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Effect of Gravity and Throughflow on Double Diffusive Convection in a Couple Stress Fluid Saturated Porous Media

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ABSTRACT

We have investigated effect of throughflow and gravity modulation on double diffusive convection with couple-stress fluid saturated porous media. Applying the Landau model, we have derived finite amplitude of couple-stress convection in the presence of gravity modulation. The presence of a couple-stress parameter produces both diminishing and enhancing heat mass transfer in the layer. To present the results we have used Mathematica to obtain the Nusselt number and Sherwood numbers numerically. Further, it is shown that, throughflow and modulation of gravity controls double diffusive convection through convective amplitude and alter transport phenomenon.

1. Introduction

The concept of convective instability in a porous medium has been well investigated by Vafai [1,2], Ingham and Pop [3], Pop and Ingham [4], Nield and Bejan [5], and Vadász [6]. In both thermal and engineering sciences, the idea of managing convective instabilities is really an important one. Convection can be controlled by using factors like as temperature, gravity, rotating, and magnetic force modulations. According to Davis [7], depends on the exact tuning of the magnitude and amplitude of the modulation, the dynamics of stabilisation and destabilisation may result in dramatically different changes in behaviour.

If an induced modulation can destabilise an otherwise stationary state, then heat, mass, and momentum transmission are significantly improved. In an induced modulation can stabilise an otherwise unsteady state, so that higher efficiency is obtained in many processing methods. In present study, we take into account the effects of fluid motion (where continuously vibrating porosity can produce time-periodically gravitational modulation with this case). Thus, for related gravity modulation, according to research by Gresho and Sani [8] and Clever *et al.,* [9], gravity modulating influences the entire amount of fluid, and based on the magnitude and frequency of the forcing, it might be either stabilizing or destabilizing effect. Similar research related to gravity modulation was

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carried out by Bhadauria *et al.,* [10], Yang [11], Bhadauria [12], Malashetty and Padmavathi [13], Bhadauria *et al.,* [14], Bhadauria and Kiran [15,16], Bhadauria *et al.,* [17], and Kiran [18,19].

In the concept of convection, the double-diffusive convection refers to the process of coupled heat and mass transport that is enhanced by buoyancy forces. In this situation, the gradient of mass resistance and the gradient of temperature are independent. The double diffusive convection with porous layer is widely investigated with a presence of consistent concentration and temperature gradient by Nield and Bejan [5] and Shivakumara and Sumithra [21] for some of the practical problems, for example, mantle flow in the Earth's crust, seawater flow, in devising an affective technique of discarding of useless products and removal of energy, and in the applications of engineering. Siddheshwar *et al.,* [22] studied the gravitational modulation and temperature influence on double diffusive convection with porous layer. They observed that both modulations are to be performed together to increase or decrease heat/mass transport in system, going to consider weak nonlinear theory on stationary mode. Further, Bhadauria [23] studied the additional internal heating influence and anisotropy of Siddheshwar *et al.,* [22], observed that the internal heating and anisotropy can also be exploited to increase or decrease mass and heat transmission for the system. Furthermore, Malashetty *et al.,* [24] investigated by considering the linear theory which influence of rotations on double-diffusive convection at the convectional onset and also for convection of finite magnitude they consider nonlinear theory. Further, in investigation by Bhadauria and Kiran [25], they determined that, with the proper ranges of modulation parameters, gravity modulation is employed to manage the nonlinear and temperature destabilization of the problems.

Wooding [27], Homsy and Sherwood [28], and Sutton [29], were examined the throughflow effect on the onset of convective in a vertical porous media. Shivakumara [30] and Nield [31] proved that if a little quantity of throughflow which have destabilising effect where we consider if boundaries are various types and a scientific clarification have been presented. Additionally, they observed that the influence of throughflow is not always stabilising and depending on their characteristics of the boundaries. The influence of throughflow and internal heating on the onset of convection with porous media was examined by Khalili and Shivakumara [32]. Though there is no internal heating source, they found that throughflow does not destabilize the system when the similar kinds of boundaries are exit. Moreover, various kinds of hydrodynamic boundary situations, Khalili and Shivakumara [33] explored the throughflow effect with stability of double diffusive convection with in porous medium. When the lowest and the higher boundaries are of the same type, throughflow is seen to be destabilizing even if they are of the same type, and when the boundaries are of various types, it is found to be both stabilizing and destabilizing, depending on its direction. Throughflow with porous medium is managed by the Darcy-Forchheimer equation and the Beavers-Joseph condition was enforced at the intersection of the porous medium and fluid layers were investigated by Khalili and Shivakumara [33]. Hill [34] observed linear and weak nonlinear temperature instabilities of horizontal throughflow with that of fluid-saturated of porous media, where Hill *et al.,* [35] consideration of densities with exponential temperature expanded the result for penetrative flow.

Convective instability for porous layer with an inclination temperature gradient and horizontal throughflow was studied by Brevdo and Ruderman [36,38]. For horizontal porous layer, double-diffusive convection was studied by Shivakumara and Nanjundappa [39] and used the extended Forchheimer - Darcy model. Gudekote *et al.,* [51] investigated the properties of mass transmission are evaluated with slip boundary conditions at the walls, those of heat transfer are studied with convective conditions. Baliga *et al.,* [52] in their findings show that thermal and velocity slips have increasingly negative effects on temperature and pressure rise. According to Vaidya *et al.,* [53] the findings reported here, Newtonian, dilatant and pseudoplastic fluid models' flow amounts are considerably impacted by the existence of variable viscosity along porous parameters, and with slip parameter. Further, the analysis also demonstrates that the presence of the trapping phenomena is enhanced by a rise the value of viscoelastic fluid and porous parameters. Osman *et al.,* [54] they have shown the flow of free convection -across an infinitely inclined plate using magnetohydrodynamics (MHD). The Laplace transform approach has been used to examine the impacts of velocity, temperatures, and concentrations. Although it is commonly believed that the boundaries of porous media are either impermeable or porosity, actually they make excellent conductors of heat and solute concentration. For a single component system, they noticed that small amounts of throughflow in either direction destabilize the mechanism regardless of the nature of the boundaries. Throughflow occurrences causes peak hydrologic and the permeability of the geological medium determines the flow rates. Understanding the research of throughflow under gravity modulation is necessary for this problem. Moreover, in the literature it shows that there is no much more study on temperature instability in which modulation considers for convective instability there in the nonlinear mode of convection, together including vertical throughflow. This paper's goal is to analyze the weak nonlinear stability analysis on porous material with same temperature gradient and solutal concentration gradients in order to maintain vertical throughflow. To derive an analytic expression, we used the non-autonomous Complex Ginzburg-Landau Equation (CGLE), for both Nusselt number and Sherwood number to calculate finite amplitude.

2. Governing Equation

We consider a non-Newtonian fluid-saturated infinitely extended horizontally porous media bounded within two boundaries that are completely free - free at z = 0 and z = d as heated from the bottom. ΔT is fixed variation in temperature and ΔS is concentration all over the porous media. We have used the reference in Cartesian terms with the origin at the bottom as well as z-axis moving upwards in a vertical direction. Its schematic diagram is shown in Figure 1.

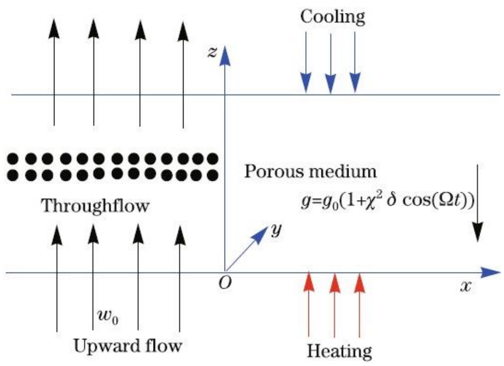


Fig. 1. Sketch of physical problem

In this paper we consider the throughflow in both vertical and horizontal directions. Furthermore, we consider these assumptions are taken under Darcy Brinkman law and the Oberbeck Boussinesq approximations, the flow model's corresponding equations are provided by Bhadauria and Kiran [49] and Kiran and Bhadauria [50].

$$\nabla.\vec{q}=0, \tag{1}$$

$$\frac{\rho_0}{\varepsilon}\frac{\partial\vec{q}}{\partial t}-\rho\vec{g}+\nabla P=-\frac{\mu}{K}\vec{q}+\frac{\mu_c}{K}\nabla^2\vec{q}, \tag{2}$$

$$\gamma \frac{\partial T}{\partial t} + (\vec{q}.\nabla)T = k_T \nabla^2 T, \tag{3}$$

$$\frac{\partial S}{\partial t} + (\vec{q}.\nabla)S = k_s \nabla^2 S, \tag{4}$$

$$\rho = \rho_0 \left[1 - \alpha_T (T - T_0) + \beta_s (S - S_0)\right], \tag{5}$$

where K is permeability, $\vec{q}$ is velocity, μ is viscosity, P is pressure, μ_c is couple stress fluid, k_T is the parameter of thermal expansion, density is ρ, S_0 = concentration, T_0 = temperature for which $\rho = \rho_0$ is the standard density and the heat capacity ratio are equal to γ (here γ is taken unity for simplicity). The following are the temperature and periodic gravitational field produced by externally

$$\vec{g} = g_0 (1 + \delta_2 \cos(\omega t))\hat{k}, \tag{6}$$

$$\begin{aligned} T &= T_0 + \Delta T && \text{at} \quad Z = 0, \\ T &= T_0 && \text{at} \quad Z = d, \\ S &= S_0 + \Delta S && \text{at} \quad Z = 0, \\ S &= S_0 && \text{at} \quad Z = d, \end{aligned} \tag{7}$$

where δ_2 is magnitude of gravity modulation and ω is frequency of modulation and ΔT is difference in temperature whereas ΔS is the solute difference.

Therefore, in this stage, basic state is considered quiescent, with the following quantities

$$\vec{q} = ((0, 0, w_0(z)), \ \rho = \rho_b(z), \ P = P_b(z), \ T = T_b(z,t), \ S = S_b(z,t) \tag{8}$$

Substituting Eq. (8) into Eq. (1) to Eq. (5), obtained following expressions, for basic state of pressure, temperature and concentration

$$\frac{dp_b}{dz} = \frac{\mu}{K} w_0 - \rho_b g, \tag{9}$$

$$w_0 \frac{dT_b}{dz} = k_T \frac{d^2 T_b}{dz^2}, \tag{10}$$

$$w_0 \frac{dS_b}{dz} = k_S \frac{d^2 S_b}{dz^2}, \tag{11}$$

$$\rho_b = \rho_0 \left[1 - \alpha_T (T_b - T_0) + \beta_S (S_b - S_0)\right]. \tag{12}$$

The amplitude solution of the Eq. (10) and Eq. (11) when subjected to thermal boundaries' condition in Eq. (7) is provided by

$$T_b = T_0 + \Delta T \frac{e^{Pez} - e^{Pe}}{1 - e^{Pe}}, \tag{13}$$

$$S_b = S_0 + \Delta S \frac{e^{\left(Pe\Gamma^{-1}\right)z} - e^{\left(Pe\Gamma^{-1}\right)}}{1 - e^{\left(Pe\Gamma^{-1}\right)}}, \tag{14}$$

Perturbations of finite-amplitude are introduced to the solution of the basic state are

$$\left.\begin{aligned} \vec{q} &= q_b + q', \\ P &= P_b + P', \\ \rho &= \rho_b + \rho', \\ T &= T_b + T', \\ S &= S_b + S'. \end{aligned}\right\} \tag{15}$$

Since, we introduce convection stream function which is two-dimensional i.e., ψ as $(u', 0, w') = \left(\frac{\partial \psi}{\partial z}, 0, -\frac{\partial \psi}{\partial x}\right)$ which satisfy Eq. (1) and following are physical factors that are not dimensional which are rescaled by

$$x^* = \frac{x}{d}, y^* = \frac{y}{d}, z^* = \frac{z}{d},\ p' = \frac{\mu k_T}{K} p^*,\ t' = \frac{d^2}{k_T} t^*,\ q' = \frac{k_T}{d} q^*,$$

$T' = \Delta T\, T^*, S' = \Delta S\, S^*,\ \psi = k_T \psi^*,$ and $\Omega = \frac{k_T}{d^2} \Omega *.$

Substituting Eq. (15) in the Eq. (1) to Eq. (5). We obtain the resulting not dimensional governing model while dropping its asterisk by using the dimensionless variables stated above and eliminating the pressure term

$$\left(\nabla^2 - C\nabla^4 + \frac{1}{\Pr}\frac{\partial}{\partial t}\nabla^2\right)\psi = \left(Rs\frac{\partial S}{\partial x} - Ra\frac{\partial T}{\partial x}\right)\left(1 + \chi^2 \delta \cos(\Omega t)\right), \tag{16}$$

$$-\frac{dT_b}{dz}\frac{\partial \psi}{\partial x} + \left(\frac{\partial}{\partial t} - \nabla^2 + Pe\frac{\partial}{\partial z}\right)T = \frac{\partial(\psi, T)}{\partial(x, z)}, \tag{17}$$

$$-\frac{dS_b}{dz}\frac{\partial \psi}{\partial x} + \left(\frac{\partial}{\partial t} - \Gamma\nabla^2 + Pe\Gamma^{-1}\frac{\partial}{\partial z}\right)S = \frac{\partial(\psi, S)}{\partial(x, z)}, \tag{18}$$

where $Pe = \frac{w_0 d^2}{k_T}$ is Peclet number, $\mathrm{Pr}_D = \frac{\phi v d^2}{K k_T}$ is Prandtl Darcy number, $Ra = \frac{\alpha_T g \Delta T d K}{v k_T}$ is thermal Rayleigh number, $g_m = (1+\delta_2 \cos(\omega t))\vec{k}$, $Rs = \frac{\beta_S g \Delta S d K}{v k_S}$; is the solutal Rayleigh number, and $C = \frac{\mu_c}{\mu d^2}$, is the couple stress parameter, Γ is diffusivity ratio, $\Gamma = \frac{k_S}{k_T}$. Considering small change of time t and re-arranging it $\tau = \varepsilon^2 t$, the system convection in a stationary mode going to be discussed. The linear and non-linear system of Eq. (16) to Eq. (18) may be represented in the matrix form as follows

$$\begin{bmatrix} \nabla^2 - C\nabla^4 & R_0 \frac{\partial}{\partial x} & -R_S \frac{\partial}{\partial x} \\ -\frac{dT_b}{dz}\frac{\partial}{\partial x} & \left(\frac{\partial}{\partial \tau} - \nabla^2 + Pe\frac{\partial}{\partial z}\right) & 0 \\ -\frac{dS_b}{dz}\frac{\partial}{\partial x} & 0 & \left(\frac{\partial}{\partial \tau} - \Gamma\nabla^2 + Pe\Gamma^{-1}\frac{\partial}{\partial z}\right) \end{bmatrix} \begin{bmatrix} \psi \\ T \\ S \end{bmatrix} = \begin{bmatrix} \frac{-\varepsilon^2}{\mathrm{Pr}_D}\frac{\partial}{\partial \tau}\nabla^2 \\ \frac{-\varepsilon^2}{\mathrm{Pr}_D}\frac{\partial T}{\partial \tau} + \frac{\partial(\psi, T)}{\partial(x,z)} \\ \frac{-\varepsilon^2}{\mathrm{Pr}_D}\frac{\partial S}{\partial \tau} + \frac{\partial(\psi, S)}{\partial(x,z)} \end{bmatrix} \quad (19)$$

To evaluate the solution of this Eq. (19), the impermeable stress-free heat transfer boundary condition is used by Bhadauria and Kiran [16], Bhadauria *et al.,* [17], and Kiran [18,19].

$\psi = 0$ and $T = 0$ for Z = 0 and Z = 1 (20)

3. Heat Transport and Stationary Instability

For determining the answer and to find nonlinearity with following asymptotic solutions are given in Bhadauria and Kiran [16], Bhadauria *et al.,* [17], and Kiran [18,19] in the above Eq. (19)

$$\left.\begin{aligned} Ra &= R_0 + \varepsilon^2 R_2 + ... \\ \psi &= \varepsilon\psi_1 + \varepsilon^2\psi_2 + \varepsilon^3\psi_3 + ... \\ T &= \varepsilon T_1 + \varepsilon^2 T_2 + \varepsilon^3 T_3 + ... \\ S &= \varepsilon S_1 + \varepsilon^2 S_2 + \varepsilon^3 S_3 + ... \\ \delta &= \delta_0 + \varepsilon\delta_1 + \varepsilon^2\delta_2 + \varepsilon^3\delta_3 + ... \end{aligned}\right\} \quad (21)$$

In the absence of gravitational modulations, R_0 would be the critical Rayleigh number where convection starts. The statement δ is suitable with a basic state solution such that if δ_0 disappears at the lower order (following Bhadauria and Kiran [16]). Further in addition, δ_1 vanishes, the equations that were derived in order $\varepsilon \, and \, \varepsilon^2$ shows that the solution has a singularity. These findings show that gravity modulation effects must be provided at an early stage $\delta = \varepsilon^2 \delta_2$ for enabling consistency [16]. Now system will be studied for different orders of ε.

3.1 First Order System

The system uses the following format at the lowest level

$$\begin{bmatrix} \nabla^2 - C\nabla^4 & R_0 \frac{\partial}{\partial x} & -R_S \frac{\partial}{\partial x} \\ -\frac{dT_b}{dz}\frac{\partial}{\partial x} & \left(\frac{\partial}{\partial \tau} - \nabla^2 + Pe\frac{\partial}{\partial z}\right) & 0 \\ -\frac{dS_b}{dz}\frac{\partial}{\partial x} & 0 & \left(\frac{\partial}{\partial \tau} - \Gamma\nabla^2 + Pe\Gamma^{-1}\frac{\partial}{\partial z}\right) \end{bmatrix} \begin{bmatrix} \psi_1 \\ T_1 \\ S_1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \tag{22}$$

Lowest-order solution according to initial conditions Eq. (20) evaluated as follows

$$\psi_1 = A\sin(k_c x)\sin(\pi z), \tag{23}$$

$$T_1 = -\frac{4k_c\pi^2 A}{c(4\pi^2 + Pe^2)}\cos(k_c x)\sin(\pi z), \tag{24}$$

$$S_1 = -\frac{4k_c\pi^2 A}{c(4\pi^2 + (Pe\Gamma^{-1})^2)}\cos(k_c x)\sin(\pi z), \tag{25}$$

where $c = k_c{}^2 + \pi^2$ is wave number. Onset of stationary convection is quantitatively determined by using value of Critical Rayleigh number with the related wave number and expressions are given by

$$R_0 = \frac{(c - c^2 C)}{4\pi^2 k_c{}^2} + \frac{Rs(4\pi^2 + Pe^2)}{(4\pi^2 + (Pe\Gamma^{-1})^2)}, \tag{26}$$

3.2 System of Second Order

Now, the system adopts the following form

$$\begin{bmatrix} \nabla^2 - C\nabla^4 & R_0 \frac{\partial}{\partial x} & -R_S \frac{\partial}{\partial x} \\ -\frac{dT_b}{dz}\frac{\partial}{\partial x} & \left(\frac{\partial}{\partial \tau} - \nabla^2 + Pe\frac{\partial}{\partial z}\right) & 0 \\ -\frac{dS_b}{dz}\frac{\partial}{\partial x} & 0 & \left(\frac{\partial}{\partial \tau} - \Gamma\nabla^2 + Pe\Gamma^{-1}\frac{\partial}{\partial z}\right) \end{bmatrix} \begin{bmatrix} \psi_2 \\ T_2 \\ S_2 \end{bmatrix} = \begin{bmatrix} R_{21} \\ R_{22} \\ R_{23} \end{bmatrix} \tag{27}$$

The following terms of RHS in present system were given below

$$R_{21} = 0, \tag{28}$$

$$R_{22} = \frac{\partial(\psi, T)}{\partial(x, z)}, \tag{29}$$

$$R_{23} = \frac{\partial(\psi, S)}{\partial(x, z)}, \tag{30}$$

The solutions of second-order subjected to initial conditions as in Eq. (20) of system are given by

$$\psi_2 = 0, \tag{31}$$

$$T_2 = \frac{-2k_c{}^2\pi^3}{c(4\pi^2 + Pe^2)^2} A^2 \sin(2\pi z) + \frac{-Pek_c{}^2\pi^2}{c(4\pi^2 + Pe^2)^2} A^2 \cos(2\pi z), \tag{32}$$

$$S_2 = \frac{-2k_c{}^2\pi^3}{c(4\pi^2 + (Pe\Gamma^{-1})^2)^2} A^2 \sin(2\pi z) + \frac{-Pek_c{}^2\pi^2}{c(4\pi^2 + (Pe\Gamma^{-1})^2)^2} A^2 \cos(2\pi z). \tag{33}$$

For convection in a stationary mode, the horizontally averaged Nusselt number is Nu and Sherwood number Sh, calculated as follows

$$Nu = 1 + \left[\frac{k_c}{2\pi} \int_0^{\frac{2\pi}{k_c}} \frac{\partial T_2}{\partial z} dx \right] \div \left[\frac{k_c}{2\pi} \int_0^{\frac{2\pi}{k_c}} \frac{\partial T_b}{\partial z} dx \right] = 1 + \frac{4\pi^4 k_c{}^2 (e^{Pe} - 1)}{cPe(4\pi^2 + Pe^2)^2} A^2. \tag{34}$$

$$Sh = 1 + \left[\frac{k_c}{2\pi} \int_0^{\frac{2\pi}{k_c}} \frac{\partial S_2}{\partial z} dx \right] \div \left[\frac{k_c}{2\pi} \int_0^{\frac{2\pi}{k_c}} \frac{\partial S_b}{\partial z} dx \right] = 1 + \frac{4\pi^4 k_c{}^2 (e^{Pe\Gamma^{-1}} - 1)}{cPe\Gamma^{-1}(4\pi^2 + (Pe\Gamma^{-1})^2)^2} A^2. \tag{35}$$

In the situation of a porous media which is isotropic in the absence of fluid flow, the following results are found in Eq. (26) Eq. (34) and Eq. (35) are presented by Bhadauria *et al.,* [14], Bhadauria and Kiran [15], and Siddheshwar *et al.,* [22].

3.3 System of Third Order

Now for this point system takes the form as

$$\begin{bmatrix} \nabla^2 - C\nabla^4 & R_0 g_m \frac{\partial}{\partial x} & -R_S g_m \frac{\partial}{\partial x} \\ -\frac{dT_b}{dz}\frac{\partial}{\partial x} & \left(\frac{\partial}{\partial \tau} - \nabla^2 + Pe\frac{\partial}{\partial z}\right) & 0 \\ -\frac{dS_b}{dz}\frac{\partial}{\partial x} & 0 & \left(\frac{\partial}{\partial \tau} - \Gamma\nabla^2 + Pe\Gamma^{-1}\frac{\partial}{\partial z}\right) \end{bmatrix} \begin{bmatrix} \psi_3 \\ T_3 \\ S_3 \end{bmatrix} = \begin{bmatrix} R_{31} \\ R_{32} \\ R_{33} \end{bmatrix} \tag{36}$$

Here terms of RHS were given by

$$R_{31} = -\frac{1}{\Pr}\frac{\partial}{\partial s}\left(\nabla^2\psi_1 - C\nabla^4\psi_1\right) + Rs\delta_2\cos(\omega\tau)\frac{\partial S_1}{\partial x} - \left(R_2 + R_0\delta_2\cos(\omega\tau)\right)\frac{\partial T_1}{\partial x}, \tag{37}$$

$$R_{32} = -\frac{\partial T_1}{\partial \tau} + \frac{\partial T_2}{\partial z}\frac{\partial \psi_1}{\partial x}, \tag{38}$$

$$R_{32} = -\frac{\partial S_1}{\partial \tau} + \frac{\partial S_2}{\partial z}\frac{\partial \psi_1}{\partial x}, \tag{39}$$

Now, putting first-order and second-order solutions are into the following Eq. (37), Eq. (38) and Eq. (39) and easily we get the expressions for R_{31}, R_{32} and R_{33}. Under solvability condition we get Ginzburg-Landau equation for existence of 3rd order system. The Ginzburg-Landau expression is given by

$$Q_1\frac{dA(\tau)}{d\tau} - Q_2(\tau)A(\tau) + Q_3A(\tau)^3 = 0, \tag{40}$$

Where these coefficients are as follows

$$Q_1(\tau) = \left(\frac{c - c^2C}{\Pr_D} + \frac{4R_0\pi^2{k_c}^2}{c^2(4\pi^2 + Pe^2)} - \frac{4R_S\pi^2{k_c}^2}{c^2(4\pi^2 + (Pe\Gamma^{-1})^2)},\right),$$

$$Q_3 = \frac{4R_0\pi^4{k_c}^4}{c^2(4\pi^2 + Pe^2)^2} + \frac{4R_S\pi^4{k_c}^4}{c^2(4\pi^2 + (Pe\Gamma^{-1})^2)^2},$$

$$Q_2(\tau)=\left(\frac{4\pi^2 k_c{}^2 R_0}{c^2(4\pi^2+Pe^2)}\left[1+\delta_2\cos(\Omega\tau)\right]-\frac{4\pi^2 k_c{}^2 R_S}{c^2(4\pi^2+(Pe\Gamma^{-1})^2)}\left[\delta_2\cos(\Omega\tau)\right]\right).$$

Eq. (40) is also known as Bernoulli equation, because of its non-autonomous structure, finding an analytical solution is very difficult in the presence of modulation. As a result, it was numerically solved by using Mathematica 12.0 built-in function ND Solution, when necessary initial condition at $A_0=a_0$ where a_0 is defined as present initial convection magnitude. Its analytic solution of Eq. (40) for such an un-modulated case is as follows

$$A=\frac{1}{\sqrt{\left(\frac{Q_3}{Q_2}+C_1 e^{\left[-\frac{2Q_2}{Q_1}\tau\right]}\right)}}, \tag{41}$$

where Q_1, Q_3 as same in Eq. (40),

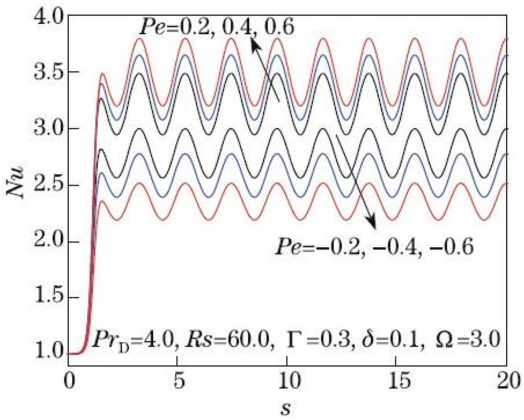
$Q_2(\tau)=\left(\frac{4R_2\pi^2 k_c{}^2}{c^2(4\pi^2+Pe^2)}\right)+\left(\frac{4R_2\pi^2 k_c{}^2}{c^2(4\pi^2+(Pe\Gamma^{-1})^2)}\right)$, and C_1 is an integration constant which appears in Eq. (41), which can be obtained by adopting appropriate boundary conditions.

4. Results and Discussions

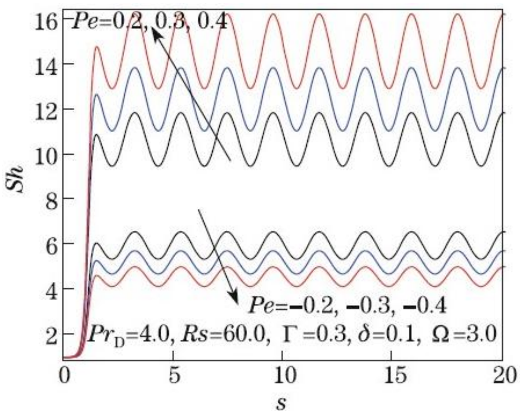
A cellular preservation of constant convection has been created for a horizontal porous material is consistently heated both from below and above it is cooled. This occurs when the Rayleigh number exceeds a critical point. In order to calculate the amplitude of convection, one must analyse the weak-nonlinear interaction of fluid motion in relation to temperature and concentration. Here, a technique is described for measuring frequency of convection's and examining heat and mass transmission. Further, an investigation is done into the cumulative effects of vertical throughflow and gravitational modulation on heat exchange in an infinite horizontally couple stress fluid with a saturated porous media. Throughflow allows temperature gradient and solutal gradient to change with linear to nonlinear for reference to the height of the porosity. By using CGLE, a nonlinear phenomenon stability examination was performed to observe the effects of gravitational modulation and vertical throughflow with heat and mass transports of system. The Dacry-Brinkman model is considered, where it is supposed that the porous medium is densely packed. It has been observed that the values of Pe (Peclet number), Pr_D (Prandtl number) and diffusivity ratio Γ must be considering to satisfy and the values of $\delta\, and\, \Omega$ are mentioned to be small. To this maximum mass and heat transfer is possible, for lowest values of amplitude and frequencies. For stabilizations or destabilization in system, a small amount of throughflow is required. For this, the values for Pe are taken around 0.1. The expressions given in Eq. (34) and Eq. (35) are the numerical obtained results for $Nu\, and\, Sh$ with solving Eq. (39) and were presented in Figure 2 to Figure 8 which illustrate how each parameter effects with heat and mass transfers, wherein the plots of Nu versus the slow time s and Sh versus the slow time s are presented. From the figures, it can be examined that, the values of Nusselt number and Sherwood number starts with one and remains constant for quite some time which shows the convection state, and on further enhancing become constant. Therefore, it obtained steady state. The Critical Rayleigh number (Eq. (26)) enhances with values of Pe, and therefore from the throughflow direction which is independent. Therefore, the effective length of scale is lower than that of porous layer's thickness. As a result, the Rayleigh number as substantially smaller than its actual value. Therefore, when the throughflow strength increases, for the onset of convection it required a large value of the Rayleigh number, these results were observed by Khalili and Shivakumara [33] for the free – free boundaries. In this case, the outcomes were opposite which are obtained by Nield [31] a fluid layer for the small amount of throughflow.

According to Reza and Gupta [42] when the throughflow velocity increases, a temperature of boundary which forms for one of the boundaries. As a result, the stratified layer's effective thickness is reduced while maintaining its characteristic temperature constant. Therefore, it is naturally follows that the critical value of Rayleigh number increases with the increasing value of Pe. The maximum temperature can be found where the vertical velocity is most perturbed, which increases the amount of energy supplied for destabilization. It is shown that a similar situation occurs in solutal concentration, which shows conduction state of concentration provided in Eq. (14) is nonlinear.

To know about their effect with stability of the system, the basic state temperature and concentration distribution is calculated for given values of Pe and Γ which is graphically displayed in Figure 2 and Figure 3. With an increase of Peclet number Pe, the basic state distributions for throughflow become nonlinear and separate from one another. In comparison to the temperature with decreases in Γ. It is observed that the values of $Nu\, and\, Sh$ start with one and showing the system in conduction state, while, increasing with the time s, and then become oscillatory. According to Figure 2(a) and Figure 2(b), the effect of upward throughflow ($Pe > 0$) increases, whereas the effect of downward throughflow ($Pe < 0$) decreases the heat and mass transmission in the given system. Further, in Figure 3(a) and Figure 3(b) shows that the influence of diffusivity ratio Γ is to decreases heat and mass transfer in the model. These results are as same as the results obtained by Bhadauria [23] and Bhadauria and Kiran [25]. In Figure 5(a) and Figure 5(b) the effect of the solutal Rayleigh number Rs is to increase heat and mass transfer.

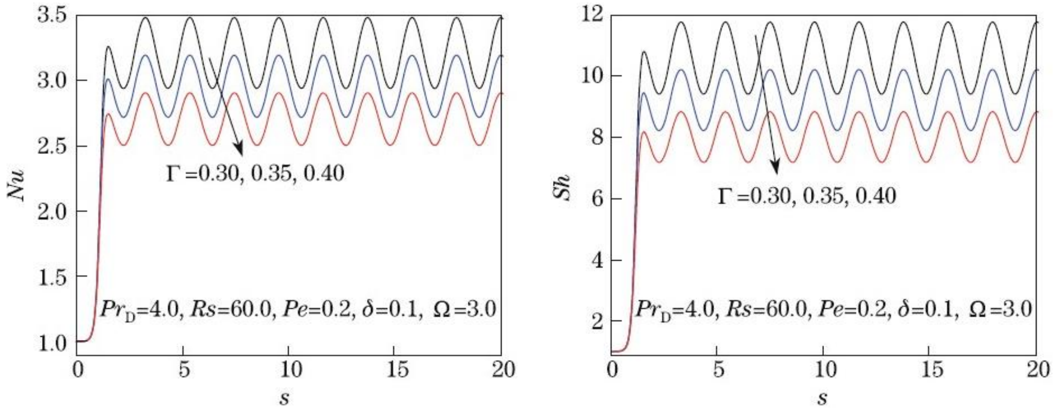


(a)



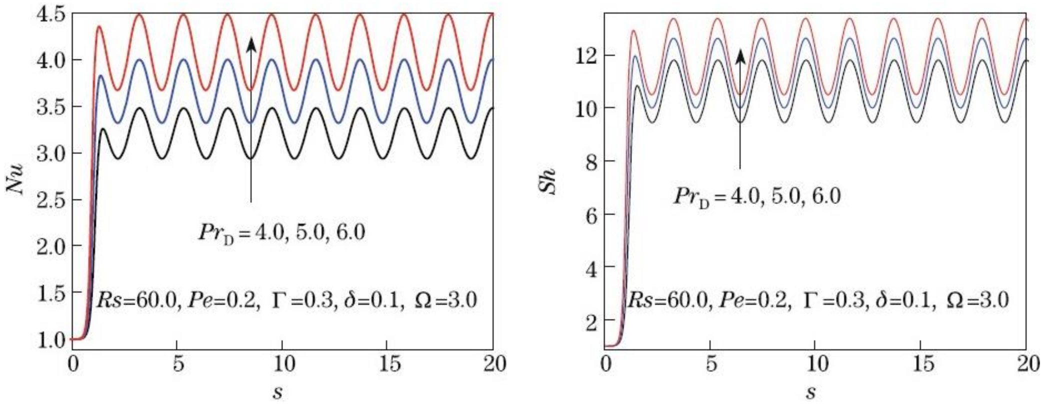
(b)

Fig. 2. Influence of Peclet number on heat and mass transfer; (a) Nu, (b) Sh



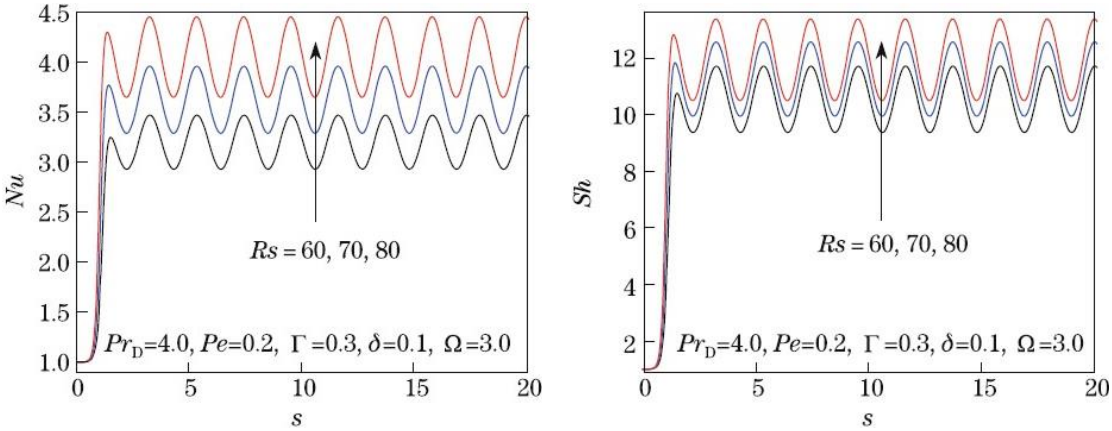
(a) (b)

Fig. 3. Influence of diffusivity ratio on heat and mass transfer; (a) Nu, (b) Sh



(a) (b)

Fig. 4. Influence of Vadasz number Pr_D on heat and mass transfer; (a) Nu, (b) Sh

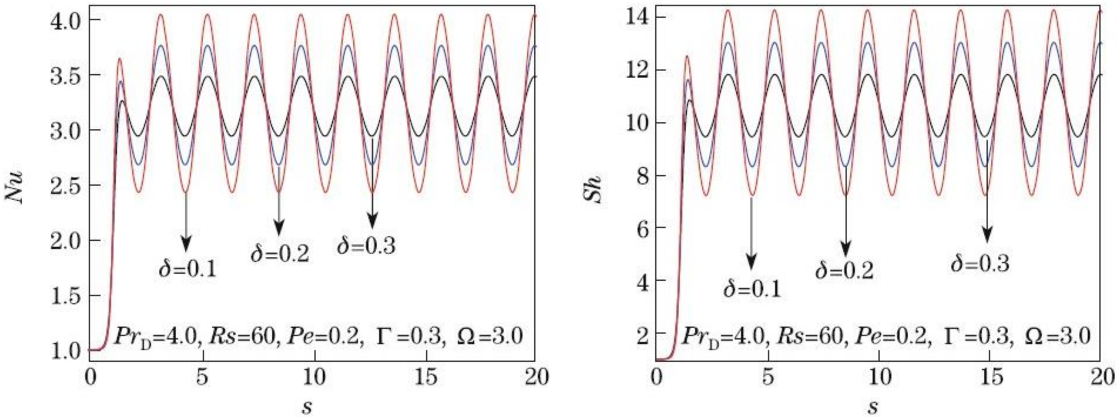


(a) (b)

Fig. 5. Influence of solutal Rayleigh number on heat and mass transfer; (a) Nu, (b) Sh

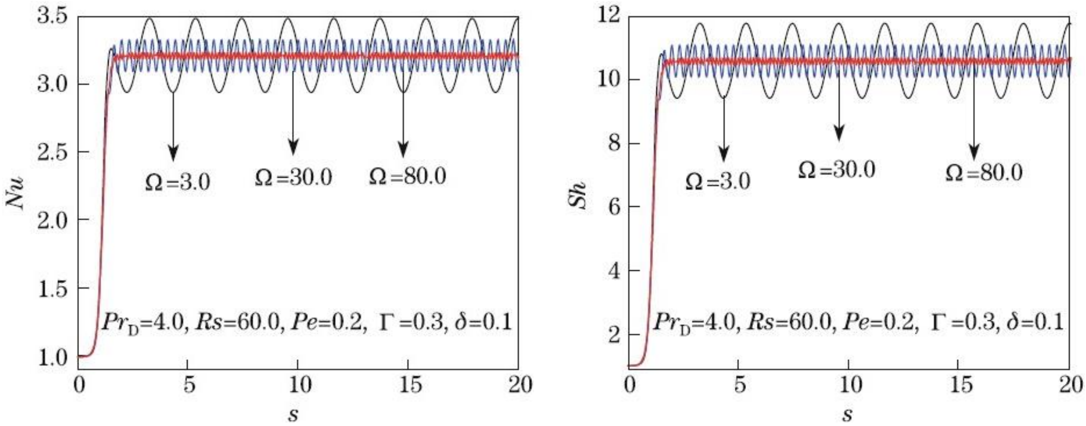
Furthermore, in Figure 6, the consequence of the gravity modulation's amplitude is to increase the magnitudes of Nu and Sh, increasing the heat and mass transfers. Additionally, it can be shown from Figure 7 that the magnitudes of $Nu\,(Nusslt\ number)\ and\ Sh\,(Sherwood\ number)$, decrease as the value of Ω, increases, and the frequency of the modulating has the effect of reducing the mass and heat transfer. Gravitational modulation's influence on thermal instability fully disappears at higher frequencies. These results support those of the linear theory researchers Malashetty and

Padmavathi [13] and Venezian [48], they found that the correction to the critical Rayleigh number caused by thermal and gravitational modulations almost disappears with higher frequencies.



(a) (b)

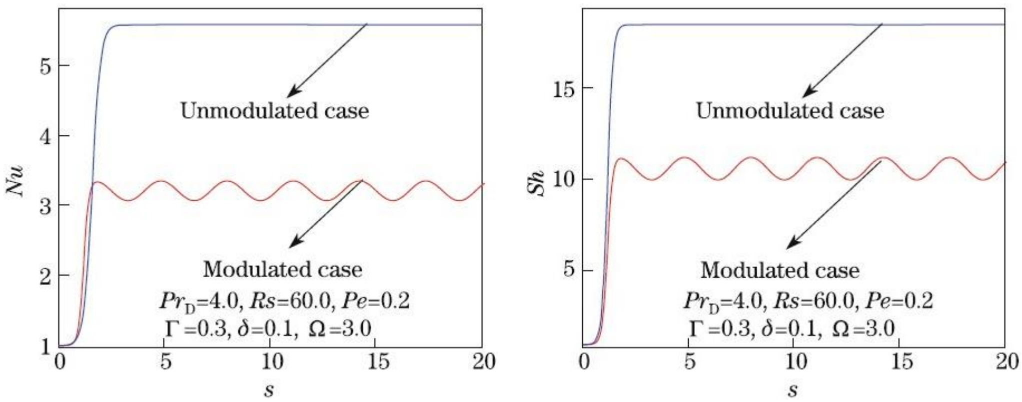
Fig. 6. Influence of amplitude modulation on heat and mass transfer; (a) Nu, (b) Sh



(a) (b)

Fig. 7. Influence of frequency of modulation on heat and mass transfer; (a) Nu, (b) Sh

In Figure 8 which has been compared the analytical and numerical solutions for the both unmodulated case and modulated case, it is seen in the graph that the Nusselt and Sherwood numbers for the unmodulated case are larger than those for the modulated case. In this study, the modulating flows transport less heat than while in a compression with unmodulated flows.



(a) (b)

Fig. 8. Comparison between modulated and unmodulated system; (a) Nu, (b) Sh

5. Conclusions

A weak nonlinear stability analysis is conducted in order to determine the Ginzburg-Landau equation by combining the effects of through-flow and gravity modulation on double diffusive convection with porous media.

i. Rate of Heat and mass transfer are increased by upward throughflow (Pe > 0), and decreased by downward throughflow (Pe 0). As a result, throughflow has dual characteristics of heat and mass transfer.
ii. Mass and heat transfer rate can be enhanced by increasing amplitude δ of modulation.
iii. As its value increases the frequency Ω of modulation decreases heat and mass transfer.
iv. Heat and mass transfer in a system can be effectively controlled through throughflow and gravity modulation.
v. There is less heat and mass transport by gravity-modulated systems compared to their corresponding unmodulated flows when there is nonlinear fluid flow.
vi. There is a similarity between gravity modulated flows and lower boundary temperature modulated flows.

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