Journal of Advanced Research in Numerical Heat Transfer

Journal homepage:
https://semarakilmu.com.my/journals/index.php/arnht/index
ISSN: 2735-0142

Computational Investigation of Brownian Motion and Thermophoresis Effect on Blood-based Casson Nanofluid on a Non-linearly Stretching Sheet

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ARTICLE INFO

Article history:
Received 23 December2023
Received in revised form 20 January 2024
Accepted 21 Februrary 2024
Available online 30 March 2024

Keywords:
Boundary layer flow; Thermophoresis; Brownian motion; Partial slip; Runge-Kutta method

ABSTRACT

The present article investigates numerically the impacts of thermophoresis and Brownian motion on boundary layer nanofluid flow across a porous, non-linearly stretching sheet at a fixed temperature of the surface and under partial slip. Using the similarity variables, non-linear ordinary differential equations are obtained from the dictating partial differential equations, and then these non-linear ordinary differential equations are converted to a first order system of differential equations. The obtained first order system is then executed using bvp4c module in MATLAB along with the shooting technique. The numerical solutions obtained are scrutinized through graphs. It is observed that, as Nt and Nb levels rise, the temperature rises, and the thermal boundary layer thickens. Additionally, it has been found that the energy distribution expands, and nanoparticle concentration falls with rising values of the thermophoresis parameter.

1. Introduction

Nanofluids are engineered colloidal suspensions containing nanoparticles (typically with dimensions less than 100 nanometers) dispersed in a base fluid, such as water, oil, or ethylene glycol. These nanoparticles can be metallic, metal oxide, carbon-based, or other materials, depending on the desired properties of the nanofluid. The addition of nanoparticles to the base fluid imparts unique characteristics to the nanofluid, making it suitable for various applications. One of the primary advantages of nanofluids is their significantly enhanced thermal conductivity compared to the base fluid. This property is particularly useful in heat transfer applications, such as cooling in electronics, automotive engines, and industrial processes [1]. Nanofluids are utilized to improve heat transfer efficiency in heat exchangers, radiators, and other thermal management systems. The enhanced thermal conductivity of nanofluids allows for better heat dissipation, leading to improved performance and energy efficiency.

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https://doi.org/10.37934/arnht.18.1.4967

Nanofluids can exhibit unique optical and electronic properties based on the type of nanoparticles used. For instance, gold or silver nanoparticles can impart plasmonic properties, making nanofluids useful in applications like sensing and imaging. Nanofluids have potential applications in the biomedical field, including drug delivery systems and diagnostic imaging [2]. Magnetic nanoparticles in nanofluids can be used for targeted drug delivery under the influence of external magnetic fields.The addition of nanoparticles can influence the rheological properties of nanofluids, including changes in viscosity and shear-thinning behavior. Challenges in nanofluid research include issues related to stability, aggregation, and the potential for clogging in certain applications. Understanding the long-term stability and behavior of nanofluids is crucial for their successful implementation [3].

Thermophoresis refers to the motion of particles in a fluid due to a temperature gradient. When it comes to nanofluid, which are fluids containing nanoparticles (typically with dimensions less than 100 nanometers), thermophoresis can have several effects. Nanoparticles in a nanofluid may migrate towards regions of higher or lower temperature, depending on the nature of the thermophoretic force. This migration can potentially enhance heat transfer characteristics in the fluid, leading to improved thermal conductivity. The presence of nanoparticles in a fluid can significantly enhance the effective thermal conductivity of the nanofluid compared to the base fluid [4]. Thermophoretic effects can further influence the distribution of nanoparticles in the fluid, impacting the overall thermal performance. Thermophoresis may contribute to the agglomeration of nanoparticles, which can affect their dispersion stability and lead to sedimentation [5]. The addition of nanoparticles might alter the viscosity of the nanofluid, influencing its flow characteristics. Nanofluids with thermophoretic effects find applications in various heat transfer systems, such as cooling in electronics, solar collectors, and heat exchangers. In biological and medical applications, thermophoresis in nanofluids can be utilized for targeted drug delivery or imaging. Modeling thermophoresis in nanofluids can be complex and may require sophisticated numerical simulations to capture the interactions between nanoparticles and temperature gradients accurately. Experimental techniques, such as particle tracking, can be employed to study the thermophoretic behavior of nanoparticles in nanofluids under different temperature conditions [6].

Brownian motion is the random motion of particles suspended in a fluid due to collisions with the fluid molecules. In the context of nanofluids, which consist of nanoparticles dispersed in a base fluid, Brownian motion can have several significant effects. Sarfraz *et al.,* [7] studied thermophoresis and Browian motion for nanofluid flow over cylinder. Brownian motion plays a crucial role in preventing the agglomeration or sedimentation of nanoparticles in the nanofluid. The constant random motion of nanoparticles disrupts their tendency to settle or cluster together, maintaining a more uniform dispersion in the fluid. Brownian motion increases the effective thermal conductivity of nanofluids by promoting better dispersion of nanoparticles. The continuous motion of nanoparticles helps in spreading them evenly throughout the fluid, enhancing their interaction with the base fluid and improving thermal conductivity [8]. Brownian motion can lead to an increase in the apparent viscosity of nanofluids, especially at high nanoparticle concentrations. The random movement of nanoparticles hinders the flow of the fluid, resulting in an increase in viscosity, which can influence the fluid's rheological properties. Brownian motion is more pronounced for smaller particles due to their higher diffusivity. Smaller nanoparticles experience more significant Brownian motion, which can impact their dispersion behaviour and heat transfer characteristics differently compared to larger nanoparticles. Brownian motion contributes to the diffusion of nanoparticles in the fluid. This diffusion is a result of the random movements of particles, allowing them to move from regions of higher concentration to regions of lower concentration, contributing to the overall transport phenomena in the nanofluid. Brownian motion affects the measurement and characterization of nanofluids [9].

Nanoparticles in the fluid can affect the thermal and rheological properties of the nanofluid, influencing the heat transfer characteristics. The Brownian motion and thermophoretic motion of nanoparticles may need to be considered, especially if their effects are significant at the length scales of the flow . One of the primary motivations for using nanofluids in such scenarios is the potential enhancement of heat transfer. The increased thermal conductivity of the nanofluid, due to the presence of nanoparticles, can lead to improved heat dissipation and more efficient cooling.

When a fluid containing nanoparticles flows over a linearly stretching sheet, it forms a boundary layer near the surface improved heat transfer over the stretching sheet [10,11]. Cortell [12] described the viscous flow and heat transfer over stretching sheets. Chandrashekhar *et al.,* [13] discussed flow of nanofluid flowing over Riga sheet. Nanoparticles in the fluid contribute to increased thermal conductivity, which is advantageous for applications requiring efficient heat dissipation. A numerical simulation of heat transfer for nanofluids on non-linearly extending sheets has been performed by Rana and Bhargava [14]. Das [15] explained the impact of slip and suction on nanofluid flow onto an extending sheet. Mathematical models, such as the boundary layer equations, are often employed to analyze linear stretching flows with nanofluids [16]. Analytical and numerical methods help in solving the governing equations to understand the flow and heat transfer characteristics [17,18].

In cases of non-linear stretching, where the sheet accelerates, the flow dynamics become more complex. The changing velocity and temperature profiles, especially in the stretching direction, are influenced by both the stretching rate and the presence of nanoparticles [19,20]. Sk *et al.,* [21] described the impact of magnetic field over a non-linearly stretching sheet for nanofluids. Non-linear stretching introduces transient behaviour in the flow, which can have implications for heat transfer processes. Nanofluids may exhibit unique behaviour during transient phases, and their effects need to be considered in the analysis [22]. Non-linear stretching may result in non-uniform stretching rates along the sheet. This non-uniformity can affect the distribution of nanoparticles in the nanofluid, influencing the local heat transfer rates. Non-linear stretching problems often require sophisticated numerical simulations due to the complexity of the governing equations. Computational fluid dynamics (CFD) methods are commonly used to model and understand the flow and heat transfer characteristics in non-linear stretching scenarios. Brownian motion and thermophoresis of nanoparticles play a role in their distribution and behavior near stretching sheets [23]. The impact of these motions on the stability and efficiency of nanofluid flows needs to be considered. In this regard, Pantokratoras [24] explored the common error made during the investigation of boundary layer flows. It is crucial to conduct a thorough review of the assumptions, numerical methods, and experimental setups, and to validate the results against known benchmarks or experimental data to avoid the error. Four usual errors which arises in boundary layer flows have been well described by Pantokratoras [25].

Understanding the behavior of nanofluids in both linear and non-linear stretching scenarios is relevant to various applications, including industrial processes, coating applications, and material manufacturing as well as other biomedical applications [26-29]. Research in this area contributes to the understanding of how nanofluids influence the flow and heat transfer characteristics in different stretching scenarios, providing insights for applications in various industries. The current problem is designed to explore the flow of blood with nanoparticles on a stretching sheet which brings out the transient behaviour due to non-linear stretching. The originality of the current research is the inclusion of slip parameter, thermophoresis and Brownian motion effects for blood based nanofluid.

The numerical solution using bvp4c module of MATLAB with shooting technique is carried out to obtain results. The current research opens new opportunities in industrial and biomedical research.

2. Modeling of the problem

The significance of non-linearly stretching a sheet in the context of blood flow lies in its potential to better capture the complex and dynamic nature of blood vessel deformation. Non-linear stretching allows for a more realistic representation of the elasticity and compliance of blood vessels, which play crucial roles in determining blood flow patterns. By incorporating non-linear stretching, one can obtain more accurate simulations that consider the intricate biomechanical behavior of blood vessels under varying physiological conditions. This approach is particularly valuable in medical and biological research, providing insights into the effects of non-uniform stretching on blood flow characteristics and facilitating a deeper understanding of vascular dynamics and related health implications.

In this study, we will explore the two-dimensional, steady boundary layer flow of nanofluid induced by a stretching surface, assuming that the fluid behaves as a continuum and possesses incompressible and non-Newtonian characteristics. Additionally, external magnetic field B is applied transverse to the flow and the Reynold’s number is taken too small such that the induced magnetic field is negligible as compared to the external magnetic field which is given by $B = B_0 x^{\frac{n-1}{2}}$, where B_0 is initial strength of magnetic field. The stretching of the sheet initiates the Casson nanofluid's flow past a stretching sheet which is flowing with velocity $u_b = ax^n$ in which a is a fixed number, parameter n is stretching parameter which changes non-linearly and x represents the co-ordinate which is determined along the extending surface. The flow is considered in the upper half plane above x-axis such that y is perpendicular to stretching surface as shown in Figure 1. The Casson nanofluid's Cauchy stress tensor rheological equation is given by

$$\tau = \tau_0 + \mu\gamma, \tag{1}$$

$$\tau_{ij} = \begin{cases} 2\left(\mu_B + \dfrac{p_y}{\sqrt{2\pi}}\right)e_{ij}, & \pi > \pi_C \\ 2\left(\mu_B + \dfrac{p_y}{\sqrt{2\pi_C}}\right)e_{ij}, & \pi < \pi_C \end{cases}. \tag{2}$$

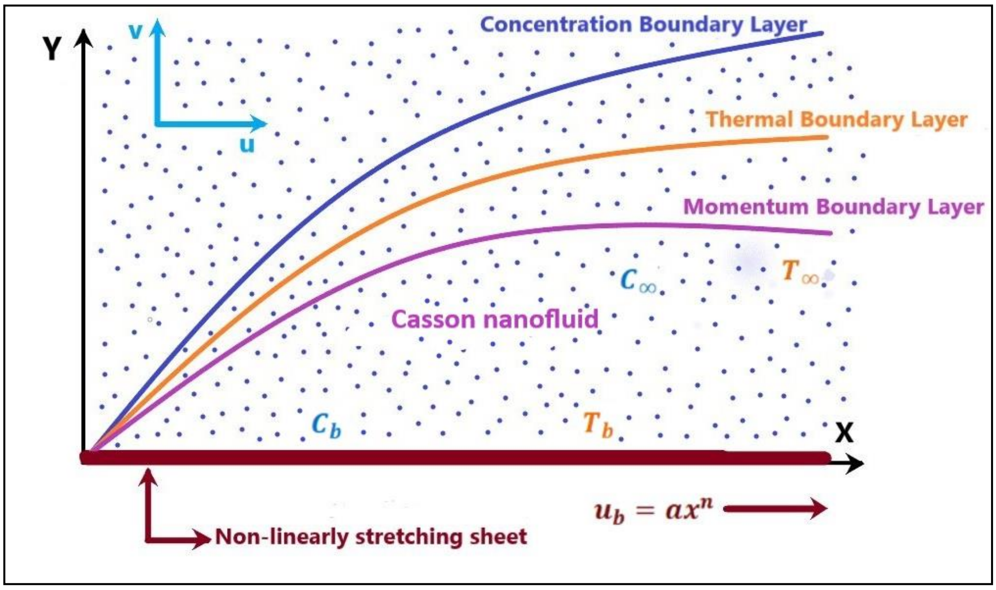

Fig. 1. Casson Nanofluid Boundary layer flow model

The approximation of boundary layer under the given assumptions is dictated by the following differential equations [10,26]:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{3}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\nu\left(1+\frac{1}{\beta}\right)\frac{\partial^2 u}{\partial y^2}-\frac{\sigma B(x)^2}{\rho}u, \tag{4}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}+\tau\left[D_B\left(\frac{\partial C}{\partial y}\right)\left(\frac{\partial T}{\partial y}\right)+\left(\frac{D_T}{T_\infty}\right)\left(\frac{\partial T}{\partial y}\right)^2\right], \tag{5}$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=\frac{D_T}{T_\infty}\frac{\partial^2 C}{\partial y^2}+D_B\frac{\partial^2 C}{\partial y^2}, \tag{6}$$

under boundary conditions

$$\left.\begin{array}{ll} u=u_b+u_s, v=\pm v_b, T=T_b, C=C_b & \text{when } y=0 \\ u\to 0, T\to T_\infty, C\to C_\infty & \text{when } \; y\to\infty \end{array}\right\}. \tag{7}$$

Here (u, v, 0) represents fluid velocity, ν represents kinematic viscosity, α represents thermal diffusivity, β represents Casson fluid parameter, $\tau=\dfrac{(\rho c)_p}{(\rho c)_f}$ represents the proportion between nanoparticle and fluid's heat capacities, ρ_f represents denseness of base fluid, C represents nanoparticle volumetric fraction, ρ_p represents density of nanoparticles, D_B represents Brownian diffusion coefficient, D_T represents Thermophoretic diffusion coefficient, v_b represents suction and u_s represents slip velocity which is proportional to wall shear stress at $y=0$.

Using the following similarity functions,

$$\left.\begin{array}{c} \eta = y\sqrt{\dfrac{a(n+1)}{2v}}x^{\frac{n-1}{2}}, u = ax^n f'(\eta) \\ v = -\sqrt{\dfrac{av(n+1)}{2}}x^{\frac{a-1}{2}}\left(f + \left(\dfrac{n-1}{n+1}\right)\eta f'\right) \\ \theta(\eta) = \dfrac{T-T_\infty}{T_b - T_\infty}, \phi(\eta) = \dfrac{C-C_\infty}{C_b - C_\infty} \end{array}\right\} \tag{8}$$

the non-linear coupled differential equations are created from the boundary layer equations (4), (5), (6) and are given by

$$\left(1+\frac{1}{\beta}\right)f''' + ff'' - \left(\frac{2n}{n+1}\right)f'^2 - M^2 f' = 0 \tag{9}$$

$$\frac{1}{Pr}\theta'' + f\theta' + Nb\theta'\phi' + Nt\theta'^2 = 0 \tag{10}$$

$$\phi'' + \text{Lef}\,\phi' + \frac{Nt}{Nb}\theta'' = 0 \tag{11}$$

and boundary conditions in (7) becomes

$$\left.\begin{array}{l} f = f_w, f' = 1+\zeta_p f'', \theta = 1, \phi = 1 \quad \text{when} \quad \eta = 0 \\ \quad f' \to 0, \theta \to 0, \phi \to 0 \text{ when} \quad \eta \to \infty \end{array}\right\} \tag{12}$$

Here derivatives w.r.t η are denoted by prime, $Le = \dfrac{\alpha}{D_B}$ represents Lewis number, $Pr = \dfrac{v}{\alpha}$ represents Prandtl number, $M = \dfrac{\sigma B_0^{\,2}}{\rho a}$ constitute the magnetic parameter, $Nt = \dfrac{(\rho c)_\beta D_T\left(T_b - T_\infty\right)}{(\rho c)_f vT_\infty}$ is thermophoresis parameter, $Nb = \dfrac{(\rho c)_p D_B\left(C_b - C_\infty\right)}{(\rho c)_f}$ is Brownian motion parameter, $\zeta_p = l\sqrt{\dfrac{a(n+1)}{2x}}x^{n-1}$ is slip parameter and $f_w = -v_0\sqrt{\dfrac{2}{av(n+1)}}$ is suction parameter.

It is important to highlight that $f_w = 0$ in the case of an impermeable surface. For scenarios involving suction, $f_w > 0$, and for instances of fluid injection on a permeable sheet, $f_w < 0$.

Incorporating the engineered physical parameters, parameters of interest for study, Nusselt number, Sherwood number, and skin friction, for the posed problem are as follows:

$$\left.\begin{aligned} Nu\,\mathrm{Re}_x^{-\frac{1}{2}} &= -\theta'(0) \\ Sh\,\mathrm{Re}_x^{-\frac{1}{2}} &= -\phi'(0) \\ C_f\,\mathrm{Re}_x^{\frac{1}{2}} &= \left(1+\frac{1}{\beta}\right) f''(0) \end{aligned}\right\}. \tag{13}$$

where Re_x is the local Reynold's number.

3. Methodology of the Solution

The complex nature of current problem is aligned with the solution strategy used in bvp4c module. The bvp4c module in MATLAB is specifically tailored for solving BVPs, where the solution needs to satisfy conditions at both the initial and final points. It is designed to handle systems of non-linear ODEs, making it suitable for a broad class of problems where the relationship between variables is non-linear. It internally employs a shooting technique to solve the BVP. This method iteratively adjusts the initial conditions to satisfy both the boundary conditions and the ODE system to provide accurate solutions for a variety of non-linear problems.

To execute bvp4c in the current problem, Eq. (9)–(11) are converted into a system of first order differential equations by the following substitutions:

$$\left.\begin{aligned} f = y_1,\, f' = y_1' = y_2,\, f'' = y_2' = y_3,\, f''' = y_3', \\ \theta = y_4, \theta' = y_4' = y_5, \theta'' = y_5', \\ \phi = y_6, \phi' = y_6' = y_7, \phi'' = y_7' \end{aligned}\right\} \tag{14}$$

The obtained first order system is represented in the matrix form as follows:

$$\begin{pmatrix} y_1' \\ y_2' \\ y_3' \\ y_4' \\ y_5' \\ y_6' \\ y_7' \end{pmatrix} = \begin{pmatrix} y_2 \\ y_3 \\ \dfrac{-ff'' + \left(\dfrac{2n}{n+1}\right) f'^2 + M^2 f'}{\left(1+\dfrac{1}{\beta}\right)} \\ y_5 \\ \Pr * \left(-f\theta' - Nb\theta'\phi' - Nt\theta'^2\right) \\ y_7 \\ -Lef\phi' - \dfrac{Nt}{Nb}\theta'' \end{pmatrix} \tag{15}$$

with boundary conditions

$$\left.\begin{array}{c} y_1(0) = f_w, y_2(0) = 1 + \zeta_p y_3(0), y_1(0) = 1, y_6(0) = 1 \\ y_2(\infty) = 0, y_1(\infty) = 0, y_6(\infty) = 0 \end{array}\right\} \tag{16}$$

The numerical computation of the non-linear boundary value problem (BVP) presented in Eq. (9)–(11) has been carried out using the bvp4c module in MATLAB along with the shooting technique. The shooting technique facilitates the guess for obtaining missing initial conditions. Accuracy of numerical values is taken upto order 10^{-6}.

4. Numerical Results and Discussion

The current research is carried out to understand the influences of thermophoresis and Brownian motion on velocity and temperature profiles of blood-based Casson nanofluid. The impact of slip parameter on the flow is also measured for better analysis of behaviour of Casson-nanofluid. The non-linear stretching of sheet is undertaken on which nano-blood is flowing. The governing PDEs of the flow are first transformed to a set of non-linear differential equations. These non-linear differential equations models a first-order system of differential equations which is then operated in bvp4c solver of MATLAB to give numerical results and graphs. The numerical solutions for concentration of nanoparticles, temperature and velocity have been graphically represented in Figures 2-13 for various values of suction parameter, slip parameter and non-linear stretching parameter in order to clearly understand the current issue. The values of various parameters is taken as $f_w = 0.2$, $Le = 5.0$, $\zeta = 1$, $Pr = 25$, $Nb = 0.5$, $Nt = 0.5$ and $\alpha = 2.0$ unless explicitly mentioned.

Figure 2 illustrates the impact of the velocity slip parameter on the velocity field in the presence or absence of a magnetic field. The observation indicates that the velocity slip has the effect of reducing the thickness of the hydrodynamic boundary layer. Generally, an increase in the slip parameter enhances the surface slip at the wall, resulting in a smaller amount of penetration into the fluid from the stretching surface. Under a no-slip condition where $\zeta \to 0$, the slip velocity at the wall is zero, making the fluid velocity at the wall equal to the velocity of the stretching surface, denoted as $f'(0) = 1$, as confirmed by Figure 3. The figure also depicts that the velocity component at the wall decreases with an increase in the slip parameter, both in the presence and absence of a magnetic field. Additionally, this velocity asymptotically approaches zero at the edge of the hydrodynamic boundary layer.

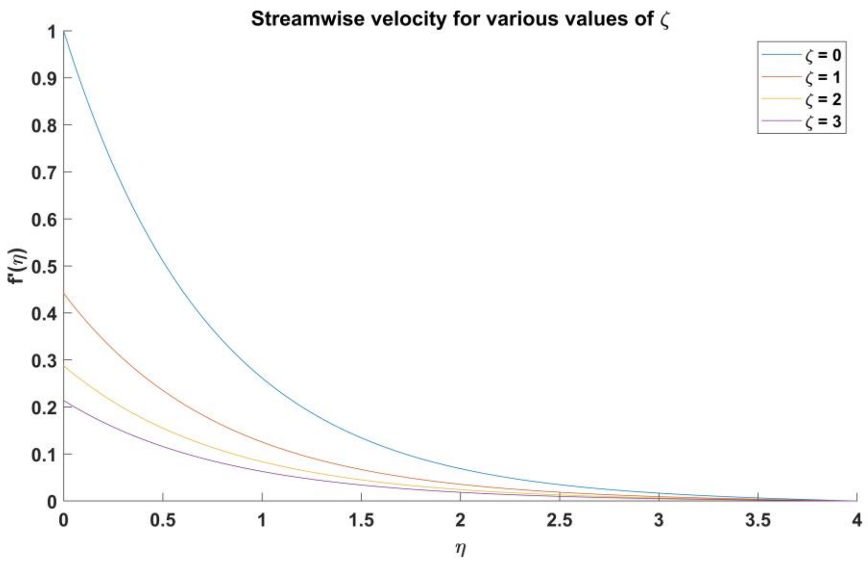

Fig. 2. Stream-wise velocity of nanofluid for various values of slip parameter

Figure 3 describes the impact of velocity slip on temperature profiles, revealing that an increase in the velocity slip parameter (ζ) results in elevated fluid temperatures within the boundary layer region in the presence and absence of magnetic field. Consequently, the thermal boundary layer thickness increases. It is noteworthy that the influence of ζ on temperature distribution is particularly noticeable in the presence of a magnetic field. Figure 4 depicts the impact of slip velocity on nanoparticle concentration. Figure 4 shows that, a rise in the values of parameter of slip causes an increment in concentration of nanoparticles In general, when the species concentration at the boundary wall exceeds that of the free stream, there is a noticeable gradual decrease in the concentration profile toward the free stream. Conversely, if the species concentration at the plate surface is lower than the concentration in the free stream, the observed effect is the opposite.

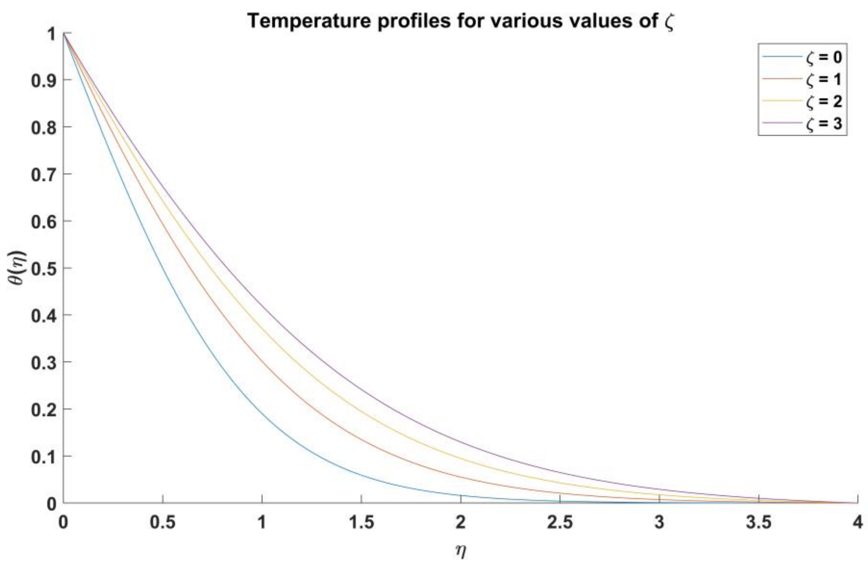

Fig. 3. Temperature profiles of nanofluid for various values of slip parameter

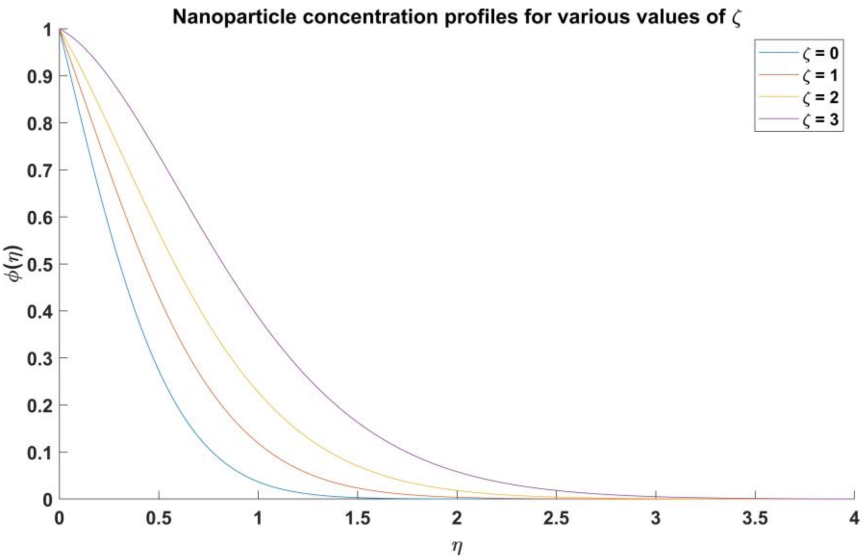

Fig. 4. Concentration profiles of nanofluid for various values of slip parameter

The variation in the stream-wise velocity for different non-linearly stretching sheet parameter n values is shown in Figure 5. The rise in this non-linear parameter is correlated with a reduction in the stream-wise velocity. This implies that the stretching process or the conditions associated with the parameterization are causing a slowdown in the flow along the direction of the flow. As a result, increase in the non-linear parameter of stretching n causes the width of momentum boundary layer to shrink. Figure 6 shows relationship between temperature change and stretching parameter n values. A higher value of the stretching parameter implies a more intense or pronounced stretching of the surface. As the surface undergoes increased stretching, it introduces more energy into the fluid flow near the surface. This stretching action tends to alter the flow patterns, influencing the velocity distribution in the boundary layer. This in turn enhances the momentum transfer and fluid motion near the surface, leading to a thicker boundary layer.

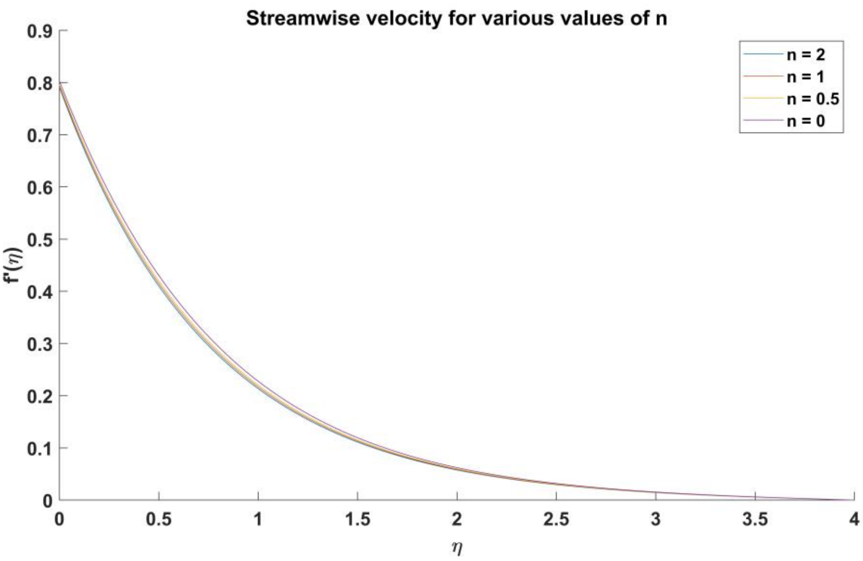

Fig. 5. Stream-wise velocity of nanofluid for various values of non-linear stretching parameter

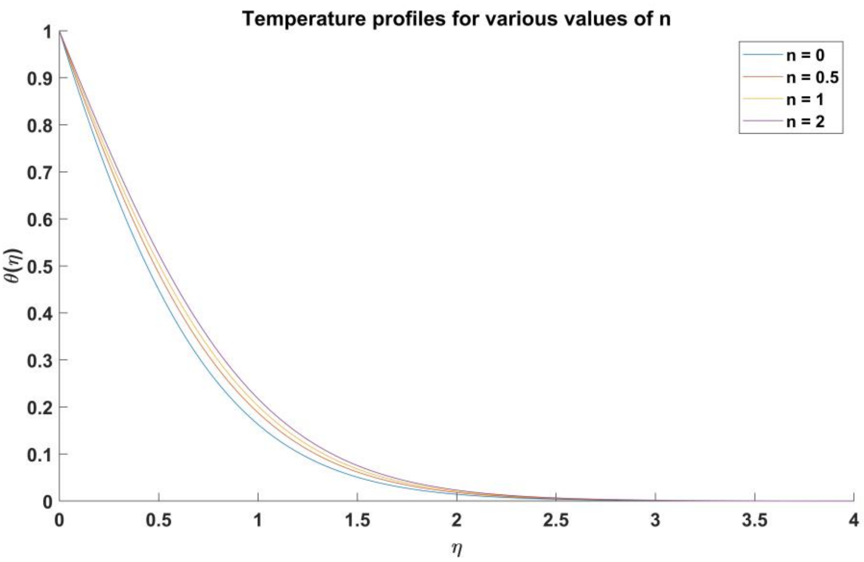

Fig. 6. Temperature profiles of nanofluid for various values of non-linear stretching parameter

The nanoparticle concentration profile changes are described in Figure 7 as a function of distinct values of stretching parameter n . Even if impact is negligible at fixed surface temperature, one can observe that concentration rises with rising values of stretching parameter. This is due to the reason that the stretching action can induce enhanced mixing, increased fluid motion, and altered velocity profiles in the vicinity of the stretched surface. This causes higher shear stresses within the fluid near the stretched surface. Enhanced shear stresses promote better mixing of the fluid, leading to increased transport of species, such as the concentration of a substance.

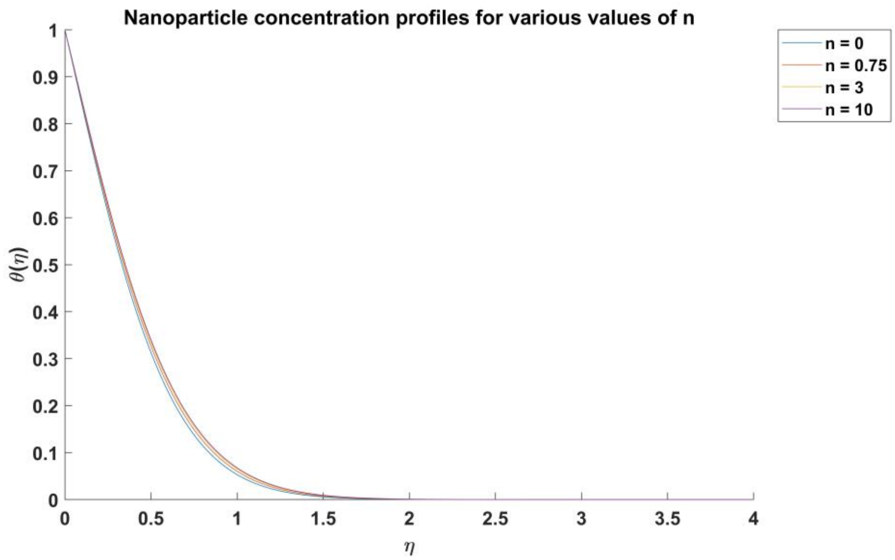

Fig. 7. Concentration profiles of nanofluid for various values of non-linear stretching parameter

The influence of the Brownian motion parameter (Nb) on temperature profiles is depicted in Figure 8. When considering a non-zero constant value of η , the temperature distribution within the boundary layer increases as the values of Nb grow. Beyond a certain distance from the wall, the fluid temperature gradually approaches zero asymptotically. Consequently, an increase in the Nb value generally results in an increased thermal diffusivity (D_B), contributing to the augmentation of the nanofluid temperature. The dimensionless nanoparticle concentration patterns are shown in Figure 9 corresponding to distinct values of Brownian motion parameter Nb. Brownian motion causes nanoparticles to move randomly and rapidly in all directions within the fluid. This random movement prevents particles from settling or aggregating, promoting a more uniform dispersion of nanoparticles throughout the fluid. Hence, an increase of fluid concentration in the region of the boundary layer is caused by an increase in the Brownian motion parameter.

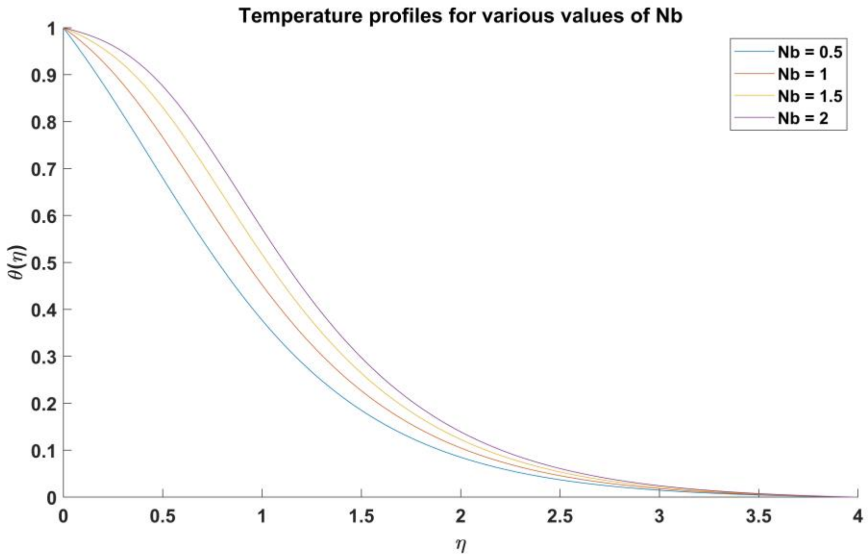

Fig. 8. Temperature profiles of nanofluid for various values of Brownian motion parameter

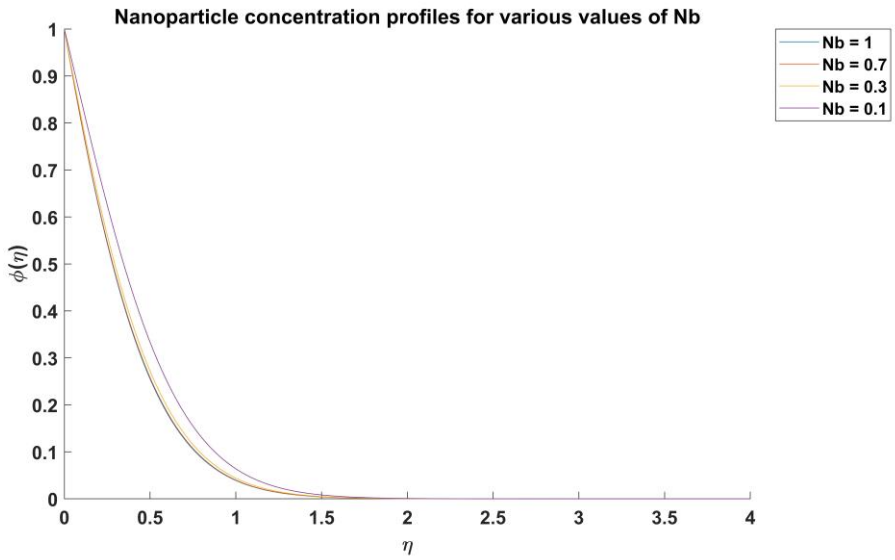

Fig. 9. Concentration profiles of nanofluid for various values of Brownian motion parameter

The impact of the thermophoresis parameter (Nt) on temperature profiles is depicted in Figure 10, and it is noted that its behavior is similar to that of Nb. Consequently, the thickness of the boundary layer increases and asymptotically approaches zero with distance from the boundary. This occurrence can be elucidated by the fact that increasing values of Nt result in an elevation of the velocity of Brownian motion for both nanoparticle and blood. Consequently, the kinetic energy at the molecular and nanoparticle levels rises, leading to an overall increase in nanofluid temperature. Figure 11 illustrates the distribution of nanoparticle concentration concerning η for different values of the thermophoresis parameter (Nt). The figure indicates that the nanofluid concentration rises as Nt values increase, reaching a peak near the plate's wall. This observation aligns with the fact that an increase in Nt corresponds to an enhancement in thermal diffusivity.

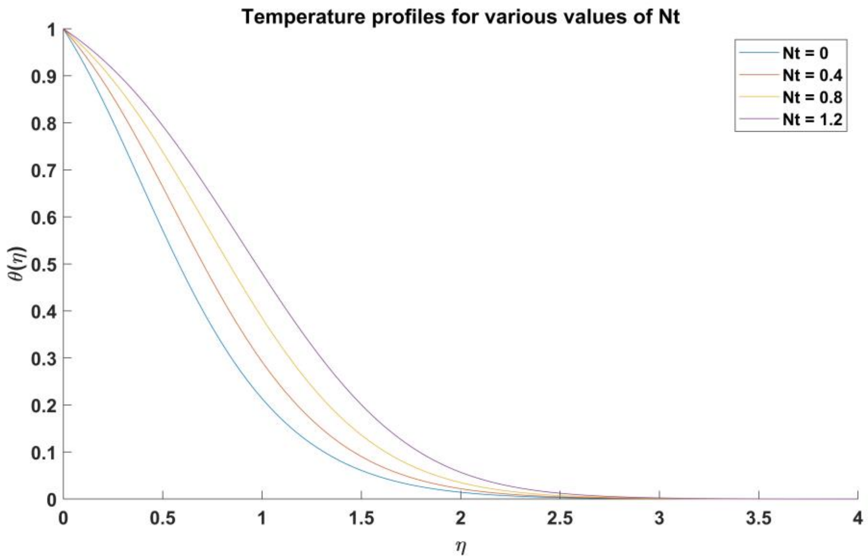

Fig. 10. Temperature profiles of nanofluid for various values of thermophoresis parameter

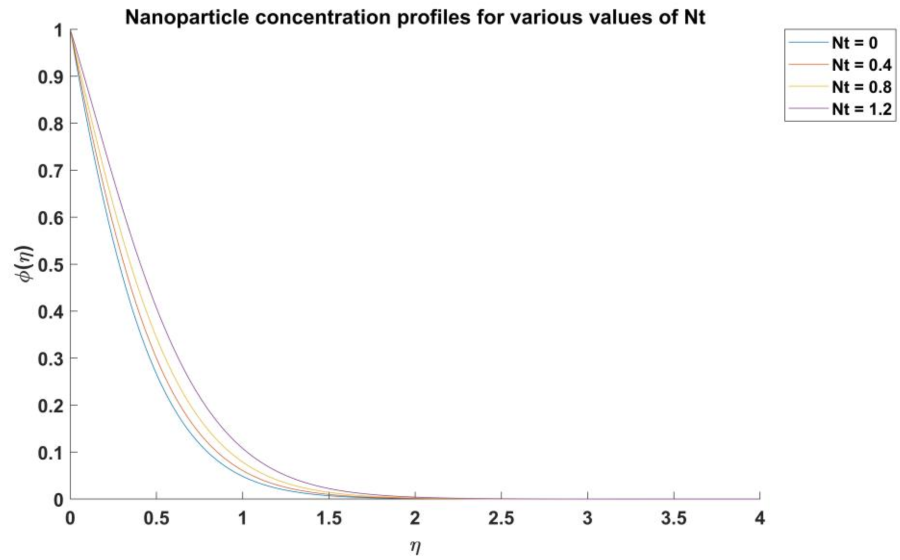

Fig. 11. Concentration profiles of nanofluid for various values of thermophoresis parameter

The influence of the Lewis Number (Le) on nanoparticle concentration is depicted in Figure 12. It is evident that the concentration distribution diminishes with an increase in the Lewis number. This effect is particularly pronounced for lower values of Le. Consequently, the Lewis number is anticipated to substantially modify the concentration boundary layer. Additionally, it is worth noting that the presence of a magnetic field parameter contributes to a higher nanoparticle concentration distribution in the boundary layer region. This occurrence is attributed to the action of the Lorentz force, which generates a force opposing the direction of fluid flow. Figure 13 demonstrates how the temperature profiles are influenced by the magnetic field parameter M . It is evident that as the magnetic field parameter elevates, an increase in the fluid temperature is noticed. This is due to the reason that the magnetic field causes the magnetic nanoparticles within the nanofluid to experience movement and rotation. This movement generates friction and collisions among the nanoparticles, leading to an increase in temperature through the conversion of kinetic energy into thermal energy. Also, the applied magnetic field induces a Lorentz force in the electrically conducting nanofluid. This force results in the conversion of electrical energy into heat, contributing to an overall temperature rise in the fluid. Consequently, the thickness of the thermal boundary layer expands as the magnetic parameter M escalates.

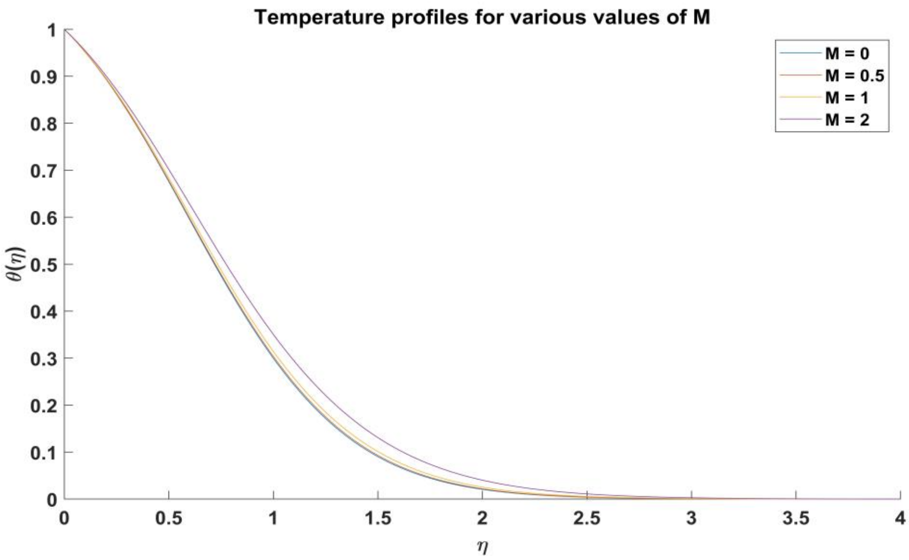

Fig. 12. Temperature profiles of nanofluid for various values of magnetic parameter

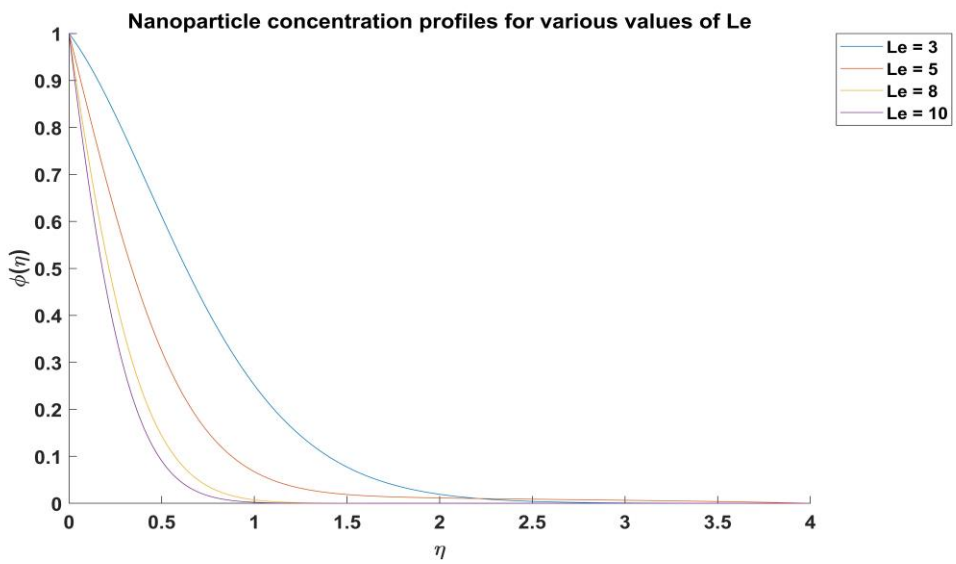

Fig. 13. Concentration profiles of nanofluid for various values of Lewis number

The engineered parameters, Nusselt number, Skin friction coefficient and Sherwood number exhibits a number of biomedical applications. The Nusselt number indicates how efficiently heat is transferred between the blood and the surface of the sheet. An increase in the Nusselt number suggests enhanced convective heat transfer, which could be beneficial for applications such as thermal therapies or temperature regulation in medical devices. Also, the skin friction coefficient reflects the resistance or drag experienced by the blood as it flows over the non-linearly stretching sheet. Higher skin friction coefficients indicate increased resistance to blood flow, potentially influencing aspects like arterial health and flow patterns. In the scenario of blood flow, the Sherwood number describes the effectiveness of transport of substances between the blood and the surface. The increase in Sherwood number suggests enhanced mass transfer, which could be relevant in applications such as drug delivery or nutrient exchange. The numerical values of Nusselt number, skin friction coefficient and Sherwood number obtained in the current study are represented in Table 1.

These parameters provide valuable insights into the heat transfer, shear forces, and mass transfer associated with the fluid dynamics of blood. The understanding these parameters is crucial for applications ranging from medical devices to therapeutic interventions where precise control over heat transfer, shear forces, and mass transfer is essential for optimizing performance and patient outcomes.

Table 1
Impact of magnetic parameter and slip velocity on Nusselt number, skin friction coefficient and Sherwood number

M	ζ	$Nu\,\mathrm{Re}_x^{-\frac{1}{2}}$	$Sh\,\mathrm{Re}_x^{-\frac{1}{2}}$	$C_f\,\mathrm{Re}_x^{\frac{1}{2}}$
	0	1.486423	2.013456	1.234512
0	0.3	1.365487	1.843214	0.843267
	0.7	1.227508	1.678123	0.617981
	0	1.428764	1.947245	1.473709
0.5	0.3	1.314956	1.782490	0.953862
	0.7	1.153289	1.610904	0.674032

To validate the accuracy of our numerical findings, benchmarking of present results is done with the existing Nusselt number values across various Prandtl (Pr) and non-linear stretching parameter (n) values by specifically taking $Nt = 0, Nb = 0, Le = 0, M = 0, f_w = 0$. These comparisons are made with the values obtained by Cortell [12], Rana and Bhargava [14], Das [15] and Sk *et al.,* [21] as summarized in Table 2. The results obtained in the current study are in good agreement with those of the aforementioned works, particularly in the context of fluid flow induced by a permeable stretching sheet. This alignment validates the application of our present numerical code for the current model.

Table 2
Comparison of Nusselt number and Skin friction coefficient corresponding to different values of Pr and n keeping $Nt = 0,\ Nb = 0,\ Le = 0,\ M = 0,\ f_w = 0$.

Pr	n	Cortell [12]	Rana and Bhargava [14]	Das [15]	Sk *et al.,* [21]	Current results
1	0.2	0.610262	0.6113	0.610571	0.610214466	0.611246
	0.5	0.595277	0.5967	0.595719	0.595222443	0.596972
	1.5	0.574537	0.5768	0.574525	0.574769900	0.575236
5	0.2	1.607175	1.5910	1.60713	1.607780982	1.598231
	0.5	1.586744	1.5839	1.58619	1.586776166	1.580425
	1.5	1.557463	1.5496	1.55719	1.557688631	1.553871

4. Concluding remarks

The problem of naofluid flow through a non-linear porous stretched sheet at a certain temperature of surface under the effect of partial slip is investigated computationally. To inspect the effects of various controlling factors on the characteristic of flow of fluid, a parametric study is conducted. From the current investigation, the following conclusions may be drawn:

(i) The stream-wise velocity of the nanofluid falls as the slip parameter and non-linear stretching parameter are increased.
(ii) With the increase in slip parameter and non-linear stretching parameter, the thermal boundary layer thickness increases.
(iii) Concentration of nanoparticles rises with increasing values of Brownian motion parameter. However, it decreases for rising values of thermophoresis parameter.
(iv) Increase in Brownian motion parameter and thermophoresis parameter increases the temperature while decrease in temperature of nanofluid is evident with decrease in Brownian motion parameter and thermophoresis parameter.
(v) Elevation in the values of magnetic parameters tends to increase the temperature of the nanofluid ultimately increasing the thickness of the boundary layer.
(vi) The increase in values of Lewis number substantially decreases the concentration of nanofluid.

Nomenclature

f_w	Suction parameter
ζ	Slip parameter
η	Dimensionless similarity variable
ϕ	Dimensionless concentration
θ	Dimensionless temperature
μ	Viscosity of fluid
τ	Cauchy stress tensor
ν	Kinematic fluid viscosity
ρ_f	Density of fluid
α	Thermal diffusivity

$(\text{a}\,\rho)_f$	Heat capacity of nanofluid
$(\text{a}\,\rho)_p$	Heat capacity of fluid
C_f	Skin friction coefficient
κ	Thermal conductivity
C_∞	Ambient nanoparticles volume fraction
T_∞	Temperature far away from the sheet
D_T	Thermophoresis diffusion coefficient
D_B	Brownian diffusion coefficient
f	Dimensionless stream function
u_b	Velocity of stretching sheet
T_b	Uniform temperature at the stretching sheet
C_b	Volume fraction of nanoparticles on stretching surface
a, b, c	Constants
Sh	Local Sherwood number
Nu	Local Nusselt number
Nt	Thermophoresis parameter
Nb	Brownian motion parameter
Le	Lewis number
$\Pr$	Prandtl number
p	Pressure of the fluid
β	Casson fluid parameter
T	Temperature of the fluid
u, v	Velocity in x and y directions

Acknowledgement
The authors express their gratitude for the financial support received from Universiti Teknikal Malaysia Melaka (UTeM).

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