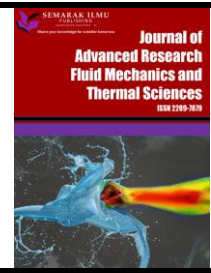




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# Carbon Nanotubes (CNTs) Nanofluids Flow and Heat Transfer under MHD Effect over a Moving Surface

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### ABSTRACT

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The 2D steady flow of CNTs nanofluids and heat transfer over the moving plate through a uniform free stream under the effect of magnetohydrodynamics (MHD) is studied. The movement of plate is presumed in the opposite or same direction. A mathematical modelling that is governed by a system of partial differential equations (PDEs) subjected to boundary conditions is transformed into a system of nonlinear ordinary equations (ODEs). An attempt at finding the expected outcomes is successfully executed by solving ODEs system using MATLAB bvp4c solver. The effect of various parameters such as magnetic field, CNTs volume fraction and moving parameter on velocity and temperature profile, skin friction, Nusselt number and heat transfer rate are investigated numerically. The results are presented using tabular and graphical illustration. From the results, the increment of magnetic field into the flow will increase in both the skin friction and the heat transfer coefficient. Besides that, non-unique solutions are obtained when the plate moves in the range of  $\lambda_c \leq \lambda < 0$  and  $M < 0.1$ . For comparing the performance of base fluid, kerosene works better than water as a base liquid. We are also showing that SWCNTs is more effective both in the skin friction and the heat transfer coefficient compare to MWCNTs.

## 1. Introduction

Heat transfer has been an important topic debating in thermal engineering. Demand for enhanced the thermal system has increased significantly over two decades and has generated many ideas including to use nanofluids. Numerous studies have been conducted in order to improve the performance of heat transfer on various applications using nanofluid from different views and models [1–6]. The term of nanofluids refers to the combination of any nanoscale materials in a base fluid [7]. From the previous literature, Aziz *et al.*, [8] found that 87% of nanofluids preparation used water as the base fluid. Nanofluids are employed in various thermal applications such as in the automotive industry [9]. A promising of providing high thermal conductivity is key to the synthesising

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of high-impact nanofluids. Carbon materials (such as carbon nanotubes (CNTs)), metal materials (such as copper (Cu)), semiconductor material (such as gallium nitride (GaN)) and polymer materials (such as polylactic acid (PLA)) have become the most favourable selection for preparing nanofluids [10]. Among them, CNTs nanofluids are considered as the best nanofluids as they are having the highest thermal conductivity compare to other nanoparticles [11–12]. Jiang *et al.*, [13] suggested that CNTs became one of the effective materials providing higher thermal conductivity of nanofluids compared to other nanoparticles.

CNTs consist of carbon nanoparticles with common range in diameter from 0.5nm to 1.5nm and above 100nm for single-walled carbon nanotubes (SWCNTs) and multi-walled carbon nanotubes (MWCNTs), respectively. Basically, CNTs are constructed and rolled into a cylindrical shape by using one-dimensional graphene structures [12]. To produce CNTs nanofluids, CNTs are immersed in a base fluid such as water, oil and ethylene glycol. The usage of CNTs is to meet the demand for various applications including bio sensors (mostly on electrochemical) [14–17], medical applications [18] and nanogenerators technology [19]. Increasing demand for CNTs with forecasting to elevate around 2,215.5 Million by 2032 [20] is due to guarantee for providing high quantity of thermal conductivity, ranging from 0.1 W/mK to 6600 W/mK [21] and high current carrying capability [22]. The thermal conductivity acts a vital role in enhancing on the heat transfer coefficient by increasing the rate of heat transfer [23]. Besides that, CNTs have worked perfectly in nanotechnology by improving the performance of heat transfer with a great thermal and optical properties [24]. An addition, the application of nanomaterials has been involving with the low cost production and making Ibrahim *et al.*, [25] classified it as an ideal working fluid. Ibrahim *et al.*, [25] concluded that nanofluids has been working at the low cost production but having a supreme properties of specific heat and lower viscosity compare to the conventional fluids. These reasons have sufficiently caused many researchers such as Danish *et al.*, [22] to nominate CNTs to be the best future transmission network of power system. CNTs have been getting attention from many researchers for over three decades since the first discovery of CNTs by Iijima in 1991 [26] for MWCNTs and followed by SWCNTs in 1993 [27]. Historically, Ferrier and Honeychurch [28] stated that the earliest development of CNTs was initiated by Radushkevich and Lukyanovich and in 1976 by Oberlin *et al.*, Based on [28], six methods can be performed for synthesising CNTs which are arc discharge, laser ablation, chemical vapour deposition, plasma enhanced, liquid electrolysis, and controlled flame environments.

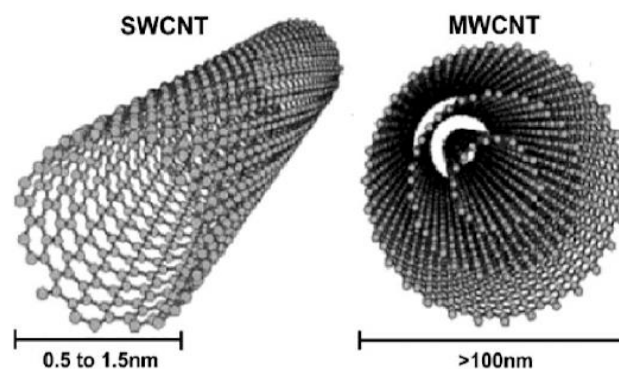


Fig. 1. Structure SWCNT and MWCNT [29]

The investigation of boundary layer flow and heat transfer of CNTs have been conducted by many researchers using different physical parameters and surfaces. Khan *et al.*, [30] was believed to be the first researcher discovered the fluid flow and heat transfer of CNTs with presence of Navier slip effect and invariable heat flux over the fixed plate. By taking into account of different parameters such as skin friction and Nusselt numbers they [30] found that a positively agreement of skin friction and

Nusselt numbers with the available data. For the fluid flow over a moving surface, Norzawary *et al.*, [31] became one of the first researchers to explore and extend the study of boundary layer flow of CNTs passing this surface. To govern the model, Norzawary *et al.*, [31] inspired the work from Bachok *et al.*, [32]. From this analysis, a dual solution was found when the plate moved reversely to the free stream. In the term of skin friction coefficient and local Nusselt numbers, they also found that MWCNTs less efficient than SWCNTs.

Magnetohydrodynamic (MHD) can represent as magneto-fluid dynamics where it can be used to investigate on electrically and conductivity accompanying fluids in magnetic field [33]. Magnetic field will create a positive response to the local skin friction where the local skin friction coefficient will rise significantly in fluid. Based on the experiment work completed by Masaaki *et al.*, [34], the magnetic field will give a positive impact to the coefficient of heat transfer. The heat transfer coefficient will increase when the magnetic field rise in fluid. Ferdows *et al.*, [35] shared the same view as Masaaki *et al.*, by concluding that the magnetic field can give the positive impact to the rate of heat transfer and temperature.

The boundary layer flow and heat transfer of CNTs nanofluids with the presence of MHD has been reviewed experimentally and theoretically by many investigators such as [36-38]. Mittal *et al.*, studied MHD flow with Cu-water nanofluid. Besides that, the numerical analysis on CNTs nanofluids flow in the existence of magnetic field have studied for various surfaces such as [39-42] for vertical plate, moving disk, stretching/shrinking sheet and thin needle, respectively. The boundary layer flow of CNTs nanofluids over the moving plate was discovered by Anuar *et al.*, [43] by focussing on the slip effect. From this work, they found that two major findings. First, they noticed that non-unique solutions appeared when the plate moved reversely to the free stream. Second, MWCNTs were less competent compared to SWCNTs both in the skin friction and heat transfer performance. Norzawary *et al.*, [31] also concentrated on CNTs flow past the moving plate by comparing the performance of CNTs using different base fluids. However, to date, we have found that no research has been conducted on exploring the boundary layer flow of CNTs past a moving surface in the presence of MHD. This gap allows us to extend the work from [32] and [31] to study the effect of MHD in CNTs nanofluids flow past the moving surface. We will propose to insert the influence of MHD in the momentum equation. By taking this effect, we will expect to have significant results both on the skin friction and heat transfer coefficient.

## 2. Mathematical Formulation

This model deals with two dimensional (2D) steady flow past a moving plate in the presence of magnetic field. This flow is categorised as incompressible and laminar flow that passes SWCNTs and MWCNTs. Water and kerosene are selected to be the base fluid. The decision of selecting different base fluids is made based on the experiment performed by Yilmaz *et al.*, [44] where they observed that the base fluids can affect the performance of heat transfer process. The thermal properties of CNTs and based liquids are depicted in Table 1 and Table 2.

**Table 1**  
 Thermophysical properties of CNTs and base fluids [30]

Physical Properties	Nanoparticles		Base Fluid	
	SWCNTs	MWCNTs	Water	Kerosene
$\rho$ (kg/m <sup>3</sup> )	2,600	1,600	997	783
$C_p$ (J/kg K)	425	796	4,179	2,090
$k$ (W/m K)	6,600	3,000	0.613	0.145

**Table 2**  
 Thermo-physical properties of CNTs Nanofluids [45]

Properties	CNTs Nanofluids
Dynamic viscosity, $\mu_{nf}$	$\mu_{nf} = \frac{\mu_f}{(1 - \phi)^{2.5}}$
Density, $\rho_{nf}$	$\rho_{nf} = (1 - \phi)\rho_f + \phi\rho_{CNT}$
Effective kinematic viscosity, $\nu_{nf}$	$\nu_{nf} = \frac{\mu_{nf}}{\rho_{nf}}$
Thermal conductivity, $k_{nf}$	$k_{nf} = k_f \left( \frac{1 - \phi + 2\phi \frac{k_{CNT}}{k_{CNT} - k_f} \ln \frac{k_{CNT} + k_f}{2k_f}}{1 - \phi + 2\phi \frac{k_f}{k_{CNT} - k_f} \ln \frac{k_{CNT} + k_f}{2k_f}} \right)$
Heat capacity, $(\rho C_p)_{nf}$	$(\rho C_p)_{nf} = (1 - \phi)(\rho C_p)_f + \phi(\rho C_p)_{CNT}$
Thermal diffusivity, $\alpha_{nf}$	$\alpha_{nf} = \frac{k_{nf}}{(\rho C_p)_{nf}}$

By extending and modifying the works from [23] and [31], the boundary layer equations for this model can be constructed as

$$\frac{\partial(u)}{\partial x} + \frac{\partial(v)}{\partial y} = 0 \tag{1}$$

$$u \frac{\partial(u)}{\partial x} + v \frac{\partial(v)}{\partial y} = \frac{\mu_{nf}}{\rho_{nf}} \frac{\partial^2(u)}{\partial y^2} + \frac{\sigma}{\rho_{nf}} B^2 (U - u), \tag{2}$$

$$u \frac{\partial(T)}{\partial x} + v \frac{\partial(T)}{\partial y} = \alpha_{nf} \frac{\partial^2(T)}{\partial y^2}. \tag{3}$$

This model is incorporated with the boundary conditions where these conditions are reasonably important for governing our model and completing our computational process wisely [46]. In addition, Bursten [47] believed that specifying boundary conditions can limit the scope of a model to a limited subclass of systems to which a set of governing equations applied. The complete boundary conditions can be written as follows

$$u = U_w = \lambda U, v = 0, T = T_w \text{ at } y = 0, u \rightarrow U_\infty, T \rightarrow T_\infty, \text{ as } y \rightarrow \infty. \tag{4}$$

The velocity components are represented by  $u$  and  $v$  where these components move along the coordinate cartesian  $(x, y)$  directions. The  $x$ -axis is determined along the plate and the  $y$ -axis is normal to the  $x$ -axis. The term of  $\mu_{nf}/\rho_{nf}$  can be set as  $\nu_{nf} = \mu_{nf}/\rho_{nf}$ , where  $\nu_{nf}$  is the effective kinematic viscosity. The uniform velocity of free stream flow  $U$  can be written as  $U = U_w + U_\infty$  and  $T$  represents the temperature of nanofluids and  $B$  is the magnetic field applied to the fluid flow.  $T_w$  defines as the constant value of temperature and  $T_\infty$  is assigned as the temperature in the ambient fluid. The term of  $\lambda$  and  $\sigma$  is the velocity parameter and the Stefan-Boltzmann constant, respectively. In this model, the subscripts  $f$ ,  $nf$  and  $CNT$  can be referred as fluid, nanofluid and carbon nanotube,

respectively. The thermal correlation of nanofluids for viscosity,  $\mu_{nf}$ , density,  $\rho_{nf}$  and thermal diffusivity,  $\alpha_{nf}$  can be written as follows

$$\mu_{nf} = \frac{\mu_f}{(1 - \phi)^{(2.5)}}, \rho_{nf} = (1 - \phi)\rho_f + \phi\rho_{CNT}, \alpha_{nf} = \frac{k_{nf}}{(\rho C_p)_{nf}}, \quad (5)$$

where  $\phi$  is the nanoparticle volume fraction,  $(\rho C_p)_{nf}$  is the heat capacity of nanofluids and  $k_{nf}$  is the thermal conductivity of nanofluids. From [48],  $(\rho C_p)_{nf}$  and  $k_{nf}$  can be illustrated as

$$k_{nf} = k_f \left( \frac{1 - \phi + 2\phi \frac{k_{CNT}}{k_{CNT} - k_f} \ln \frac{k_{CNT} + k_f}{2k_f}}{1 - \phi + 2\phi \frac{k_f}{k_{CNT} - k_f} \ln \frac{k_{CNT} + k_f}{2k_f}} \right), (\rho C_p)_{nf} = (1 - \phi)(\rho C_p)_f + \phi(\rho C_p)_{CNT}, \quad (6)$$

where  $k_f$  and  $k_{CNT}$  are the thermal conductivity of fluids and carbon nanotubes, respectively, and  $(\rho C_p)_f$  and  $(\rho C_p)_{CNT}$  are the heat capacity of fluids and carbon nanotubes, respectively. The term of the kinematic viscosity of nanofluids,  $k_{nf}$  in Eq. (6) can be arranged as

$$\frac{k_{nf}}{k_f} = \left( \frac{1 - \phi + 2\phi \frac{k_{CNT}}{k_{CNT} - k_f} \ln \frac{k_{CNT} + k_f}{2k_f}}{1 - \phi + 2\phi \frac{k_f}{k_{CNT} - k_f} \ln \frac{k_{CNT} + k_f}{2k_f}} \right)$$

where  $\frac{k_{nf}}{k_f}$  is describing the effect of space distribution of CNTs on the thermal conductivity [48].

To reduce the partial differential equations (PDEs) Eq. (1), Eq. (2) and Eq. (3) to the dimensionless ordinary equations (ODEs), we will use the similarity variables [43] as follows

$$\eta = y \sqrt{\frac{U}{v_f x}}, \psi = \sqrt{v_f x U} f(\eta), \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty} \quad (7)$$

where  $\psi$  represents the stream function with  $u = \frac{\partial \psi}{\partial y}$  and  $v = -\frac{\partial \psi}{\partial x}$ . The non-dimension form of ODEs will help us to reduce the number and complexity of variables and decrease the difficulty of defining the governing equations of the phenomena investigated [49].

Utilizing the similarity variables (from Eq. (7)) to transform PDEs (Eq. (2) and Eq. (3)) into ODEs with respect to  $\eta$  in the following form

$$\frac{1}{(1 - \phi)^{(2.5)} + \left(1 - \phi + \frac{\phi \rho_{CNT}}{\rho_f}\right)} f'''' + \frac{1}{2} f f'' + M(1 - f') = 0 \quad (8)$$

$$\frac{1}{Pr} \frac{1}{\left[1 - \phi + \frac{\phi(\rho C_p)_{CNT}}{(\rho C_p)_f}\right]} \theta'' + \frac{1}{2} f \theta' = 0 \quad (9)$$

where  $M = \frac{\sigma B^2}{U \rho_{nf}}$  defines as the magnetic field and  $Pr = \frac{\nu_f}{\alpha_f}$  refers the Prandtl number and  $\nu_f$  is the effective kinematic viscosity of fluid, subject to the transformation boundary conditions as follows

$$f'(0) = \lambda, f(0) = 0, \theta(0) = 1, f'(\eta) \rightarrow (1 - \lambda), \theta(\eta) \rightarrow 0, \text{ as } \eta \rightarrow \infty \quad (10)$$

As our main goal is to study the influence of different parameters on the performance of heat transfer such as velocity and temperature then we define the skin friction coefficient,  $C_f$ , and the local Nusselt number,  $Nu_x$ , as follows

$$C_f = \frac{\tau_w}{\rho_f U^2}, Nu_x = \frac{x}{k_f (T_w - T_\infty)} \quad (11)$$

where  $\tau_w$  and  $q_w$  is the surface shear stress and the heat flux, respectively. The term of  $\tau_w$  can be expressed as

$$\tau_w = \mu_{nf} \left( \frac{\partial u}{\partial y} \right) \text{ at } y = 0, q_w = -k_{nf} \left( \frac{\partial T}{\partial y} \right) \text{ at } y = 0 \quad (12)$$

From Eq. (7), we will set

$$\frac{\partial T}{\partial y} = \sqrt{\frac{U}{\nu_f x}} \theta' \text{ at } y = 0 \quad (13)$$

Substituting Eq. (12) and Eq. (13) into Eq. (11) resulting

$$C_f Re_x^{1/2} = \frac{1}{(1 - \phi)^{(2.5)}} f''(0), Nu_x Re_x^{-1/2} = - \left( \frac{k_{nf}}{k_f} \right) \theta'(0) \quad (14)$$

where  $Re_x = \frac{U_x}{\nu_f}$  is the local Reynold number.

### 3. Result and Discussion

Analytical solutions for ODEs system in Eq. (8) and Eq. (9) through a set of initial values and boundary conditions as mentioned in Eq. (10) is solved numerically using bvp4c MATLAB package. Before we start to investigate the effect of various parameter such as  $M$ ,  $\phi$  and  $\lambda$  on velocity and temperature profile, skin friction and the rate of heat transfer, we perform a comparison of previous solutions made by Bachok *et al.*, [32] without taking the influence of  $M$ , where  $M = 0$ . The value of  $\phi$  are set at  $\phi = 0, 0.1, 0.2$  while the value of  $\lambda$  varies in the range of  $-1 < \lambda < 1$ . The comparison data for selected values of  $\phi$  and  $\lambda$  using SWCNT/water for values of the reduced skin friction coefficient  $f''(0)$  are depicted in Table 3. From Table 3, we can see the current results seem to be perfectly similar with the previous literature [32].

**Table 3**

Variation values of  $f''(0)$  using different values of  $\phi$  and  $\lambda$  for SWCNTs/water under zero magnetic field ( $M = 0$ )

$\phi$	$\lambda$	Bachok <i>et al.</i> , [32]		Present Result		
		First Solution	Second Solution	First Solution	Second Solution	
0	-0.5	0.3979	0.1710	0.3979	0.1710	
	-0.4	0.4357	0.0834	0.4357	0.0823	
	-0.3	0.4339	0.0367	0.4339	0.0367	
	-0.2	0.4124	0.0114	0.4124	0.0114	
	-0.1	0.3774	0.0010	0.3774		
	0.0	0.3321		0.3321		
	0.5	0.0000		0.0000		
	1.0	-0.4438		-0.4438		
	1.0	-0.5			0.3735	0.1615
		-0.4			0.4114	0.0787
-0.3				0.4098	0.0347	
-0.2				0.3895		
-0.1				0.3564		
0.0				0.3136		
0.5				0.0000		
1.0				-0.4191		
0.2		-0.5			0.3460	0.1488
		-0.4			0.3789	0.0725
	-0.3			0.3774	0.0092	
	-0.2			0.3587		
	-0.1			0.3282		
	0.0			0.2888		
	0.5			0.0000		
	1.0			-0.3861		

After meeting our expectation on numerical results of CNTs nanofluids flow without the existence of magnetic field, then we continue to explore more on the flow under magnetic field effect and other parameters such as CNTs volume fraction,  $\phi$ , and moving parameter,  $\lambda$ , graphically. First, we observe the variation values of the reduced skin friction,  $f''(0)$  and the reduced on the heat transfer coefficient,  $-\theta'(0)$  of SWCNT/water with different values of magnetic field,  $M$  and moving parameter,  $\lambda$ . The value of  $M = 0$  indicates that the flow is without the influence of magnetic field in the fluid flow. We also assume that the plate is moving to the free stream in opposite ( $\lambda < 0$ ) or same ( $\lambda > 0$ ) direction. These phenomena can be illustrated in Figure 2 and 3 where the value of  $\phi$  and Pr are fixed at 0.1 and 6.2, respectively. From these figures, we can see the range of single solutions can be detected when  $\lambda > 0$  and  $M > 0.1$  while for  $\lambda_c \leq \lambda < 0$  and  $M < 0.1$  the non-unique solutions (dual solutions) will emerge. However, we discover that no solution produced in the range of  $\lambda_c > \lambda$ . The term of  $\lambda_c$  can be described as the critical value of moving parameter that is essentially for exposing either solutions are dual solutions or no solution exists. Besides that, the rise in  $M$  will increase the value of  $f''(0)$  and  $-\theta'(0)$  and slow down the process of separation of thermal boundary layer.

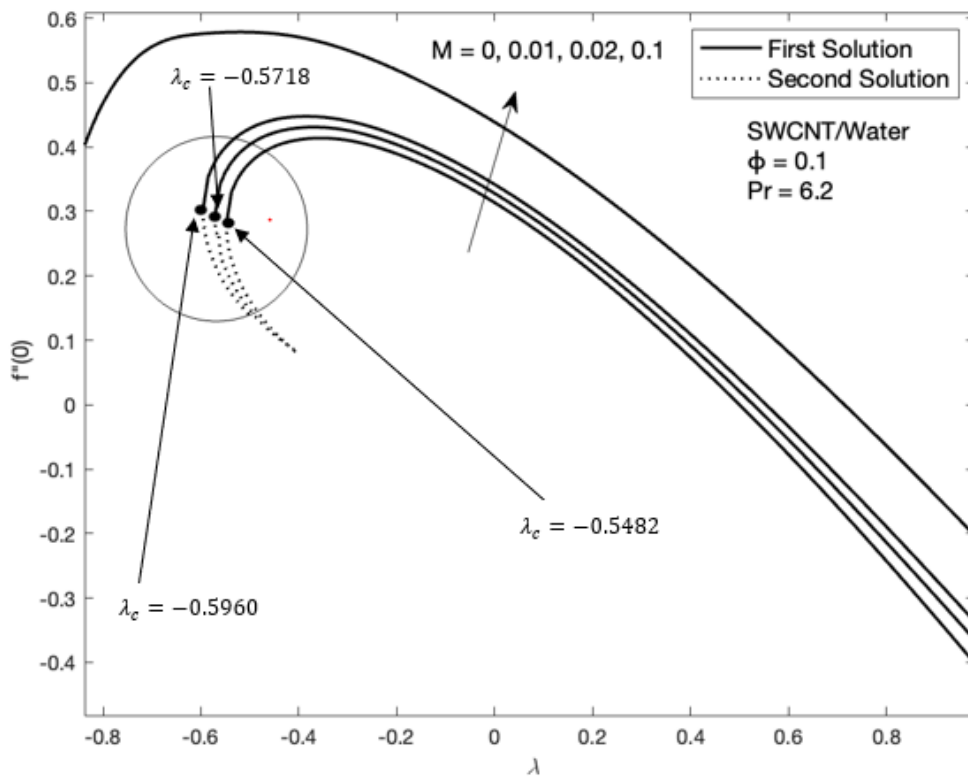


Fig. 2. Variation of  $f''(0)$  with different  $M$  and  $\lambda$  for SWCNT/water

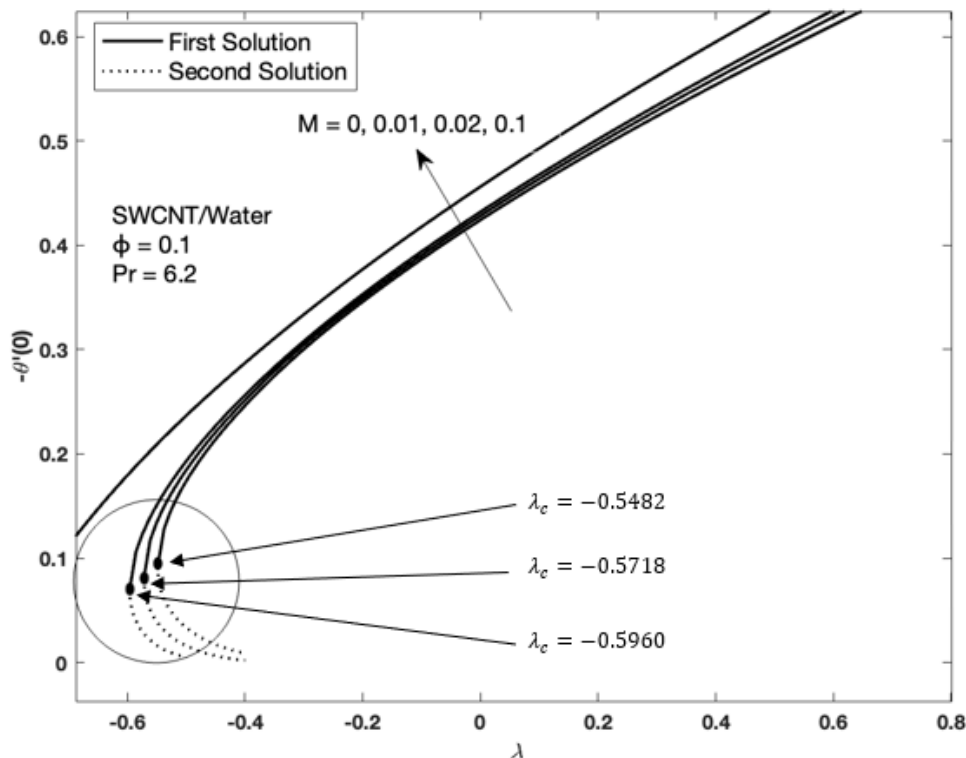
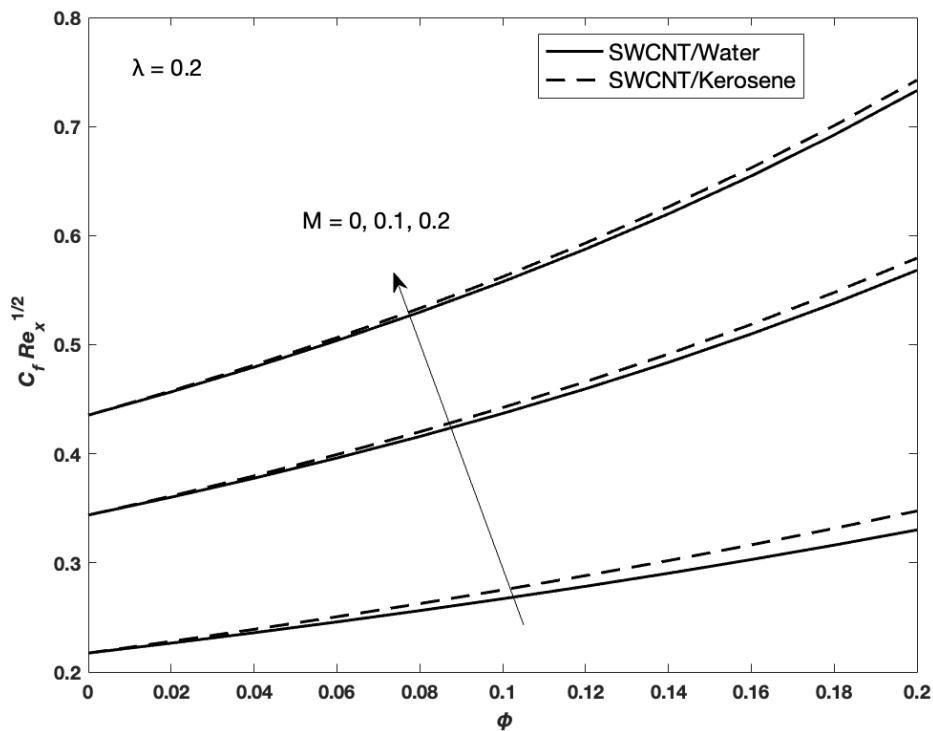


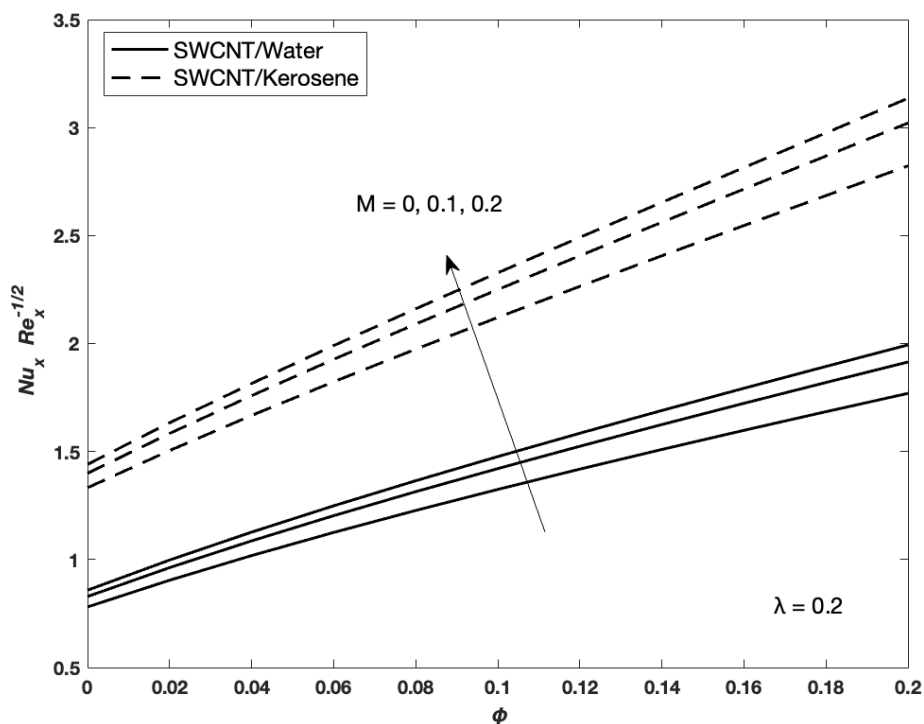
Fig. 3. Variation of  $-\theta'(0)$  with different  $M$  and  $\lambda$  for SWCNT/water

To make a decision based on the type of base fluid, we do analysis on the effect of  $M$  for SWCNT in water and kerosene by applying a variety number of  $\phi$  where  $\phi$  is arranged in the range of  $0 \leq \phi \leq 0.2$ . By referring on Figure 4 and 5, we can see obviously that SWCNT/kerosene is more effective than SWCNT/water both in skin friction coefficient and local Nusselt number.





**Fig. 4.** Effect of different values of  $M$  on the skin friction coefficient using various  $\phi$  for SWCNT/water and SWCNT/kerosene



**Fig. 5.** Effect of different values of  $M$  on the Nusselt number using various  $\phi$  for SWCNT/water and SWCNT/kerosene

A comparison performance of SWCNT and MWCNT can be shown in Figure 6 and 7. In this case we use water as the base fluid and take on different values of  $M$  and  $\phi$  in order to study the influence of  $M$  and  $\phi$  for SWCNT/water and MWCNT/water. The range of  $M$  and  $\phi$  are varied in the following

sequence:  $M = 0, 0.1, 0.2$  and  $0 \leq \phi \leq 0.2$ . By referring on Figure 6 and 7, we can see obviously that SWCNT is more effective than MWCNT in both skin friction coefficient and local Nusselt number.

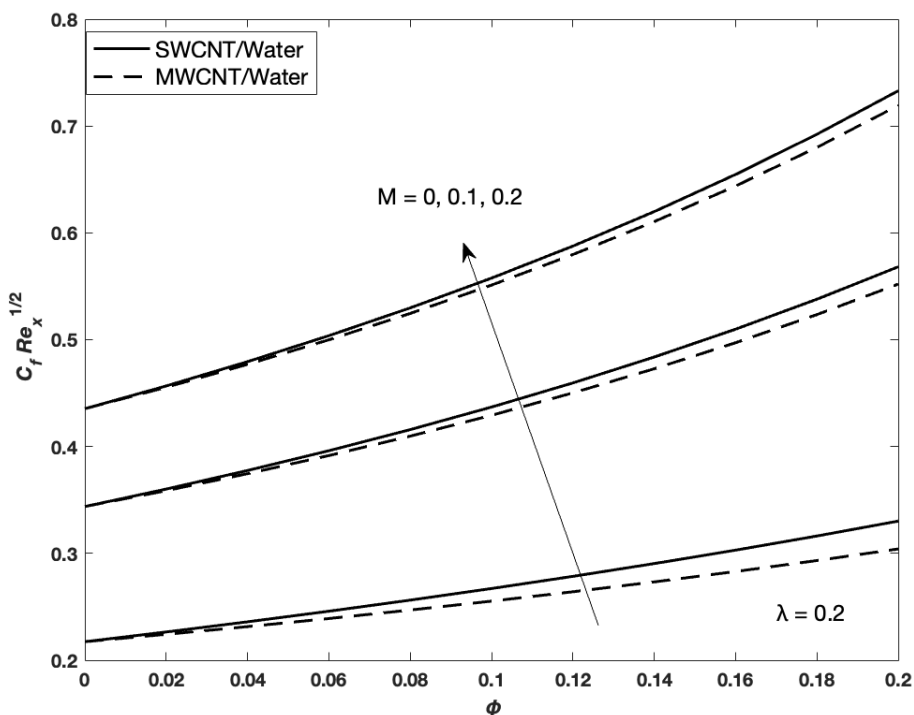


Fig. 6. Effect of different values of  $M$  on the skin friction coefficient using various  $\phi$  for SWCNT/water and MWCNT/water

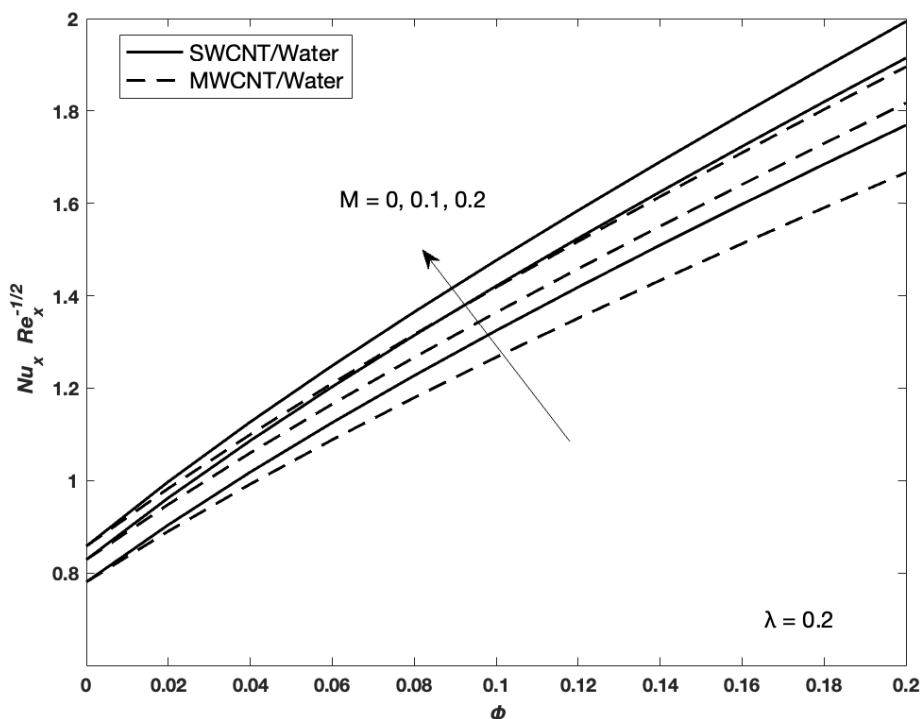


Fig. 7. Effect of different values of  $M$  on the Nusselt number using various  $\phi$  for SWCNT/water and MWCNT/water

The investigation of the velocity and temperature profile for SWCNT/water with  $\phi = 0.1$  can be viewed in Figure 8 and Figure 9. From these figures, when  $M$  is set at  $M = 0.01$  and the range of  $\lambda$  is  $-0.5 \leq \lambda \leq -0.4$ , we can see that the thickness of momentum boundary and thermal boundary layer for second solution is bigger than first solution.

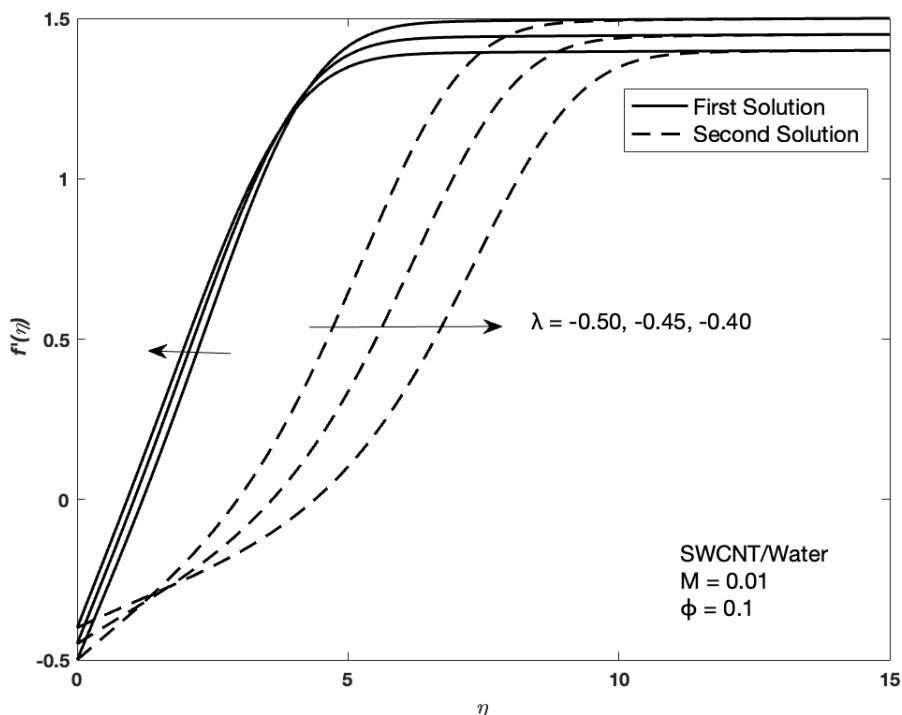


Fig. 8. Effect of different values of  $\lambda$  on the velocity profile using various  $\eta$  and for SWCNT/water

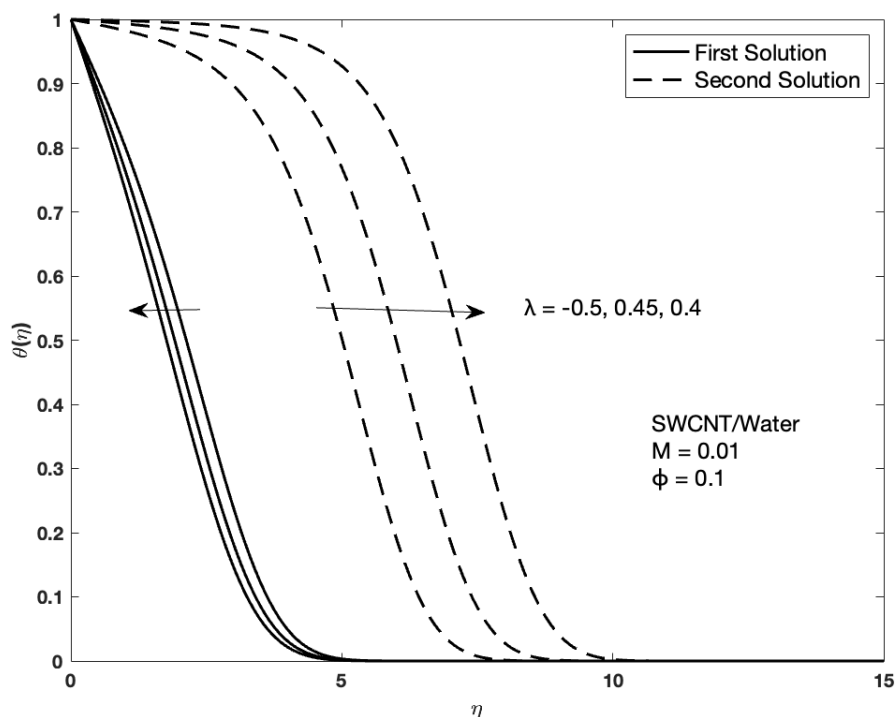


Fig. 9. Effect of different values of  $\lambda$  on the temperature profile using various  $\eta$  and for SWCNT/water

#### 4. Conclusion

From this present paper, we have investigated theoretically and numerically CNTs nanofluids flow and the properties of heat transfer over a moveable plate. From our analysis, we can summarize that

- i. The duality solutions appear for the range of  $\lambda_c \leq \lambda < 0$  and at  $M < 0.1$ .
- ii. Employing magnetic field into CNTs nanofluids can increase in both the skin friction and heat transfer coefficient.
- iii. Kerosene will perform better as a base fluid compare to water in term of the skin friction coefficient and heat transfer coefficient.
- iv. The performance of SWCNT is higher than MWCNT both in the skin friction coefficient and heat transfer coefficient.

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