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Weak Nonlinear Oscillatory Double Diffusive Convection in a Viscoelastic Fluid-Saturated Porous Layer Under Gravity Modulation

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ABSTRACT

A thermal instability caused by buoyancy force is investigated in an initially quiescent indefinitely extended horizontal porous layer saturated with non - Newtonian fluid. The characteristics of fluid motion are explained by using Modified Darcy's law. Here, we considered the time periodic gravity field, and its effect on the system has been investigated. For the oscillatory mode of convection, a weakly non-linear stability analysis has been performed to evaluate Heat and Mass transfers in terms of the Nusselt number and Sherwood Number. To compute the results, the complex non-autonomous Ginzburg – Landau equation is used. It has been studied how viscoelastic fluid relaxation and retardation times impacts on heat and mass transmission. Further, the research confirms that heat and mass transfer can be successfully regulates by a technique that is external to the system. Finally, it has been discovered that excessive stability delays the onset of convection, hence increasing in the heat and mass transmission.

1. Introduction

The problem of convective instability of a horizontal fluid saturated porous layer has been investigated extensively, and is well documented by Lowrie [1] explained the viscoelastic behavior of fluids which is a major rheological phenomenon in the asthenosphere and in the deeper mantle. The dynamic of stabilization and destabilization may lead to dramatic changes of behavior depending on the proper tuning of the amplitude and frequency of the modulation. If an imposed modulation can destabilize an otherwise stable state, then there can be a major enhancement of heat/mass/momentum transport. If an imposed modulation can stabilize an otherwise unstable state, then higher frequencies can be attained in various processing techniques.

Furthermore, Herbert [2] and Green III [3] were the first authors to investigate oscillatory convection in a regular viscoelastic fluid of the Oldroyd form under the presence of infinitesimal perturbations. Vest and Arpaci [4] analyzed the over-stability in a viscoelastic fluid layer heated from below, and identified the condition for the onset of thermal expansion overstability. By considering

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two sets of boundary conditions Bhatia and Steiner [5] studied the convective instability in a rotating viscoelastic fluid layer. They evaluated that it has stabilizing effect in the case of Newtonian fluid while it has destabilizing effects in case with rotation.

After that, Bhatia and Steiner [6] have studied how a magnetic field affected the oscillatory convection in a layer of viscous fluid. Rosenblat [7] studied the onset of convective flow in a medium of viscoelastic fluid heated from below. Various models were used by Rudraiah *et al.,* [8]; Rudraiah *et al.,* [9] to investigate the onset of oscillatory convection for porous media saturated with viscoelastic fluid. Moreover, the experiment has been developed by Martinez-Mardones *et al.,* [10] introducing the binary aspect to the viscoelastic fluids. Kim *et al.,* [11] investigated the thermal destabilization of viscoelastic fluids in porous media, using nonlinear analysis for transfer of heat and linear analysis to determine the stability criterion for convective flow. Later, considering linear theory, Yoon *et al.,* [12] investigated the onset of oscillatory convective flow in a horizontal porous medium which is saturated by viscoelastic fluid. Malashetty *et al.,* [13] conducted a linear stability study to examine the impact of temperature modulation on the onset of convection in a horizontal layer and anisotropic porous layer saturated by a viscoelastic fluid. Tan and Masuoka [14] used a simplified Darcy-Brinkman-Maxwell method to investigate the stability of a Maxwell fluid with porous media and discovered the condition for the onset of oscillatory convection. Moreover, in a porous layer that was heated from below, research on the double - diffusive convection with Maxwell fluid was examined by Wang and Tan [15]. Malashetty *et al.,* [16,17] considered the linear and non-linear-instability for onset of double-diffusive flow for binary viscoelastic fluid-saturated anisotropic and isotropic porous media. A rotating viscoelastic fluid-saturated porous medium that has been heated from below has been analyzed and determined for linear and nonlinear stability by Malashetty *et al.,* [18].

In a binary viscoelastic fluid-saturated anisotropic for rotating porous medium, Malashetty *et al.,* [19] investigated the onset of double-diffusive convection using both studies of linear and weakly nonlinear stability. The momentum equation is calculated by using the modified Darcy law for the Oldroyd types of viscous incompressible fluid. By the use of linear and nonlinear concepts, Kumar and Bhadauria [20] investigated the double diffusive convection and its relationship to rotation for horizontal porous medium with viscoelastic fluid. Furthermore, there is a good amount of nonlinear studies for stationary convection under gravity modulation in the presence of Newtonian fluid with and without porous media available; Vanishree and Siddheshwar [21], Saravanan and Arunkumar [22], Saravanan and Sivakumar [23,24], Bhadauria *et al.,* [25,28-30], Siddheshwar *et al.,* [26,27], and Srivastava *et al.,* [31]. To the best of authors' information, there are no much more studies on double diffusive oscillatory convection under gravity modulation. Kiran *et al.,* [32] shown that the effects of amplitude and frequency of modulation on heat transport have been analyzed and depicted graphically. The study shows that the heat transport can be controlled effectively by a mechanism that is external to the system. Further, it is also found that heat transfer is more in oscillatory mode of convection rather than in stationary mode of convection Kanafiah *et al.,* [33] examine the flow of Brinkman-viscoelastic fluid in the boundary layer region. The flow over a Horizontal Circular Cylinder (HCC) is investigated. In the presence of the Brinkman and viscoelastic parameters influences the velocity behaviour of fluid, with a tendency to decrease the velocity distribution of fluid. Furthermore, both parameters have the potential to decrease the skin friction coefficient. The dynamics of the non - Newtonian viscoelastic fluid with micro-rotation at a boundary layer of a horizontal circular cylinder are presented. Results are validated before the velocity and micro-rotation profiles are examined and the effect of material, viscoelastic and magneto hydrodynamic parameter on the flow is discussed by Aziz *et al.,* [34]. Usman *et al.,* [35] investigated the performance of viscoelastic nano-liquid film sprayed over a stretched cylinder. It also explores the effects of activation energy and the assessment of entropy on heat and mass flow. The temperature increases with the Brownian motion parameter while it decreases with the increasing of Prandtl number, film thickness parameter and thermophoresis parameter.

Abidin *et al.,* [36] discovered the effects of strain retardation and thermal anisotropy parameter slow down the formation of heat transfer when their values are increased and stabilized the system. Meanwhile, the stress relaxation, Darcy-Prandtl, and mechanical anisotropy parameter enhanced the heat transfer mechanism rapidly in the convection when the values are increased thus destabilize the system. Bakar and Roslan [37] aim to analyse numerically the effect of internal heat generation or absorption in a two-dimensional (2D) horizontal cavity to the fluid flow and heat transfer process. The influences of heat generation or absorption parameters are investigated in terms of the flow, heat transfer, and Nusselt number. This inspired us to research a weak nonlinear thermal instability of double diffusive convection for non - Newtonian fluid saturated porous medium under gravity modulation, and calculate the Nusselt number and Sherwood number in terms of the amplitude of convection by solving the complex Ginzburg–Landau equation.

2. Governing Equation

Consider a non-Newtonian fluid saturated infinitely extended horizontally porous media bounded within two boundaries that are completely free; free at z = 0 and z = d as heated from the bottom. We have used the reference in Cartesian terms with the origin at the bottom as well as z-axis moving upwards in a vertical direction. Its schematic diagram is shown in Figure 1.

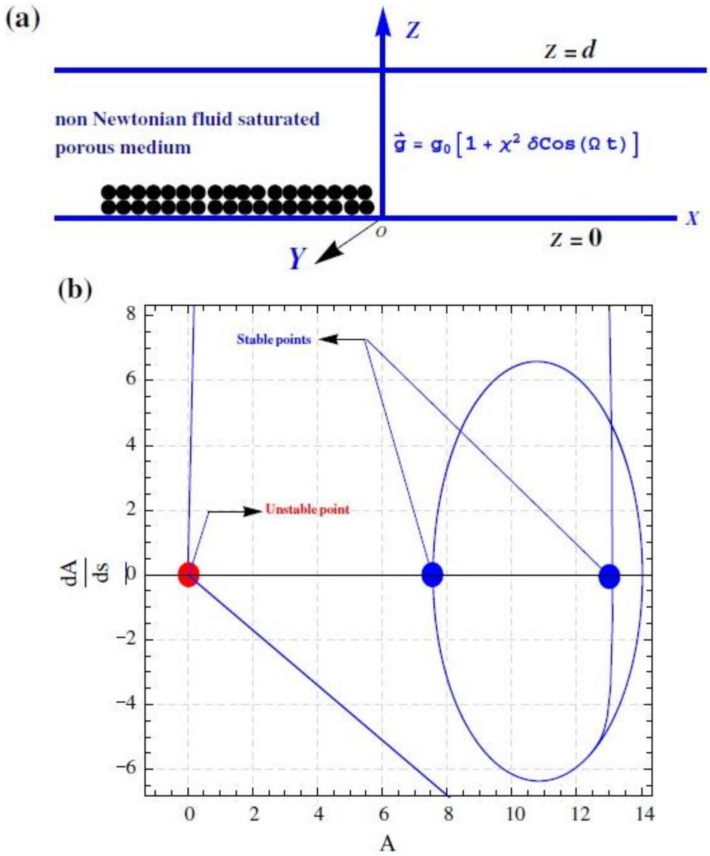


Fig. 1. (a) Physical configuration of the problem, (b) Supercritical pitchfork bifurcation diagram

The Boussinesq approximation is applied to this problem, and a modified Darcy's model is used [38]. The governing equations of flow, temperature and concentration fields are written as

$$\nabla.\vec{q} = 0, \tag{1}$$

$$\left(\lambda \frac{\partial}{\partial t} + 1\right)\left(-\nabla P + \rho \vec{g}\ \right) - \frac{\mu}{K}\left(\varepsilon \frac{\partial}{\partial t} + 1\right)\vec{q} = 0. \tag{2}$$

$$\frac{\partial T}{\partial t} + (\vec{q}.\nabla)T = k_T \nabla^2 T, \tag{3}$$

$$\frac{\partial S}{\partial t} + (\vec{q}.\nabla)S = k_s \nabla^2 S, \tag{4}$$

$$\rho = \rho_0\left[1 - \alpha_T(T - T_0) + \beta_s(S - S_0)\right], \tag{5}$$

where K is permeability, $\vec{q}$ is velocity, μ is viscosity, P is pressure, k_T is the parameter of thermal expansion, density is ρ, S_0 is the concentration, T_0 is the temperature for which $\rho = \rho_0$ is the standard density. The following are the temperature, concentration and periodic gravitational field produced by externally:

$$\vec{g} = g_0(1 + \chi^2 \delta \cos(\Omega t))\hat{k}, \tag{6}$$

$$\left.\begin{array}{lll} T = T_0 + \Delta T & \text{at} & Z = 0, \\ T = T_0 & \text{at} & Z = d, \\ S = S_0 + \Delta S & \text{at} & Z = 0, \\ S = S_0 & \text{at} & Z = d, \end{array}\right\} \tag{7}$$

where δ is magnitude of gravity modulation and Ω is frequency of modulation and χ is the smallest of amplitude of modulation.

ΔT is difference in temperature across porous media whereas ΔS is the solute difference across porous media.

Therefore, in this stage, the basic state is considered quiescent, with the following quantities

$$\vec{q}_b = 0,\ \ \rho = \rho_b(z),\ \ P = P_b(z),\ \ T = T_b(z,t)\ ,\ S = S_b(z,t) \tag{8}$$

Substituting Eq. (8) into Eq. (1) to Eq. (5), obtained following expressions, for basic state of pressure, temperature and concentration

$$\frac{dp_b}{dz} = -\rho_b g, \tag{9}$$

$$k_T \frac{d^2 T_b}{dz^2} = 0, \tag{10}$$

$$k_S \frac{d^2 S_b}{dz^2} = 0, \tag{11}$$

$$\rho_b = \rho_0 \left[1 - \alpha_T (T_b - T_0) + \beta_S (S_b - S_0)\right]. \tag{12}$$

The solution of the Eq. (10) and Eq. (11) when subjected to thermal boundaries' condition in Eq. (7) is provided by

$$T_b = T_0 + \Delta T \left(1 - \frac{z}{d}\right), \tag{13}$$

$$S_b = S_0 + \Delta S \left(1 - \frac{z}{d}\right), \tag{14}$$

The finite-amplitude disturbances are introduced to the solution of the basic state is superposed in the following form

$$\left.\begin{aligned} \vec{q} &= q_b + q', \\ P &= P_b + P', \\ \rho &= \rho_b + \rho', \\ T &= T_b + T', \\ S &= S_b + S'. \end{aligned}\right\} \tag{15}$$

When introducing the Eq. (15) and temperature of basic state and concentration fields have given by Eq. (13) and Eq. (14) in Eq. (1) to Eq. (5). Since, we use convection stream function which is two-dimensional i.e., ψ as $(u', 0, w') = \left(\frac{\partial \psi}{\partial z}, 0, -\frac{\partial \psi}{\partial x}\right)$ and following are physical factors that are not dimensional which are rescaled by

$$x^* = \frac{x}{d}, y^* = \frac{y}{d}, \; z^* = \frac{z}{d}, \; p' = \frac{\mu k_T}{K} p^*, \; t' = \frac{d^2}{k_T} t^*, \; q' = \frac{k_T}{d} q^*,$$

$T' = \Delta T \, T^*$, $S' = \Delta S \, S^*$, $\psi = k_T \psi^*$, and $\Omega = \dfrac{k_T}{d^2} \Omega *$.

Obtain the resulting non-dimensional governing model while dropping its asterisk by using the dimensionless variables stated above and eliminating the pressure term

$$\left(\varepsilon \frac{\partial}{\partial t} + 1\right) \nabla^2 \psi + \left(\lambda \frac{\partial}{\partial t} + 1\right)\left(Ra_D \frac{\partial T}{\partial x} - Rs \frac{\partial S}{\partial x}\right)\left(1 + \chi^2 \delta \cos(\Omega t)\right) = 0, \tag{16}$$

$$\frac{\partial \psi}{\partial x}+\left(\frac{\partial}{\partial t}-\nabla^2\right)T=\frac{\partial(\psi,T)}{\partial(x,z)}, \tag{17}$$

$$\frac{\partial \psi}{\partial x}+\left(\frac{\partial}{\partial t}-\Gamma\nabla^2\right)S=\frac{\partial(\psi,S)}{\partial(x,z)}, \tag{18}$$

where $Ra_D=\frac{\alpha_T g\Delta TdK}{vk_T}$ is thermal Darcy-Rayleigh number, $Rs=\frac{\beta_S g\Delta SdK}{vk_S};$ is the solutal Rayleigh number, Γ is diffusivity ratio, $\Gamma=\frac{k_S}{k_T}$ and $v=\frac{\mu}{\rho_0}$ is the kinematic viscosity. Considering small change of time t & re-arranging it $\tau=\varepsilon^2 t,$ the system convection in a stationary mode going to be discussed.

To evaluate the solution of this, the impermeable stress-free heat transfer boundary condition is used.

$\psi=0$ and $T=0$ for Z = 0 and Z = 1 (19)

We propose a small perturbation parameter χ that shows derivation from the critical state of onset of convection, then the variables with weak nonlinear state may be expandable as power series of χ as

$$\left.\begin{aligned} Ra_D &= R_0+\chi^2 R_2+\ldots. \\ \psi &= \chi\psi_1+\chi^2\psi_2+\chi^3\psi_3+\ldots. \\ T &= \chi T_1+\chi^2 T_2+\chi^3 T_3+\ldots.. \\ S &= \chi S_1+\chi^2 S_2+\chi^3 S_3+\ldots.. \end{aligned}\right\} \tag{20}$$

In the absence of gravitational modulations, R_0 would be the value of critical Darcy-Rayleigh number where convection starts.

3. Bifurcation of Periodic Solution

For the bifurcation method to allow for the projected frequency, we propose the fast time scale of time *τ* and the slow time scale of *s*.

As a result, the time variable is scaled such that $\frac{\partial}{\partial t}=\frac{\partial}{\partial t}+\chi^2\frac{\partial}{\partial s}$. For that, in the solution of first-order problem, the non-linear term will be vanishing in the energy equation; In view of over-stability, the first-order problem can be reduced to the linear stability problem.

3.1 First Order System

The system uses the following format at the lowest level

$$\begin{bmatrix} \left(\varepsilon\frac{\partial}{\partial\tau}+1\right)\nabla^4 & R_0\left(\lambda\frac{\partial}{\partial\tau}+1\right)\frac{\partial}{\partial x} & -R_S\left(\lambda\frac{\partial}{\partial\tau}+1\right)\frac{\partial}{\partial x} \\ \frac{\partial}{\partial x} & \left(\frac{\partial}{\partial\tau}-\nabla^2\right) & 0 \\ \frac{\partial}{\partial x} & 0 & \left(\frac{\partial}{\partial\tau}-\Gamma\nabla^2\right) \end{bmatrix}\begin{bmatrix}\psi_1\\ T_1\\ S_1\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix} \tag{21}$$

Lowest order solution according to initial conditions Eq. (19) evaluated as follows

$$\psi_1=\left(A(s)e^{i\omega t}+\overline{A}(s)e^{-i\omega t}\right)\sin(ax)\sin(\pi z), \tag{22}$$

$$T_1=\left(B(s)e^{i\omega t}+\overline{B}(s)e^{-i\omega t}\right)\cos(ax)\sin(\pi z), \tag{23}$$

$$S_1=\left(C(s)e^{i\omega t}+\overline{C}(s)e^{-i\omega t}\right)\cos(ax)\sin(\pi z), \tag{24}$$

To undefined amplitude are the functions of the slow scale which are given by the following relations

$$B(s)=-\frac{a}{(c+i\omega)}A(s), \tag{25}$$

$$C(s)=-\frac{a}{(\Gamma c+i\omega)}A(s), \tag{26}$$

where $c=a^2+\pi^2$. Onset of stationary convection is quantitatively determined by using value of Darcy-Rayleigh number with the related wave number and expressions are given by

$$R_0^{st}=\frac{c^2}{a^2} \tag{27}$$

$$a=\pi \tag{28}$$

These are the standard conclusions reached by Horton and Rogers Jr [39], and Lapwood [40]. As a result, the critical wave numbers as well as the Darcy-Rayleigh number for oscillatory convection are described in the following

$$R_0^{Osc}=\frac{\left(c\delta_R^2-c\pi Pe\right)\left(1-c\right)-\omega^2}{a^2}-\frac{aRs\left(\omega^2-\left(\delta_R^2-c\Gamma\right)\left(c\Gamma-\pi Pe\Gamma^{-1}\right)\right)}{\omega^2+c\Gamma^2+\pi^2Pe^2\left(\Gamma^{-1}\right)^2}, \tag{29}$$

$$\omega^2 = \frac{aRs\pi^2 Pe\left(\left(1-\Gamma^{-1}\right)+\delta_R^2 - c^2\Gamma\right)}{c\left(\delta_R^2+\pi Pe\right)-c^2} - c^2\Gamma^2 - \pi^2 Pe^2\left(\Gamma^{-1}\right)^2, \tag{30}$$

which match the conclusions found by Kim *et al.,* [11]. In stationary mode the critical Darcy–Rayleigh number and corresponding wave number do not depend on *(λ, ε)* but, in oscillatory mode it is dependant. Further, we observe that overstability for a certain wave number a can only exist if the following inequality holds

$$\lambda > \varepsilon + \frac{1}{c} \tag{31}$$

The non-dimensional frequency of the neutral oscillatory mode is

$$\omega^2 = \frac{c(\lambda-\varepsilon)-1}{\lambda\varepsilon} \tag{32}$$

3.2 System of Second Order

Now, the system adopts the following form

$$\begin{bmatrix} \left(\varepsilon\frac{\partial}{\partial\tau}+1\right)\nabla^4 & R_0\left(\lambda\frac{\partial}{\partial\tau}+1\right)\frac{\partial}{\partial x} & -R_S\left(\lambda\frac{\partial}{\partial\tau}+1\right)\frac{\partial}{\partial x} \\ \frac{\partial}{\partial x} & \left(\frac{\partial}{\partial\tau}-\nabla^2\right) & 0 \\ \frac{\partial}{\partial x} & 0 & \left(\frac{\partial}{\partial\tau}-\Gamma\nabla^2\right) \end{bmatrix} \begin{bmatrix} \psi_2 \\ T_2 \\ S_2 \end{bmatrix} = \begin{bmatrix} R_{21} \\ R_{22} \\ R_{23} \end{bmatrix} \tag{33}$$

The following terms of RHS in present system were given below

$$R_{21} = 0, \tag{34}$$

$$R_{22} = \frac{\partial(\psi_1, T_1)}{\partial(x,z)}, \tag{35}$$

$$R_{23} = \frac{\partial(\psi_1, S_1)}{\partial(x,z)}, \tag{36}$$

The solutions of second order subjected to initial conditions as in Eq. (19) of system are given by

$$\psi_2 = 0, \tag{37}$$

$$\left(\frac{\partial}{\partial\tau}-\nabla^2\right)T_2=\frac{\partial\left(\psi_1,T_1\right)}{\partial\left(x,z\right)}, \tag{38}$$

$$\left(\frac{\partial}{\partial\tau}-\Gamma\nabla^2\right)S_2=\frac{\partial\left(\psi_1,S_1\right)}{\partial\left(x,z\right)}, \tag{39}$$

The second order results are produced by taking the first-order solution. From the above relations, one can deduce that the temperature, velocity, and solutal fields have the components of the frequency 2ω and are independent of the rapid time scale. In the second order system, we introduce the temperature and solutal concentration parameters as follows

$$T_2=\left(T_{20}+T_{22}e^{2i\omega\tau}+\overline{T_{22}}e^{-2i\omega\tau}\right)\sin\left(2\pi z\right), \tag{40}$$

$$S_2=\left(S_{20}+S_{22}e^{2i\omega\tau}+\overline{S}_{22}e^{-2i\omega\tau}\right)\sin\left(2\pi z\right), \tag{41}$$

where $\left(T_{20},T_{22}\right)$ temperature field of the terms, $\left(S_{20},\,S_{22}\right)$ solutal field of the terms, which have the frequency 2ω and are independent of the rapid time scale, respectively.

$$T_{20}=\frac{a}{8\pi}\left\{A(s)\overline{B}(s)+\overline{A}(s)B(s)\right\},\qquad \psi_{20}=0, \tag{42}$$

$$T_{22}=\frac{a\pi}{8\pi^2+4i\omega}A(s)B(s) \tag{43}$$

and

$$S_{20}=\frac{a}{8\pi\Gamma}\left\{A(s)\overline{C}(s)+\overline{A}(s)C(s)\right\}, \tag{44}$$

$$S_{22}=\frac{a\pi}{8\pi^2\Gamma+4i\omega}A(s)C(s). \tag{45}$$

The horizontally averaged Nusselt number for the oscillatory mode of convection is defined by

$$Nu(s)=1-\chi^2\left(\frac{\partial T_2}{\partial z}\right)_{z=0} \tag{46}$$

By using the expressions of T_2, given in Eq. (40), one can simplify Eq. (46) as

$$Nu(s)=1+\left(\frac{ca^2}{2(c^2+\omega^2)}+\frac{2\pi^2a^2}{\sqrt{64\pi^4+16\omega^2}\sqrt{c^2+\omega^2}}\right)\left|A(s)\right|^2. \tag{47}$$

The horizontally averaged Sherwood number for the oscillatory mode of convection is defined by

$$Sh(s) = 1 - \chi^2 \left(\frac{\partial S_2}{\partial z} \right)_{z=0} \tag{48}$$

By using the expressions of S_2, given in Eq. (41), one can simplify Eq. (48) as

$$Sh(s) = 1 + \left(\frac{ca^2}{2(\Gamma^2 c^2 + \omega^2)} + \frac{2\pi^2 a^2}{\sqrt{64\pi^4 + 16\omega^2}\sqrt{\Gamma^2 c^2 + \omega^2}} \right) |A(s)|^2 . \tag{49}$$

3.3 System of Third Order

Now for this point system takes the form as

$$\begin{bmatrix} \left(\varepsilon \frac{\partial}{\partial \tau} + 1 \right) \nabla^2 & R_0 \left(\lambda \frac{\partial}{\partial \tau} + 1 \right) \frac{\partial}{\partial x} & -R_S \left(\lambda \frac{\partial}{\partial \tau} + 1 \right) \frac{\partial}{\partial x} \\ \frac{\partial}{\partial x} & \left(\frac{\partial}{\partial \tau} - \nabla^2 \right) & 0 \\ \frac{\partial}{\partial x} & 0 & \left(\frac{\partial}{\partial \tau} - \Gamma \nabla^2 \right) \end{bmatrix} \begin{bmatrix} \psi_3 \\ T_3 \\ S_3 \end{bmatrix} = \begin{bmatrix} R_{31} \\ R_{32} \\ R_{33} \end{bmatrix} \tag{50}$$

where terms of RHS are given by

$$R_{31} = \varepsilon \frac{\partial}{\partial s} \left(\nabla^4 \psi_1 \right) - R_0 \lambda \frac{\partial}{\partial s} \left(\frac{\partial T_1}{\partial x} \right) - \left(R_2 + R_0 \chi^2 \cos(\Omega \tau) \right) \left(\lambda \frac{\partial}{\partial t} + 1 \right) \frac{\partial T_1}{\partial x} + R_s \lambda \frac{\partial}{\partial s} \left(\frac{\partial S_1}{\partial x} \right), \tag{51}$$

$$R_{32} = -\frac{\partial T_1}{\partial \tau} + \frac{\partial T_2}{\partial z} \frac{\partial \psi_1}{\partial x}, \tag{52}$$

$$R_{32} = -\frac{\partial S_1}{\partial \tau} + \frac{\partial S_2}{\partial z} \frac{\partial \psi_1}{\partial x}, \tag{53}$$

Now, putting firs -order and second order solutions are into the following Eq. (51), Eq. (52), and Eq. (53) and easily we get the expressions for R_{31}, R_{32} and R_{33}. Now, under solvability condition we get Ginzburg-Landau equation for existence of third order system. The Ginzburg-Landau expression which indicates how the amplitude changes over time $A(s)$ of the convection cell which is given by

$$\frac{dA(s)}{d\tau} - \gamma^{-1}(s) F(s) A(s) + \gamma^{-1} k |A(s)|^2 A(s) = 0, \tag{54}$$

where, coefficients are as follows

$$\gamma^{-1} = \left[\varepsilon c^2 + a^2\lambda\left(\frac{R_0}{(c+i\omega)} - \frac{R_S}{(\Gamma c+i\omega)}\right) + \frac{R_0 a^2(1+i\omega\lambda)}{(c+i\omega)^2} - \frac{R_S a^2(1+i\omega\lambda)}{(\Gamma c+i\omega)^2}\right],$$

$$F(s) = \frac{a^2 R_0(1+i\omega\lambda)}{(c+i\omega)}\left[1+\chi^2\cos(\Omega\tau)\right] + \frac{a^2 R_s(1+i\omega\lambda)}{(\Gamma\delta^2+i\omega)}\chi^2\cos(\Omega\tau),$$

$$k = \left(\frac{a^4 c R_0(1+i\omega\lambda)}{4(c^2+\omega^2)(c+i\omega)} + \frac{a^4\pi^2 R_0(1+i\omega\lambda)}{(8\pi^2+4i\omega)(c+i\omega)^2} - \frac{a^4 c R_S(1+i\omega\lambda)}{4(\Gamma^2 c^2+\omega^2)(\Gamma c+i\omega)} - \frac{a^4\pi^2 R_S(1+i\omega\lambda)}{(8\pi^2\Gamma+4i\omega)(\Gamma c+i\omega)^2}\right),$$

where $A(s)$ should be written in the phase-amplitude form which is shown below

$$A(s)=|A(s)|e^{i\theta}. \tag{55}$$

Now, while substituting the value of $A(s)$ into Eq. (54), we get the equation for amplitude $|A(s)|$ of convection as given below

$$\frac{d|A(s)|^2}{ds} - 2p_r|A(s)|^2 + 2l_r|A(s)|^4 = 0, \tag{56}$$

$$\frac{d\left(ph\left(A(s)\right)\right)}{ds} = p_i - l_i|A(s)|^2, \tag{57}$$

where

$$\gamma_1^{-1}F(s) = p_r + ip_i, \gamma_1^{-1}k_1 = l_r + il_i,$$

where in which the imaginary and real parts are represented by subscripts i and r respectively, and the phase represents *ph* (⋅) As Eq. (54) is also known as Bernoulli equation, because of its non-autonomous structure, finding an analytical solution is very difficult in the presence of modulation. As a result, it was numerically solved by using Mathematica 12.0 built-in function ND Solution, when necessary initial condition at $A(s)=a_0$ where a_0 is defined as present initial convection magnitude. Without loss of generality, $R_2 = R_0$ is taken as given in the calculations, and to reduce the number of parameters to the minimum.

4. Results and Discussions

In this work, it carried outs a study of heat and mass transport for double diffusive oscillatory convection in a horizontal porous layer saturated with viscoelastic fluid in the presence of gravity modulation. So, order to explain how relaxational parameters effects on *λ*, *ε*, the frequency Ω and the amplitude *δ* of modulation on heat and mass transfer, we plot the graphs of Nusselt and

Sherwood numbers against time *s*. It is found that the relation Eq. (32) leads to an interesting result; that for a horizontal porous layer heated underneath. Also, when the relaxation parameter λ is higher than the retardation parameter ε then, the oscillatory type of destabilization possible. Furthermore, it is clear from the equation Eq. (29) that the oscillatory convection is dependent on both the relaxation and retardation times. In Figure 2 and Figure 3, the marginal stability curves for both the stationary and oscillatory modes are plotted. Curves that illustrate the interchange of stabilities and over-stability at the marginal stage are sketched for comparison. As a function of wave number which represents the solid curve with the Rayleigh number in the presence of oscillatory convection, while for stationary convection which is represented by the broken curves. To show the impacts of both the relaxation and the retardation parameters on the onset of convection, we plot the graphs of the Rayleigh number v/s the wave number. Figure 2 and Figure 3 show a bifurcation point on the stationary Newtonian curve, explaining how the marginal overstability curve differs from it and in this case, we noticed that, stationary convection is present at the point where the onset of convection starts.

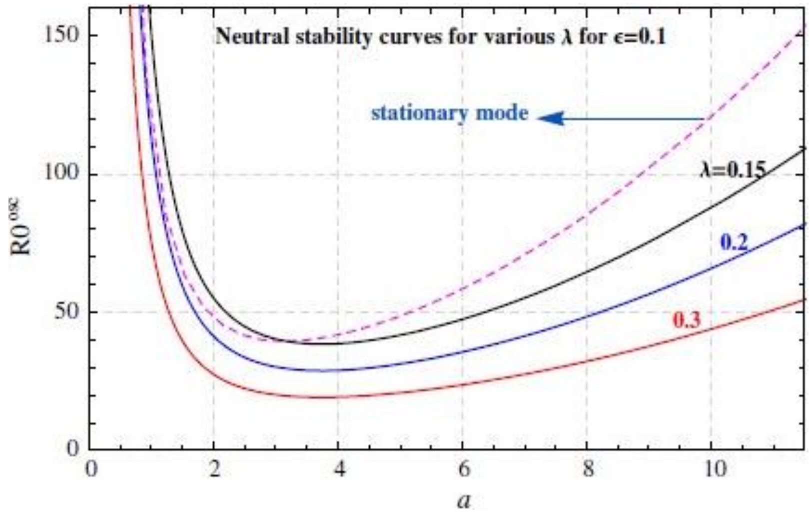


Fig. 2. Effect of λ on $R0^{osc}$ for fixed value of ε

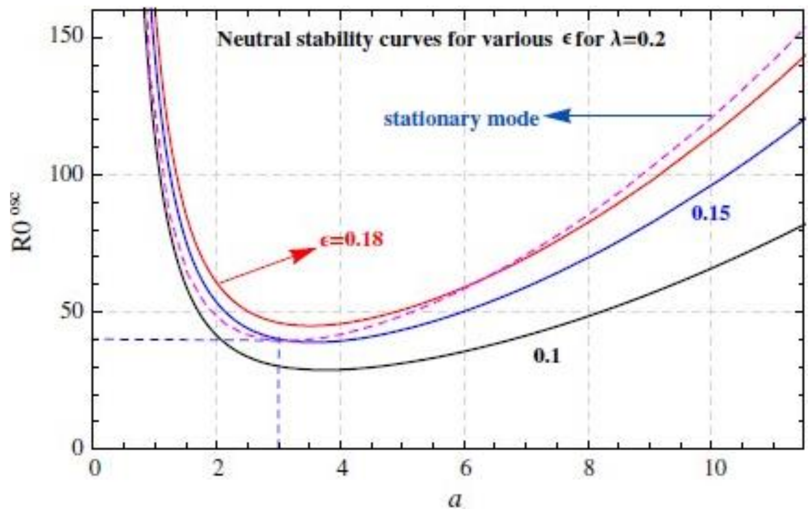


Fig. 3. Effect of ε on $R0^{osc}$ for fixed value of λ

To investigate the impact of the fluid's relaxation time on the onset of overstability, for constant values of ε, it is clear from Figure 2 that the critical Rayleigh number diminishes as the relaxation λ time increases, showing that the system becomes unstable as the relaxation time increases. Furthermore, the impact of retardation time ε on the onset of overstability is shown in Figure 3, it is noted that the critical Rayleigh number increases with increasing retardation time and that viscoelastic fluids with larger values of retardation time indicates overstability at higher Rayleigh numbers. Thus, the impact of increasing retardation time has a stabilising effect on the system. From the Eq. (32) the effect of time relaxation and time retardation parameter with critical value of dimensionless frequency in presence of marginal oscillatory modes.

Figure 4 and Figure 5 show the obtained results as the square of frequency against the square of wave number, the critical value of the frequency increases with increasing relaxation time Figure 4 but with decreasing retardation time Figure 5.

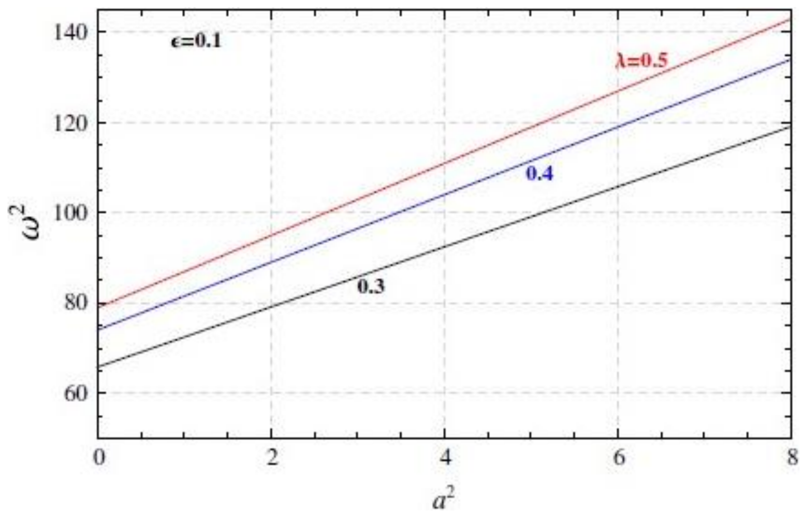


Fig. 4. Effect of λ on ω^2 for fixed value of ε

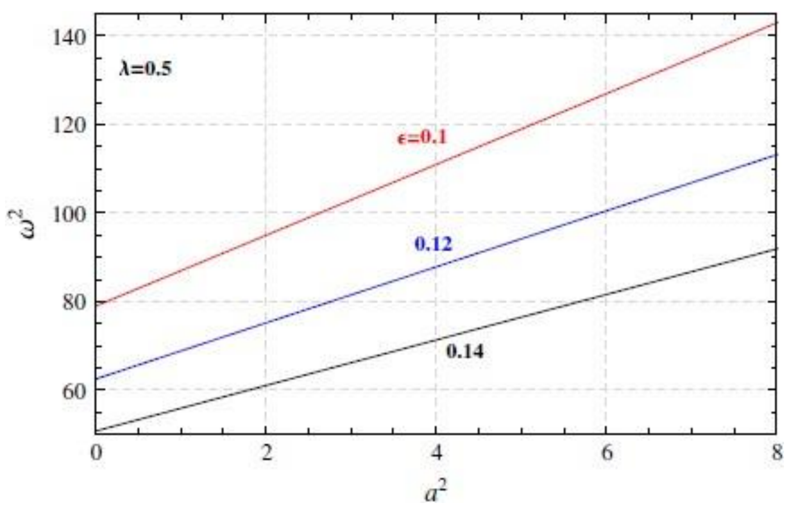
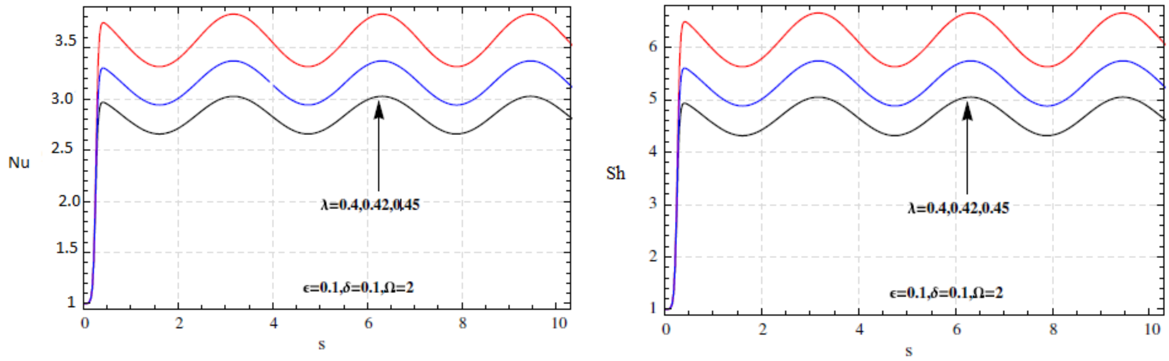


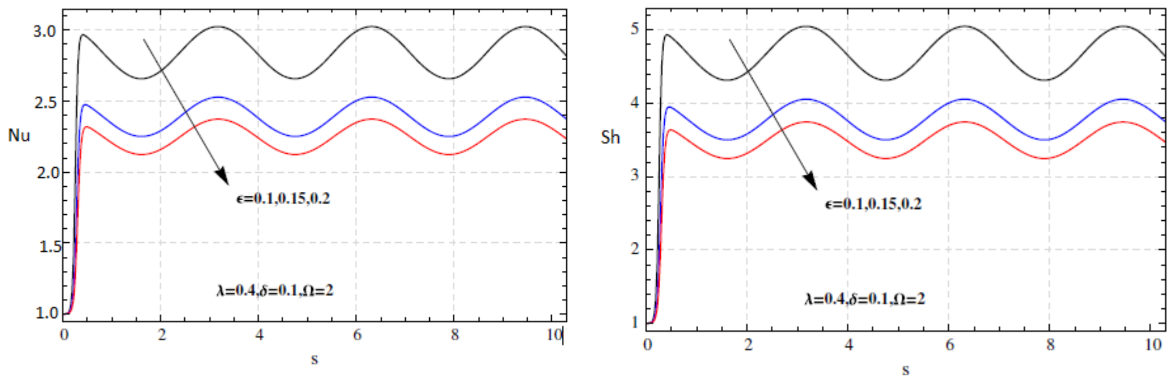
Fig. 5. Effect of ε on ω^2 for fixed value of λ

The corresponding results to the gravity modulation have been plotted in Figure 6 to Figure 10, where the graphs of *Nu* and *Sh* with respect to the slow time *s* is presented. It is shown that the value of *Nu* and *Sh* starts with 1, by showing the conduction state initially where the heat and mass transfer across with porous medium is taking place in the presence of conduction when *s* is small. The values of *Nu* and *Sh* increases for intermediate values of *s* thus showing that convection is in progress and finally when s is very large, the oscillatory state is achieved. As in Figure 6(a) and Figure 6(b), the effect of an increment in the value of relaxation parameter λ is destabilizing as the value of *Nu and Sh* increases on increasing λ. Further, the effect of retardation parameter ε is found to stabilize the system as the heat transfer decreases on increasing ε, given in Figure 7(a) and Figure 7(b).



(a) (b)

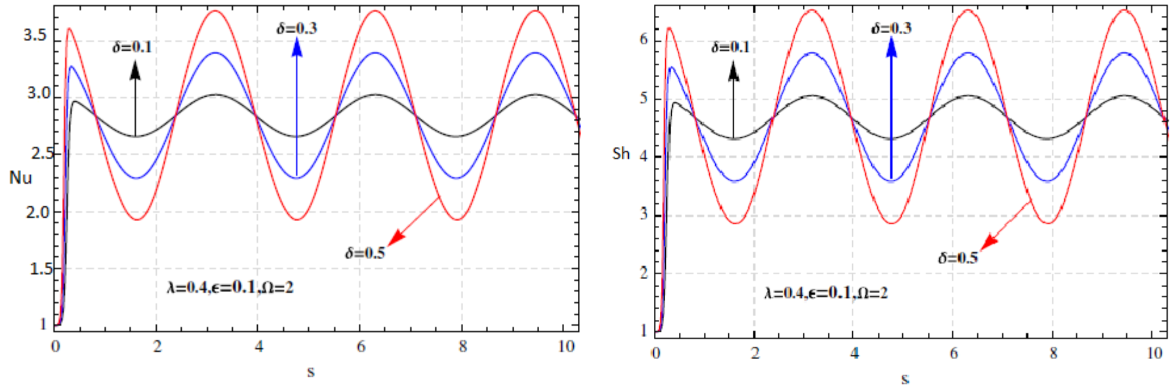
Fig. 6. Effect of λ on (a) Nu, and (b) Sh, for fixed values of other parameters



(a) (b)

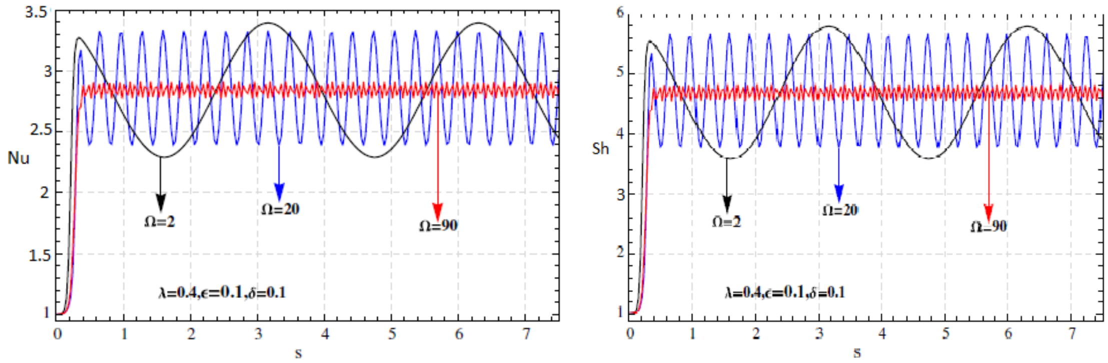
Fig. 7. Effect of ε on (a) Nu, and (b) Sh, for fixed values of other parameters

The effects of frequency Ω and the amplitude of modulation δ on heat and mass transfer are given in Figure 8 and Figure 9. In Figure 8(a) and Figure 8(b), one can see that an increment in amplitude of modulation increases the magnitude of *Nu and Sh*, thus enhances the heat and mass transfer and advancing the onset of convection. An opposite effect is obtained in the case of frequency of modulation as Ω increases given in Figure 9(a) and Figure 9(b). Hence, we found that the effect of gravity modulation decreases as the frequency of modulation increases, and finally when Ω is very large, the effect of modulation disappears altogether, thus confirming the results of Venezian [41] and Bhadauria and Kiran [42]. In Figure 10(a) and Figure 10(b), we compare the results of oscillatory and stationary instabilities. It is found that heat and mass transfer is more in oscillatory mode of convection than in stationary mode. This implies that oscillatory instability sets in before the stationary instability. Similar results have also been obtained by Kim *et al.,* [11], and Rajib and Layek [43].



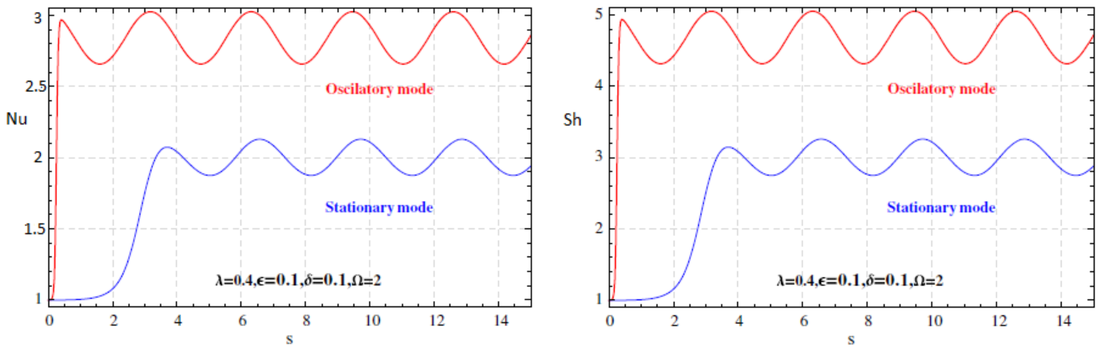
(a) (b)

Fig. 8. Effect of δ on (a) Nu, and (b) Sh, for fixed values of other parameters



(a) (b)

Fig. 9. Effect of Ω on (a) Nu, and (b) Sh, for fixed values of other parameters



(a) (b)

Fig. 10. (a) and (b) Comparison of stationary and oscillatory mode of convection

In Figure 11 and Figure 12, the stream lines and the corresponding isotherms are depicted for gravity modulation, respectively, at s = 0.0, 0.12, 0.14, 0.15, 0.16, 0.17 for λ = 0.4, ε = 0.1, δ = 0.1, Ω= 2.0 and χ = 0.5. From the figures, we found that initially when the time is small, the magnitude of streamlines is also small given in Figure 11(a) and Figure 11(b), and isotherms are straight, that is the system is in conduction state Figure 12(a) and Figure 12(b). However, as time increases, the magnitude of streamlines increases and the isotherms loses their evenness. This shows that the convection is taking place in the system. Convection becomes faster on further increasing the value of time s. However, the system achieves the study state beyond s = 0.16 as there is no change in the streamlines and isotherms (Figure 11(d) to Figure 11(f) and Figure 12(d) to Figure 12(f)).

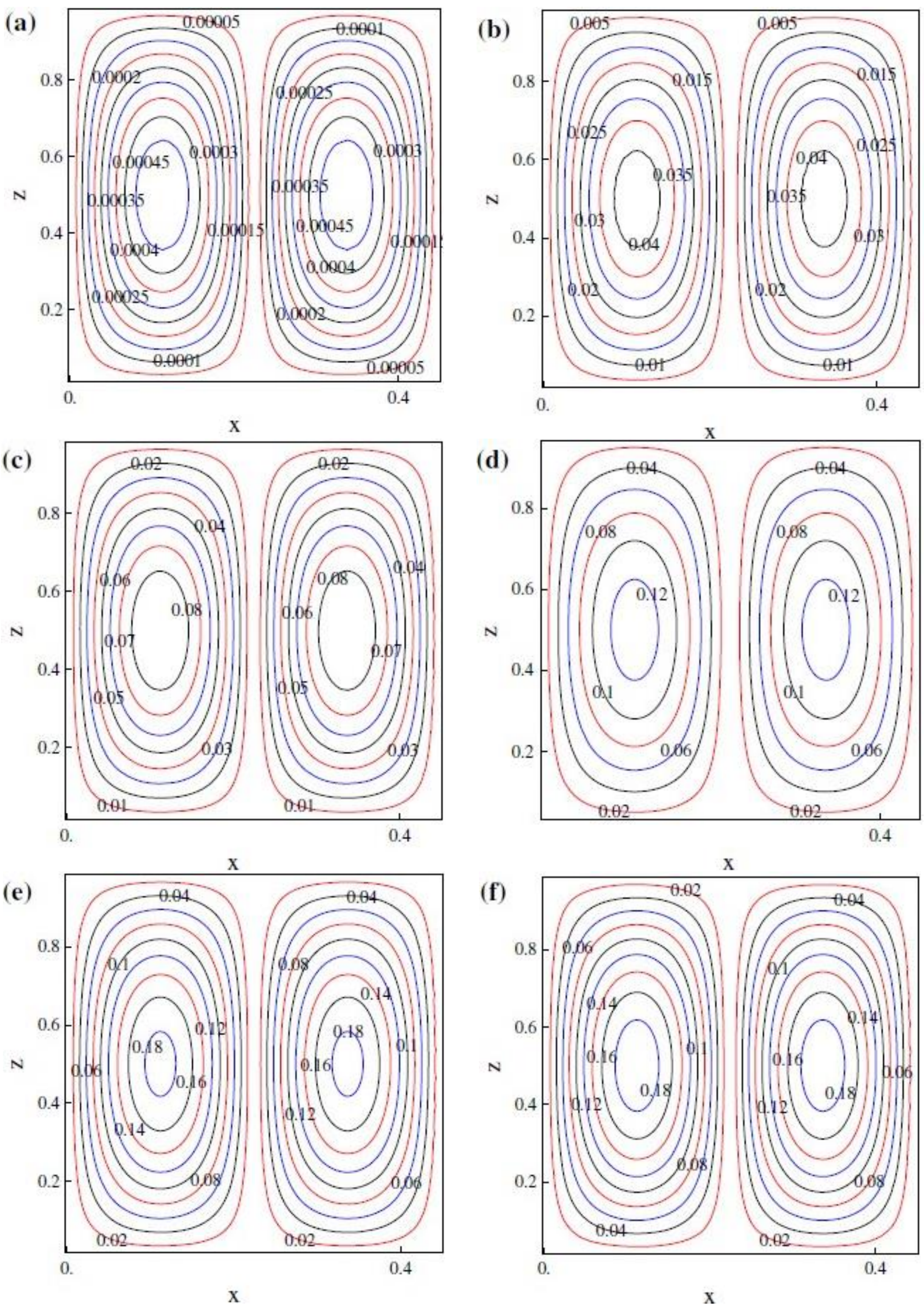


Fig. 11. Streamlines for various values of times (a) $s = 0.0$, (b) $s = 0.12$, (c) $s = 0.14$, (d) $s = 0.15$, (e) $s = 0.16$, (f) $s = 0.17$

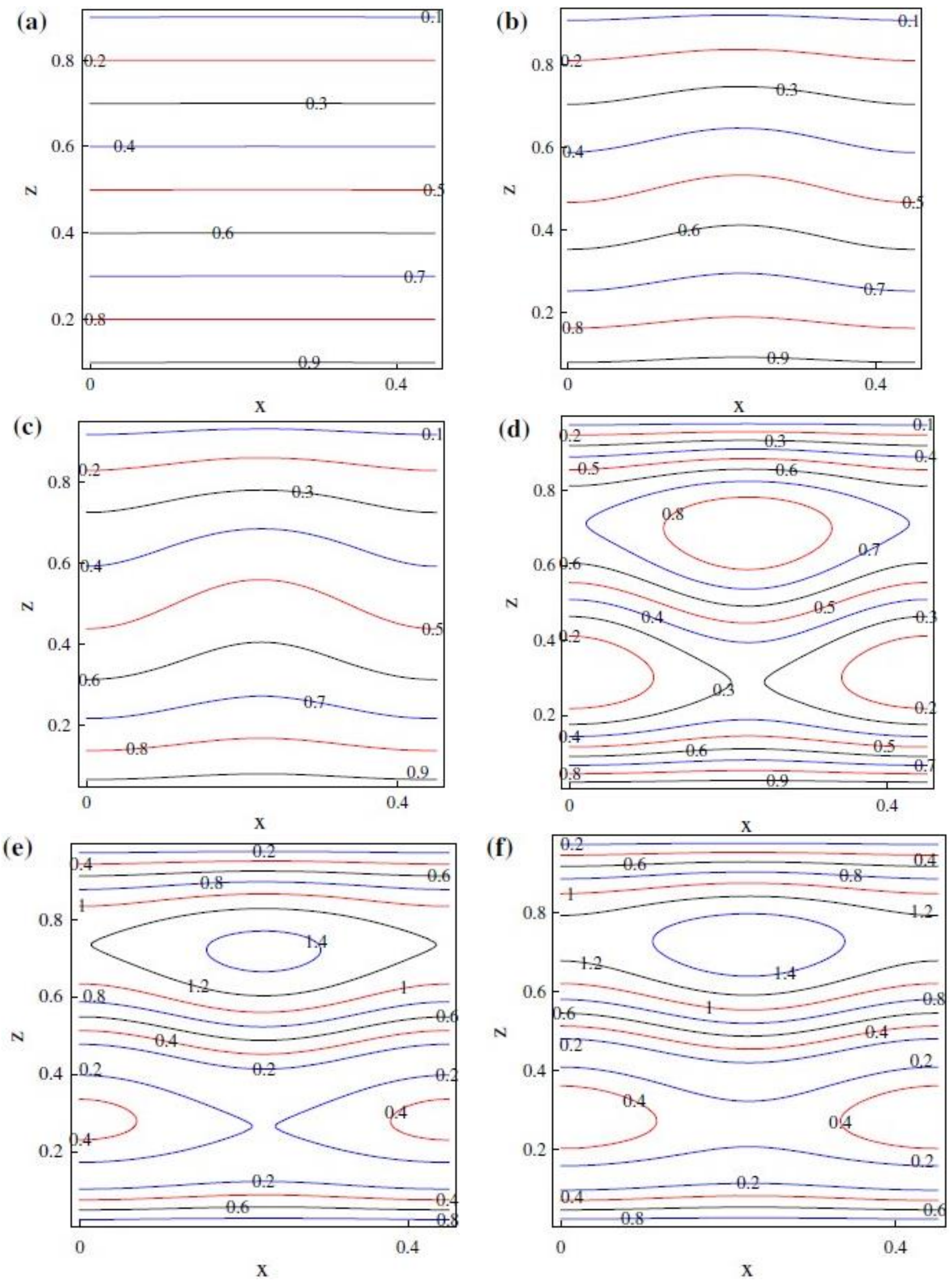


Fig. 12. Isotherms for various values of times (a) s = 0.0, (b) s = 0.12, (c) s = 0.14, (d) s = 0.15, (e) s = 0.16, (f) s = 0.17

5. Conclusions

Weakly nonlinear stability analysis was employed to investigate the impact of gravity modulation on the overstability of Bénard-Darcy convection, leading to the complex Ginzburg-Landau amplitude equation. The following results are obtained

(i) Impact of relaxation time λ is to develop the onset of convection, and hence increases the heat and mass transmission.

(ii) Effect of retardation time ε is to defer the onset of convection, and hence decreases the heat and mass transport.
(iii) The oscillatory critical Rayleigh–Darcy number depends on λ, ε, but in stationary case, it is independent.
(iv) An increment in the amplitude δ of modulation is to advance the convection, and hence heat and mass transfer.
(v) As its value of the heat and mass transfer increases the frequency Ω of modulation decreases.

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