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Mathematical Model Represents the Effect of Flexible Endoscopy on Suspension Fluid Flow

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ABSTRACT

In this manuscript, peristaltic transport induced by a sinusoidal traveling wave in the case for a viscous incompressible Newtonian fluid mixed with rigid spherical particles in the presence of a flexible inner tube, where the inner tube is also moving with a sinusoidal traveling wave of moderate amplitude is studied. The governing equations of the mixture (fluid-particle suspension) are written in two-dimensional cylindrical coordinates. The long-wavelength approximation is used to simplify the system of equations ($\delta << 1$). The velocities distribution for both fluid and particles are obtained and evaluated numerically with discussion for special cases. The flow rate, pressure drop, friction forces and shear stress at the outer and inner walls of tubes are derived and represented graphically. In the urinary system, peristalsis is due to involuntary muscular contractions of the ureter wall which drives urine from the kidneys to the bladder through the ureters. A mathematical analysis of peristaltic flow with application to the ureter in presence of flexible endoscopy (Peristaltic Endoscope) is taken as a real application in this study. Finally, conclusions of the research and recommendations for future work are discussed. The results obtained may be relevant to the transport of other physiological fluids and industrial applications in which peristaltic pumping is used.

1. Introduction

The modeling of two-phase flows has a tremendous variety of engineering and scientific applications, e.g., pollution dispersion in the atmosphere, fluidization processes and aerosol deposition in spray medication. The sedimentation of particles in a liquid is of interest in many chemical engineering processes, where erythrocyte sedimentation has become a standard clinical test such as kidney stones in the ureter Jimenez-Lozano *et al.,* [14], embryo implantation in the ureter and blood flows capillaries Srivastava and Srivastava [34]. The studies which related to the dusty gases were concerned by Saffman who carried out pioneering work on the stability of a laminar flow of a dusty gas by introducing relaxation parameter. By describing the motion of a gas carrying small dust

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particles. The equations are derived to satisfy small disturbances of a steady laminar flow. The effect of the dust is described by two parameters; the concentration of dust and a relaxation time Saffman [29]. The problem of the flow of a conducting dusty gas occupying a semi infinite space in the presence of a transverse magnetic field was studied by Baral [5] while Rao [26] studied the unsteady flow of a dusty viscous liquid in a channel and a pipe. Also discussed the unsteady flow of a dusty viscous liquid through a circular cylinder under the influence of a pressure gradient varying in magnitude but not in direction.

In Biological system of hydrodynamics, the continuum theory of mixtures is very helpful because it provides useful information about various diverse subjects like movement of microorganisms, deposition of different particle in the respiratory tract, rheology of blood and diffusion of proteins. About this topic there are a lot of studies which take the peristaltic transport on the walls Aarts and Ooms [1], Antanovskii and Ramkissoon [2], El-Misery *et al.,* [8], Hung and Brown [12], Mekheimer *et al.,* [21], Srivastava and Saxena [34], Srivastava and Srivastava [36] and Srivastava and Srivastava [37].

Also, this transport has great importance in a wide range of areas like fluidization, flow in rocket tubes, combustion, paint spraying, petroleum industry and in the purification of crude oil. The basic principles of peristaltic explained by Fung and Yih [9], Hung and Brown [12], Jaffrin and Shapiro [13], Medhavi and Singh [18], Medhavi and Singh [19], Muthuraj *et al.,* [23], Shapiro *et al.,* [33], Srivastava and Srivastava [36], Srivastava and Srivastva [38] and Takabatake and Ayakawa [41].

Mekheimer *et al.,* [20] discuss the peristaltic transport of a particle-fluid mixture through the gap between eccentric cylinders take the outer flexible cylinder with a peristaltic wave on its wall and the inner one is rigid uniform tube moving with constant velocity.

The flow between coaxial tubes where outer tube (uniform or nonuniform) are found in many applications such as the endoscopy problem, catheterized artery and exerted natural gases Srivastava and Srivastava [40] and Nagarani *et al.,* [24].

In modern medicine endoscopy and catheters have many uses which enable the physician to look inside closed tubes (gastrointestinal tract, respiratory tract, the urinary tract). There are many topics about the use and effects of catheters on flows such as Back [3], Back *et al.,* [4], MacDonald [15], Medhavi [17], Roos and Lykoudis [28], Sankar and Hemlatha [30] and Sarkar and Jayaraman [31]. All of these topics, the catheter or the inner tube was supposed to be rigid. Currently, endoscopes are either semi-rigid or completely rigid tubes that can only be inserted with the exertion of considerable force by a physician. As a consequence, they are liable to damage the internal vessels of the body, and may also induce patient discomfort during insertion. Even catheters, which are more flexible, can do damage when pushed into the different vessels of the body Grove and Pevec [10] and Wu *et al.,* [44]. Mangan *et al.,* [16] have successfully demonstrated a prototype of flexibly endoscope that could move through tubes and around curves using peristaltic locomotion. A device that could move through curving and tortuous spaces would find many applications in medicine and in industry. It is also possible to use the device as a catheter for infusion or withdrawal of fluids.

Rachid *et al.,* [25], Rachid and Ouazzani [26], treated with this problem under the consideration of flexible inner tube. Recently, there are explanation to peristaltic induced flow of blood through narrow catheterized arteries as a view of the observations of Srivastava [39]. Mekheimer and Ramadan [22] studied the peristaltic flow of Jeffrey nanofluid containing motile gyrotactic microorganisms through an endoscope with non-linear thermal radiation, magnetic field, and viscous dissipation.

With the above presentation in mind, the researchers try to introduce new case of annular flexible tubes the goal of this investigation to study the effect of inner flexible tube on the axial velocities, pressure drop and frictional force of particle-fluid suspension in a circular tube moving by peristaltic transport in the presence of inner tube where inner wall is flexible moving with a sinusoidal wave. In this paper the results will be represent graphically and numerically taken experimental data into account and discuss this results.

Medhavi [17], Saxena and Srivastava [32] and Shapiro *et al.,* [33] are special cases of this problem. The researcher suggested here this formulation may be applied to explain the inserted flexible endoscopy into gastrointestinal, respiratory or the urinary tract.

2. Formulation of the Problem

Consider the flow of a particle-fluid suspension through the gap between two coaxial peristaltic tubes where the sinusoidal waves travelling down their walls, (see Figure 1) [45].

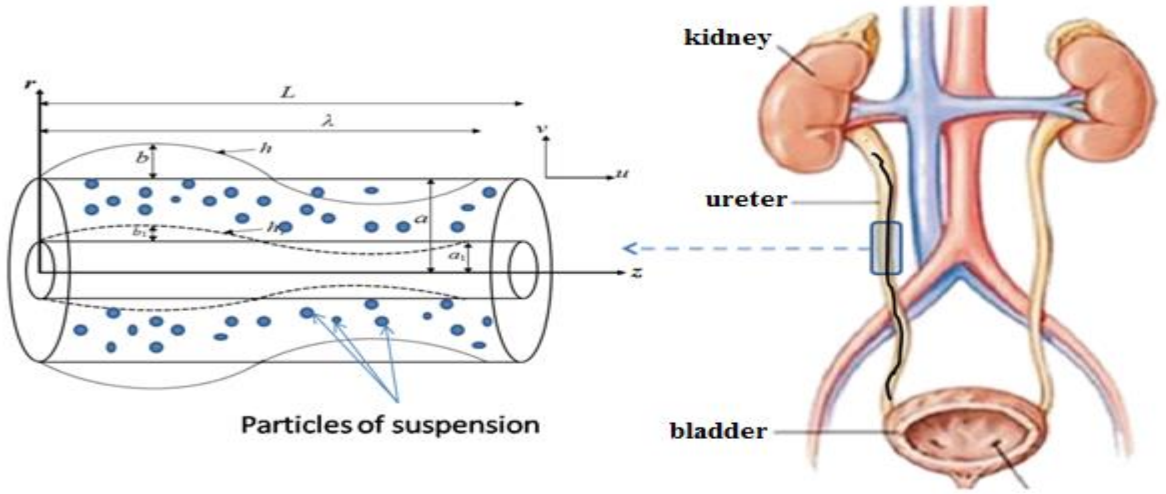

Fig. 1. Geometry of the problem

Consider the flow of a particle-fluid mixture in a circular cylindrical tube of radius (a) with an inserted tube of radius (a_1). The outer tube is flexible (sinusoidal peristaltic waves) takes the form

$$H(z,t) = a + b\sin\frac{2\pi}{\lambda}(z-ct), \tag{1}$$

where b is amplitude of the wave on outer tube, λ is wavelength, c is speed of the wave propagation, t is time and (r,z) are (radial coordinate, axial coordinate). Also, the inner tube wall is flexible (sinusoidal peristaltic waves) takes the form

$$H_1(z,t) = a_1 + b_1\sin\frac{2\pi}{\lambda}(z-ct). \tag{2}$$

where b_1 is amplitude of the wave on the inner tube; $0 \le b \le a - a_1$ and $b_1 < b$. The gap between two coaxial tubes fill with particle-fluid suspension which governing by the following system of equations Srivastava and Srivastva [38]

Fluid phase:

$$(1-C)\rho_f\left[\frac{\partial u_f}{\partial t}+u_f\frac{\partial u_f}{\partial z}+v_f\frac{\partial u_f}{\partial r}\right]= -(1-C)\frac{\partial P}{\partial z}+(1-C)\mu_s(C)\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u_f}{\partial r}\right)+\frac{\partial^2 u_f}{\partial z^2}\right]+CS(u_p-u_f), \quad (3)$$

$$(1-C)\rho_f\left[\frac{\partial v_f}{\partial t}+u_f\frac{\partial v_f}{\partial z}+v_f\frac{\partial v_f}{\partial r}\right]= -(1-C)\frac{\partial P}{\partial r}+(1-C)\mu_s(C)\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_f}{\partial r}\right)-\frac{v_f}{r^2}+\frac{\partial^2 v_f}{\partial z^2}\right]+CS(v_p-v_f), \quad (4)$$

$$\frac{1}{r}\frac{\partial}{\partial r}[r(1-C)v_f]+\frac{\partial}{\partial z}[(1-C)u_f]=0. \quad (5)$$

Particle phase:

$$C\rho_p\left[\frac{\partial u_p}{\partial t}+u_p\frac{\partial u_p}{\partial z}+v_p\frac{\partial u_p}{\partial r}\right]=-C\frac{\partial P}{\partial z}+CS(u_f-u_p), \quad (6)$$

$$C\rho_p\left[\frac{\partial v_p}{\partial t}+u_p\frac{\partial v_p}{\partial z}+v_p\frac{\partial v_p}{\partial r}\right]=-C\frac{\partial P}{\partial r}+CS(v_f-v_p), \quad (7)$$

$$\frac{1}{r}\frac{\partial}{\partial r}(rCv_p)+\frac{\partial}{\partial z}(Cu_p)=0. \quad (8)$$

where
C is volume fraction of the particulate phase (concentration),
(ρ_f, ρ_p) actual density of the fluid and particles respectively,
$((1-C)\rho_f, (C)\rho_p)$ is fluid phase and particulate phase densities, respectively,
P is pressure,
$\mu_s(C) \equiv \mu_s$ is mixture viscosity and
S is drag coefficient of interaction for the force exerted by one phase on the other. Concentration of particles is small enough to neglect the interaction between them. The volume fraction density C of the particles is constant Srivastava and Srivastava [35].

$$\mu_s(C)=\frac{\mu_0}{1-mC}, \quad (9)$$

where $m = 0.07exp\left[2.49C + \frac{1107}{T}e^{-1.69C}\right]$ with μ_0 is fluid viscosity (suspending medium), T temperature of the mixture measured in absolute scale (k) and μ_s is reasonably accurate up to $C = 0.6$ Charm and Kurland [7], Srivastava and Srivastava [36]. The drag coefficient of interaction is given by as Tam [42]

$$S = \frac{9}{2}\frac{\mu_0}{a_0^2}\left[\frac{4 + 3[8C - 3C^2]^{\frac{1}{2}} + 3C}{(2 - 3C)^2}\right] \tag{10}$$

where a_0 radius of a particle. The boundary conditions of motion are given by

$$\left.\begin{aligned} u_f &= 0, \quad v_f = \frac{\partial H(z,t)}{\partial t} \quad \text{at}: \quad r = H(z,t), \\ u_f &= 0, \quad v_f = \frac{\partial h_1(z,t)}{\partial t} \quad \text{at}: \quad r = H_1(z,t). \end{aligned}\right\} \tag{11}$$

introducing the following dimensionless variables

$$\left.\begin{aligned} &r' = \frac{r}{a}, \quad z' = \frac{z}{\lambda}, \quad u_f' = \frac{u_f}{c}, \quad u_p' = \frac{u_p}{c}, \quad v_p' = \frac{\lambda v_p}{ac}, \quad v_f' = \frac{\lambda v_f}{ac}, \\ &t' = \frac{ct}{\lambda}, \quad P' = \frac{a^2 P}{\lambda c \mu_0}, \quad S' = \frac{Sa^2}{\mu_0}, \quad \mu = \frac{\mu_s}{\mu_0}, \quad H' = \frac{H}{a}, \quad H_1' = \frac{H_1}{a}. \end{aligned}\right\} \tag{12}$$

Using (12) in (3-8) and (11) then dropping (‘) the system of motion become, for fluid phase

$$\begin{aligned} &(1-C)\delta Re\left[\frac{\partial u_f}{\partial t} + u_f\frac{\partial u_f}{\partial z} + v_f\frac{\partial u_f}{\partial r}\right] = \\ &-(1-C)\frac{\partial P}{\partial z} + (1-C)\mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u_f}{\partial r}\right) + \delta^2\frac{\partial^2 u_f}{\partial z^2}\right] + CS(u_p - u_f), \end{aligned} \tag{13}$$

$$\begin{aligned} &(1-C)\delta^3 Re\left[\frac{\partial v_f}{\partial t} + u_f\frac{\partial v_f}{\partial z} + v_f\frac{\partial v_f}{\partial r}\right] = \\ &-(1-C)\frac{\partial P}{\partial r} + (1-C)\mu\delta^2\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial(rv_f)}{\partial r}\right) + \delta^2\frac{\partial^2 v_f}{\partial z^2}\right] + CS\delta^2(v_p - v_f), \end{aligned} \tag{14}$$

$$\frac{1}{r}\frac{\partial}{\partial r}[r(1-C)v_f] + \frac{\partial}{\partial z}[(1-C)u_f] = 0. \tag{15}$$

and for particular phase

$$C\left(\frac{\rho_p}{\rho_f}\right)\delta Re\left[\frac{\partial u_p}{\partial t}+u_p\frac{\partial u_p}{\partial z}+v_p\frac{\partial u_p}{\partial r}\right]=-C\frac{\partial P}{\partial z}+CS(u_f-u_p), \tag{16}$$

$$C\left(\frac{\rho_p}{\rho_f}\right)Re\delta^3\left[\frac{\partial v_p}{\partial t}+u_p\frac{\partial v_p}{\partial z}+v_p\frac{\partial v_p}{\partial r}\right]=-C\frac{\partial P}{\partial r}+CS\delta^2(v_f-v_p), \tag{17}$$

$$\frac{1}{r}\frac{\partial}{\partial r}(rCv_p)+\frac{\partial}{\partial z}(Cu_p)=0. \tag{18}$$

where $Re=\dfrac{\rho_f ca}{\mu_0}$ is Reynolds number, $\delta=\dfrac{a}{\lambda}$ is wave number.

Using the wave frame (moving with the speed c):

$$z'=z-ct \quad , \quad r'=r,$$
$$u'=u-c \quad , \quad v'=v.$$

Substitution this frame in (1,2) and dropping (‘)

$$h(z)=a+b\sin\frac{2\pi}{\lambda}z,$$

$$h_1(z)=a_1+b_1\sin\frac{2\pi}{\lambda}z.$$

with the boundary conditions:

$$\left.\begin{aligned} u_f&=-1, \quad v_f=-\frac{dh(z)}{dz} \quad \text{at}: \quad r=h(z)=1+\phi\sin 2\pi z,\\ u_f&=-1, \quad v_f=-\frac{dh_1(z)}{dz} \quad \text{at}: \quad r=h_1(z)=\varepsilon+\phi_1\sin 2\pi z. \end{aligned}\right\} \tag{19}$$

where $\phi=b/a$, $\phi_1=b_1/a$ are the amplitude ratio for the outer and inner tubes respectively and $\varepsilon=a_1/a$ is eccentricity parameter.

3. Method of Solution

Using the long wavelength approximation $\delta \ll 1$ Shapiro *et al.,* [33]. From (14) $\dfrac{\partial P}{\partial r}=0$ the pressure is not a function of r then $\dfrac{\partial P}{\partial z}\equiv\dfrac{dP}{dz}$, as consequence (13) and (16) can be written as:

$$(1-C)\frac{dP}{dz}=(1-C)\mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u_f}{\partial r}\right)\right]+CS(u_p-u_f), \tag{20}$$

$$C\frac{dP}{dz} = CS(u_f - u_p). \tag{21}$$

Using of Eq. (20) and Eq. (21) with boundary conditions Eq. (19) the expressions of the velocity profiles u_f and u_p given as:

$$u_f = -1 - \frac{(dP/dz)}{4\mu(1-C)}\left\{h^2 - r^2 + (h^2 - h_1^2)\frac{\ln\left(\frac{r}{h}\right)}{\ln\left(\frac{h}{h_1}\right)}\right\}, \tag{22}$$

and

$$u_p = -1 - \frac{(dP/dz)}{4\mu(1-C)}\left\{h^2 - r^2 + (h^2 - h_1^2)\frac{\ln\left(\frac{r}{h}\right)}{\ln\left(\frac{h}{h_1}\right)} + \frac{4\mu(1-C)}{S}\right\}. \tag{23}$$

The instantaneous non-dimensional volumetric flow rate: ($q = \frac{q'}{\pi a^2 c}$; q' being the flow rate in wave frame of reference which is the same as laboratory frame of reference) which calculated in fluid-particle suspension as:

$$q = 2(1-C)\int_{h_1}^{h} r u_f\, dr + 2C\int_{h_1}^{h} r u_p\, dr. \tag{24}$$

Substitution from Eq. (22) and Eq.(23) in Eq.(24) the total flow rate q take the form

$$q = -(h^2 - h_1^2) - \frac{(h^2 - h_1^2)}{8\mu(1-C)}\frac{dP}{dz}\left[\chi(z)\right]. \tag{25}$$

where

$\chi(z) = h^2 + h_1^2 + \beta - \frac{(h^2 - h_1^2)}{\ln\left(\frac{h}{h_1}\right)}$ is non-dimensional suspension parameter

with $\beta = \frac{8\mu C(1-C)}{S}$.

Eq. (25) can be written as:

$$-\frac{dP}{dz}=\frac{8\mu(1-C)(q+h^2-h_1^2)}{(h^2-h_1^2)\chi(z)}. \tag{26}$$

To find the non-dimensional mean volumetric flow rate (Q) over one period of the wave as in Shapiro *et al.,* [33].

$$Q=q+1+\frac{1}{2}(\phi^2-\phi_1^2)-\varepsilon^2. \tag{27}$$

Using Eq. (27) in Eq. (26) yields the mathematical relation for non-dimensional pressure drop ΔP across on wavelength, (which is the same in wave or laboratory frame of reference) and then it take the form:

$$\Delta P=\int_0^1\left(-\frac{dP}{dz}\right)dz=2\mu(1-C)\left\{(Q-1-\frac{1}{2}(\phi^2-\phi_1^2)+\varepsilon^2)I_1+I_2\right\}. \tag{28}$$

with: $I_1=4\int_0^1\frac{dz}{(h^2-h_1^2)\chi(z)},\qquad I_2=4\int_0^1\frac{dz}{\chi(z)}.$

The friction force (at the wall) of the outer and inner tubes $F_o=F_o'/\pi\lambda c\mu_0$, $F_i=F_i'/\pi\lambda c\mu_0$ in (non-dimensional forms) obtained as:

$$\begin{aligned}F_0&=\int_0^1 h^2\left(-\frac{dP}{dz}\right)dz\\&=2\mu(1-C)\left\{\left(Q-1-\frac{1}{2}(\phi^2-\phi_1^2)+\varepsilon^2\right)I_3+I_4\right\},\end{aligned} \tag{29}$$

$$\begin{aligned}F_i&=\int_0^1 h_1^2\left(-\frac{dP}{dz}\right)dz\\&=2\mu(1-C)\left\{(Q-1-\frac{1}{2}(\phi^2-\phi_1^2)+\varepsilon^2)I_5+I_6\right\}.\end{aligned} \tag{30}$$

with: $I_3=4\int_0^1\frac{dz}{\left(1-\frac{h_1^2}{h^2}\right)\chi(z)},\qquad I_4=4\int_0^1\frac{h^2dz}{\chi(z)},$

$$I_5=4\int_0^1\frac{dz}{(\frac{h^2}{h_1^2}-1)\chi(z)},\qquad I_6=4\int_0^1\frac{h_1^2dz}{\chi(z)}.$$

By substitution Eq. (28) in Eq. (29) and Eq. (30) the pressure-flow rate and the friction forces-flow rate relationships are thus given in the following forms:

$$Q = 1 + \frac{1}{2}(\phi^2 - \phi_1^2) - \varepsilon^2 - \frac{I_2}{I_1} + \frac{\Delta P}{2\mu(1-C)I_1}, \tag{31}$$

$$F_o = 2\mu(1-C)\left\{ I_4 - \frac{I_2 I_3}{I_1} + \frac{\Delta P I_3}{2\mu(1-C)I_1} \right\}, \tag{32}$$

$$F_i = 2\mu(1-C)\left\{ I_6 - \frac{I_2 I_5}{I_1} + \frac{\Delta P I_5}{2\mu(1-C)I_1} \right\}. \tag{33}$$

The pressure rise $(-\Delta P)$ for zero time mean flow is obtained from Eq. (28):

$$(-\Delta P)_{Q=0} = 2\mu(1-C)\left\{ (1 + \frac{1}{2}(\phi^2 - \phi_1^2) - \varepsilon^2) I_1 - I_2 \right\}. \tag{34}$$

The time-mean flow for zero pressure drop $\Delta P = 0$ is given by applied on Eq. (31):

$$Q = 1 + \frac{1}{2}(\phi^2 - \phi_1^2) - \varepsilon^2 - \frac{I_2}{I_1}. \tag{35}$$

The shear stress at walls of the outer and inner tubes given by (respectively):

$$T_1 = -\mu \left.\frac{\partial u_f}{\partial r}\right|_{r=h}, \tag{36}$$

$$T_2 = -\mu \left.\frac{\partial u_f}{\partial r}\right|_{r=h_1}. \tag{37}$$

Using Eq. (22) in Eq. (36), Eq. (37) to get the following mathematical formula for T_1 and T_2 respectively:

$$T_1 = \frac{(dP/dz)}{4(1-C)}\left\{ -2h + \frac{(h^2 - h_1^2)}{h \ln\left(\frac{h}{h_1}\right)} \right\}, \tag{38}$$

$$T_2 = \frac{(dP/dz)}{4(1-C)}\left\{ -2h_1 + \frac{(h^2 - h_1^2)}{h_1 \ln\left(\frac{h}{h_1}\right)} \right\}. \tag{39}$$

In the absence of peristaltic on the inner tube expressions for ΔP, F_o, and F_i in Eq. (28), Eq. (29) and Eq. (30) can be written as:

$$\Delta P = 2\mu(1-C)\left\{(Q-1-\frac{1}{2}\phi^2)l_1 + l_2\right\}, \tag{40}$$

$$F_o = 2\mu(1-C)\left\{(Q-1-\frac{1}{2}\phi^2)l_2 + l_3\right\}, \tag{41}$$

$$F_i = 2\mu(1-C)\left\{(Q-1-\frac{1}{2}\phi^2)l_4 + l_5\right\}. \tag{42}$$

with: $l_1 = 4\int_0^1 \frac{dz}{(h^2-\varepsilon^2)\chi(z)}$, $\quad l_2 = 4\int_0^1 \frac{dz}{(1-\frac{\varepsilon^2}{h^2})\chi(z)}$, $\quad l_3 = 4\int_0^1 \frac{h^2 dz}{(1-\frac{\varepsilon^2}{h^2})\chi(z)}$,

$l_4 = 4\int_0^1 \frac{dz}{(\frac{h^2}{\varepsilon^2}-1)\chi(z)}$, $\quad l_5 = 4\int_0^1 \frac{\varepsilon^2 dz}{(1-\frac{\varepsilon^2}{h^2})\chi(z)}$.

These results are the same as Medhavi [17].
Also, in the absence of inner tube (i.e $\varepsilon,\phi_1 \to 0$) the formulas of ΔP and F_o in Eq. (28) and Eq. (29) can be written as:

$$\Delta P = 2\mu(1-C)\left\{(Q-1-\frac{1}{2}\phi^2)R_1 + R_2\right\}, \tag{43}$$

$$F_o = 2\mu(1-C)\left\{(Q-1-\frac{1}{2}\phi^2)R_3 + R_4\right\}. \tag{44}$$

with: $R_1 = 4\int_0^1 \frac{dz}{(h^4+\beta h^2)}$, $\quad R_2 = 4\int_0^1 \frac{dz}{(h^2+\beta)}$, $\quad R_3 = 4\int_0^1 \frac{dz}{(h^2+\beta)}$, $\quad R_4 = 4\int_0^1 \frac{dz}{(1+\frac{\beta}{h^2})}$.

These results are the same as Saxena and Srivastava [32]. Farther more, in the peristaltic pump of Newtonian fluid in tube (i.e $C,\varepsilon,\phi_1 \to 0$)

$$\Delta P = \frac{8}{(1-\phi^2)^{7/2}}\left\{Q(1+\frac{3\phi^2}{2}) + \phi^2\frac{(\phi^2-16)}{4}\right\}, \tag{45}$$

$$F_o = \frac{8}{(1-\phi^2)^{3/2}}\left\{(Q-1-\frac{\phi^2}{2}) + (1-\phi^2)^{3/2}\right\}. \tag{46}$$

which give the same results as in Shapiro *et al.,* [33]. In the next section, the effects of parameters on the flow will be discussed.

4. Graphical Results and Discussion

Using a computer programmer to evaluate the system of flow for an incompressible Newtonian fluid between two axial flexible cylinders pumping by peristaltic motion on the walls using long wavelength approximation. The research analysis results on fluid and particular phases for axial velocity, pressure drop, friction force and shear stress on inner and outer walls. Particles' radius are taken at the range $a_1 = (0.1:0.2)$. Figure 2 and 3 indicate how the axial velocity of fluid u_f influenced by both ϕ (amplitude ratio of the outer tube) and h and ϕ_1 (amplitude ratio of the inner tube) and h_1 but the effects are different in two cases . u_f increases by increasing ϕ and h but decreases by increasing ϕ_1 and h_1. The same parameters are represent in Figure 4 and 5 to investigate effects on the axial velocity of the particles u_p. These figures show that the parameters have the same manner to u_p as in u_f the axial velocity of particles u_p influenced by both ϕ and h and ϕ_1 and h_1. It could be observed that u_p increases by increasing ϕ and h but decreases by increasing ϕ_1 and h_1.

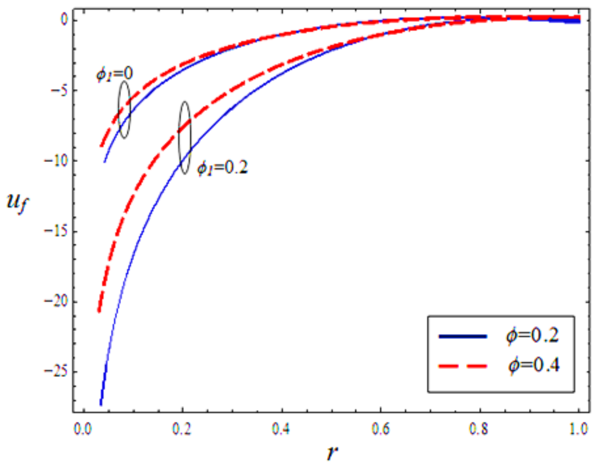

Fig. 2. u_f with r for different ϕ and ϕ_1, ($\varepsilon = 0.4$, $C = 0.4$, $Q = 0.6, h = 1 + \phi$, $h_1 = \varepsilon + \phi_1$)

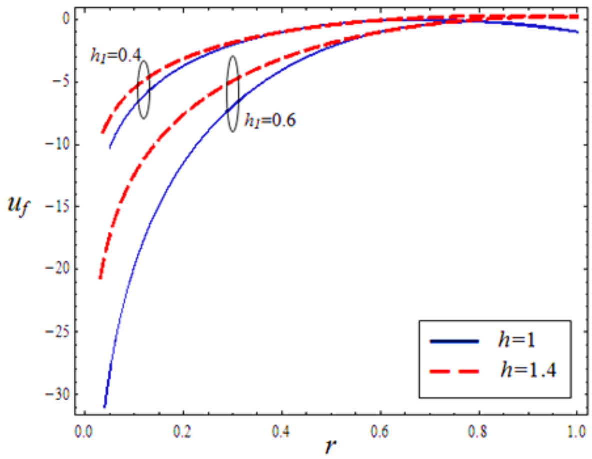

Fig. 3. u_f with r for different values of h and h_1, ($\phi = 0.4$, $\phi_1 = 0.2$, $Q = 0.6, C = 0.4, \varepsilon = 0.4$)

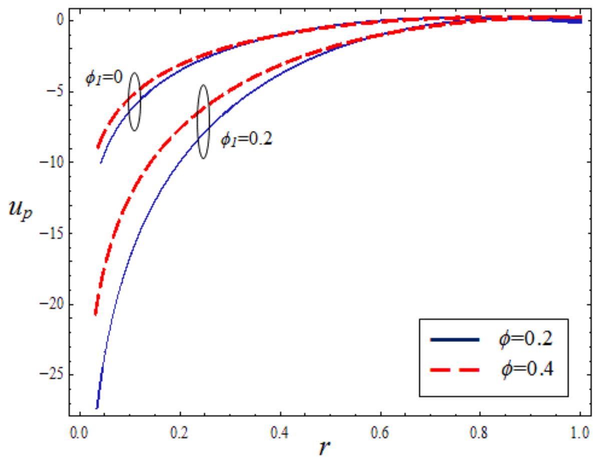

Fig. 4. u_p with r for different values of ϕ and ϕ_1 ($\varepsilon = 0.4$, $C = 0.4$, $Q = 0.6$, $h = 1+\phi$, $h_1 = \varepsilon + \phi_1$)

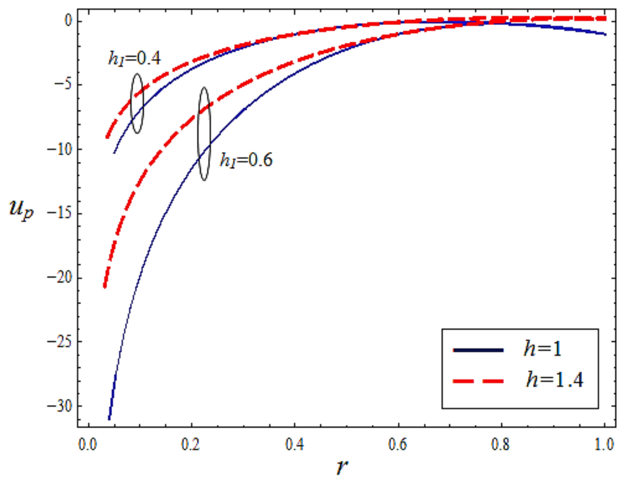

Fig. 5. u_p with r for different value of h and h_1, ($\phi = 0.4$, $\phi_1 = 0.2$, $Q = 0.6$, $C = 0.4$, $\varepsilon = 0.4$)

For any given set of parameters, a liner relationship between the flow rate and the pressure drop is observed in Figure 6 and 7. In these two figures, the pressure drop ΔP increases with the flow rate Q which implies that (an increase in the flow rate reduces the pressure rise **)** $-\Delta P$ **()** thus the maximum flow rate is accrued at zero pressure rise and the maximum pressure is achieved at zero flow rate**.**

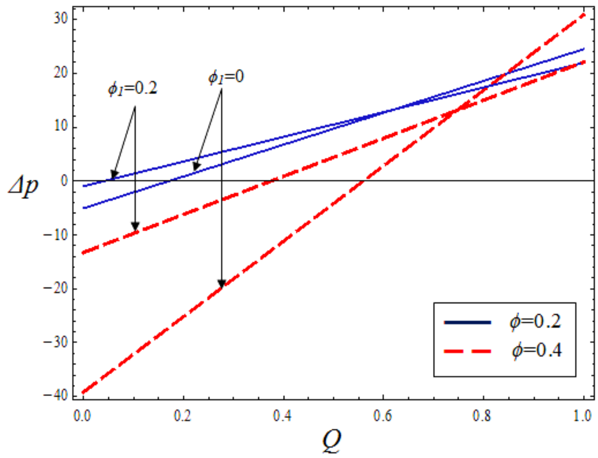

Fig. 6. ΔP with Q for different value of ϕ and ϕ_1, ($\varepsilon = 0.2, C = 0.4$)

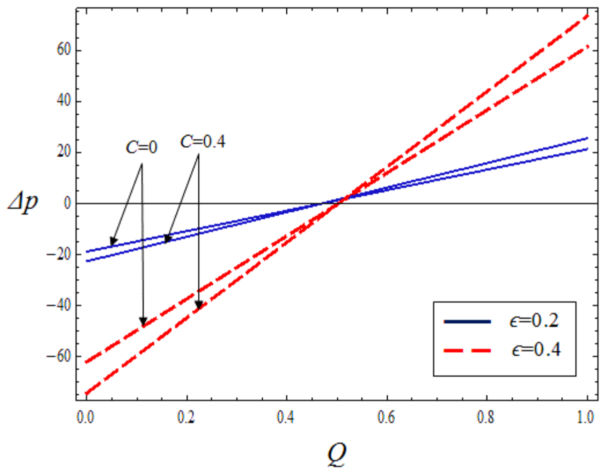

Fig. 7. ΔP with Q for different value of ε and C ($\phi = 0.4$, $\phi_1 = 0.1$)

It could be observed in Figure 6 and 7 that the pressure drop ΔP increases with ϕ (amplitude ratio of the outer tube), ϕ_1 (amplitude ratio of the inner tube), C (particle concentration) and ε (eccentricity parameter).

The friction force at the tube wall F_o discussed in Figure 8 and 9, F_o decreases with increasing amplitude ratio ϕ, ϕ_1 and ε while F_o increases with particle concentration C.

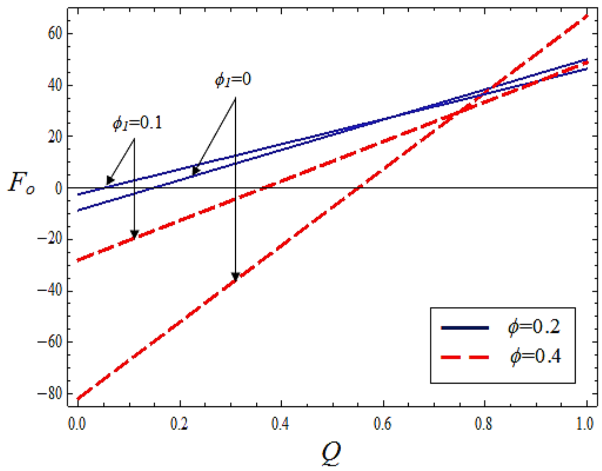

Fig. 8. F_o with Q for different values of ϕ and ϕ_1 ($\varepsilon = 0.4$, $C = 0.4$)

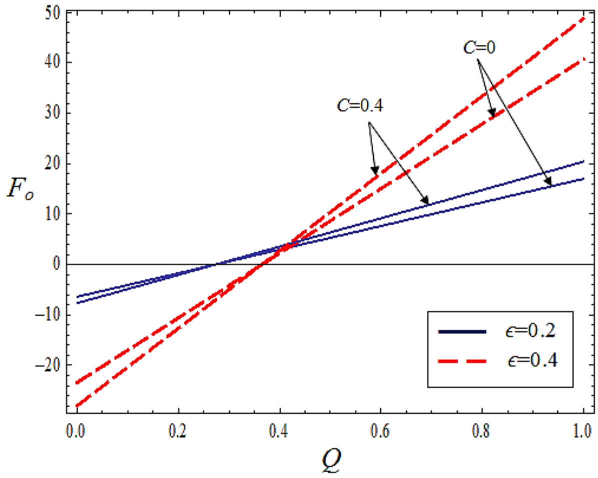

Fig. 9. F_o with Q for different value of ε and C ($\phi = 0.4$, $\phi_1 = 0.1$)

Figure 10 and 11 represent how friction force on the inner tube F_i effected by various parameters as same as on the outer tube that F_i decreases with increasing amplitude ratio ϕ, ϕ_1 and ε while F_i increases with particle concentration C.

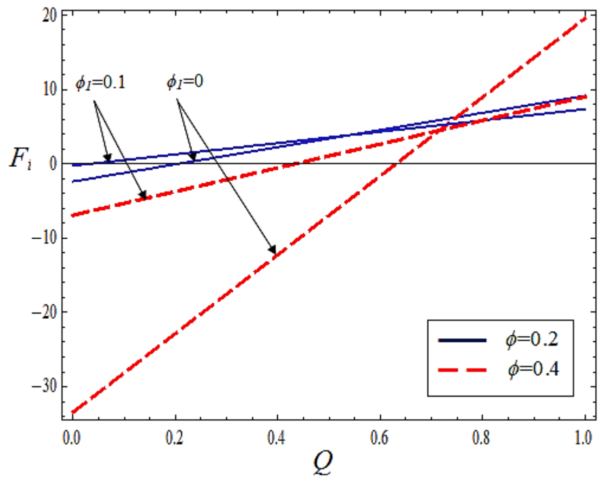

Fig. 10. F_i with Q for different values of ϕ and ϕ_1 ($\varepsilon = 0.4, C = 0.4$)

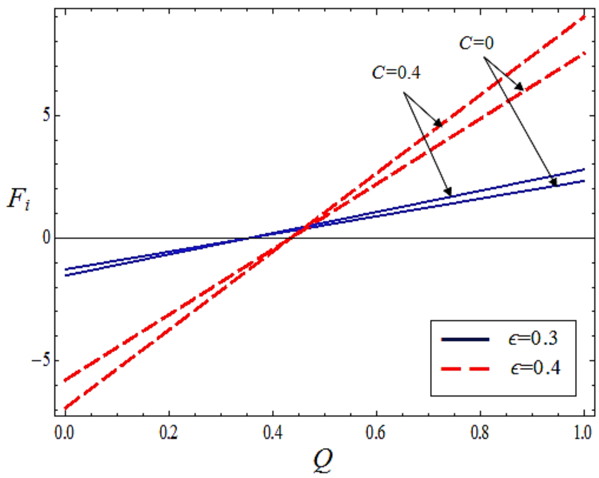

Fig. 11. F_i with Q for different values of ε and C, ($\phi = 0.4, \phi_1 = 0.1$)

Shear stress on the outer tube T_1 are represented in Figure 12-14 in which the shear stress increases by increasing the flow rate Q while the effect of both ϕ and ϕ_1 are not the same which appeared in Figure 12 T_1 decreases by increasing ϕ and increases by increasing ϕ_1. In Figure 13 effects of both h and h_1 on T_1 which represent T_1 decreases by increasing h and increases by increasing h_1. Figure 14 show that T_1 increases by increasing both ε and C.

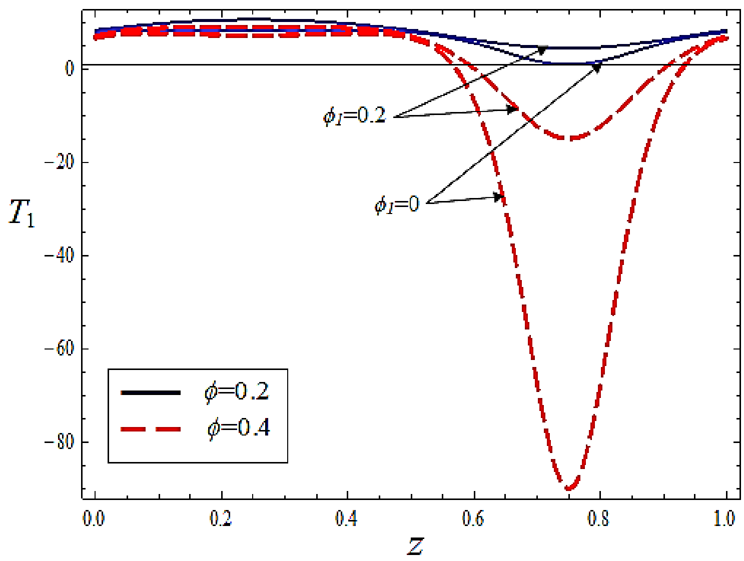

Fig. 12. T_1 with z for different values of ϕ and ϕ_1 for $(\varepsilon = 0.4, C = 0.4, Q = 0.4)$

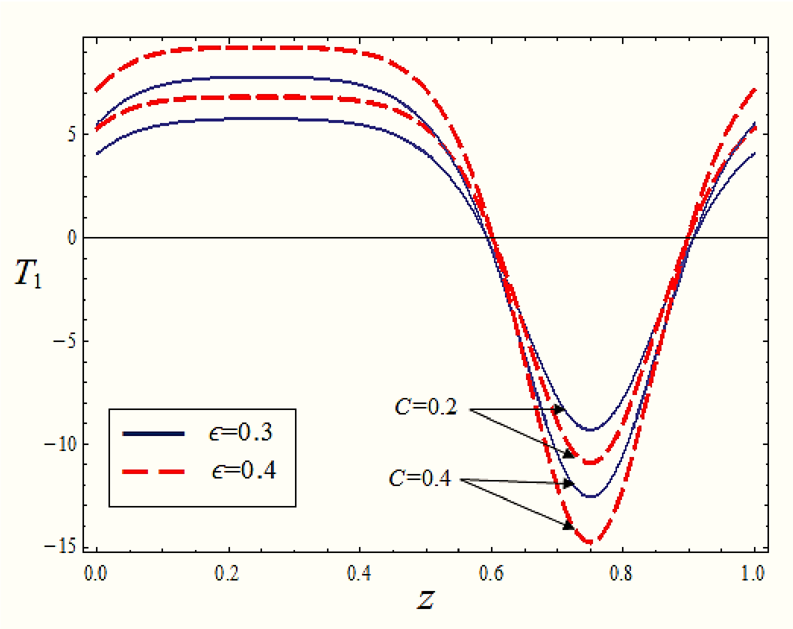

Fig. 13. T_1 with z for different values of ε and C, for ($\phi = 0.4$, $\phi_1 = 0.2$, $Q = 0.4$)

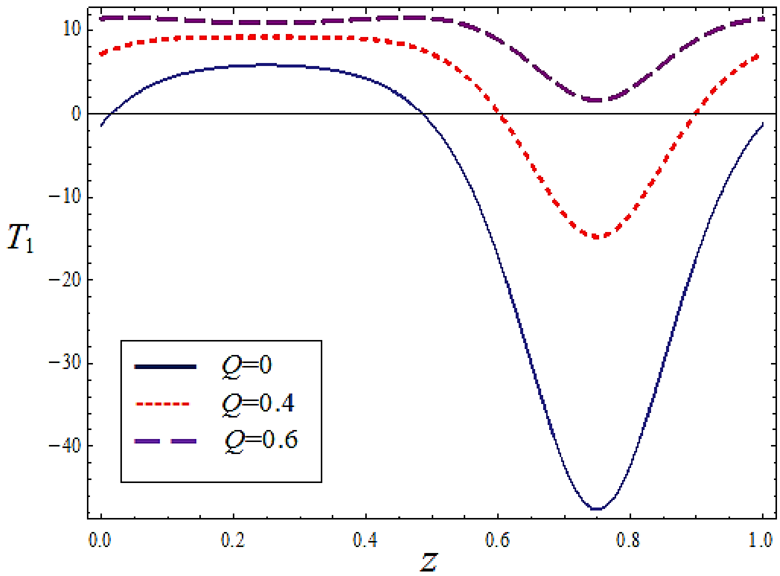

Fig. 14. T_1 with z for different values of Q ($\phi = 0.4$, $\phi_1 = 0.2$, $\varepsilon = 0.4$, $C = 0.4$)

Also, shear stress on the inner tube T_2 are represented in Figure 15-17 in which the shear stress decreases by increasing the flow rate Q while in Figure 15 effects of both ϕ and ϕ_1 on T_2. T_2 increases by increasing ϕ and decreases by increasing ϕ_1. In Figure 16 effects of both h and h_1 on T_2 which represent T_2 increases by increasing h and decreases by increasing h_1. Figure 17 show that T_2 decreases by increasing both ε and C.

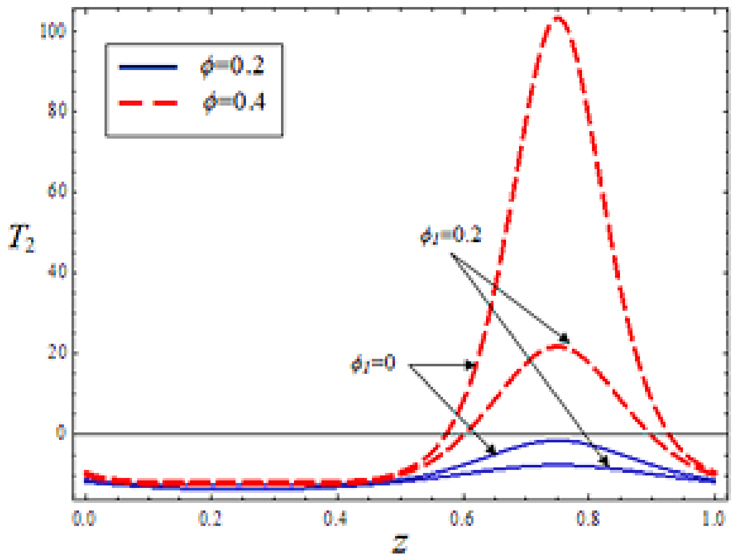

Fig. 15. T_2 with z for different values of ϕ and ϕ_1, for ($\varepsilon = 0.4$, $C = 0.4$, $Q = 0.4$)

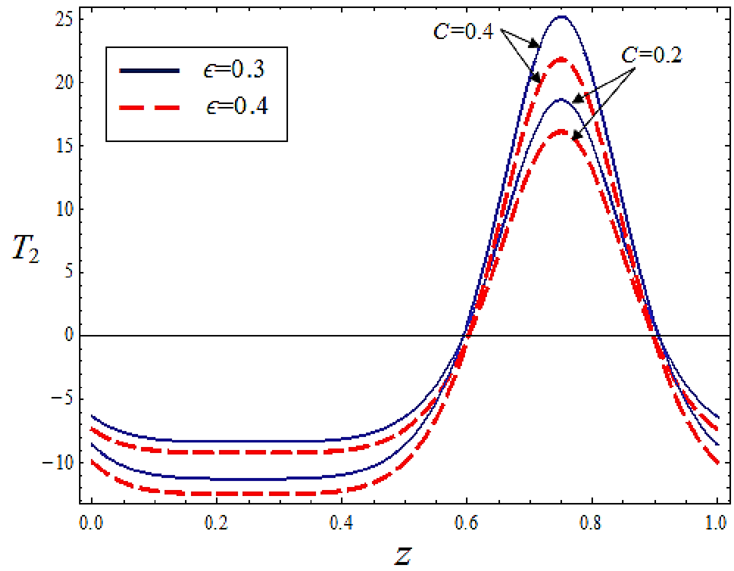

Fig. 16. T_2 with z for different values of ε and C, for ($\phi = 0.4$, $\phi_1 = 0.2$, $Q = 0.4$)

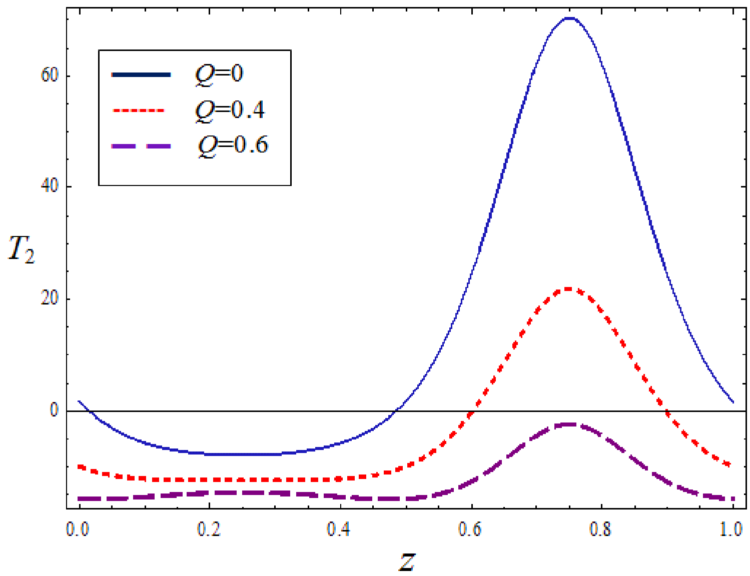

Fig. 17. T_2 with Z for different values Q, for ($\phi = 0.4$, $\phi_1 = 0.2$, $\varepsilon = 0.4$, $C = 0.4$)

5. Real Life Application

The urinary system consists of the kidneys, ureters, bladder and urethra. It can be divided into two sections, an upper (kidneys to ureters) and a lower (bladder to urethra) sections. Under normal conditions, urine flows from the kidneys to the bladder by peristaltic action of the ureters wall. In an adult human, typical ureter dimensions and bolus velocity, as suggested for biomechanical modeling by Boyarski *et al.,* [6], are: $a = 2.5mm, \lambda = 120mm,$ and $c = 30mm/s$. Urine characteristics are similar to that of water so that the density is $\rho_f = 1000kg/m^3$ at $T = 298K^o$ and viscosity

$\mu_o = 0.00089\ Ns/m^2$. The present study can be applied to discuss the effect of flexible endoscopy has inside the ureter. The process occurs at isothermal conditions. The major assumption in the present analysis is a two-dimensional geometry with the walls of ureter and endoscopy being represented as an infinite train of sinusoidal waves. The fluid is incompressible and Newtonian, peristalsis occurs with solid particles as in the fluid dispersion/aggregation of CaO. In a diluted urine mixture, the particle's diameter of CaO crystals is approximately $2a_0 = 10.2\,\mu m$ Guerra *et al.*, [11], and the average density of these crystals is approximately $\rho_p = 1960 kg/m^3$ Walton et al., [43]. The value of $Q_0 \equiv (Q)_{\Delta P=0}$ (free pumping) is of particular physiological relevance since it represents peristalsis without a prescribed pressure gradient it be represented in Table 1 flow rate for different types of endoscopy.

Table 1
Comparison between the time-mean flow rate of suspension fluid during the presence of flexible endoscopy for different radii a_1 and wave amplitudes b_1 and the time-mean flow rate of suspension fluid during the presence of traditional endoscopy

Radius of endoscopy $a_1(mm)$	Amplitude of endoscopy $b_1(mm)$	The time-mean flow rate $\pi a^2 c Q_0 \equiv Q_{\Delta p=0}$	The time-mean flow rate in the absence of b_1 $\pi a^2 c Q_0 \equiv Q_{\Delta p=0.b_1=0}$
0.25	0.01	217.234	218.858
0.3	0.02	232.887	237.268
0.8	0.03	243.461	251.254
1.00	0.04	255.997	268.138
1.25	0.05	275.608	293.607
1.5	0.06	298.619	323.221
1.6	0.08	301.481	335.403

6. Conclusion

An incompressible suspension Newtonian fluid between two axial flexible cylinders pumping by peristaltic motion on the walls is studied. Our system of equations is solved using long-wavelength approximation. The major outcomes of the investigation are featured underneath

i. u_f and u_p increases by increasing ϕ (amplitudee ratio for the outer tube) but decreases by increasing ϕ_1 (amplitudee ratio for the inner tube).
ii. The flow rate influenced by existing peristaltic wave on the inner tube.
iii. The pressure drop, pressure rise are disused in the presence of flexible inner tube it found that the pressure drop ΔP increases with increasing ϕ (amplitude ratio of the outer tube), ϕ_1 (amplitude ratio of the inner tube), C (particle concentration) and ε (eccentricity parameter)..
iv. The friction force on inner and outer tubes are decreases with increasing ampitude ratio for the outer and inner tube ϕ and ϕ_1 respectively.
v. The effect of flexible inner tube on shear stress: T_1 (shear stress on the outer tube) decreases by increasing ϕ and increases by increasing ϕ_1, T_2 (shear stress on the inner tube) increases by increasing ϕ and decreases by increasing ϕ_1.

vi. The present analysis can also be reduced to Newtonian fluid by taking C=0, as special case of our study.
vii. This study could be applied on ureter tract in the presence of flexible endoscopy.
viii. Real data has been used to calculate the effect of flexible endoscopy on the flow in ureter tract.
ix. Finally, this work can be developed by studying other types of fluids or studying effects of new physical quantities on the flow.

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