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Development of Friction Factor-Generalized Equation for Multi-Shapes Microchannel

Bahaa Abdelhamid Ismail Abdelhamid Mourad[1], Ernie Mat Tokit[1,2,*], Mohammad O. Hamdan[3], Fatimah Al-Zahrah Mohd Sa'at[1,2]

[1] Fakulti Teknologi dan Kejuruteraan Mekanikal, Universiti Teknikal Malaysia Melaka, 76100 Durian Tunggal, Melaka, Malaysia
[2] Centre for Advanced Research on Energy, Universiti Teknikal Malaysia Melaka, 76100 Durian Tunggal, Melaka, Malaysia
[3] Department of Mechanical Engineering, College of Engineering, American University of Sharjah, United Arab Emirates

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ABSTRACT

The major goal of this research is to use computational fluid dynamics, *CFD* to derive a generalized friction factor formula for a smooth micro-size channel in a laminar flow regime. Based on the number of channel sides and the Reynolds number value, this generated formula can be used to precisely calculate the friction factor. The microchannel shapes are correlated to the friction factor constant as a function of the number of channel sides, *N*, starting from infinity for a circular microchannel, down to 3 numbers of *N*, for a triangular microchannel. The following microchannel shapes with a constant hydraulic diameter of 104 µm are selected, which are triangle, square, pentagon, hexagonal, octagonal, decagon, and circle. The developed equation works for a fully developed laminar flow range up to Reynolds number of 2000. Moreover, the current work results are further compared with the previous results of microchannels for verification within acceptable value of 0.07% to 0.48%.

1. Introduction

Moody [1] and his team developed a dimensionless parameter relation investigating the fully developed flow in circular pipes. This relationship has been used to determine the friction factor, head-loss, and pipe surface roughness over a range of Reynolds number, *Re*.

An extensive review by Steinke and Kandlikar [2] revealed the causes of deviations in friction factor analysis. Discrepancies in experimental research were exhibited because of the fabrication uncertainties leading to inaccuracy of channel geometry measurements, surface roughness measurements, developing and fully developed regions, and inlet and outlet losses that are unaccounted for. They concluded that, given the developed flow region and channel geometry, the conventional friction factor equation is still applicable for microchannels. Some researchers have used the Moody chart to determine the Darcy friction factor for non-circular channels by

* *Corresponding author.*
E-mail address: ernie@utem.edu.my

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approximating the diameter with a hydraulic diameter using the well-known relation 64/*Re* for laminar flow [3-6]. However, the flow for a microchannel with a smaller aspect ratio is mainly laminar. The Darcy friction factor values determined from the Moody chart need attention for non-circular microchannels. In addition, they did not consider the accurate calculation of the flow entrance length in microchannel but instead assumed a developed flow relation [4].

Wu and Cheng [7] have experimentally investigated the friction factor for laminar flow in smooth trapezoidal microchannels with hydraulic diameters between 25.9 to 291 µm and Re up to 3200. They found that, the experimental data matches the analytical solutions within a range of $\pm$11% for a fully developed flow, this proves the validity of the Navier-Stokes equations under these circumstances. Results further demonstrate that, for a given hydraulic diameter, microchannels of different geometrical shapes have varying friction factors. Gunnasegaran *et al.,* [8] investigated the influence of the aspect ratio of a rectangular microchannel on the values of the friction factor under laminar conditions. It is found that the friction factor increases with the aspect ratio along with the Poiseuille number (*f*.Re). Moreover, it is found that the tip angle of a triangular microchannel has a significant effect on the friction constant due to the change in flow distribution. Park and Punch [9], based on their experimental work, concluded that, that the traditional theory of fully developed flow is suitable for microchannels within the range of *Re* < 800 and have a hydraulic diameter not exceeding 307 µm as per the scope of their experiment.

Bayraktar and Pidugu [10] conducted review research on microchannels liquid flow. They reported that microchannels' pressure drop and friction factor need more investigation due to the inconsistent published experimental and numerical results. They attributed the inconsistency of the friction factor and pressure drop due to the uncertainties in the measurements of the microchannel's actual size and wall roughness. In addition, they reported that neglection of the entrance and exit effect of the microchannel is one of the reasons for the inconsistent results. Weilin *et al.,* [11] investigated the flow characteristics of a trapezoidal shape microchannels with a hydraulic diameter between 51 to 169 µm and Re up to 2000. The friction factor results in laminar steady-state flow conditions deviating from the conventional theory. The microchannel friction factor is found to be greater than that predicted by the standard laminar theory. A corrected roughness-viscosity model was proposed by them to precisely interpret the friction coefficient value. Judy *et al.,* [12] have experimentally investigated the *f*.Re values of circular and square stainless-steel microchannels within a hydraulic diameter range of 15 to 150µm. They have found that, the *f*.Re value depends on the diameter of the channel, and the values found in this investigation deviated from the classical published values of 64 and 56.9 for circular and square microchannels, respectively, for an Re range of up to 2300. However, the experimental data matches the Stokes flow theory for circular and square fused silica microchannels. Peng and Peterson [13] have experimentally investigated the thermal performance of rectangular microchannels and the friction factor in relation to the channel width and height. The friction factor of the microchannel, with a hydraulic diameter of 0.133 to 0.367µm, was found to decrease as the minimum to maximum height and width ratio approach 0.5. It was also found that the friction factor values match the conventional theory for laminar flow but deviate for transitional and turbulent flow regimes. Celata *et al.,* [14] have investigated experimentally the friction factor of circular microchannels for laminar and turbulent flow. They concluded that, the friction factor is above the classical value of 64/Re for a smaller diameter microchannel, at 126 µm. They correlated this effect to the uncertainty of the roughness measurements and geometric deformation of circular channels. They also reported that, the transitional *Re* is between 2000 and 3000. Aniskin *et al.,* [15] studied the friction factor in a circular microchannel using the two-channel method for a circular microchannel with multiple hydraulic diameters and lengths under laminar flow conditions. They reported that, the results agree with the theoretical predictions for developed flow as long as the channel length is 120 times the diameter of the channel. Li *et al.,* [16] found that the friction coefficient values are higher than the classical values when comparing the theoretical friction factor value of 64/*Re* with the numerical values of rectangular microchannels with hydraulic diameters ranging from 50 to 200µm. In addition, they illustrated that the channel's roughness leads to an increase in friction factor for *Re* up to 100.

The study of the friction factor highlights the accuracy of using the conventional equation in determining and optimizing the thermal performance, pumping power, and overall system performance of the microchannel [17-24]. Chen *et al.,* [25] studied the thermal efficiency using the Nusselt number as an indicator for thermal performance, and microchannels of three different shapes (triangular, rectangular, and trapezoidal) under fully developed flow. The triangular microchannel was found to have the highest thermal efficiency and the lowest pumping power compared to other shapes. Qu and Mudawar [26] investigated numerically and experimentally the heat transfer properties and pressure drop of a laminar flow microchannel. They indicated that the conventional Navier-Stokes equations are applicable to microchannels. Wang *et al.,* [27] concluded that, the Navier-Stokes equations could be used for determining the fluid flow and thermal behavior of rectangular, trapezoidal, and triangular microchannels with hydraulic diameters up to 349 µm using numerical studies. It was also found that increasing the number of channels results in lower thermal resistance at the expense of more pumping power. The rectangular microchannel with an aspect ratio of 8.904 to 11.442 has the lowest thermal resistance and pressure drop compared to other shapes. However, the overall thermal performance was the lowest for the triangular microchannel, unlike the trapezoidal and rectangular shape microchannel. Khan *et al.,* [28] reported that for circular microchannels under a single-phase flow and an Re in the range of 304 to 2997, the experimental data agree with the proposed correlation of friction factor for laminar and transitional flows. They also demonstrated that the temperature profile depends on the hydraulic diameter and the total length of the channel. Mirmanto [29] reported via adiabatic and non-adiabatic experimental work that the friction factor for a microchannel is independent of the inlet temperature and hydraulic diameter, and that the friction factor values obtained match with the conventional theory for a Re up to 3430 and an inlet temperature up to 90 ^{o}C with and without heat flux. Additionally, he reported that the hydraulic diameter has a substantial impact on the pressure drop, but not on the friction factor.

In the light of the above literature review, the previous studies focus mainly on three shapes of microchannels: circular, rectangular, and triangular. However, a few studies considered other shapes that can be used for different purposes, such as electronic cooling. In addition, it is observed that the published relations for determining the friction factor are limited to the abovementioned shapes. It is also observed that the Navier-Stokes equations are applicable to microchannels. To the best of the authors' information, there is no unified equation to get the friction factor of different microchannel shapes. This is the primary motivational point for this study. Computational fluid dynamics, *CFD* is used to develop the generalized equation for determining the Darcy friction factor for smooth microchannels with different shapes under laminar flow conditions.

2. Methodology

The methods in this paper are explained in detail to give the insight view of the investigation of this study.

2.1 Validation

The numerical CFD model has been compared with Toh *et al.,* [6] model, as depicted in Figure 1. The predicted friction factor trend using CFD matches with the CFD values of Toh *et al.,* [6] with an average discrepancy of 2%. A relatively higher deviation of 3% is observed at the lower *Re*. This slight deviation is likely due to the meshing quality with the use of 51,200.0 elements, while the number of elements used in this study is 3,192,798.0 for rectangular microchannel. This can be considered as validation for the numerical method used in this study.

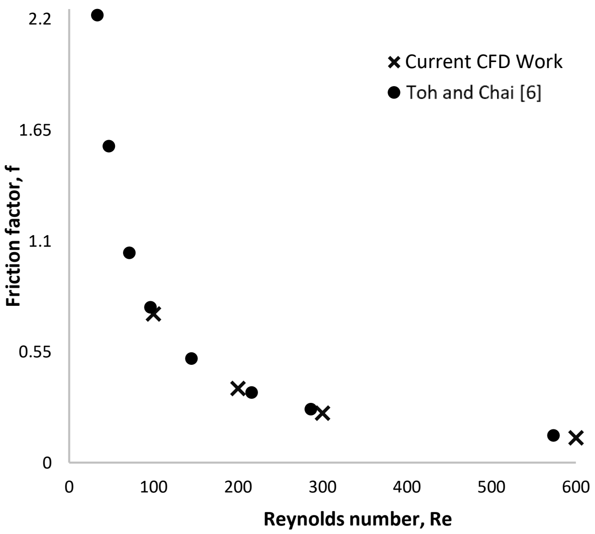

Fig. 1. Comparison of the predicted friction factor at various Reynolds numbers with the previous work

2.2 Model Geometry

The 3D model geometry is developed using the ANSYS FLUENT software, based on the previous work model, and without heat flux at the base of the microchannel [6]. Other model shapes designed in the current study have the same hydraulic diameter (D_h = 104 µm), meshing quality, and element size.

2.3 Mesh Independence Test

The accuracy of different mesh elements on the CFD solution is tested through a mesh independence study. An element size of 0.005mm is selected based on the outcome of mesh validation at the center point velocity as shown in Figure 2.

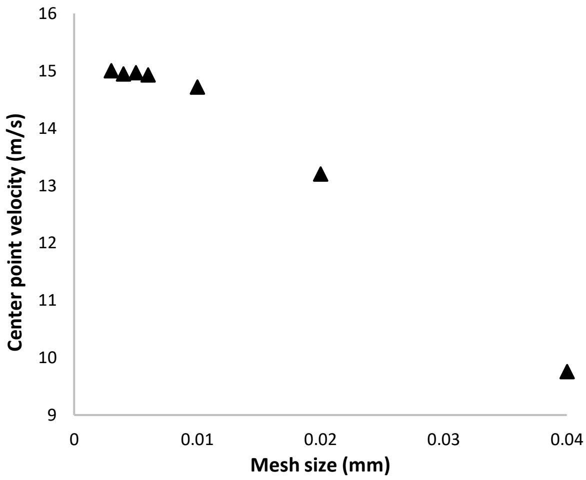

Fig. 2. Center point velocity for different model mesh sizes

2.4 Governing Equations

The continuity and momentum equations are presented in Eq. (1) and Eq. (2) respectively.

Continuity equation:

$$\nabla\vec{V} = 0 \tag{1}$$

where divergence of velocity, $\nabla\vec{V}$.

Momentum equation:

$$\rho\left(\vec{V}.\nabla\vec{V}\right) = -\nabla P + \nabla(\mu_f\,.\nabla\vec{V}) \tag{2}$$

where fluid's density, ρ, and fluid's dynamic viscosity, μ_f and pressure, *P* respectively.

2.5 Boundary Conditions

The water flow through the smooth microchannel is assumed to be steady and incompressible. The inlet velocity is based on *Re* range of 100 to 2000. The water inlet temperature is constant and set at 20 ^oC. The atmospheric pressure is assigned as the outlet pressure. No heating is assigned at the bottom wall, and a no-slip condition is applied at the adjacent walls.

2.6 Numerical Setup

The algorithm used in the *CFD* involves pressure-velocity coupling with continuity converging residuals of 1e^{-6} and 1e^{-12} for continuity and velocity equations.

3. Results and Discussion

First, the friction factor is numerically calculated using Fluent for three microchannel shapes: a rectangular microchannel with a unity aspect ratio, a circular microchannel, and an isosceles triangle-shaped microchannel. For each case, the entrance length for the developed region is calculated using CFD by observing the pressure drop through the channel's length. The hydraulic diameters for all the tested shapes are the same (0.104 mm). For verification purposes, the friction factor is calculated at various *Re* (from 100 to 2000) based on Eq. (3). All values are in good agreement with the previously published data by Çengel, and Ghajar [30]. Then, based on this verification, the friction factor for the pentagon, hexagonal, octagonal, and decagon microchannels of the same hydraulic diameter is determined. Table 1 illustrates the findings of the CFD study. The static pressure drop and the average velocity are calculated for each case using average surface integrals in the Fluent software, considering the developed flow conditions.

$$f = \frac{2\Delta P D_h}{l\rho v_{avg}^2} \tag{3}$$

where the Darcy friction factor, f, pressure drop, ΔP , hydraulic diameter, D_h, channel's length, l, fluid's density, ρ and average velocity, v_{avg} respectively.

Table 1
Friction factor values for different microchannel shapes

Re	Channel's cross sectional area						
	Triangle	Square	Pentagon	Hexagon	Octagonal	Decagon	Circle
	f.Re						
100	53.04	56.88	58.63	59.93	61.44	62.13	64.01
300	53.04	56.94	58.63	59.93	61.44	62.14	64.01
900	53.10	56.93	58.66	59.92	61.43	62.14	64.02
1200	53.30	56.93	58.65	59.94	61.44	62.17	64.00
2000	53.32	56.92	58.65	59.94	61.44	62.18	64.00

Figure 3 depicts the friction factor curve for multiple microchannel shapes starting from a triangular shape duct with three sides until a circular shape with an infinite number of sides.

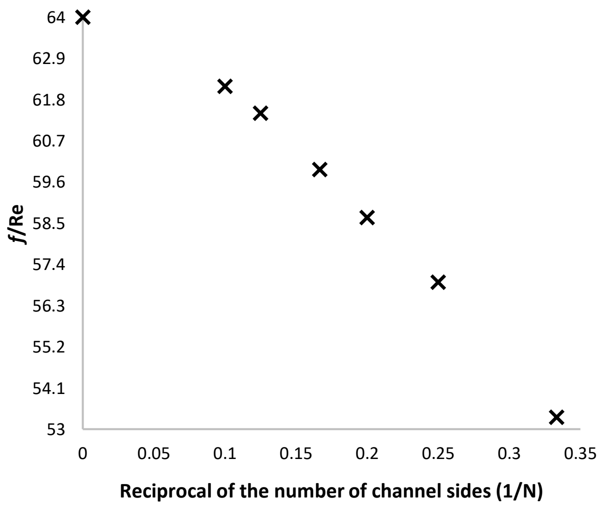

Fig. 3. Friction factor values for multiple microchannels with different shapes

As mentioned before, the main goal of this work is to ease the friction factor calculation for microchannels. Therefore, the following formula (Eq. (4)) is developed based on Figure 3 using a MATLAB equation solver with a curve fitting option. The developed equation has a coefficient of determination, R^2 of 0.998, which shows a high level of accuracy of the developed formula.

$$f = \frac{64.169 - \left(-6.367 * \left(1 - e^{\left(3.029 * \frac{1}{N}\right)}\right)\right)}{Re} \quad (4)$$

where the Darcy friction factor, f, the number of channel's sides, N, *and Reynolds number,* Re respectively.

The logarithmic formula is based on the number of channel sides and the Re inputs. Figure 4 represents the output of the equation with a comparison of available published data [30]. The current results (both trend and values) are in good agreement with the published ones within the range of 0.07% to 0.49%.

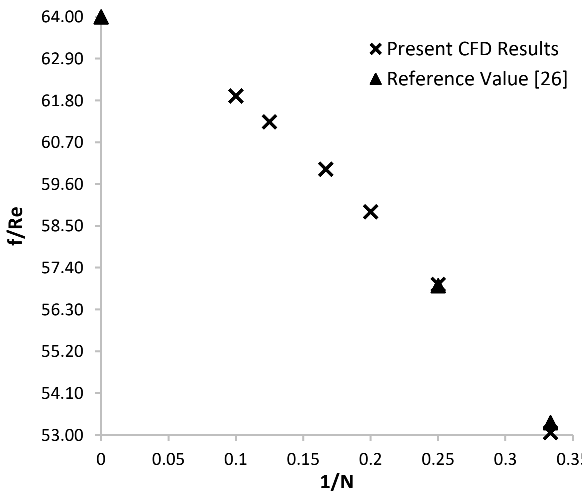

Fig. 4. Friction factor values determined using the developed generalized equation

4. Conclusion

The findings of this work provide researchers with a generated formula that can be utilized for determining the friction factor of microchannels with a constant hydraulic diameter of 104 µm, of different shapes which has been developed for laminar flow for Re range of 100 to 2000. The developed friction factor is correlated as a function of the number of channel sides, N, starting from infinity for a circular microchannel, down to 3 numbers of N, for a triangular microchannel. The deviations with the published work were predicted to lie within 0.07% to 0.49%.

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