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Nonlinear Convective Flow of Williamson Fluid Over a Slendering Stretching Sheet with Melting Heat Transfer

Mocherla Gayatri^{1,*}, Konda Jayarami Reddy², Bommanna Lavanya³

¹ Department of Mathematics, Research Scholar, Koneru Lakshmaiah Education Foundation, Vaddeswaram, Andhra Pradesh, 522302, India

² Department of Mathematics, Koneru Lakshmaiah Education Foundation, Vaddeswaram, Andhra Pradesh, 522302, India

³ Department of Mathematics, Manipal Institute of Technology, Manipal Academy of Higher education, Manipal, Karnataka, 576104, India

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ABSTRACT

The main concern of this paper is to analyze the nonlinearity of the Grashof numbers (thermal and diffusion) on the Williamson fluid flow across a stretching sheet with non-uniform thickness with the consideration of melting heat transfer. The modelled equations are non-dimensionalized with the utilization of similarity transformations. We used shooting technique based on Runge-Kutta 4th order method to unriddle the ensuing equations. Influence of key parameters on the common profiles i.e., velocity, temperature and concentration are deliberated with the support of graphs. And also, we examined friction factor, local Sherwood and Nusselt numbers against the same parameters and displayed the results in tabular form. It is seen that Weissenberg number and wall thickness parameters rise the magnitude of friction factor. It is additionally qualified to specify that the temperature and concentration profiles looking high in the absence of melting parameter contrasted with the presence of melting parameter.

1. Introduction

During the most recent couple of years, analysts appear to be keen on investigating liquefying heat exchange because of across the board utilizations of freezing and melting in magma solidification, potential heat thermal energy storage, casting of an assembling procedure, welding process, and crystal growth and so on. So extensive measure of test and hypothetical research has been completed to decide the part of natural convection in the dynamics of heat transfer attached to solidification or melting impact. With a specific end goal to control the major parameters, for example, the speed of manufacture, rate of deformities and additionally the impact on the final properties of items and the likelihood of harm of the contact surface between the wall and phase change material, the forecast of temperature appropriation, solidification or melting rate is critical in some cutting-edge advancements. Initially, Epstein and Cho [1] introduced an exact closed- form solution for the case of porous media. They noticed that the influence of the Prandtl number on the

* Corresponding author

Email address: gayatri.mocherla@gmail.com

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normalized Nusselt number disappears as it becomes large. Bachok *et al.*, [2] used to Runge-Kutta-Fehlberg method to solve the equations which are raised in the problem of two-dimensional stagnation point flow across a horizontal stretching/shrinking sheet in the presence of melting heat transfer. They found that melting parameter reduces the skin friction coefficient. Ishak *et al.*, [3] examined the melting phenomena on the forced convective flow across a moving surface. They observed that melting strength lessen the rate of heat transfer at the solid-fluid interface. By considering viscous dissipation, the characteristics of melting heat transfer on the stagnation-point flow of Jeffrey fluid were analysed by Mustafa *et al.*, [4] and concluded that the influence of melting parameter and the Prandtl number is quite opposite on temperature and velocity profiles. Hayat *et al.*, [5-7] investigated stagnation-point flows of different fluids across a stretching surface by viewing several parameters including Soret and Dufour along with melting heat transfer. Some of their findings are (a) Prandtl enhances the rate of heat transfer (b) melting parameter lessen the magnitude of local Sherwood number (c) mixed convection parameter improves the velocity profile. Das *et al.*, [8] used MAPLE 17 to obtain the numerical results for the problem of Jeffrey fluid flow across a stretching surface with the consideration of some parameters including surface slip and melting heat transfer and found that Jeffrey fluid enhances the temperature compare to Newtonian fluid. By considering stretching sheet, Mabood and Antonio and Gireesha *et al.*, [9-10] studied the flow of nanofluid in the presence of various parameters including melting and thermophoresis. The part of their results and discussion contains many points including Prandtl number enhances the rate of mass transfer and thermophoresis shows the same kind of behaviour on the Nusselt number. Mabood and Das, Krishnamurthy et al [11-12] reported that the rate of mass transfer is high at the wall in case of no-slip condition and non-linear stretching parameter enhances the friction factor in the problem of two dimensional MHD flow of a fluid across a stretching sheet by viewing chemical reaction and melting parameter. Animasaun [13] used shooting technique based on fourth order R-K Gill method to solve equations (transformed) which are raised in the problem of mixed convective stagnation micropolar fluid flow across a stretching sheet by considering variable thermal conductivity along with melting and other parameters and noticed that variable thermal conductivity lessen the local Nusselt number. Hashim [14] and Sheikholeslam *et al.*, [15-16] studied the characteristics melting heat transfer of various fluids across various channels including stretching plate. They witnessed that temperature improved with bigger Prandtl number and melting parameter raises the nanofluid concentration.

Thermal radiation is developed by the movement of charged particles in subject. Generally, their movement is in a shape of waves (electromagnetic) and look like as if they were from room heater and electric burner. With regards to space innovation and in the procedures including high temperatures, the impact of radiation assumes an indispensable job. Initially, by employing Rosseland approximation to alter the energy equation, Viskanta and Grosh [17] studied the influence of thermal radiation on an incompressible flow and concluded that radiation parameter enhances the thickness of the thermal boundary layer. Gupta and Gupta [18] elucidated the influence of radiation on the mixed convective fluid flow within an open-ended vertical channel and observed that the radiation applies a balancing out impact on the flow which is reliable with the way that radiation will in general decrease the impact of regular convection. B.Lavanya [18] studied MHD Rotating Flow Through a Porous Medium with Heat and Mass Transfer. The study of the interaction of thermal radiation with the natural convection was conducted by Ali [19] in the domain of semi-infinite flat plate. Hossain and Takhar [20] noticed that the Prandtl number lessens the local shear stress in the three convective modes (free, forced and mixed) in their work on the boundary layer flow across a vertical flat plate in the presence of thermal radiation. Israel-Cookey *et al.*, [21] and Raptis *et al.*, [22] explained the MHD flow across different plates by considering several parameters

including Grashoff number along with thermal radiation. Darcy parameter lessens the velocity and radiation parameter enhances the normalized similarity temperature are some of their observations. With the aid of Runge-Kutta fourth order based shooting method, Cortell [23] resolved the transmuted equations for the fluid flow across a flat sheet and mentioned that wall thickness parameter lessens the velocity. By considering vertical porous plate, Mohamed and Abo-Dahab [24] and Mukhopadhyay [25] demonstrated different flows by viewing various parameters including unsteadiness, thermal radiation and chemical reaction. Some of their results are unsteadiness parameter enhances the heat transfer rate and Schmidt number lessens the velocity. M.Nagasasikala et.al [26] studied Heat and Mass Transfer of a MHD Flow of a Nanofluid Through a Porous Medium in an Annular, Circular Region with Outer Cylinder Maintained at Constant Heat Flux. Bhattacharyya and Layek [27] provided dual solutions in the analysis of stagnation point flow across a shrinking sheet with thermal radiation. Later several researchers [29-35] contributed their work of various fluid flows in the domain of different surfaces in the presence of thermal radiation along with some other parameters.

The present work is partly taken from Hayat *et al.*, [33] and we have extended it by considering nonlinear convection parameter together with chemical reaction and thermal radiation. Transformed the governing equations as ordinary differential equations with the aid of similarity variables and then the equations resolved by using Runge-Kutta fourth order method based shooting technique. We have provided solutions in two cases i.e., in the presence and absence of melting heat transfer and used plots to display them. We have also analyzed the friction factor, heat and mass transfer rates in tabular form against the same parameters for which we have graphs.

2. Mathematical Analysis

A two-dimensional Williamson fluid flow across a stretching sheet with non-uniform thickness in the manifestation of thermal radiation and chemical reaction is examined. An inhomogeneous magnetic field of strength B_0 is carried out normal to the sheet. We have analyzed the flow with melting heat transfer. Furthermore, we assume that the stretching velocity is $U_w(x) = A(x+q)^m f'$ in the x-direction, $y = \varepsilon(x+q)^{1-m/2}$ (where ε is small so that the surface is sufficiently thin) and considered Reynolds's number as much as less to push away the stimulated magnetic field.

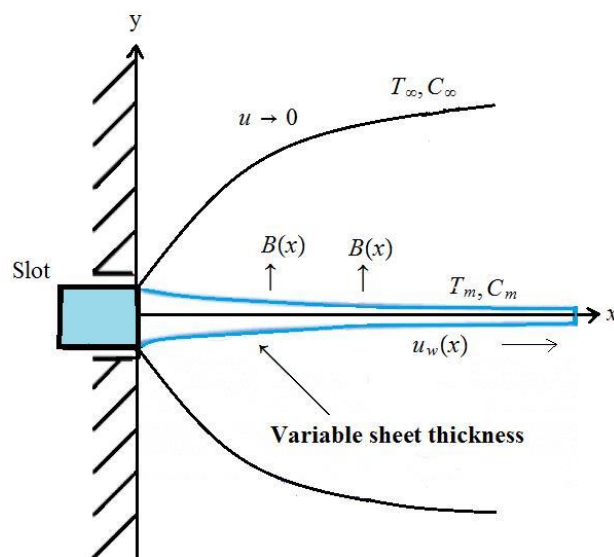


Fig. 1. Physical model of the problem

In 1929, Williamson proposed a model to examine the pseudoplastic fluid flow. Not like the other models, this model considers the minimum and maximum viscosities of the fluid which are needed for pseudoplastic fluids. The underlying equations of the Williamson fluid model are established as [34]:

$$S = -pI + \tau \tag{1}$$

$$\tau = \left[\mu_{\infty} + \frac{(\mu_0 - \mu_{\infty})}{1 - \Gamma \dot{\gamma}} \right] A_1 \tag{2}$$

Where p is the pressure, I is the identity vector, τ is the extra stress tensor, μ_0 and μ_{∞} are the limiting viscosities at zero and at infinite shear rate, $\Gamma > 0$ is the time constant, A_1 is the first Rivlin-rickson tensor.

$\dot{\gamma}$ is defined as follows:

$$\dot{\gamma} = \sqrt{\frac{1}{2} \pi}, \tag{3}$$

Where $\pi = trace(A_1^2)$,

$$\dot{\gamma} = \left[\left(\frac{\partial u}{\partial x} \right)^2 + \frac{1}{2} \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right)^2 + \left(\frac{\partial v}{\partial y} \right)^2 \right]^{1/2} \tag{4}$$

If $\mu_{\infty} > 0$ and $\Gamma \dot{\gamma} < 1$, then (1.2) can be written as

$$\tau = \left[\frac{\mu_0}{1 - \Gamma \dot{\gamma}} \right] A_1 \tag{5}$$

With the help of binomial expansion, (1.5) can be expressed as

$$\tau = \mu_0 \left[1 + \Gamma \dot{\gamma} \right] A_1 \tag{6}$$

With these assumptions, the governing equations [33] are

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{7}$$

$$\left. \begin{aligned}
 u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} &= \nu \frac{\partial^2 u}{\partial y^2} + \sqrt{2\nu}\Gamma \left(\frac{\partial u}{\partial y} \right) \left(\frac{\partial^2 u}{\partial y^2} \right) + g\beta_T (T - T_m) + g\beta_{1T} (T - T_m)^2 \\
 &+ g\beta_C (C - C_m) + g\beta_{1C} (C - C_m)^2 - \frac{\sigma B^2(x)}{\rho} u
 \end{aligned} \right\} \quad (8)$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{k}{\rho c_p} \frac{\partial^2 T}{\partial y^2} + \frac{\mu}{\rho c_p} \left((T_\infty - T_m) \left(\frac{\partial u}{\partial y} \right)^2 \right) + \frac{1}{\rho c_p} \frac{16\sigma^* T_\infty^3}{3k^*} \frac{\partial^2 T}{\partial y^2} \quad (9)$$

$$u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D_m \frac{\partial^2 C}{\partial y^2} - k_0 (C - C_m) \quad (10)$$

And the right boundary conditions which are suitable to the present problem are mentioned as

$$\left. \begin{aligned}
 u = 0, k \left(\frac{\partial T}{\partial y} \right) &= \rho [\gamma^* + c_s (T_m - T_0)] v, T = T_m, C = C_m \text{ at } y = 0 \\
 u \rightarrow 0, T \rightarrow T_\infty, C \rightarrow C_\infty &\text{ as } y \rightarrow \infty
 \end{aligned} \right\} \quad (11)$$

where u, v are the velocity elements in parallel to x and y directions respectively, ν is the kinematic viscosity, Γ is the dimensional Williamson parameter, g is the gravity field, β_T is the volumetric coefficient of thermal expansion, T_m is the melting surface temperature, β_C is the volumetric coefficient of diffusion expansion, σ is the electrical conductivity, ρ is the density of the fluid, $B(x)$ is the dimensional magnetic field parameter, k is the thermal conductivity, C_p is the specific heat capacitance, μ is the dynamic viscosity, σ^* is the Stefan-Boltzmann constant, k^* is mean absorption coefficient, C is the dimensional concentration parameter, D_m is the molecular diffusivity, k_0 is the dimensional chemical reaction parameter, γ^* is the latent heat, C_s is the heat capacity of the solid, T_0 is the temperature of the solid.

The needed transformations to decrypt the regulating equations as a set of non-linear ordinary differential equations are given below:

$$\left. \begin{aligned}
 \zeta &= y \sqrt{\left(\frac{m+1}{2} \right) \frac{A}{\nu}} (x+q)^{m-1/2}, \\
 u &= A(x+q)^m \frac{dF}{d\zeta}, v = -\sqrt{\left(\frac{m+1}{2} \right) A\nu} (x+q)^{m-1/2} [F(\zeta) + \left(\frac{m-1}{m+1} \right) \zeta F'(\zeta)], \\
 \Theta(\zeta) &= \frac{T - T_m}{T_\infty - T_m}, \Phi(\zeta) = \frac{C - C_m}{C_\infty - C_m}
 \end{aligned} \right\} \quad (12)$$

By using Eq. (12), Eq. (8) – Eq. (10) transmuted as the following equations:

$$\left[1 + \Lambda \frac{d^2 F}{d\zeta^2}\right] \frac{d^3 F}{d\zeta^3} + f \frac{d^2 F}{d\zeta^2} - \frac{2m}{m+1} \left(\frac{dF}{d\zeta}\right)^2 + \frac{2}{m+1} \left[Gr\Theta + Gr_1(\Theta^2) + Gc\Phi + Gc_1(\Phi)^2 - M \frac{dF}{d\zeta}\right] = 0 \quad (13)$$

$$\left(1 + \frac{4}{3R}\right) \frac{d^2 \Theta}{d\zeta^2} + Pr \left[f \frac{d\Theta}{d\zeta} + Ec \left(\frac{d^2 F}{d\zeta^2}\right)^2 \right] = 0 \quad (14)$$

$$\frac{d^2 \Phi}{d\zeta^2} + Sc \left(f \frac{d\Phi}{d\zeta} - \frac{2}{m+1} Kr\Phi \right) = 0 \quad (15)$$

and the boundary conditions (11) changed as

$$\left. \begin{aligned} \left. \begin{aligned} \frac{dF}{d\zeta} \Big|_{\zeta=0} &= 0, Me \frac{d\Theta}{d\zeta} \Big|_{\zeta=0} + Pr \left[F(0) + \frac{m-1}{m+1} \delta \right] = 0, \Theta(0) = 0, \Phi(0) = 0, \\ \frac{dF}{d\zeta} \Big|_{\zeta=\infty} &= 0, \Theta(\infty) = 1, \Phi(\infty) = 1 \end{aligned} \right\} \end{aligned} \right\} \quad (16)$$

where $\delta = \varepsilon \sqrt{\frac{m+1}{2} \frac{A}{\nu}}$ is the wall thickness parameter. Eq. (13) – (15) can be transformed into the system of dimensionless expressions by utilizing $F(\zeta) = f(\zeta - \delta) = f(\varsigma)$, $\Theta(\zeta) = \theta(\zeta - \delta) = \theta(\varsigma)$, $\Phi(\zeta) = \phi(\zeta - \delta) = \phi(\varsigma)$.

The resultant equations are

$$\left[1 + \Lambda \frac{d^2 f}{d\varsigma^2}\right] \frac{d^3 f}{d\varsigma^3} + f \frac{d^2 f}{d\varsigma^2} - \frac{2m}{m+1} \left(\frac{df}{d\varsigma}\right)^2 + \frac{2}{m+1} \left[Gr\theta + Gr_1(\theta^2) + Gc\phi + Gc_1(\phi)^2 - M \frac{df}{d\varsigma}\right] = 0 \quad (17)$$

$$\left(1 + \frac{4}{3R}\right) \frac{d^2 \theta}{d\varsigma^2} + Pr \left[f \frac{d\theta}{d\varsigma} + Ec \left(\frac{d^2 f}{d\varsigma^2}\right)^2 \right] = 0 \quad (18)$$

$$\frac{d^2 \phi}{d\varsigma^2} + Sc \left(f \frac{d\phi}{d\varsigma} - \frac{2}{m+1} Kr\phi \right) = 0 \quad (19)$$

And the resultant boundary conditions are

$$\left. \begin{aligned} \left. \begin{aligned} \frac{df}{d\varsigma} \Big|_{\varsigma=0} &= 0, Me \frac{d\theta}{d\varsigma} \Big|_{\varsigma=0} + Pr \left[f(0) + \frac{m-1}{m+1} \delta \right] = 0, \theta(0) = 0, \phi(0) = 0, \\ \frac{df}{d\varsigma} \Big|_{\varsigma=\infty} &= 0, \theta(\infty) = 1, \phi(\infty) = 1 \end{aligned} \right\} \end{aligned} \right\} \quad (20)$$

Here Weissenberg parameter Λ , thermal Grashof number Gr , nonlinear thermal Grashof number Gr_1 , diffusion Grashof number Gc , nonlinear diffusion Grashof number Gc_1 , Magnetic field parameter M , radiation parameter R , Prandtl number Pr , Eckert number Ec , Schmidt number Sc , chemical reaction parameter Kr , melting heat transfer parameter Me are defined as follows:

$$\left. \begin{aligned} \Lambda &= \Gamma \sqrt{(m+1) A \frac{(x+q)^{3m-1}}{\nu}}, Gr = \frac{g \beta_T (T_\infty - T_m)}{A^2 (x+q)^{2m-1}}, Gr_1 = \frac{g \beta_{1T} (T_\infty - T_m)^2}{A^2 (x+q)^{2m-1}}, \\ Gc &= \frac{g \beta_C (C_\infty - C_m)}{A^2 (x+q)^{2m-1}}, Gc_1 = \frac{g \beta_{1C} (C_\infty - C_m)^2}{A^2 (x+q)^{2m-1}}, M = \frac{\sigma B_0^2}{A \rho}, R = \frac{kk^*}{4\sigma^* T_\infty^3}, \\ Pr &= \frac{\mu c_p}{k}, Ec = \frac{A^2 (x+q)^{\frac{5m-1}{2}}}{c_p}, Sc = \frac{\nu}{D_m}, Kr = \frac{k_0}{A(x+q)^{m-1}}, Me = \frac{c_p (T_\infty - T_m)}{[\gamma^* + c_s (T_m - T_0)]} \end{aligned} \right\} \quad (21)$$

The usual non-dimensionalization parameters of interest, friction factor C_f , the local Nusselt and Sherwood numbers Nu and Sh are defined as the following:

$$\left. \begin{aligned} C_f &= (\text{Re}_x)^{-0.5} \sqrt{\frac{m+1}{2} \frac{d^2 f}{d\zeta^2}} \Big|_{\zeta=0}, Nu = -(\text{Re}_x)^{0.5} \sqrt{\frac{m+1}{2} \frac{d\theta}{d\zeta}} \Big|_{\zeta=0} \\ Sh &= -(\text{Re}_x)^{0.5} \sqrt{\frac{m+1}{2} \frac{d\phi}{d\zeta}} \Big|_{\zeta=0} \end{aligned} \right\} \quad (22)$$

Where Re_x is defined as $\text{Re}_x = \frac{U_w(x)}{\nu} (x+q)$

3. Numerical Procedure

Numerical solutions of profoundly nonlinear customary differential Eq. (17) – Eq. (19) related with the boundary conditions (20) are determined by shooting technique dependent on Runge-Kutta fourth order method. The fundamental step of computational scheme incorporates the accompanying focuses.

Reduce Eq. (17) – Eq. (19) into system of 1st order equations by utilizing reasonable substitution characterized as follows:

$$\left. \begin{aligned} f_1 &= f, \\ f_2 &= \frac{df_1}{d\zeta} = \frac{df}{d\zeta}, \\ f_3 &= \frac{df_2}{d\zeta} = \frac{d^2f}{d\zeta^2}, \\ f_4 &= \theta(\zeta), \\ f_5 &= \frac{df_4}{d\zeta} = \frac{d\theta}{d\zeta}, \\ f_6 &= \phi(\zeta), \\ f_7 &= \frac{df_6}{d\zeta} = \frac{d\phi}{d\zeta} \end{aligned} \right\} \quad (23)$$

By using this, we can write

$$\left. \begin{aligned} \frac{df_1}{d\zeta} &= f_2, \\ \frac{df_2}{d\zeta} &= f_3, \\ \frac{df_3}{d\zeta} &= \left(\frac{1}{1+\Lambda f_3} \right) \left(\frac{2m}{m+1} f_2^2 - f_1 f_3 - \frac{2}{m+1} (Gr f_4 + Gr_1 f_4^2 + Gc f_6 + Gc_1 f_6^2 - M f_2) \right) \\ \frac{df_4}{d\zeta} &= f_5, \\ \frac{df_5}{d\zeta} &= - \frac{Pr}{\left(1 + \frac{4}{3} R \right)} [f_1 f_5 + Ec f_3^2], \\ \frac{df_6}{d\zeta} &= f_7, \\ \frac{df_7}{d\zeta} &= -Sc \left(f_1 f_5 - \frac{2}{m+1} Kr f_6 \right) \end{aligned} \right\} \quad (24)$$

We can use the same procedure for the boundary conditions mentioned in (20). After the conversion, we can apply the bvp5c MATLAB package to get the appropriate results.

4. Results and Graphs

The set of ordinary differential Eq. (17) – Eq. (19) with respect to the boundary conditions (20) are settled numerically by applying shooting technique based on Runge-Kutta fourth order method. We exhibited the impact of non-dimensional parameters, for example Gr_1, Gc_1, R , on velocity, concentration and temperature using plots and the impact of the same parameters on friction factor, local Nusselt number and Sherwood numbers using tables. In the present investigation, we examine the results in two cases i.e., presence and absence of melting surface and we have obtained solutions by keeping $Me = 1.1$ and $Me = 0$.

Figure 2-4 show the effect of the magnetic field on the common profiles (velocity, temperature, concentration) for both $Me = 1.1$ and $Me = 0$. From Figure 2-4, it is evident that the strength of magnetic field is to diminish the velocity, temperature and concentration fields. This reduction can be attributed to the fact that the magnetic field provides a resisting type force known as the Lorentz force. This force tends to lessen the motion of fluid and as a consequence, the velocity depreciates. The temperature and concentration fields are high in $Me = 0$ when compared to $Me = 1.1$. Figure 5 and 6 represent the plots of temperature and velocity with variation in R . The temperature increases rapidly throughout the boundary layer owing to the release of energy in thermal boundary layer for increasing values of the thermal radiation. In physically as rising values of thermal radiation encourages the generation of thermal heat energy in the flow, due to this boosting phenomena in velocity and temperature. The impact of Eckert number that it improves the thermal boundary layer, lessens the concentration and shows mixed performance in velocity is perceived in Figure 7-9. Since as improving Eckert number the dissipation among the particles is increase this helps to improve the boundary layer near the surface velocity and surface temperature. But due to unsteadiness domination, we saw decrement in diffusion boundary layer.

Figure 10-12 are plotted to demonstrate the influence of the chemical reaction on the temperature, concentration and velocity. It displays that the concentration, velocity and temperature fields are reduced with incrementing values of chemical reaction. The concentration profiles are high in the non-melting surface condition when compared with melting surface condition. Figure 13-15 observed the temperature and concentration fields are decreased and mixed performance in velocity fields respectively with improving values of Weissenberg number. As we expected, when we incorporate the non-Newtonian fluid, it improves the pressure on the flow. But in this study, due to domination of nonlinear convection in the flow, initially we saw depreciation after reaches certain limit enhancement in the flow. In Figure 16-18 shows the influence of wall thickness on velocity, temperature and concentration field in both cases. As rising of wall thickness improves the thickness of the boundary layer, wall thickness helps to depreciate the temperature and concentration boundary, whereas it shows mixed behaviour on velocity. The impact of nonlinear thermal convection on velocity, temperature and concentration fields is depicted in Figure 19 - 21. Note that improving Gr_1 shows encouragement in all the profiles ($f'(\zeta), \theta(\zeta), \phi(\zeta)$) of the flow. In the presence of melting surface condition, the momentum boundary layer is initially and at the end of the surface is high compared to the absence of melting surface condition. Figure 22-24 depicts the $f'(\zeta), \theta(\zeta), \phi(\zeta)$ profiles against the nonlinear diffusion convection parameter. We observed that the nonlinear diffusion convection parameter improves concentration in the flow, and furthermore this helps to boost the momentum, diffusion and concentration boundary layers for both the cases.

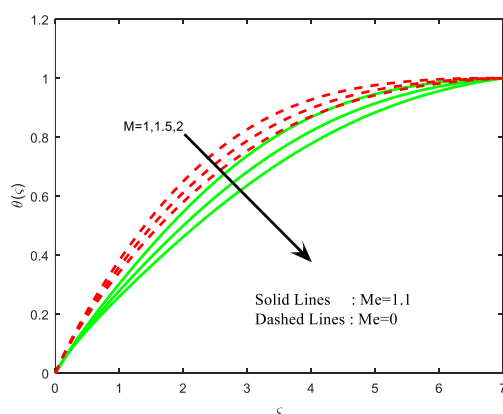


Fig. 2. Impact of M on $\theta(\zeta)$

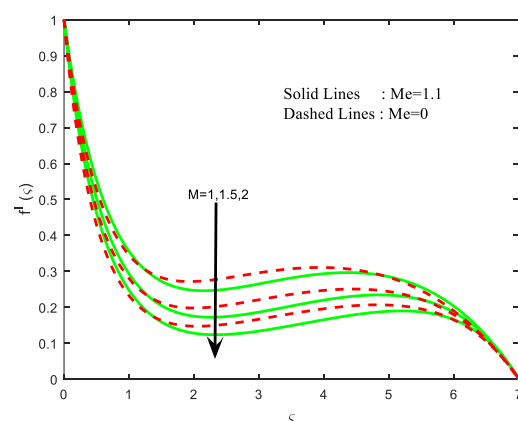


Fig. 3. Impact of M on $f'(\zeta)$

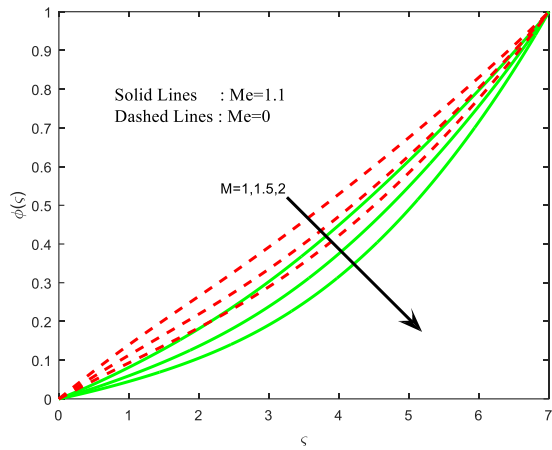


Fig. 4. Impact of M on $\phi(\zeta)$

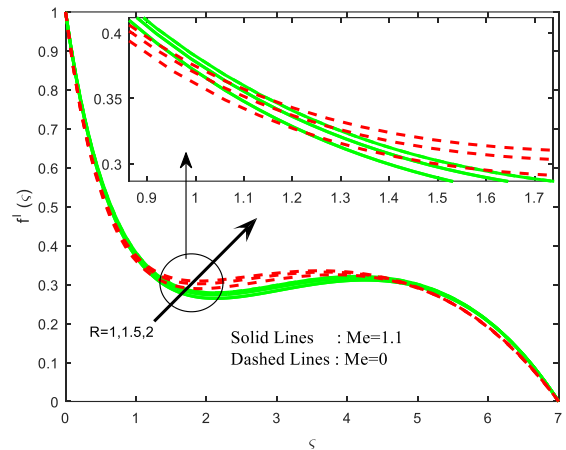


Fig. 5. Impact of R on $f'(\zeta)$

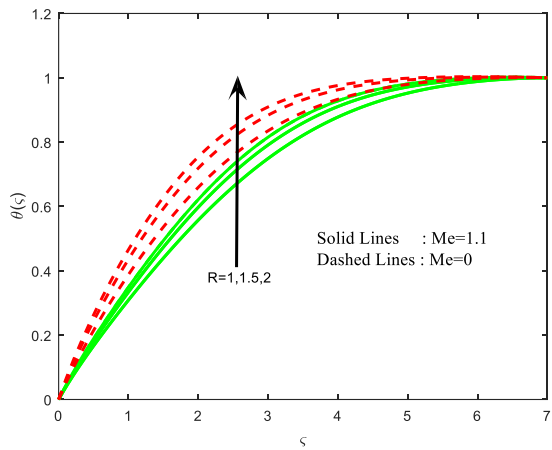


Fig. 6. Impact of R on $\theta(\zeta)$

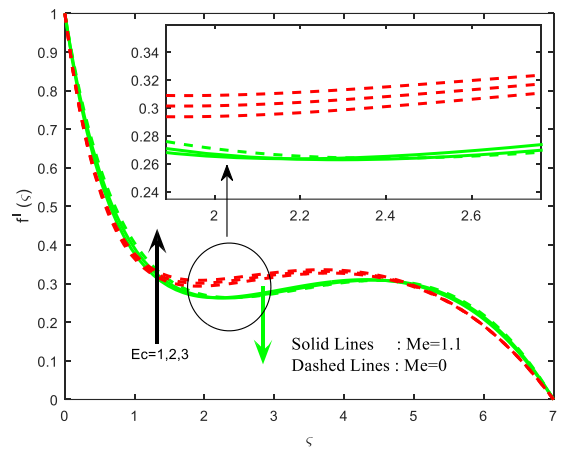


Fig. 7. Impact of Ec on $f'(\eta)$

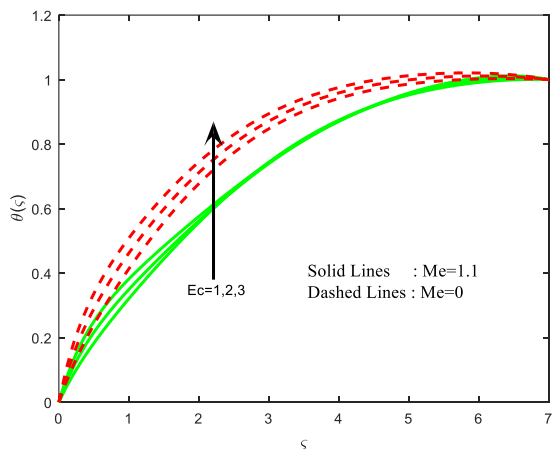


Fig. 8. Impact of Ec on $\theta(\zeta)$

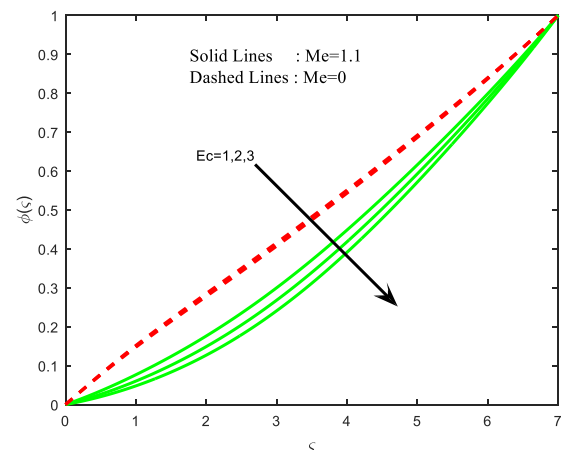


Fig. 9. Impact of Ec on $\phi(\zeta)$

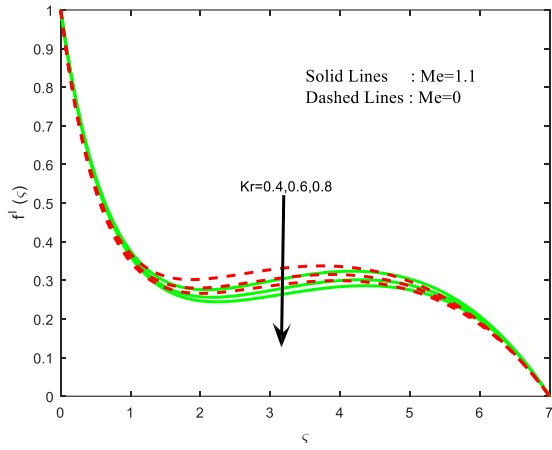


Fig. 10. Impact of Kr on $f'(\zeta)$

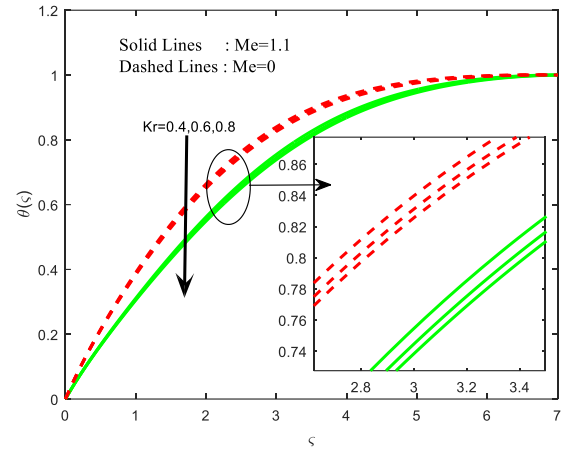


Fig. 11. Impact of Kr on $\theta(\zeta)$

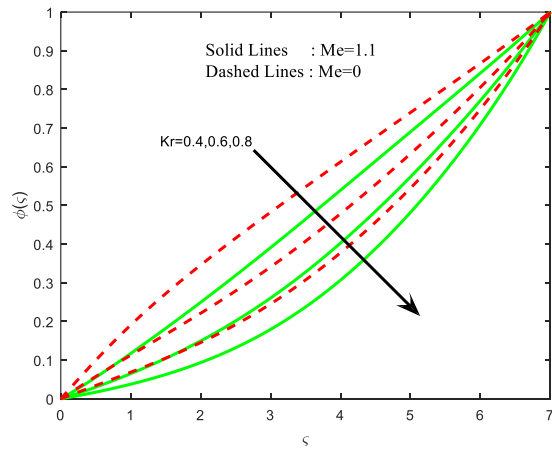


Fig. 12. Impact of Kr on $\phi(\eta)$

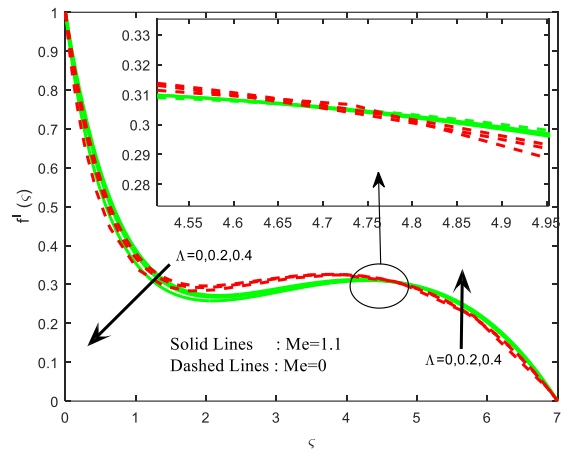


Fig. 13. Impact of Λ on $f'(\zeta)$

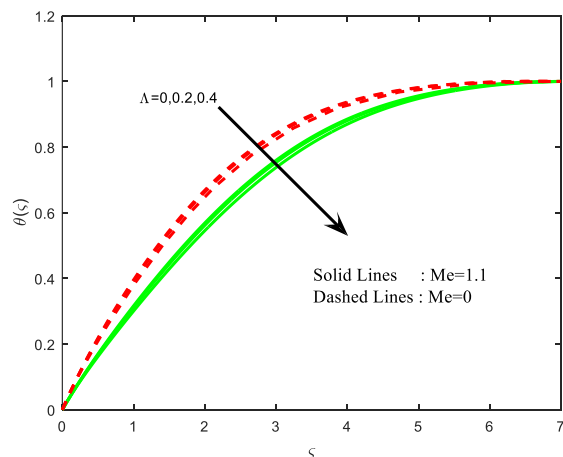


Fig. 14. Impact of Λ on $\theta(\eta)$

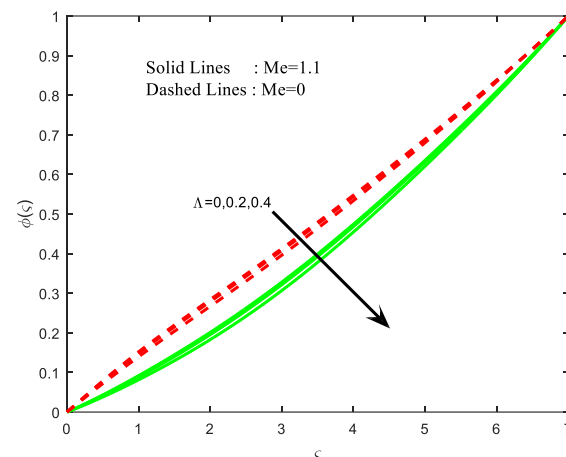


Fig. 15. Impact of Λ on $\phi(\zeta)$

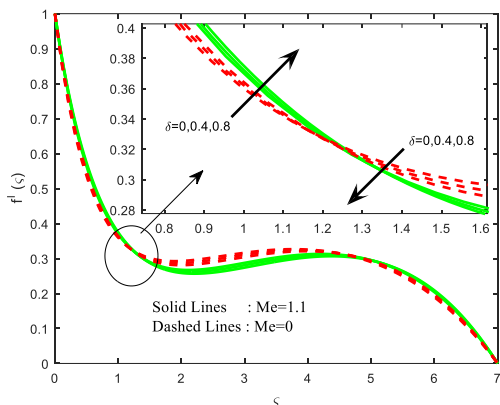


Fig. 16. Impact of δ on $f'(\zeta)$

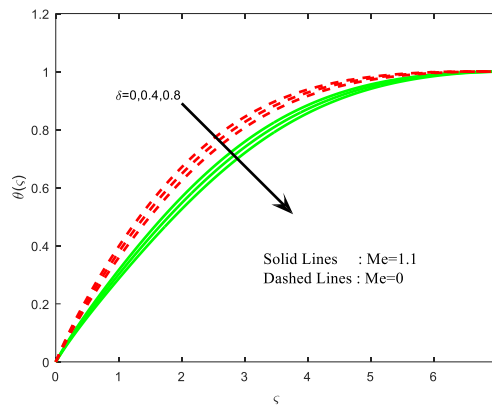


Fig. 17. Impact of δ on $\theta(\eta)$

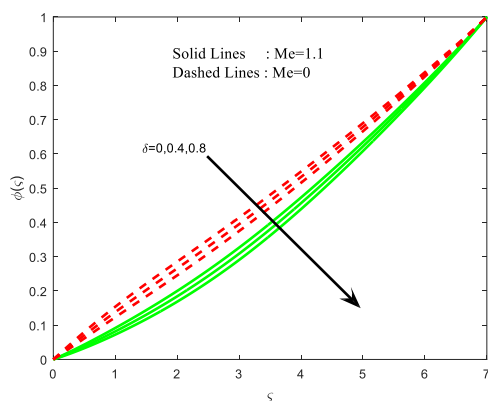


Fig. 18. Impact of δ on $\phi(\zeta)$

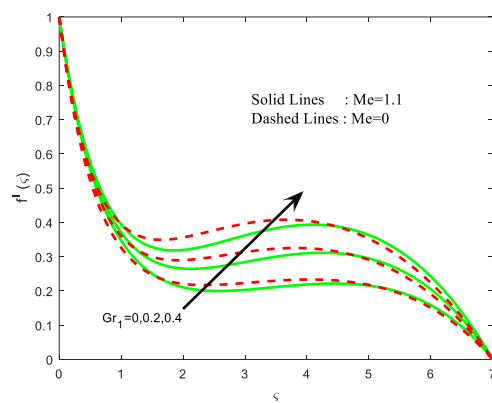


Fig. 19. Impact of Gr_1 on $f'(\zeta)$

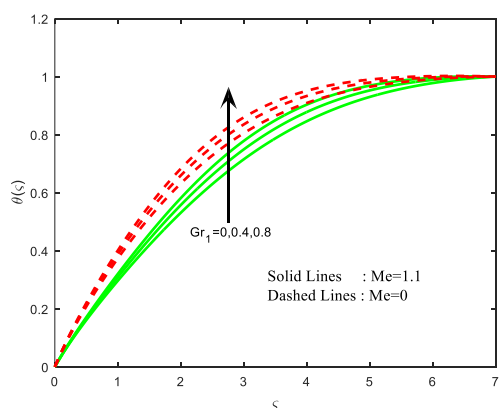


Fig. 20. Impact of Gr_1 on $\theta(\eta)$

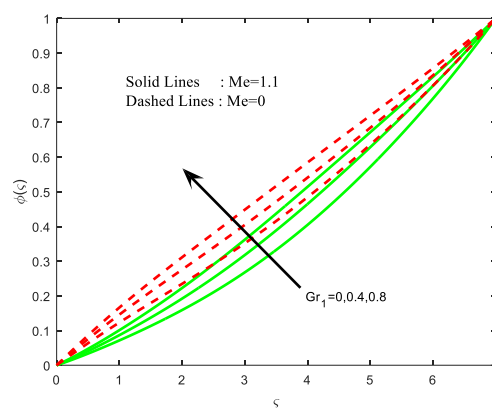


Fig. 21. Impact of Gr_1 on $\phi(\zeta)$

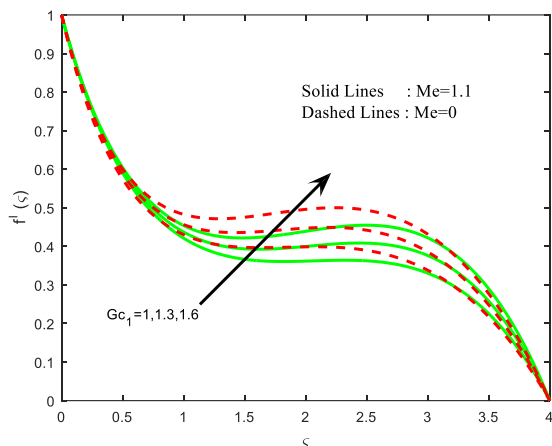


Fig. 22. Impact of Gc_1 on $f'(\zeta)$

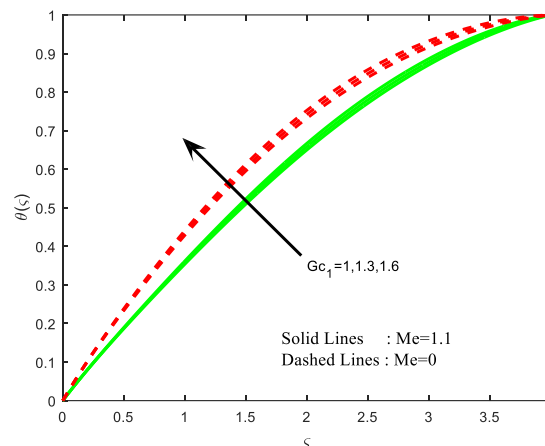


Fig. 23. Impact of Gc_1 on $\theta(\eta)$

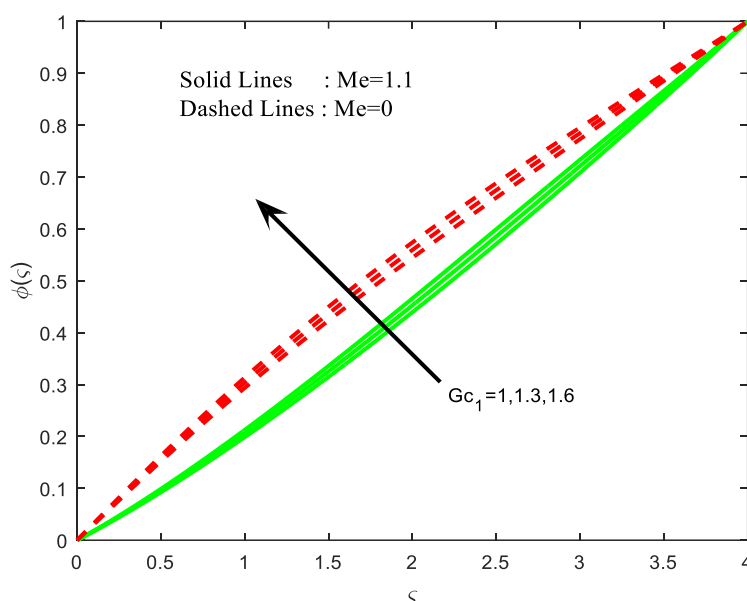


Fig. 24. Impact of Gc_1 on $\phi(\zeta)$

Table 1 and 2 showed the impact of the relevant parameters on friction factor, local Nusselt number (heat transfer rate) and Sherwood numbers (mass transfer rate) for two cases. From the table 1, we can state that all the previously mentioned parameters with the exception of M, Kr, δ , diminish the heat transfer rate. In the meantime, all the said parameters aside from Gr_1, Gc_1 , improve the mass transfer rate. It is seen that Weissenberg number and wall thickness parameters rise the magnitude of friction factor. Table 2 demonstrates that the parameters are having the exact behaviour as in Table 1. It is worthy to make reference that all quantities are less in case of the absence of melting parameter contrasted with its presence.

Table 1

Values of friction factor, Nusselt number and Sherwood number for Diverse Parameters in the presence of melting heat transfer

M	R	Ec	Kr	Λ	δ	Gr_1	Gc_1	$f''(0)$	$-\theta'(0)$	$-\phi'(0)$
1								-1.900469	-0.551932	-0.099007
1.5								-2.207576	-0.551776	-0.072095
2								-2.501244	-0.555224	-0.054031
	1							-1.836544	-0.552391	-0.105666
	1.5							-1.805724	-0.602878	-0.101068
	2							-1.787654	-0.633146	-0.098330
		1						-1.775874	-0.674736	-0.089038
		2						-1.671252	-0.896842	-0.064192
		3						-1.584621	-1.092837	-0.047314
			0.4					-1.828968	-0.553866	-0.144891
			0.6					-1.842069	-0.551205	-0.078024
			0.8					-1.849315	-0.549453	-0.044033
				0				-1.604460	-0.547565	-0.112665
				0.2				-1.739231	-0.550337	-0.108287
				0.4				-2.252550	-0.560047	-0.098569
					0			-1.860016	-0.568205	-0.113976
					0.4			-1.813333	-0.536752	-0.097754
					0.8			-1.767675	-0.506025	-0.083108
						0		-1.869692	-0.542231	-0.087389
						0.2		-1.836544	-0.552391	-0.105666
						0.4		-1.803907	-0.561232	-0.122778
							1	-1.754774	-0.597266	-0.238598
							1.3	-1.735415	-0.598973	-0.246911
							1.6	-1.714996	-0.601037	-0.255287

Table 2

Values of friction factor, Nusselt number and Sherwood number for Diverse Parameters in the absence of melting heat transfer

M	R	Ec	Kr	Λ	δ	Gr_1	Gc_1	$f''(0)$	$-\theta'(0)$	$-\phi'(0)$
1								-2.183678	-0.733625	-0.208244
1.5								-2.556111	-0.739621	-0.167080
2								-2.924451	-0.748703	-0.136975
	1							-2.107201	-0.732795	-0.217947
	1.5							-2.090616	-0.843194	-0.223122
	2							-2.080351	-0.916374	-0.226058
		1						-2.098654	-0.915622	-0.219796
		2						-2.081611	-1.270611	-0.223448
		3						-2.064682	-1.611772	-0.227039
			0.4					-2.093522	-0.734144	-0.288883
			0.6					-2.117411	-0.731634	-0.165725
			0.8					-2.131092	-0.729779	-0.098119
				0				-1.769926	-0.722445	-0.227774
				0.2				-1.957891	-0.728265	-0.221676
				0.4				-2.065072	-0.726504	-0.207535
					0			-2.143380	-0.756335	-0.235258
					0.4			-2.071767	-0.709568	-0.201435
					0.8			-2.003030	-0.664083	-0.170795
						0		-2.164174	-0.721275	-0.183538
						0.2		-2.107201	-0.732795	-0.217947
						0.4		-2.051755	-0.742190	-0.248589
							1	-1.987204	-0.773723	-0.439687
							1.3	-1.945373	-0.775808	-0.455278
							1.6	-1.902355	-0.778522	-0.470711

4. Conclusions

The present study addresses the nonlinear convection on Williamson fluid over variable thickness surface with melting transfer parameter. To control the thermal and diffusion boundary, we also included the chemical reaction, thermal radiation and viscous dissipation. The key observations of this investigation can be described as:

- The temperature increases rapidly throughout the boundary layer owing to the release of energy in thermal boundary layer is higher the $Me = 0$ case when compared to $Me = 1.1$ for increasing values of all governing parameters.
- The wall thickness and Weissenberg number are showed mixed performance on velocity profile due to nonlinear convection in the flow.
- The rate of heat transfer is high in melting surface condition when compared its absence. This result helps us to conclude that, for heating treatment melting surface is useful.
- The time required for the execution is lesser in $Me = 0$ case when compared to $Me = 1.1$ case in the presence of Eckert number, chemical reaction, nonlinear convection and wall thickness parameters.

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