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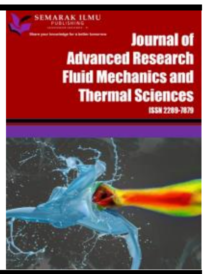

Core Variation of a Non-Newtonian Fluid in an Annular Cylinder

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ABSTRACT

The objective of the paper is to investigate the variation of core thickness of a non-Newtonian fluid in an annular cylinder at the entrance region. It is derived numerically, with no prior assumptions on the form of the Velocity profile. The thickness of the core is examined at each cross section of annuli for various values of Bingham number and aspect ratio by applying mass balance equation. The effect of various parameters on the flow rate are obtained graphically. The examination incorporates Bingham plastic fluids, Casson and Herschel-Bulkley fluid. The administering equations are solved using an iterative procedure by converting non-linear algebraic equation, expressing the variation in pressure in terms of core thickness obtained using the mass balance equation and plug core velocities along the boundary layers. The core thickness is obtained numerically by solving this equation using an iterative procedure. MATLAB programming tool has been employed to get the graphical variation of core thickness for different values of aspect ratio. The comparisons have been made between the obtained results and the results of Casson and Herschel-Bulkley fluids. Also, we find excellent agreement between our numerical results and the results available in literature.

1. Introduction

The entrance region flow in channels finds its applications in the field of engineering such as capillary membrane oxygenators, processing of polymers, haemodialyzers, nuclear reactors and hydro-mechanical machinery. The knowledge of control of flow rate in a pipe facilitates the optimal design and layout of the flow networks. Many scientists have contributed their efforts towards better understanding of the non –Newtonian flow. The fluids included in the majority of realistic models are complex non-Newtonian fluids. Their complex rheological characteristics cannot be adequately represented by a single model.

Various types of non-Newtonian fluid have been studied using different mathematical models. This type of installation may result in a significant pressure drop in the channel since the fluid's

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behaviour in the entrance region has a key role in the overall length of the channel. Recently, the study of non-Newtonian fluids with variable viscosities has gained increasing attention. Previous authors have analysed such non – Newtonian fluids flow at the entrance region by changing the geometry and considered the core formation at the entrance region [1-10]. Kandasamy and Pai [11] have analysed the flow of Casson fluid in a circular tube at the entrance region. Ree-Eyring non-Newtonian model was used by Vaidya *et al.,* [12] to study the blood flow in small arteries by taking convective and wall properties into consideration. Prasad *et al.,* [13] investigated the effects of properties of the fluid on the flow and transfer of heat of an electrically conducting fluid over a stretching sheet by varying the thickness in the presence of a transverse magnetic field. The paper also examines the peristaltic transport of an incompressible non-Newtonian fluid in a porous elastic tube. The impact of slip and heat transfer on a Herschel-Bulkley fluid is taken into account by Vaidya *et al.,* [14]. Manjunatha *et al.,* [15] have investigated the effect on Jeffrey liquid's MHD peristaltic process by varying transport properties and slip situations. Rajashekhar *et al.,* [16] have analysed the effects on Casson liquid peristaltic transport in a convectively heated inclined porous tube by varying viscosity and thermal conductivity. Yusof *et al.,* [17] have analysed the steady of stagnation point flow and radiative heat transfer of a non-Newtonian fluid which is Casson fluid passing over an exponentially permeable slippery Riga plate in presence of thermal radiation, magnetic field, velocity slip, thermal slip, and viscous dissipation effects. Omar *et al.,* [18] have analysed the analytical solution of performance of unsteady Casson fluid with the inclusion of thermal radiation and chemical reaction. Harish *et al.,* [19] have numerically investigated the Thermo-Hydraulic Characteristics of Turbulent Flow in a Tube. Khan *et al.,* [20] have analysed the magnetohydrodynamic (MHD) flow of a double stratified micropolar fluid across a vertical stretching/shrinking sheet in the presence of suction, chemical reaction and heat source effects. The objective of the current work is to numerically analyse Bingham fluid core formation in an annular cylinder at the region of entrance.

2. Methodology

We are investigating the flow of Bingham fluid through an annular cylinder in the entrance region. Fluid enters a horizontal annular duct with a uniform velocity along the axial direction from a large chamber. The examination is carried out over a wide range of aspect ratios. In the annulus, boundary layer development is envisioned upon the enter of the fluid. Also, a fully developed velocity profile is observed where the boundary layer converges with the outer edge of the plug flow zone. Figure 1 depicts annuli geometry.

An annular duct of radius r_2 and r_1 is considered as depicted in the Figure 1. A laminar, isothermal Bingham liquid which is incompressible enters this duct with a constant velocity V_O from a large chamber. A cylindrical co-ordinate system (r, θ, z) is considered taking the centre of the cylinder as origin, coinciding with the z axis. V_z and V_r are the velocities in z and r directions. There is a boundary layer formation with core separation on each wall. The problem is solved considering only upper half, as the areas are symmetrical about the axis. The Figure 2 depicts the flow of fluid in different areas of annuli is divided into two regions with thickness $\delta_1(z)$, $\delta_2(z)$ in the range $r_1 \leq r \leq r_1 + \delta_1$ and $r_2 - \delta_2 \leq r \leq r_2$ respectively. These two regions are separated by plug flow region ranging from $r_1 + \delta_1$ to $r_2 - \delta_2$.

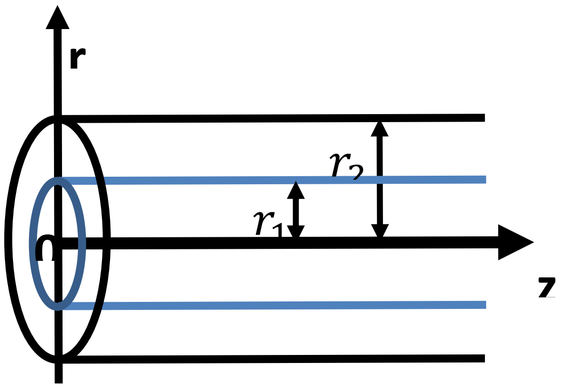

Fig. 1. Annuli Geometry

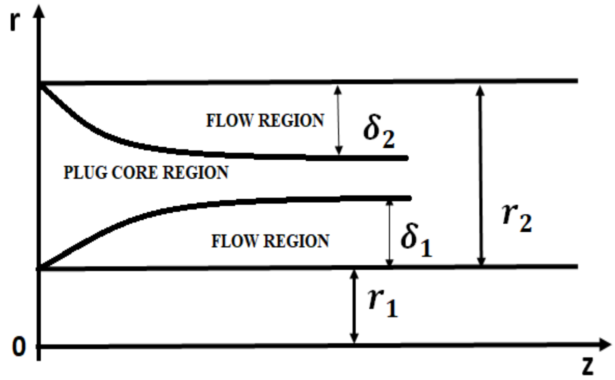

Fig. 2. Flow of fluid in different areas of annuli

Based on the boundary layer assumption and not taking into account inertia terms, the equation of motion of an isotropic, incompressible Bingham liquid can be expressed as

$$\frac{1}{\mathrm{r}}\frac{\partial(\tau\,\mathrm{r})}{\partial\mathrm{r}}=-\frac{\mathrm{dp}}{\mathrm{dz}} \tag{1}$$

Bingham fluid constitutive equation is given by

$$\tau=\tau_0+\eta_1\left|\frac{\partial v_z}{\partial r}\right|, \tau\geq\tau_0 \tag{2}$$

Where η_1 represents Bingham viscosity, and boundary conditions for this model is given by

i) The wall axial velocities are given by

$$v_z\,(z,\,r_1)=\,0,\;\; v_z\,(z,\,r_2)=0 \tag{3}$$

ii) The core velocities at the corners of the boundary layer are given by

$$v_z\,(z,r_1+\delta_1)=v_{z_{c1}},\;\; v_z\,(z,r_2-\delta_2)=v_{z_{c2}} \tag{4}$$

iii) Using the limiting conditions of shear stress at the boundaries of plug core region, Eq. (2) reduces

$$\left(\frac{\partial v_z}{\partial r}\right)=0 \text{ at } r_1+\,\delta_1 \textit{ and also at } r_2-\,\delta_2 \tag{5}$$

Non-dimensional velocity profiles in different areas are obtained from (1), (2) and using the boundary conditions (3), (4) and (5)

$$V_1=\left[\frac{1}{8}\left(r_{c_1}\right)^2 log\left(\frac{R}{\varepsilon}\right)\frac{dp}{dz}-\frac{1}{16}\frac{dp}{dz}(R^2-\varepsilon^2)+Nr_{c_1}\log\left(\frac{R}{\varepsilon}\right)-N(R-\varepsilon)\right],\varepsilon\leq R\leq r_{c_1} \tag{6}$$

$$V_2=\left[\frac{1}{8}\left(r_{c_2}\right)^2 log(R)\frac{dp}{dz}-\frac{1}{16}\frac{dp}{dz}(R^2-1)-N(R-1)-Nr_{c_2}\log R\right],r_{c_2}\;\leq R\leq 1 \tag{7}$$

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$$V_{c_1} = \begin{bmatrix} \frac{1}{8}\left(r_{c_1}\right)^2 log\left(\frac{r_{c_1}}{\varepsilon}\right)\frac{dp}{dz} - \frac{1}{16}\frac{dp}{dz}\left({r_{c_1}}^2 - \varepsilon^2\right) + Nr_{c_1}\log\left(\frac{r_{c_1}}{\varepsilon}\right) \\ -N\left(r_{c_1} - \varepsilon\right) \end{bmatrix}, r_{c_1} \leq R \leq r_{c_2} \tag{8}$$

$$V_{c_2} = \begin{bmatrix} \frac{1}{8}\left(r_{c_2}\right)^2 log\left(r_{c_2}\right)\frac{dp}{dz} - \frac{1}{16}\frac{dp}{dz}\left({r_{c_2}}^2 - 1\right) - Nr_{c_2}\log\left(r_{c_2}\right) \\ -N\left(r_{c_2} - 1\right) \end{bmatrix}, r_{c_1} \leq R \leq r_{c_2} \tag{9}$$

where non-dimensional parameters are

$$V_1 = \frac{V_{z_1}}{V_0}, V_2 = \frac{V_{z_2}}{V_0}, V_{c_1} = \frac{V_{zc1}}{V_0}, V_{c_2} = \frac{V_{zc2}}{V_0}, r_{c_1} = \frac{r_1+\delta_1}{r_2} = \varepsilon + \delta_{c_1}, r_{c_2} = \frac{r_2-\delta_2}{r_2} = 1 - \delta_{c_2},$$

$$Z = \frac{z\eta_1}{4v_0 r_2^2 \rho}, R = \frac{r}{r_2}, P = \frac{p}{v_0^2\rho}, N = \frac{\tau_0 r_2}{\eta_1 V_0} = Bingham\ Number, \varepsilon = \frac{r_1}{r_2} = Aspect\ Ratio$$

We equate, $V_{c_1} = V_{c_2}$ to find the unknowns r_{c_1}, r_{c_2}, change in pressure, core velocity and the core thickness

$$\frac{dP}{dZ} = \frac{X}{Y} \tag{10}$$

where X and Y are given by

$$X = \left[N(r_{c_1})log\left(\frac{r_{c_1}}{\varepsilon}\right) - N\left(r_{c_1} - \varepsilon\right) + N\left(r_{c_2} - 1\right) + N(r_{c_2})\log\left(r_{c_2}\right)\right]$$

$$Y = \left[\frac{1}{8}\left(r_{c_2}\right)^2 log\left(r_{c_2}\right) - \frac{1}{8}\left(r_{c_1}\right)^2\log\left(\frac{r_{c_1}}{\varepsilon}\right) + +\frac{1}{16}\left(\left(r_{c_1}\right)^2 - \varepsilon^2\right) - \frac{1}{16}\left(\left(r_{c_2}\right)^2 - 1\right)\right]$$

The integral balance equations are given by

$$\int_0^1 2VRdR = 1 \tag{11}$$

substituting (6), (7) and (8) in (11), we get

$$A_1 + B_1 + C_1 = \frac{1}{2} \tag{12}$$

$$A_1 = Nr_{c_1}\left(\frac{r_{c_1}^2}{2}\log\left(r_{c_1}\right) - \frac{r_{c_1}^2}{4} - \frac{r_{c_1}^2}{2}\log(\varepsilon) - \frac{\varepsilon^2}{2}\log(\varepsilon) + \frac{\varepsilon^2}{4} + \varepsilon^2\log(\varepsilon)\right) - N\left(\frac{r_{c_1}^3}{3} - \frac{\varepsilon r_{c_1}^2}{4} - \frac{\varepsilon^3}{6}\right) + \frac{1}{8}\frac{dP}{dZ}\left(r_{c_1}\right)^2\left(\frac{r_{c_1}^2}{2}\log\left(r_{c_1}\right) - \frac{r_{c_1}^2}{4} - \frac{r_{c_1}^2}{2}\log(\varepsilon) - \frac{\varepsilon^2}{2}\log(\varepsilon) + \frac{\varepsilon^2}{4} + \varepsilon^2\log(\varepsilon)\right) - \frac{1}{16}\frac{dP}{dZ}\left(\frac{r_{c_1}^4}{4} - \frac{r_{c_1}^2\varepsilon^2}{2} + \frac{\varepsilon^4}{2}\right)$$

$$B_1 = -N\left(-\frac{1}{6} - \frac{r_{c_2}^3}{3} + \frac{r_{c_2}^2}{2}\right) - Nr_{c_2}\left(-\frac{1}{4} - \frac{r_{c_2}^2}{2}\log\left(r_{c_1}\right) + \frac{r_{c_2}^2}{4}\right) - \frac{1}{16}\frac{dP}{dZ}\left(-\frac{1}{4} - \frac{r_{c_2}^4}{4} + \frac{r_{c_2}^2}{2}\right) + \frac{1}{8}\frac{dP}{dZ}r_{c_2}^2\left(-\frac{r_{c_2}^2}{2}\log\left(r_{c_2}\right) + \frac{r_{c_2}^2}{4} - \frac{1}{4}\right)$$

$$C_1 = \left(\frac{r_{c_2}^2}{2} - \frac{r_{c_1}^2}{2}\right)\left[Nr_{c_1}\log\left(\frac{r_{c_1}}{\varepsilon}\right) - N\left(r_{c_1} - \varepsilon\right) - \frac{1}{16}\frac{dP}{dZ}\left(r_{c_1}^2 - \varepsilon^2\right) + \frac{1}{8}\frac{dP}{dZ}r_{c_1}^2\log\left(\frac{r_{c_1}}{\varepsilon}\right)\right]$$

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Eq. (10) and Eq. (12) are numerically solved for multiple variables using Newton Raphson method to determine the values of r_{c_1}, r_{c_2} and variations in pressure for various values of Bingham number and aspect ratio.

3. Results

The core thickness of Bingham fluid through an annular cylinder at the region of entrance has been numerically obtained without taking into account the velocity profile in the developing boundary layer. Initially, the variation in pressure with respect to core thickness is derived using mass balance, momentum balance equations and plug core velocities along the boundary layers. The solution of this equation yields dP/dZ. Using the values of variation in pressure so obtained, the thickness of the core for various values of Bingham number and aspect ratio are obtained. The change in core thickness for different values of Bingham number and aspect ratios $\varepsilon = 0.3$, 0.4 and 0.5 are indicated in Figure 3, Figure 4 and Figure 5. Looking at the variation at any cross section Z, it is observed that the thickness of the core increases with the increase in Bingham number at the entrance region. Additionally, it has been found that the entrance length reduces for a given Bingham number as the thickness of the core grows along with the annuli's aspect ratio. The Figure 6 and Figure 7 show the variation in core thickness for different values of Casson number. By comparing Figure 3 to Figure 5 with Figure 6 and Figure 7, it is observed that for a particular value of Casson and Bingham number, the entrance length decreases and the core thickness increases. The core thickness also gets enhanced with the increase in the aspect ratio, for particular values of Casson and Bingham number.

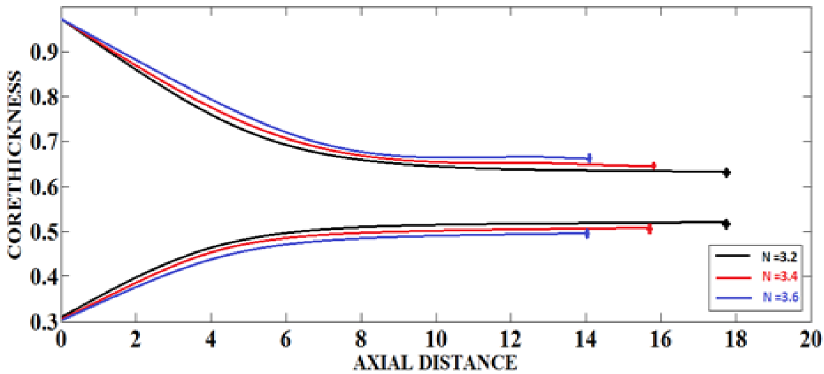

Fig. 3. The variation in Core Thickness for ε = 0.3, Bingham Number = N = 3.2, 3.4 and 3.6

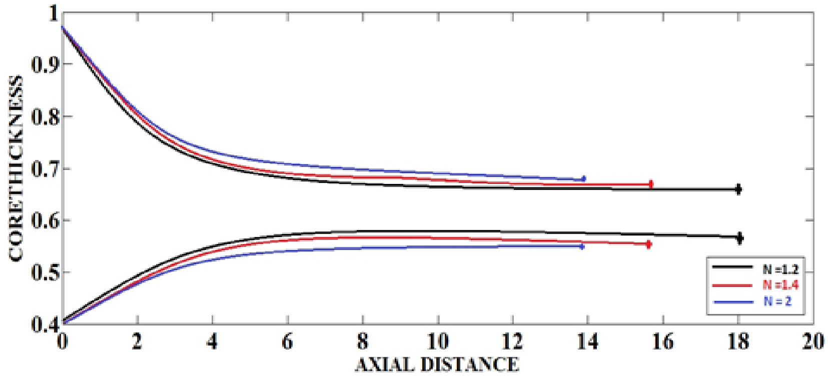

Fig. 4. The variation in Core Thickness for $\varepsilon = 0.4$, Bingham Number = N = 1.2, 1.4, 2

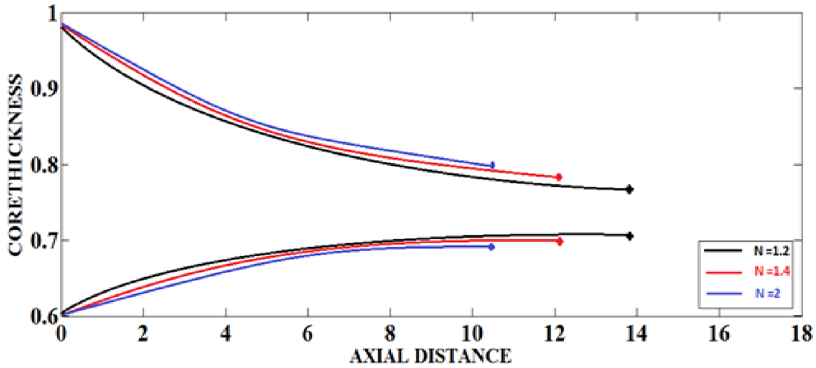

Fig. 5. The variation in Core Thickness for ε = 0.5, Bingham Number = N = 1.2, 1.4 and 2

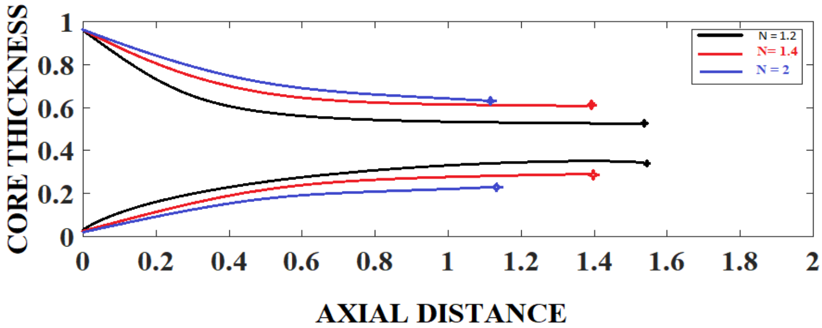

Fig. 6. The variation in Core Thickness for ε = 0.4, Casson Number = N = 1.2, 1.4 and 2

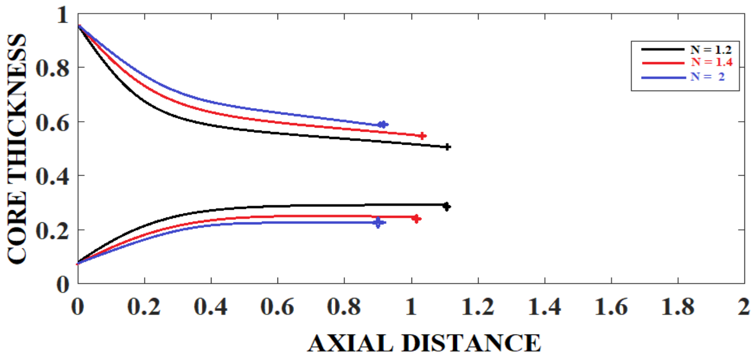

Fig. 7. The variation in Core Thickness for ε = 0.5, Casson Number = N =1.2, 1.4 and 2

It is also observed from Figure 8 and Figure 9, when the power law index is n= 0.8 for the Herschel-Bulkley fluid having same Bingham number as of Figure 4 and Figure 5, the core thickness gets enhanced with rise in Bingham number and aspect ratio. In general, it is observed that the increase in the value ε reduces the maximum growth of the inner and outer boundary layers and at the extremity of the entrance length, the plug core region thickness equals the yield value, as is physically expected. By comparing the results, we see that the value of entrance length of Casson fluid is shorter than Bingham fluid for a particular value of non-Newtonian fluid number 'N' and for all acceptable values of aspect ratio.

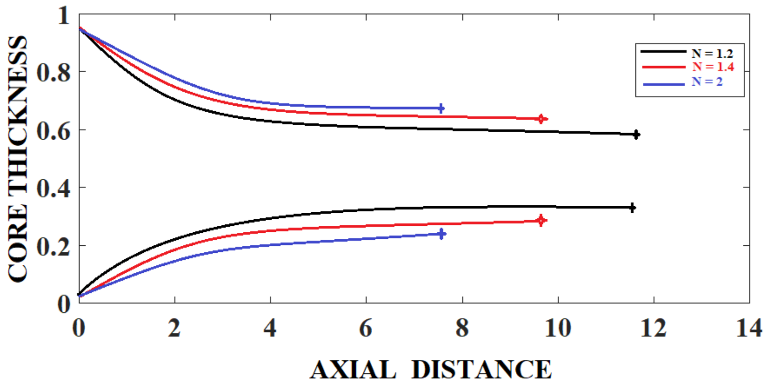

Fig. 8. The variation in Core Thickness for $\varepsilon = 0.4$, power index = 0.8, Herschel Bulkley Number = N =1.2, 1.4 and 2

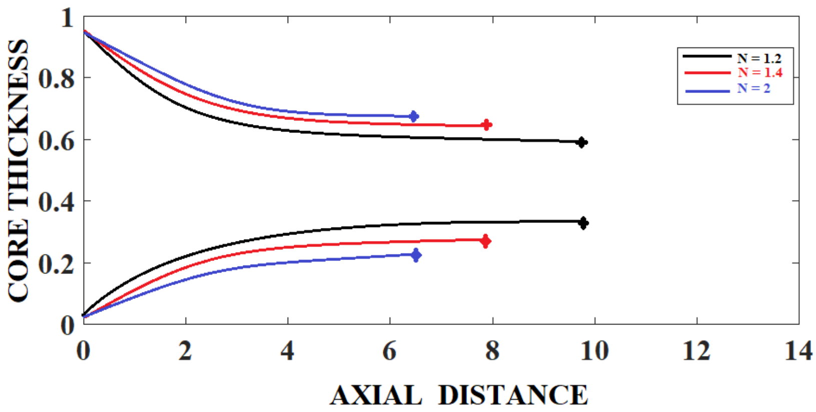

Fig. 9. The variation in Core Thickness for $\varepsilon = 0.5$, power index = 0.8, Herschel Bulkley Number = N =1.2, 1.4 and 2

4. Conclusions

The analysis carried out has the capacity to analyse the outcomes of numerous already presented models within the scope of a single investigation, viz. Bingham fluid model, Hershel Bulkley fluid model and Casson fluid model. The results of these models have been compared. From the variations, it is noteworthy to observe that the core thickness in materials with thick viscosity is high. Also, the core thickness for Casson, Herschel-Bulkley fluids in the above study is observed to be more compared to Bingham fluid.

References

[1] Batra, R. L., and A. Kandasamy. "Entrance flow of Herschel-Bulkley fluids in a duct." *Fluid Dynamics Research* 6, no. 1 (1990): 43. https://doi.org/10.1016/0169-5983(90)90037-Y

[2] Gupta, R. C. "Developing Bingham fluid flow in a channel." *Mathematical and Computer Modelling* 21, no. 8 (1995): 21-28. https://doi.org/10.1016/0895-7177(95)00037-3

[3] Dash, R. K., G. Jayaraman, and K. N. Mehta. "Estimation of increased flow resistance in a narrow catheterized artery-a theoretical model." *Journal of Biomechanics* 29, no. 7 (1996): 917-930. https://doi.org/10.1016/0021-9290(95)00153-0

[4] Gupta, Ramesh C. "On developing laminar non-Newtonian flow in pipes and channels." *Nonlinear Analysis: Real World Applications* 2, no. 2 (2001): 171-193. https://doi.org/10.1016/S0362-546X(00)00109-7

[5] Maia, M. C. A., and C. A. Gasparetto. "A numerical solution for the entrance region of non-newtonian flow in annuli." *Brazilian Journal of Chemical Engineering* 20 (2003): 201-211. https://doi.org/10.1590/S0104-66322003000200014

[6] Soares, Edson J., Mônica F. Naccache, and Paulo R. Souza Mendes. "Heat transfer to viscoplastic materials flowing axially through concentric annuli." *International Journal of Heat and Fluid Flow* 24, no. 5 (2003): 762-773. https://doi.org/10.1016/S0142-727X(03)00066-3

[7] Sayed-Ahmed, Mohamed Eissa, and Hazem Sharaf-El-Din. "Entrance region flow of a power-law fluid in concentric annuli with rotating inner wall." *International Communications in Heat and Mass Transfer* 33, no. 5 (2006): 654-665. https://doi.org/10.1016/j.icheatmasstransfer.2006.01.004

[8] Sankar, D. S., and K. Hemalatha. "Pulsatile flow of Herschel-Bulkley fluid through catheterized arteries-A mathematical model." *Applied Mathematical Modelling* 31, no. 8 (2007): 1497-1517. https://doi.org/10.1016/j.apm.2006.04.012

[9] Arthur, Emmanuel Maurice, Ibrahim Yakubu Seini, and Letis Bortey Bortteir. "Analysis of Casson Fluid Flow over a Vertical Porous Surface with Chemical Reaction in the Presence of Magnetic Field." *Journal of Applied Mathematics and Physics* 3, no. 06 (2015): 713. https://doi.org/10.4236/jamp.2015.36085

[10] Mahmood, R., N. Kousar, K. Usman, and A. Mehmood. "Finite element simulations for stationary Bingham fluid flow past a circular cylinder." *Journal of the Brazilian Society of Mechanical Sciences and Engineering* 40, no. 9 (2018): 459. https://doi.org/10.1007/s40430-018-1383-2

[11] Kandasamy, A., and Rekha G. Pai. "Entrance region flow of casson fluid in a circular tube." In *Applied Mechanics and Materials*, vol. 110, pp. 698-706. Trans Tech Publications Ltd, 2012. https://doi.org/10.4028/www.scientific.net/AMM.110-116.698

[12] Vaidya, Hanumesh, Rajashekhar Choudhari, Fateh Mebarek-Oudina, Isaac Lare Animasaun, Kerehalli Vinayaka Prasad, and Oluwale Daniel Makinde. "Combined effects of homogeneous and heterogeneous reactions on peristalsis of Ree-Eyring liquid: Application in hemodynamic flow." *Heat Transfer* 50, no. 3 (2021): 2592-2609. https://doi.org/10.1002/htj.21995

[13] Prasad, K. V., H. Vaidya, K. Vajravelu, and M. M. Rashidi. "Effects of variable fluid properties on MHD flow and heat transfer over a stretching sheet with variable thickness." *Journal of Mechanics* 33, no. 4 (2017): 501-512. https://doi.org/10.1017/jmech.2016.101

[14] Vaidya, Hanumesh, Manjunatha Gudekote, Rajashekhar Choudhari, and K. V. Prasad. "Role of slip and heat transfer on peristaltic transport of Herschel-Bulkley fluid through an elastic tube." *Multidiscipline Modeling in Materials and Structures* 14, no. 5 (2018): 940-959. https://doi.org/10.1108/MMMS-11-2017-0144

[15] Manjunatha, G., C. Rajashekhar, Hanumesh Vaidya, K. V. Prasad, O. D. Makinde, and J. U. Viharika. "Impact of variable transport properties and slip effects on MHD Jeffrey fluid flow through channel." *Arabian Journal for Science and Engineering* 45 (2020): 417-428. https://doi.org/10.1007/s13369-019-04266-y

[16] Rajashekhar, C., G. Manjunatha, Hanumesh Vaidya, B. Divya, and K. Prasad. "Peristaltic flow of Casson liquid in an inclined porous tube with convective boundary conditions and variable liquid properties." *Frontiers in Heat and Mass Transfer (FHMT)* 11, no. 35 (2018). https://doi.org/10.5098/hmt.11.35

[17] Yusof, Nur Syamila, Siti Khuzaimah Soid, Mohd Rijal Illias, Ahmad Sukri Abd Aziz, and Nor Ain Azeany Mohd Nasir. "Radiative Boundary Layer Flow of Casson Fluid Over an Exponentially Permeable Slippery Riga Plate with Viscous Dissipation." *Journal of Advanced Research in Applied Sciences and Engineering Technology* 21, no. 1 (2020): 41-51. https://doi.org/10.37934/araset.21.1.4151

[18] Omar, Nur Fatihah Mod, Husna Izzati Osman, Ahmad Qushairi Mohamad, Rahimah Jusoh, and Zulkhibri Ismail. "Analytical Solution of Unsteady MHD Casson Fluid with Thermal Radiation and Chemical Reaction in Porous Medium." *Journal of Advanced Research in Applied Sciences and Engineering Technology* 29, no. 2 (2023): 185-194. https://doi.org/10.37934/araset.29.2.185194

[19] Harish, H. V., Birlie Fekadu, and Manjunath K. "Numerical Studies on Thermo-Hydraulic Characteristics of Turbulent Flow in a Tube with a Regularly Spaced Dimple on Twisted Tape." *CFD Letters* 13, no. 8 (2021): 20-31. https://doi.org/10.37934/cfdl.13.8.2031

[20] Khan, Ansab Azam, Khairy Zaimi, Suliadi Firdaus Sufahani, and Mohammad Ferdows. "MHD flow and heat transfer of double stratified micropolar fluid over a vertical permeable shrinking/stretching sheet with chemical reaction and heat source." *Journal of Advanced Research in Applied Sciences and Engineering Technology* 21, no. 1 (2020): 1-14. https://doi.org/10.37934/araset.21.1.114