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Mathematical Solutions of Free Convection Flow of Casson Fluid in Channel with Accelerated Plate

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ABSTRACT

This research develops mathematical solutions for Casson fluid flow with free convective phenomena within channels with an accelerated plate. The study transforms the governing energy and momentum equations into the dimensionless form using appropriate variables. The Laplace transformation is used to acquire analytical solutions. Dimensionless parameters, such as the Prandtl number, Grashof number, accelerated parameter, and Casson fluid parameter, are determined, which are further investigated for their impacts on the flow and thermal behaviour. Visual representations of the mathematical results for velocity and temperature are presented using MATHCAD software through graphical plots. The study observes that fluid velocity increases with higher values of *Gr* but decreases with increased Pr. Additionally, temperature profiles exhibit a decrease with higher Prandtl numbers and an increase with time. The obtained results are validated by comparing them with published results in limiting cases, showing good agreement, and confirming the accuracy and reliability of the research outcomes.

1. Introduction

Non-Newtonian fluids find a wide range of applications in multiple domains, including biomedical, chemical engineering, and industrial settings. It is essential to comprehend their characteristics thoroughly in order to optimize systems that rely on them, such as pipelines, pumps, and mixing apparatus. Scientists are driven by the need to explore non-Newtonian fluids like the Casson fluid to expand our understanding, create predictive models, and anticipate their behavior across various flow conditions. For a considerable duration, researchers have maintained a keen interest in examining the behavior of Casson fluids in conjunction with heat transfer processes. In their work, Das *et al.,* [1] delved into the investigation of how chemical reactions, thermal radiation and Newtonian heating influence the processes of mass and heat transfer in the time dependent

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hydromagnetic Casson fluid flow over a vertical plate. Meanwhile, Kataria and Patel [2] managed to derive exact solutions for the Casson fluid that unsteadily flow over an oscillating vertical plate with natural convective phenomenon under the impact of uniform transverse magnetic field, employing the Laplace transform method. Hussanan *et al.,* [3] furnished exact solutions for the time dependent magnetohydrodynamic (MHD) heat transfer flow of a Casson fluid passing through a porous medium. They considered factors such as Newtonian heating caused by plate oscillations and the influence of buoyancy force. On a related note, Raju [4] conducted a study examining the chemical reactions, heat absorption and thermal radiation impacts on the unsteady free convective MHD flow of a Casson fluid. This investigation also encompassed mass and heat transfer phenomena which were bounded by infinitely inclined vertical plate. Mahanthesh *et al.*, [5] conducted an investigation into the nonlinear density fluctuations occurring in a moving fluid, taking into account the impacts of mass and heat transfer. This study was carried out on a non-axially rotating plate. Meanwhile, Shivhare and Kumar Deka [6] managed to derive an exact solution for transportation of the thermal and solutal with natural convection scenario for a Casson fluid that unsteadily flow along an infinite vertical plate with MHD effect. Their approach involved the utilization of the Laplace transform method and was conducted within a uniform porous medium. Heat transfer in fluid flow holds significant practical importance, especially in applications such as thermal power plants where heat is converted into electricity through fluid flow processes. However, it's worth noting that there are researchers who have explored non-thermal aspects of fluid behavior. For instance, Azmi *et al.,* [7] conducted a study specifically on Casson fluid flow, concentrating on non-thermal aspects. In their research, they examined the flow propagation of Casson fluid passing unsteadily through a cylinder while setting the boundary with slip velocity condition. Their work resulted in analytical solutions for this particular scenario. Casson fluid behavior in cylindrical domains has been explored in various studies. Dash *et al.*, [8] and Fredrickson [9] delved into investigations of Casson fluid flow in different contexts, including the flow in a pipe filled with a homogeneous porous medium and the steady-flow behavior of Casson fluid within a tube. Boyd *et al.,* [10] adopted Casson fluid modeling for blood flow, utilizing the lattice Boltzmann method to examine both steady and oscillatory blood flow patterns. Bhattacharyya *et al.,* [11] contributed to the field by discovering exact solutions for Casson fluid behavior on a permeable stretching/shrinking sheet. Mernone *et al.*, [12] focused on the peristaltic flow of Casson fluid within a two-dimensional channel, specifically without considering the influence of free convection flow effects. Azmi *et al.,* [13] investigate the impact of slip velocity on the unsteady free convection flow of a Casson fluid within a vertical cylinder. The problem was solved using the techniques of Hankel transform and Laplace transform method. Lately, researchers have shifted their focus towards simulating blood flow applications in narrow arteries by studying the Casson fluid flow in cylinders [14]. They tackled this research using the Caputo-Fabrizio fractional derivative approach. Finally, Akaje and Olajuwon [15] delved into the effects of nonlinear thermal radiation on the stagnation point of an aligned MHD Casson nanofluid flow with Thompson and Troian slip boundary conditions. They numerically solved this problem using the spectral collocation method. These studies collectively contribute to our understanding of Casson fluid dynamics in a variety of scenarios and applications.

The investigation of free convection flow in channels has become a focal point of research, garnering substantial interest due to its wide-ranging practical applications across fields such as engineering, physics, and chemistry. In this context, Ram Reddy *et al.,* [16] conducted a study that looked into the effects of Joule heating, the Hall parameter, and viscous dissipation on the phenomenon of free convection flow. Their research specifically focused on an electrically conducting Casson fluid within a vertical channel. Similarly, Mohamad *et al.,* [17] conducted an investigation into unsteady free convection flow, specifically of a Casson fluid, within a vertical channel. These studies contribute to our understanding of the complex dynamics involved in free convection flows in channel geometries. Ali *et al.*, [18] conducted research to examine how the presence of the Lorentz force affects the flow patterns of a pulsating non-Newtonian micropolar-Casson fluid within a narrowed channel. In a separate study, Abbas *et al.,* [19] investigated the flow behavior of oscillating Casson fluid with temperature-dependent viscosity in a porous channel, considering the phenomenon of velocity slip. More recently, Saleem *et al.,* [20] determined critical parameters such as the skin friction coefficient, local Nusselt number, and local Sherwood number in the context of Casson fluid flow within an irregularly shaped channel. This study also considered the influence of heat and mass transfer effects on the fluid dynamics. Furthermore, there is a group of researchers focusing on the nanofluid domain. Mahat *et al.,* [21] conducted a study on the mathematical model for free convection boundary flow on a horizontal cylinder, which they solved numerically using the Keller-box method, a finite difference scheme. Similarly, Zokri *et al.,* [22] utilized the Keller-box method to address their problem concerning free convection boundary layer flow of Jeffrey nanofluid on a horizontal cylinder, taking into account the viscous dissipation effect. Awang *et al.*, [23] delved into the effects of nanoparticle shape on aligned magnetohydrodynamics mixed convection flow of Jeffrey hybrid nanofluid over a vertical plate, solving the problem numerically using the bvp4c solver. Last but not least, Aman *et al.*, [24] investigated the heat generation effects on Maxwell nanofluid passing over an oscillating vertical plate, solving it through the well-known Laplace transform method.

Numerous studies have tackled practical challenges by seeking mathematical solutions for the flow of Casson fluids over accelerating plates. Chaudhary *et al.,* [25] explored free convection involving a viscous, incompressible fluid passing over an infinitely tall vertically accelerating plate within a porous medium, utilizing the Laplace transform. Vijayalakshmi and Florence Kamalam [26] investigated the flow characteristics of an accelerating vertical plate with uniform temperature submerged in a rotating fluid, taking thermal radiation into account. Ulhaq *et al.,* [27] saturated an incompressible MHD viscous fluid in porous medium and scrutinized the flow distribution under the influence of exponentially accelerating vertical plate and natural convection phenomenon. Asogwa [28] discovered and analysed the impacts of radiation and chemical reactions on an exponentially accelerating infinite vertical plate experiencing a constant mass flux. Adapting the Laplace transform technique to solve the free convection boundary layer flow, Raju *et al.,* [29] analysed the transmission of mass and heat with the condition of a periodically accelerating vertical surface. Other than that, Yusof *et al.,* [30] concentrate on studying the steady stagnation point flow and radiative heat transfer of Casson fluid flowing over a permeable slippery Riga plate. They numerically solved this problem using a boundary value problem solver (bvp4c).

As previously mentioned, researchers have shown significant interest in studying Casson fluid, a non-Newtonian fluid, to enhance our understanding of its behavior and develop predictive models under varying flow conditions. However, prior studies have predominantly focused on aspects such as heat transfer, magnetic field effects, and nanofluids on flat vertical plates. Surprisingly, there has been no investigation employing Laplace transform to analyze the free convection flow of Casson fluid in a channel with an accelerated plate. Therefore, this research is conducted with the aim of exploring the behavior of Casson fluid in free convection flow within a channel featuring an accelerated plate, utilizing the Laplace transform method. The primary objective of this study is to derive mathematical solutions for velocity and temperature profiles employing Laplace transform techniques and subsequently analyze their behavior concerning various physical flow parameters using MATHCAD.

2. Mathematical Formulation and Solution

In this study, the focus was on the free convection flow of Casson fluid in two vertical channels with an accelerated plate positioned at a distance, d and constant temperature, T_d^* . The channels were oriented along the *x*-axis, while the *y*-axis represented the normal position to the plate. At time $t^* > 0$, the plate $y^* = 0$ starts moving with velocity $U(t^*)$ in the *x*-axis direction and its temperature is raised or lowered T_w^* while the plate $y^* = d$ is kept fixed and is maintained at T_d^* . The geometry of the problem is presented in Figure 1 depicted below

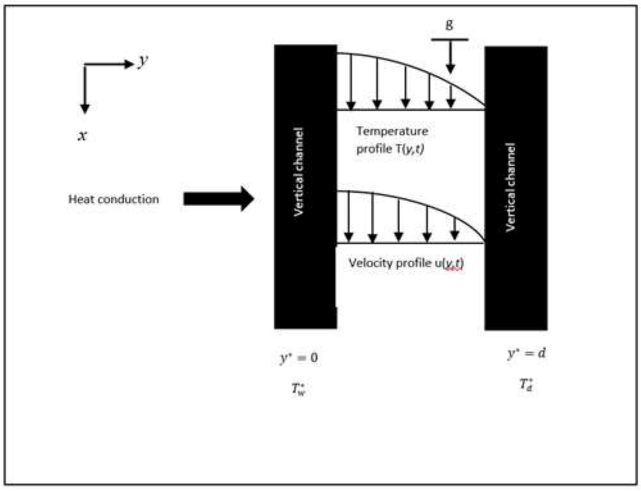

Fig. 1. Physical diagram

The governing equations for energy and momentum were derived using Boussinesq's approximation as follow (Qushairi *et al.,* [17], Azmi *et al.,* [31])

$$\rho c_p \frac{\partial T^*(y,t)}{\partial t^*} = k \frac{\partial^2 T^*(y,t)}{\partial y^{*2}} \tag{1}$$

$$\rho \frac{\partial u^*(y,t)}{\partial t^*} = \mu \left(1 + \frac{1}{\beta} \right) \frac{\partial^2 u^*(y,t)}{\partial y^{*2}} + \rho g \beta_T \left(T^* - T_d^* \right) \tag{2}$$

where T^* is the temperature of the fluid, ρ is the constant density of the fluid, u^* is the velocity in the *x*-direction, g is the acceleration due to gravity, c_p is the specific heat at constant pressure, v is the kinematic viscosity, k is the thermal conductivity of the fluid, β is the Casson fluid parameter, and β_T is the heat transfer coefficient. The dimensional initial and boundary conditions are (Seth *et al.,* [32])

$$\begin{aligned} &u(0,t) = U(t^*),\ u(y,t) \to 0 \text{ as } y \to d;\ t > 0, \\ &u(y,0) = 0;\ y > 0. \end{aligned} \tag{3}$$

and,

$$T(0,t)=T_w^*,\ T(y,t)=T_d^* \text{ as } y\to d;\ t>0, \qquad (4)$$
$$T(y,0)=T_d^*;\ y>0.$$

The dimensionless variables are defined as (Qushairi *et al.,* [17], Azmi *et al*., [31])

$$u=\frac{u^*d}{v},\ y=\frac{y^*}{d},\ t=\frac{t^*d^2}{v},\ T=\frac{T^*-T_d^*}{T_w^*-T_d^*} \qquad (5)$$

2.1 Solution

Substituting Eq. (5) into Eqs. (1-4), the dimensionless equations are obtained (by dropping the * notation)

$$\Pr\frac{\partial T}{\partial t}=\frac{\partial^2 T}{\partial y^2}. \qquad (6)$$

$$\frac{\partial u}{\partial t}=\left(1+\frac{1}{\beta}\right)\frac{\partial^2 u}{\partial y^2}+GrT. \qquad (7)$$

$$u(0,t)=F(t),\ u(y,t)\to 0 \text{ as } z\to 1;\ t>0, \qquad (8)$$
$$u(y,0)=0;\ y>0.$$

$$T(0,t)=1,\ T(y,t)=0 \text{ as } y\to 1;\ t>0, \qquad (9)$$
$$T(y,0)=0;\ y>0.$$

where parameters that involved in this study are (Seth *et al.,* [32])

$$\Pr=\frac{\mu c_p}{k},\quad Gr=\frac{g\beta_T d^3\left(T_w^*-T_d^*\right)}{v^2},\quad F(t)=\frac{U(t^*)d}{\nu} \qquad (10)$$

which are is the Prandtl number, is the Grashof number and is dimensionless accelerated parameter (Seth *et al.,* [32]). The Laplace transform technique was utilized to solve Eqs. (6) and (7) along with the initial and boundary conditions (8) and (9). The transformed equations in the s-domain are as follows:

$$\frac{d^2}{dy^2}\bar{T}(y,s)-s\Pr\bar{T}(y,s)=0. \qquad (11)$$

$$\bar{T}(0,s)=\frac{1}{s},\ \bar{T}(y,s)\to 0;\ \text{as } y\to 1. \qquad (12)$$

and

$$\frac{d^2\bar{u}(y,s)}{dy^2}-\frac{s}{b_1}\bar{u}(y,s)=\frac{-Gr\bar{T}(y,s)}{b_1}. \tag{13}$$

$$\bar{u}(0,s)=\frac{R}{s^2},\bar{u}(y,s)\to 0;\text{ as } y\to 1. \tag{14}$$

The characteristics equation will be employed to solve the homogeneous (11) and non-homogeneous (13) differential equations. The solution for Eq. (11) is given by the following expression:

$$\bar{T}(y,s)=C_1e^{\sqrt{s\Pr}y}+C_2e^{-\sqrt{s\Pr}y}. \tag{15}$$

By utilizing the boundary conditions (12) and infinite geometric series, the resulting solution is as follows

$$\bar{T}(y,s)=\sum_{n=0}^{\infty}\left(\frac{e^{(-y-2n)\sqrt{s\Pr}}-e^{(y-2-2n)\sqrt{s\Pr}}}{s}\right). \tag{16}$$

The characteristic for non-homogeneous in Eq. (13) is

$$\bar{u}(y,s)=\overline{u_c}(y,s)+\overline{u_p}(y,s). \tag{17}$$

where

$$\overline{u_c}(y,s)=C_3e^{\sqrt{\frac{s}{b_1}}y}+C_4e^{-\sqrt{\frac{s}{b_1}}y}. \tag{18}$$

and

$$\overline{u_p}=A_1e^{-y\sqrt{s\Pr}}+A_2e^{y\sqrt{s\Pr}}. \tag{19}$$

In order to solve the Eq. (17) with respect to Eqs. (18) and (19), the boundary condition (14) was applied to these functions and expressed as:

$$\bar{u}(y,s)=\frac{\mathrm{Re}^{-\left(-\sqrt{\frac{1}{b_1}}y\right)\sqrt{s}}}{s^2}-\frac{\mathrm{R}b_2+Gr}{b_2}\left(\sum_{n=0}^{\infty}\frac{e^{-\left(2n\sqrt{\frac{1}{b_1}}-\sqrt{\frac{1}{b_1}}y\right)\sqrt{s}}}{s^2}\right)+\frac{Gre^{-\left(-\sqrt{\frac{1}{b_1}}y\right)\sqrt{s}}}{s^2b_2}$$
$$+\left[\frac{\mathrm{R}b_2+Gr}{b_2}\left(\sum_{n=0}^{\infty}\frac{e^{-\left(2n\sqrt{\frac{1}{b_1}}+\sqrt{\frac{1}{b_1}}y\right)\sqrt{s}}}{s^2}\right)\right]+\sum_{n=0}^{\infty}\frac{Gre^{-\left(2\sqrt{\mathrm{Pr}}-y\sqrt{\mathrm{Pr}}+2n\sqrt{\mathrm{Pr}}\right)\sqrt{s}}}{s^2b_2}+\sum_{n=0}^{\infty}\frac{Gre^{-\left(y\sqrt{\mathrm{Pr}}+2n\sqrt{\mathrm{Pr}}\right)\sqrt{s}}}{s^2b_2}. \quad (20)$$

Then, using inverse Laplace transform for Eqs. (16) and (20), obtained as

$$T(y,t)=\sum_{n=0}^{\infty}\left[erfc\left(\frac{(y+2n)\sqrt{\mathrm{Pr}}}{2\sqrt{t}}\right)-erfc\left(\frac{(2+2n-y)\sqrt{\mathrm{Pr}}}{2\sqrt{t}}\right)\right] \quad (21)$$

and

$$
\begin{aligned}
u(y,t) = {} & R\left[\left(t+\frac{\left(-\sqrt{\frac{1}{b_1}}y\right)^2}{2}\right)erfc\frac{-\sqrt{\frac{1}{b_1}}y}{2\sqrt{t}}-\frac{-\left(-\sqrt{\frac{1}{b_1}}y\right)\sqrt{t}}{\sqrt{\pi}}e^{\frac{-\left(-\sqrt{\frac{1}{b_1}}\right)^2}{4t}}\right] \\
& -R\sum_{n=0}^{\infty}\left[\left(t+\frac{\left(2n\sqrt{\frac{1}{b_1}}-\sqrt{\frac{1}{b_1}}y\right)^2}{2}\right)erfc\frac{\left(2n\sqrt{\frac{1}{b_1}}-\sqrt{\frac{1}{b_1}}y\right)}{2\sqrt{t}}-\frac{\left(2n\sqrt{\frac{1}{b_1}}-\sqrt{\frac{1}{b_1}}y\right)\sqrt{t}}{\sqrt{\pi}}e^{\frac{-\left(2n\sqrt{\frac{1}{b_1}}-\sqrt{\frac{1}{b_1}}y\right)^2}{4t}}\right] \\
& -\frac{Gr}{b_2}\sum_{n=0}^{\infty}\left[\left(t+\frac{\left(2n\sqrt{\frac{1}{b_1}}-\sqrt{\frac{1}{b_1}}y\right)^2}{2}\right)erfc\frac{\left(2n\sqrt{\frac{1}{b_1}}-\sqrt{\frac{1}{b_1}}y\right)}{2\sqrt{t}}-\frac{\left(2n\sqrt{\frac{1}{b_1}}-\sqrt{\frac{1}{b_1}}y\right)\sqrt{t}}{\sqrt{\pi}}e^{\frac{-\left(2n\sqrt{\frac{1}{b_1}}-\sqrt{\frac{1}{b_1}}y\right)^2}{4t}}\right] \\
& +\frac{Gr}{b_2}\left[\left(t+\frac{\left(-\sqrt{\frac{1}{b_1}}y\right)^2}{2}\right)erfc\frac{-\sqrt{\frac{1}{b_1}}}{2\sqrt{t}}-\frac{-\left(-\sqrt{\frac{1}{b_1}}y\right)\sqrt{t}}{\sqrt{\pi}}e^{\frac{-\left(-\sqrt{\frac{1}{b_1}}y\right)^2}{4t}}\right] \\
& +R\sum_{n=0}^{\infty}\left[\left(t+\frac{\left(2n\sqrt{\frac{1}{b_1}}+\sqrt{\frac{1}{b_1}}y\right)^2}{2}\right)erfc\frac{2n\sqrt{\frac{1}{b_1}}+\sqrt{\frac{1}{b_1}}y}{2\sqrt{t}}-\frac{\left(2n\sqrt{\frac{1}{b_1}}+\sqrt{\frac{1}{b_1}}y\right)\sqrt{t}}{\sqrt{\pi}}e^{\frac{-\left(2n\sqrt{\frac{1}{b_1}}+\sqrt{\frac{1}{b_1}}y\right)^2}{4t}}\right] \\
& +\frac{Gr}{b_2}\sum_{n=0}^{\infty}\left[\left(t+\frac{\left(2n\sqrt{\frac{1}{b_1}}+\sqrt{\frac{1}{b_1}}y\right)^2}{2}\right)erfc\frac{2n\sqrt{\frac{1}{b_1}}+\sqrt{\frac{1}{b_1}}y}{2\sqrt{t}}-\frac{\left(2n\sqrt{\frac{1}{b_1}}+\sqrt{\frac{1}{b_1}}y\right)\sqrt{t}}{\sqrt{\pi}}e^{\frac{-\left(2n\sqrt{\frac{1}{b_1}}+\sqrt{\frac{1}{b_1}}y\right)^2}{4t}}\right] \\
& +\frac{Gr}{b_2}\sum_{n=0}^{\infty}\left[\left(t+\frac{\left(2\sqrt{\Pr}-y\sqrt{\Pr}+2n\sqrt{\Pr}\right)^2}{2}\right)erfc\frac{2\sqrt{\Pr}-y\sqrt{\Pr}+2n\sqrt{\Pr}}{2\sqrt{t}}-\frac{\left(2\sqrt{\Pr}-y\sqrt{\Pr}+2n\sqrt{\Pr}\right)\sqrt{t}}{\sqrt{\pi}}\cdot e^{\frac{-\left(2\sqrt{\Pr}-y\sqrt{\Pr}+2n\sqrt{\Pr}\right)^2}{4t}}\right] \\
& -\frac{Gr}{b_2}\sum_{n=0}^{\infty}\left[\left(t+\frac{\left(y\sqrt{\Pr}+2n\sqrt{\Pr}\right)^2}{2}\right)erfc\frac{y\sqrt{\Pr}+2n\sqrt{\Pr}}{2\sqrt{t}}-\frac{\left(y\sqrt{\Pr}+2n\sqrt{\Pr}\right)\sqrt{t}}{\sqrt{\pi}}e^{\frac{-\left(y\sqrt{\Pr}+2n\sqrt{\Pr}\right)^2}{4t}}\right]
\end{aligned}
\tag{22}
$$

Here, the constant parameters involved are

$b_2 = b_1\Pr - 1$ and $b_1 = \left(1+\frac{1}{\beta}\right)$.

3. Result and Discussion

In this study, we have employed the Laplace transform method to obtain analytical solutions for the velocity and temperature profiles in Casson fluid flow through channels with an accelerated plate, as demonstrated in equations Eq. (21) and Eq. (22). The findings are visually represented using graphs to both present and corroborate the analytical results. We have observed and analysed the influence of various parameters, including the Casson parameter (β), Grashof number (*Gr*), Prandtl number (Pr), accelerated parameter (*R*), and time parameter (*t*), on the velocity profile. Additionally, we've examined the impact of the time parameter (*t*) and Prandtl number (Pr) on the temperature profile. For numerical computations, we have chosen specific values for these parameters: β = 1.5, Gr = 5, Pr = 7, and t = 0.2, following the references of Azmi *et al*., [31] and R = 1.0, based on the work of Seth *et al*., [32].

To assess the accuracy of our current solution, we have presented and discussed the validation results in Figure 2. Notably, we observed that our solution [Eq. (22)] matches precisely with the results obtained by Aqilah [33] when the magnetic parameter (M) equals 0, and the Casson fluid parameter (β) is infinite. Consequently, this alignment of results verifies the accuracy of our present solution. Additionally, the current velocity solution has been corroborated through validation with the numerical Gaver-Stehfest algorithm [34, 35] as detailed in Table 1. This algorithm facilitates the numerical inversion of the Laplace transform, and it's computed values are employed for comparison with the exact solutions. The outcomes of this comparison exhibit a high level of agreement, with only minor discrepancies. Consequently, this confirms the validity of the derived solution.

Figure 3 demonstrates the impact of the Casson fluid parameter on the velocity profile. It is evident that an increase in β results in reduced viscosity, leading to higher fluid velocity. This implies that the fluid requires greater stress or shear rate to initiate flow. In simpler terms, as the velocity profile rises, the fluid becomes less resistant to flowing. Notably, a significantly high Casson fluid parameter ($\beta = \infty$) essentially mimics the behaviour of a Newtonian fluid. Moreover, as plasticity increases, there's a corresponding rise in Casson fluid parameter viscosity, which, in turn, leads to an increased thickness of the velocity boundary layer.

Table 1
Comparison of present velocity profile, Eq. (22) with numerical Gaver-Stehfest Algorithm, Eq. (20)

y	R	Pr	Gr	β	t	Exact solution, Eq. (22)	Numerical Gaver-Stehfest Algorithm, Eq. (20) [26,27]
0	1	7	5	0.5	1	1.0000	1.0000
0.2	1	7	5	0.5	1	0.8428	0.8478
0.4	1	7	5	0.5	1	0.6504	0.6591
0.6	1	7	5	0.5	1	0.4386	0.4486
0.8	1	7	5	0.5	1	0.2192	0.2265
1.0	1	7	5	0.5	1	0	0

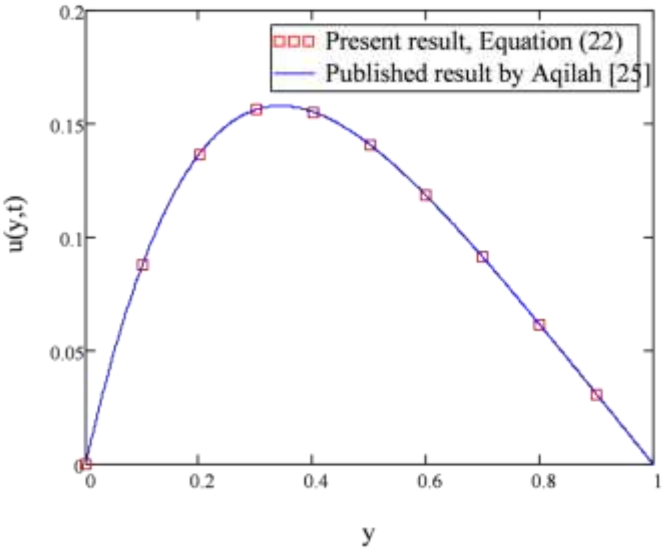

Fig. 2. Velocity profile comparison between present velocity solution, Eq. (22) and previous results by Aqilah [33]

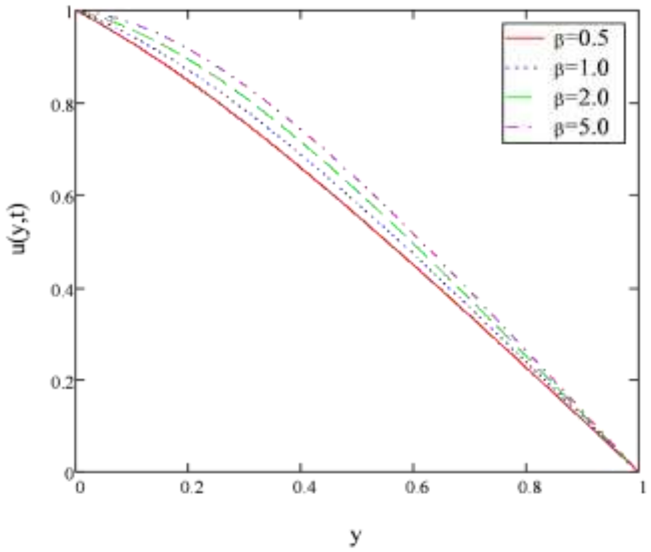

Fig. 3. *u(y,t)* plot vs *y* for several values of β with $R=1.0, Pr=7.0$, $Gr=5$, and $t=1.0$

Figures 4 and 5 offer some important insights. In Figure 4, it's clear that a higher value of parameter *R* leads to an elevated velocity profile. As acceleration intensifies, so does the fluid velocity. This indicates that the fluid is experiencing a more rapid change in its velocity over time. This effect can be attributed to the presence of the accelerated plate, which acts as an external force augmenting fluid flow. Figure 5 shows that as the Grashof number (*Gr*) increases, the velocity profile also experiences a corresponding rise. This alignment with the Grashof number demonstrates that an increase in Grashof number enhances the buoyancy force within the flow, subsequently accelerating the velocity of the fluid.

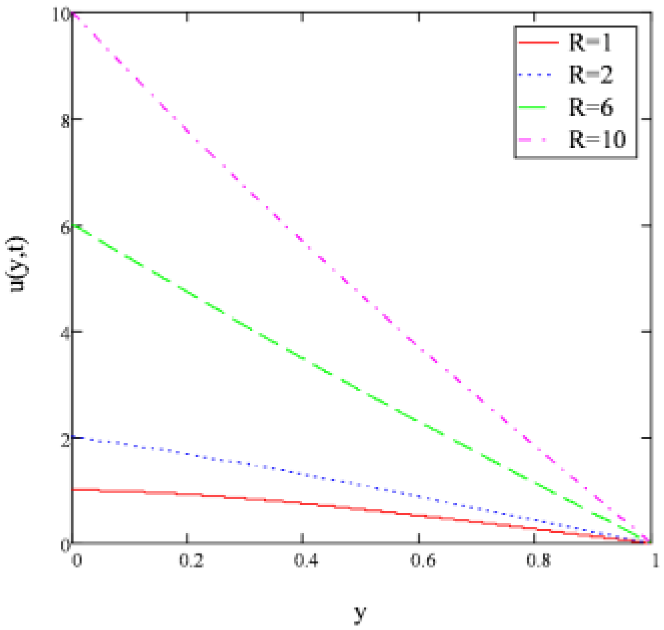

Fig. 4. *u(y,t)* plot vs *y* for several values of R with $\beta = 5.0$, $\Pr = 7.0$, $Gr = 5.0$, and $t = 1.0$.

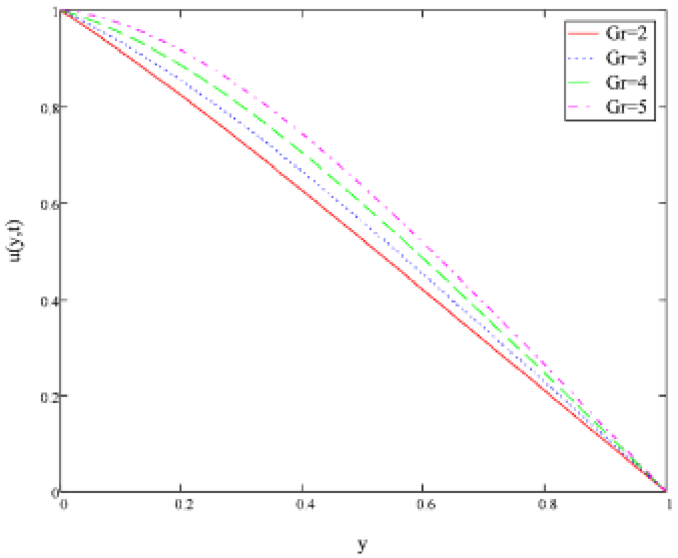

Fig. 5. *u(y,t)* plot vs *y* for several values of Gr with $\mathrm{R} = 1.0$, $\beta = 5.0$, $\Pr = 7.0$, and $t = 1.0$.

Figures 6 and 8 illustrate a noteworthy trend: as the Prandtl number (Pr) decreases, the velocity and temperature profiles increase. This phenomenon can be explained by the fact that fluids with lower Prandtl numbers possess higher thermal conductivity and thicker thermal boundary layers. Consequently, heat diffuses more rapidly from the plate compared to fluids with higher Prandtl numbers. The Prandtl number, as defined by Das *et al.,* [1], quantifies the ratio of kinematic viscosity to thermal diffusivity in the fluid. Increasing the Prandtl number reduces heat conductivity and elevates viscosity in the fluid flow. Turning to Figures 7 and 9, they provide insight into the flow's behaviour concerning the parameter *t* in velocity and temperature profiles. As the value of time (*t*) increases, both velocity and temperature exhibit a simultaneous rise. This behaviour can be attributed to an aging process wherein external energy input intensifies particle motion within the fluid, leading to increased velocity and temperature.

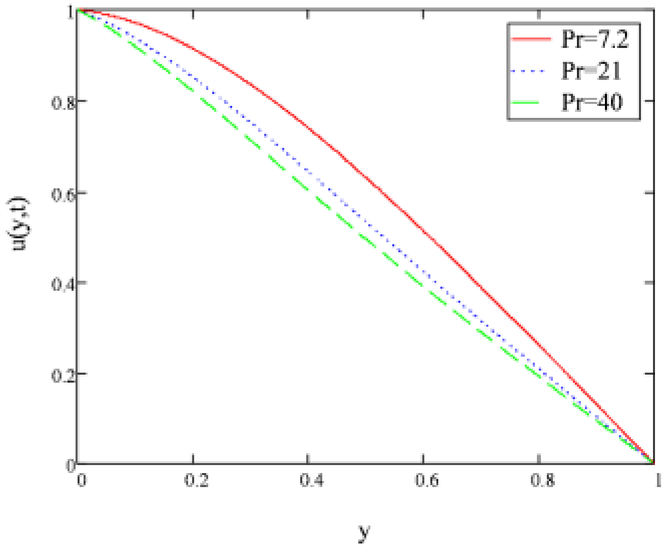

Fig. 6. u(y,t) plot vs y for several values of Pr with R $=1.0$, $\beta=5.0$, $Gr=5.0$, and $t=1.0$.

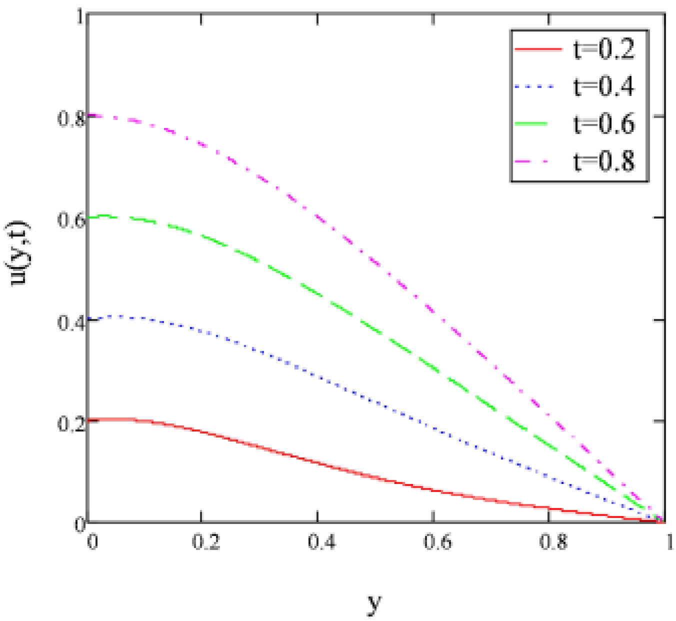

Fig. 7. $u(y,t)$ plot vs y for several values of t with R $=1.0$, $\beta=5.0$, Pr=7.0, and $Gr=5.0$.

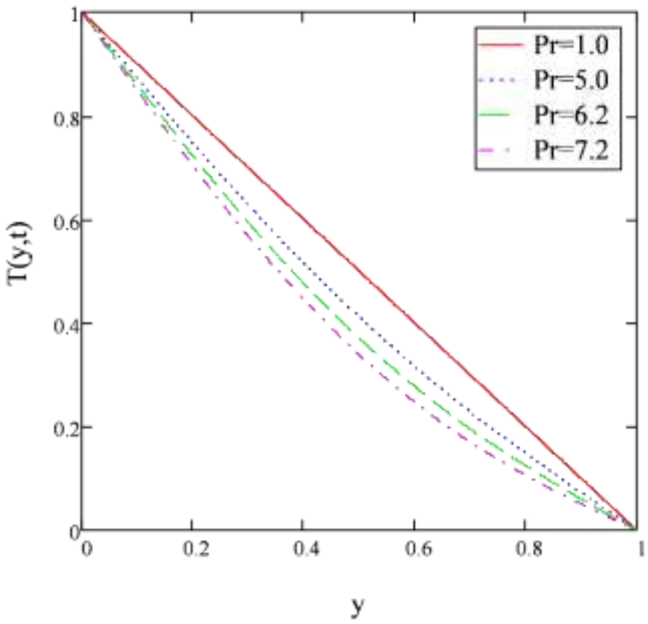

Fig. 8. T(y,t) plot vs y for several values of $\Pr$ with $t = 1.0$

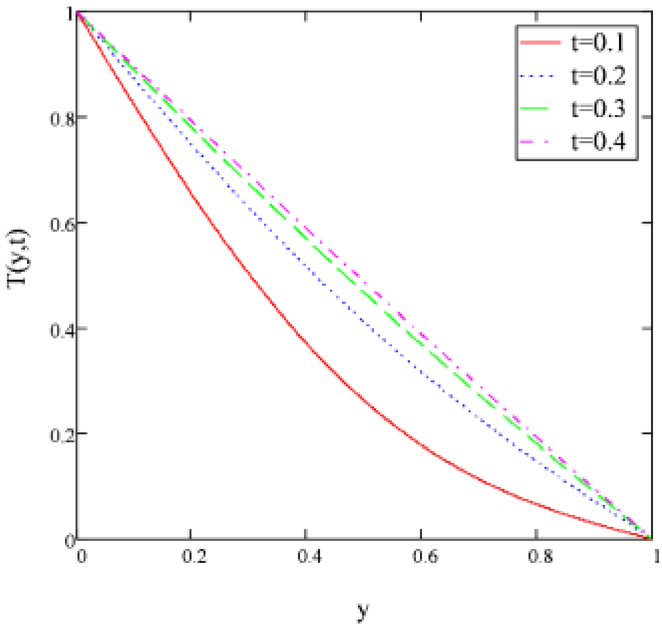

Fig. 9. T(y,t) plot vs y for several values of t with $\Pr = 1.0$

4. Conclusions

This study investigates the analytical solutions for the free convection flow of Casson fluid in channels with an accelerated plate. The problem is solved using the Laplace transform and inverse Laplace transform methods, yielding satisfactory solutions in accordance with the given initial and boundary conditions. It focuses on the characteristics of free convection in Casson fluid problems, with a particular emphasis on the influence of Casson fluid parameters and the characteristics of the accelerated plate. The present exact solution has been validated using numerical Gaver -Stehfest algorithm, whereby their comparison demonstrated a high degree of satisfaction, with minimal errors observed. Moreover, the result obtained in this paper showed:

i. The velocity profiles increase with the increasing value of Gr, β, t, and R and decreasing of Pr.
ii. The temperature profiles decrease with the higher value of *Pr* and increase with higher value of *t.*

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