuses on the heat transfer evaluation on unsteady fully developed oscillatory flow in combined convection vertical channel with non-uniform internal heating. Runge−Kutta method with shooting technique was applied to solve the dimensionless forms of the governing equations. The main results are as follows:

1. In general, since the temperature on the left channel is constant whereas the temperature on the right channel varies sinusoidally in time, the magnitude of oscillation is more severe for the graphs of Nusselt numbers on the right wall.

2. It is found that a large development of flow is caused by the plate temperature oscillation with high amplitude, therefore, the nature of the flow differs from the non-oscillating case. As a result; A wider fluctuating of the Nusselt numbers for higher amplitude values and this is more obvious at the right wall.

3. The oscillatory heat transfer for both walls generally has the highest amplitude at relatively low oscillating frequency. In such a condition, the momentum and convective flow slow down with slight modification in repetition of the thermal frequency.

The heat transfer process of combined convection and its flow pattern in a vertical channel is important, especially in environmental, industrial, and engineering applications. These results would help to overcome the concern regarding the heat transfer process in which it is difficult, expensive and time consuming.

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