

Analytical Computation of Thermal and Electrical Issues in E-Mobility Cabling Network

Mahipal Bukya^{1,2,*}, Rajesh Kumar², Akhilesh Mathur²

¹ Department of Electrical and Electronics Engineering, Manipal Institute of Technology Bengaluru, Manipal Academy of Higher Education,

Manipal, 576104, India

² Department of Electrical Engineering, Malaviya National Institute of Technology, Jaipur 302017, India

ARTICLE INFO	ABSTRACT
Article history: Received 11 November 2022 Received in revised form 9 February 2023 Accepted 17 February 2023 Available online 10 March 2023 <i>Keywords:</i> Electric vehicles; components; wiring harness; cabling network; thermal stress;	The demand for pollution-free transportation is increasing daily worldwide due to increasing concerns about global warming, greenhouse gas emissions, and the depletion of fossil fuels. In e-mobility, electric vehicles (EVs) have received massive popularity due to their performances and efficiencies compared with IC engine vehicles in recent decades. Cables play an essential role in delivering electric power to the different electrical components system from the battery source of vehicles. Electrical insulation is the backbone of the power cable, and its state is usually used to reflect the healthy condition of this cable and any electrical networks. Deterioration of the cable during normal operation is a major concern. Usually, electric cables receive less maintenance as compared to other electrical components in EVs. Due to the many stresses, the cable insulation must constantly withstand better connectivity between different parts in e-mobility. In the analytical method, the solution obtains known elementary functions by integrating a partial differential equation better to understand the variation of thermal and electrical stresses. This paper studied the high voltage cabling network of electric vehicles and the analytical method used for computing conductivity, temperature, and electrical stress distribution in electric vehicle cabling networks by including the higher vehicle ambient temperatures. The thermal and
electrical stress; Finite Difference Method	electrical stress maximizes at the conductors' surface and gradually reduces the function of the radial distance.

1. Introduction

Renewable and sustainable transportation is the solution for rapid urbanization, rising greenhouse gas emissions, and excessive resource consumption [1-2]. Day by day, the demand for e-mobility is increasing, so the requirement for onboard electrical components, such as cable harnesses, is also increasing with the rise in EVs. The e-mobility industries must be able to deliver technically excellent and reasonably priced items inventively to compete in this market. Providing comfort and safety for the passengers while also competing in weight and energy savings have been

* Corresponding author.

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E-mail address: mahipalbhukya@gmail.com

the emerging trends in the automotive industry over the past several years. The automotive power supply system has great importance regarding electrical functions and is one of the main factors in reducing manufacturing costs, including the cost of the cable harness [2-3]. The difficulties in designing vehicles are to make them lighter, cheaper, more efficient, and more practical while maintaining an ever-rising quality standard. Due to these difficulties, automobile wire harnesses must now accommodate an increasing number of conductors in a relatively constrained area [4-5]. The cable, which connects the power source and the load, must be the right size to perform at its best. Correct sizing is needed for selecting the ideal cross-sectional area considering several factors, including ambient temperature, insulation thickness, current [6-9] carrying capacity, voltage drop allowance, and other relevant factors. The correct cable size must be used in certain applications, such as high-current circuits for batteries and motors or relatively lengthy cables in bus applications. The wrong cable size could cause melting, fire, or even an explosion. The conductor's inefficiency is due to its size. Big wires need a lot of space.

Damage could occur as a result of the cable's undersize. The conductor will heat up more quickly for a given amount of current passing through a cable due to its increased resistive loss, which is a significant disadvantage. In contrast to the larger conductor, the smaller conductor will heat up to the highest temperature permitted by the lesser current. In other words, a smaller conductor results in decreased ampacity, and overall temperature affect the insulation strength. Trucks and buses will require more current to flow from the charging component than lightweight vehicles. Therefore, heavy-duty vehicles must investigate cable performance. Most of the literature worked towards cable ampacity, potential drop, and short circuit problems [10-15]. EV needed more work, especially in cable insulation design. This paper studied the possible cabling networks and proposed the openended high voltage cabling network with approximate dimensions of the connecting wires. From the Table 1, cable dimensions have been chosen based on high-power capacity vehicles for evaluating thermal and electrical field distribution using the analytical method in Matlab. The reason for using the analytical method is the solution obtained in terms of known elementary functions by integrating a partial differential equation.

2. EV Cabling Layout

Insulation and electrical conductors are two essential parts of wires. In contrast to static power system wire, which normally offers a vast region that can be used, the dynamic electric car does not give this choice. There ought to be less room for the wire. Therefore, having a large cable is not advantageous in terms of system architecture. Additionally, heavier wires increase the vehicle's overall weight. Importantly, longer cables require more conductor material, which drives up the cost of the component at first purchase [16-20].

The high voltage of automotive cable and the low voltage in the power system is comparable, the cable has distinct differences. The car cable needs to be more flexible because it is smaller physically than the distribution cables. Most automotive cables are not armored since it would make them more difficult to bend [21-26]. The majority of cables include a screening layer to reduce electromagnetic interference. When EV cable is affected by the temperature of the environment and the current flowing through it, as well as when the cables are routed in close proximity with little room for airflow, thermal performance is even more important. A comprehensive examination is needed of the thermal specifications for EV/HEV cable systems. In Figure 1 shows the maximum possible high voltage electrical wiring harnesses, which will be very useful for classifying low voltage auxiliary supply circuits for thermal and electrical field distribution.



Fig. 1. High voltage cabling network of EV

3. Theoretical Background

The cable insulation depends on conductivity, radial distances, temperatures, and electrical field. The conductivity in DC cable is traced to a thermal origin, being the consequence of thermal runaway. The causative principles of a runway under direct electric stress are more complex than those under AC. As the electric field increases, the dielectric conductivity increases, hence the temperature. A cumulative compound effect makes it reach asymptotic proportions implying the violation of Ohm's law hence an imminent breakdown [14-16].

Maxwell equation used for electric vehicle electric field, temperature, and conductivity of the cabling insulation. The well-known Arrhenius law can describe electrical conductivity as a function of the absolute temperature [14]

$$\sigma(T) = \sigma' exp \frac{-\Delta W}{kT} \tag{1}$$

where σ' is the model constant, ΔW is the activation energy for the conduction process in the insulation bulk, and k is the Boltzmann constant.





Fig. 2. Practical EV Cabling layers (a) Cross-section view of EV cable [19] (b) Geometrical structure of cable

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In general, the σ is a function of temperature and electric field. During normal operation, the cable temperature is not uniform across the insulation (decreasing from the conductor to the outer shield). For instance, the temperature distribution can be assumed as derived by the thermal-Ohms law, as

$$T(r) = \frac{T_0 - T_1}{\ln \frac{r_1}{r_0}} \ln \frac{r_1}{r} + T_1$$
(2)

where "0" and "1" indicate quantities relevant to the inner conductor(r=r0) and the external shield ($r = r_1$), respectively. The conductivity written as a function of radius r by substituting Eq. (2) into Eq. (1). The thermal continuity equation is [27-30]

$$\nabla \cdot \left(k \nabla(T) \right) + \sigma |\nabla(V)|^2 = 0 \tag{3}$$

The heat loss in the insulation under normal loading and voltage conditions, the Eq. (3) reduces to

$$\frac{1d}{rdr} \cdot kr\frac{dT}{dr} = 0 \tag{4}$$

Integrating once, we get

$$\left(kr\frac{dT}{dr}\right) = k_1 \tag{5}$$

where, k_1 is integration constant. Integrating again, we get

$$T = \frac{k_1}{k} ln(r) + k_2 \tag{6}$$

where k_2 is another integration constant.

Re-write Eq. (2) for EV cables. By considering the first boundary condition at $r = r_1$, $T = T_c$, the conductor temperature and at $r = r_2$, $T = T_{chasiss}$, the shield temperature, gives the following solution

$$T_r = \frac{T_c - T_{cs}}{\ln \frac{r_2}{r_{01}}} \ln \frac{r_2}{r} + T_{cs}$$
(7)

where T_r is the temperature at radial distance r.

This is the same as the equation suggested by EPRI, without proof. It merely implies a logarithmic distribution of temperature across the insulation. It may be noted that the directional derivative, (dT_r/dr) is always negative and that the heat flow is outward normal. Also, the temperature is a decreasing function of r.

With the boundary conditions of the second kind at r_1 , r_2 , and the first kind at r_3 , similar calculations give the temperature distribution on the chassis of vehicles,

$$T_{\rm r} = \frac{I_{\rm L}^2 R}{2\pi k_{\rm s}} \ln\left(\frac{r_3}{r}\right) + T_{\rm a}; \quad r_2 < r < r_3$$
(8)

In which, $k_{chassis}$ - thermal conductivity of the chassis, T_a - ambient temperature at chassis r_3 - temperature in the chassis at $r = r_2$ is determined to be

$$T_{r_2} = \frac{I_L^2 R}{2\pi k_s} ln\left(\frac{r_3}{r_2}\right) + T_a \tag{9}$$

Using Eq. (8) and Eq. (9) along with the boundary condition at r_1 , the temperature distribution in the insulation is obtained as under

$$T_r = \frac{I_L^2 R}{2\pi k_i} \ln\left(\frac{r_2}{r}\right) + \frac{I_L^2 R}{2\pi k_s} \ln\left(\frac{r_3}{r_2}\right) + T_a$$
(10)

The electric field in steady state can be calculated through the following Maxwell equations

$$\epsilon_0 \epsilon_r \nabla E = \rho \tag{11}$$

$$div J = 0 \tag{12}$$

$$J = \sigma E \tag{13}$$

where E and J are electric field and current density respectively, ρ is space charge density, ϵ_0 and ϵ_r are vaccum and insulation relative permittivity respectively. Substitute Eq. (5) into Eq. (4), gives.

$$\nabla . \sigma E = 0 \tag{14}$$

The electric field of inner conductor surface, E can be calculated by means of the following conditions

$$V = \int_{r_1}^{r_2} E(r). \, dr \tag{15}$$

where r_1 , r_2 are radial distances between representative points in the cable matrix as shown in the figure and V is the potential of conductor with respect to ground or outer layer of insulation.

Let V(r) be the potential function defined in the interval $[r_1, r_2]$ (closed and bounded interval). By langrage mean value theorem there exists a point, $r \in [r_1, r_2]$ such that

$$\left(\frac{dV}{dr}\right)_{r} = \frac{V(r_{2}) - V(r_{1})}{r_{2} - r_{1}} = -\frac{V_{a}}{r_{2} - r_{1}}$$
(16)

where $V(r_1)$ is the potential at the conductor-insulation boundary and is taken applied voltage V_a , the potential, $V(r_2)$ at insulation-chassis boundary is taken as zero. Therefore, there is a point in $[r_1, r_2]$, where, the stress is given by

$$E = -\left(\frac{dV}{dr}\right)_r = \frac{V_a}{r_2 - r_1} \tag{17}$$

4. Results and Discussion

Table 1

From the Table 1 considered the physical and electrical parameter of cable taken are $r_1 = 5.5$ mm, $r_2 = 6.7$ mm, $r_3 = 7$ mm, R = 0.196 ohm, $I_L = 180$ A to 720 A, $k_i = 0.34$ W/m⁰ k, $k_s = 1$ W/m⁰k and T_a k. As can be derived from the figure arbitrary fixing of temperature with a boundary condition of the first kind at r_2 would result in an optimistic maximum temperature. The optimal load current can be estimated from these curves by limiting the peak insulation temperature. For example, in the present case, if the peak temperature of the insulation is too limited to around 180° C, then optimal conductor loading can be 720 A taken from the cable catalogue Table 1.

High Voltage Cable Catalogue [13]											
Conductor construction		Insulation	Screen	Outer	Overall	Weight	Current	Max.			
Nominal	No. of	Diameter	thickness	Wire	sheet	Diameter	Approx.	Carrying	Conductor		
Cross-	Strands	of the	(Nom)	Dia	thickness			Capacity	Resistance		
Section		Single		(Nom)	(Nom)				at 20 ⁰ C		
		wire									
		(Max)									
mm ²	Nos.	mm	mm	mm	mm	mm	Kg/km	Amps	Ω/km		
1.5	19	0.33	0.35	0.10	0.45	3.9 ± 0.3	33	47	12.7		
2.5	50	0.26	0.35	0.10	0.55	5.0 ± 0.3	50	65	7.6		
4	56	0.31	0.40	0.10	0.60	5.8 ± 0.3	70	88	4.71		
6	84	0.31	0.40	0.10	0.70	6.6 ± 0.3	100	112	3.14		
10	80	0.41	0.60	0.10	0.85	8.4 ± 0.3	155	165	1.84		
16	126	0.41	0.65	0.15	0.85	9.8 ± 0.3	230	221	1.16		
25	196	0.41	0.65	0.15	0.90	11.2 ± 0.3	330	294	0.743		
35	276	0.41	0.80	0.15	0.90	12.7 ± 0.3	450	370	0.527		
50	396	0.41	0.90	0.20	0.95	14.9 ± 0.3	620	470	0.368		
70	360	0.51	1.00	0.20	1.00	17.0 ± 0.3	850	592	0.259		
95	475	0.51	1.20	0.25	1.30	19.9 ± 0.4	1145	720	0.196		
120	608	0.51	1.40	0.25	1.50	22.6 ± 0.4	1450	840	0.153		
150	756	0.51	1.50	0.25	1.60	24.9 ± 0.5	1778	975	0.122		

The distribution of electric stress, as a function of load current, calculated using Eq. (15), for a typical 1500 V, 720 A, automotive shielded cable is shown in above Figure 3. This graph illustrates how the steady state stress distribution strongly depends on the conductor's current. Additionally, under both normal and high load current conditions, the amount of stress is also higher at the insulation's outer radii. At lower conductor currents, the stress tends to be nearly uniform across the insulation. The analytical results plot for stress, temperature, and conductivity are presented and clearly show variation with radial distances at 1500V DC in Figure 3 and Figure 4. In Figure 3(a) shows the conductivity variations with respect to the electric field distribution in the insulation; the conductivity increases as the field increases. In Figure 3(b) shows the conductivity variations with respect to the temperature distribution in the insulation; the conductivity increases as the field shows the electric field variation with radial distances in the cable. In Figure 4(b) shows the temperature variation, and the temperature is maximum on the surface of the conductor. From all the simulated plots, wherever temperature and stress are maximum, need to provide good insulation to avoid the failure of the insulation.







Fig. 4. (a) Electric field distribution (b)Temperature distribution

5. Conclusions

For EV cables, the main requirement of thermal insulation materials is low thermal conductivity since heat is transferred through insulation material by conduction—the higher thermal conductivity of the insulating materials to lead greater ampacity. The increase in ambient temperature is not conductive to the heat dissipation of the cable. Therefore, the thermal conductivity of the insulating material and the ambient temperature will seriously affect the insulation of the vehicle's cable. After an exhaustive search and critical reviews of the current status of research, the area of cabling networks recognized the need to consider some of the basic tenets of the thermo- electric behavior of cabling networks under different stresses. Deterioration of the cable during normal operation is a major concern. Usually, electric cables receive less maintenance as compared to other electrical components in EVs. This paper created the cable geometry in MATLAB by using analytical equations and simulated the conductivity, temperature, and electrical stress distribution of electric vehicle cabling networks by including the higher vehicle ambient temperatures. The conductivity of the cable insulation increases with the electrical field and temperature. The thermal and electrical stress maximizes at the conductors' surface and gradually reduces the function of the radial distance. This work can open pathways for further study of stress and temperature distribution under transient

electrical and thermal conditions and suitable new insulating materials required for efficient cable and battery insulation design.

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